

$$E = \frac{1}{2} (\text{target} - \text{output})^2$$

$$\begin{aligned} J &= \text{neuron} \\ \delta_x &= \frac{dE}{dx} \cdot \frac{dx}{d\sigma_x} \\ &= \frac{dE}{d\sigma_j} \cdot \frac{d\sigma_j}{d\text{net}_j} \cdot \frac{d\text{net}_j}{dw_{ij}} \end{aligned}$$

$$\begin{aligned} \frac{d\text{net}_j}{dw_{ij}} &= \frac{\partial}{\partial w_{ij}} \left(\sum_{k=1}^n w_{kj} o_k \right) \\ &= \frac{\partial}{\partial w_{ij}} w_{ij} o_i \\ &= o_i \end{aligned}$$

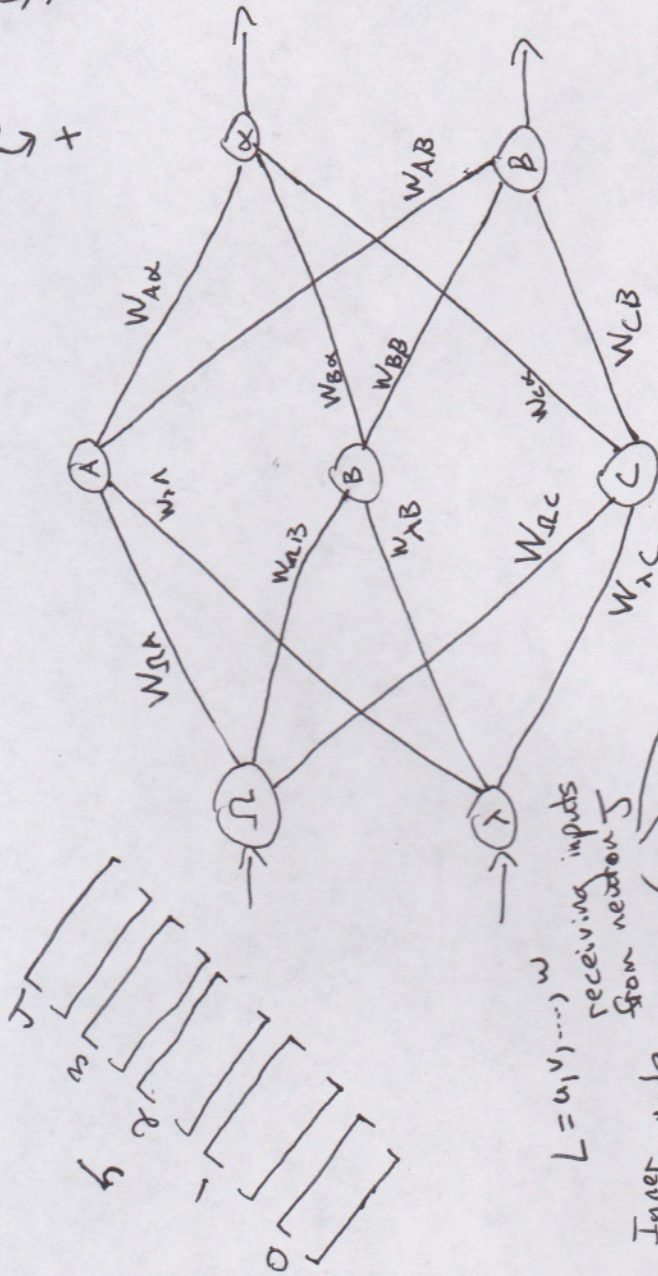
$$\boxed{\frac{\partial \sigma_j}{\partial \text{net}_j}} = \frac{\partial}{\partial \text{net}_j} (\sigma(\text{net}_j))$$

$$\text{for sigmoid} \rightarrow = \sigma(\text{net}_j) (1 - \sigma(\text{net}_j))$$

$$\frac{dE}{d\sigma_j} = \frac{\partial}{\partial \sigma_j} \left[\frac{1}{2} (t - \gamma)^2 \right] = -(t - \gamma) = (\gamma - t)$$

output node

$$\begin{aligned} \frac{\partial E}{\partial w_{ij}} &= o_i \delta_j \\ \text{with } \delta_j &= \frac{\partial E}{\partial \sigma_j} \cdot \frac{\partial \sigma_j}{\partial \text{net}_j} = \begin{cases} (\sigma_j - t_j) \sigma_j (1 - \sigma_j) & \rightarrow \text{if } j \text{ is output} \\ \left(\sum_{\text{rel}} \delta_{\text{rel}} w_{j\text{rel}} \right) \sigma_j (1 - \sigma_j) & \rightarrow \text{if } j \text{ is inner} \end{cases} \\ \Delta w_{ij} &= -\eta \frac{\partial E}{\partial w_{ij}} \end{aligned}$$



$$\text{Inner node} \quad \frac{\partial E(\sigma_j)}{\partial \sigma_j} = \frac{\partial E(\text{net}_1, \text{net}_2, \dots, \text{net}_n)}{\partial \sigma_j}$$

$$\begin{aligned} \frac{\partial E}{\partial \sigma_j} &= \sum_{\text{rel}} \left(\frac{\partial E}{\partial \text{net}_e} \cdot \frac{\partial \text{net}_e}{\partial \sigma_j} \right) \\ &= \sum_{\text{rel}} \left(\frac{\partial E}{\partial o_e} \cdot \frac{\partial o_e}{\partial \text{net}_e} w_{je} \right) \end{aligned}$$

Relu

$$f(x) = \max(x, 0)$$

$$\frac{df(x)}{dx} = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$\delta_x = (\text{out}_x - \text{target}_x) \cdot \frac{df(x)}{dx}$$

$$= (0.525 - 1) \cdot 1$$

$$= -0.475$$

$$\delta_B = (0 - 0) \cdot 0$$

$$= 0$$

$$\delta_A = \left(\sum_{\text{REL}} \delta_{\text{EL}} \omega_{\text{AE}} \right) \cdot \frac{df(x)}{dx}$$

$$= (\delta_A \cdot \omega_{AA} + \delta_B \cdot \omega_{AB}) \cdot \frac{df(x)}{dx}$$

$$= 0$$

$$\delta_B = (\delta_A \cdot \omega_{BA} + \delta_C \cdot \omega_{BB}) \cdot \frac{df(x)}{dx}$$

$$= (-0.475 \cdot 3) \cdot 1$$

$$= -0.1425$$

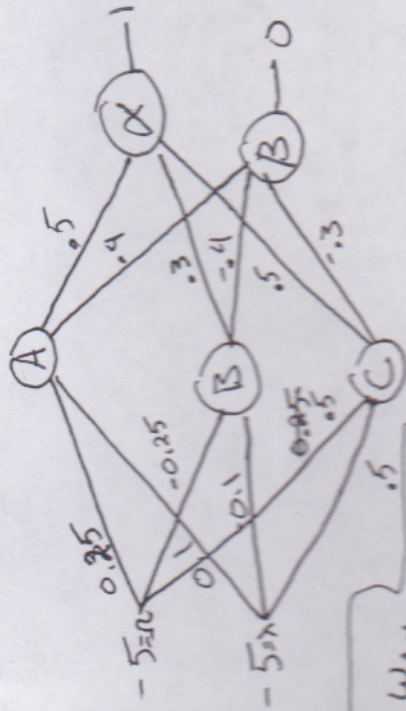
$$\delta_C = 0$$

$$\Delta \omega_{AC} = 0, \Delta \omega_{BC} = 0$$

$$\Delta \omega_{AB} = -\text{out}_A \cdot \delta_B$$

$$= 1 + 5 \cdot -0.1425 = -0.7125$$

$$\Delta \omega_{AB} = -\text{out}_A \cdot \delta_B = -0.7125$$



$$\Delta \omega_{AA} = \text{out}_A \cdot \delta_A$$

$$= 0 \cdot -0.475 = 0$$

$$\Delta \omega_{BA} = 1.75 \cdot -0.475 = -0.83125$$

$$\Delta \omega_{CA} = 0 \cdot -0.475 = 0$$

$$\Delta \omega_{AB} = 0 \cdot 0 = 0$$

$$\Delta \omega_{BB} = 0 \cdot \delta_B$$

$$\Delta \omega_{CB} = 0 \cdot 0 = 0$$

$$\Delta \omega_{AA} = -\text{out}_A \cdot \delta_A$$

$$= 0$$

$$\Delta \omega_{AA} = -\text{out}_A \cdot \delta_A$$

$$= 0$$

$$\Delta \omega_{AB} = -\text{out}_A \cdot \delta_B$$

$$= 1 + 5 \cdot -0.1425 = -0.7125$$

Forward

$$W_{\text{XA}} = 0.25$$

$$W_{\text{YA}} = 0.4$$

$$\text{net}_A = -5 \cdot 0.25 + -5 \cdot 1$$

$$= -1.75$$

$$\text{out}_A = \max(0, -1.75) = 0$$

$$W_{\text{XB}} = -0.25$$

$$W_{\text{YB}} = -0.1$$

$$\text{net}_B = -5 \cdot -0.25 + -5 \cdot 0.1$$

$$= 1.25 + -0.5 = 0.75$$

$$\text{out}_B = \max(0, 0.75) = 0.75$$

$$W_{\text{XC}} = 0.5$$

$$W_{\text{YC}} = 0.5$$

$$\text{net}_C = 0.5 \cdot -5 + 0.5 \cdot -5$$

$$= -2.5 + -2.5 = -5$$

$$\text{out}_C = \max(0, -5) = 0$$

$$W_{\text{AX}} = 0.5 \cdot 0 + 0.3 \cdot 1.75 + 0.5 \cdot 0$$

$$\text{net}_A = 0.525$$

$$\text{out}_A = \max(0, 0.525) = 0.525$$

$$\text{net}_B = 0.4 \cdot 0 + -0.4 \cdot 1.75 + 0$$

$$= -0.7$$

$$\text{out}_B = \max(0, -0.7) = 0$$