Tyler Olivieri HW4

1a) p. 66 #3

Consider the vectors in $\mathbb{R}^4$ defined by $\alpha_1 = (-1,0,1,2)$

Find a system of homogenous eq. for which $\alpha_2 = (3,4,-2,5)$

the space of solutions is exactly the $\alpha_3 = (1,4,0,9)$

subspace of $\mathbb{R}^4$ spanned by the three given vectors.

$Ax = 0$

choose A s.t. $\alpha_1, \alpha_2, \alpha_3$ are basis for nullspace of A.

let $A = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 3 & 4 & -2 & 5 \\ 1 & 4 & 0 & 9 \end{bmatrix} \xrightarrow{RREF(A)} \begin{array}{cc} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1/4 & 11/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} \beta_1 \\ \beta_2 \\ \ \end{matrix} \end{array}$

Note $2\alpha_1 + \alpha_2 = \alpha_3$

$2(-1,0,1,2) + (3,4,-2,5) = (-2,0,2,4) + (3,4,-2,5)$
$= (1,4,0,9)$

subspace spanned by $\alpha_1, \alpha_2, \alpha_3$ is spanned by

$c, d \in F$ $\quad c\beta_1 + d\beta_2 = (\underset{x}{c}, \underset{y}{d}, \underset{z}{-c + 1/4 d}, \underset{w}{-2c + 11/4 d})$

$\Rightarrow$ $z = -c + 1/4 d = -x + 1/4 y$

$w = -2c + 11/4 d = -2x + 11/4 y$

Span the subspace defined by $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}^4$

Tyler Olivieri HW4

1) a) p48-49 #7

Let be v.s of all 2×2 matrices over F.

Let W_1 be the set of matrices of the form

$\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$. Prove W_1 is a subspace of V

Clearly $W_1 \subseteq V$ because W_1 is 2×2.

I claim that W_1 is closed under addition/multiplication

let $\begin{bmatrix} x_1 & -x_1 \\ y_1 & z_1 \end{bmatrix}, \begin{bmatrix} x_2 & -x_2 \\ y_2 & z_2 \end{bmatrix} \in W_1$ $\alpha \in F$

then

$$\alpha \begin{bmatrix} x_1 & -x_1 \\ y_1 & z_1 \end{bmatrix} + \begin{bmatrix} x_2 & -x_2 \\ y_2 & z_2 \end{bmatrix} =$$

$$\begin{bmatrix} \alpha x_1 & -\alpha x_1 \\ \alpha y_1 & \alpha z_1 \end{bmatrix} + \begin{bmatrix} x_2 & -x_2 \\ y_2 & z_2 \end{bmatrix} = \begin{bmatrix} \alpha x_1 + x_2 & -\alpha x_1 - x_2 \\ \alpha y_1 + y_2 & \alpha z_1 + z_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha x_1 + x_2 & -(\alpha x_1 + x_2) \\ \alpha y_1 + y_2 & \alpha z_1 + z_2 \end{bmatrix} \in W_1$$

$\Rightarrow$ closed under add/mult.

let W_2 be the set of matrices of the form

$\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$ prove W_2 is subspace of V.

$W_2 \subseteq V$ because W_2 is 2×2.

let $\begin{bmatrix} a_1 & b_1 \\ -a_1 & c_1 \end{bmatrix}, \begin{bmatrix} a_2 & b_2 \\ -a_2 & c_2 \end{bmatrix} \in W_2$ $\alpha \in F$

$$\alpha \begin{bmatrix} a_1 & b_1 \\ -a_1 & c_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ -a_2 & c_2 \end{bmatrix} = \begin{bmatrix} \alpha a_1 & \alpha b_1 \\ -\alpha a_1 & \alpha c_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ -a_2 & c_2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha a_1 + a_2 & \alpha b_1 + b_2 \\ -\alpha a_1 - a_2 & \alpha c_1 + c_2 \end{bmatrix} = \begin{bmatrix} \alpha a_1 + a_2 & \alpha b_1 + b_2 \\ -(\alpha a_1 + a_2) & \alpha c_1 + c_2 \end{bmatrix} \in W_2$$

$\Rightarrow$ W_2 is closed under addition/multiplication.

1) b) Tyler Olivieri HW4

p 73-74 #1

1) Which ~~are~~ of the following functions T from $\mathbb{R}^2$ into $\mathbb{R}^2$ are linear transformations?

a) $T(x_1, x_2) = (1 + x_1, x_2)$ Not linear transformation $T(0) \neq 0$

$T(0) = T(0+0) = T(0) + T(0)$ = holds for linear transformation.

$T(0+0) = T(0) = T(0,0) = (1,0)$

$T(0) + T(0) = (1,0) + (1,0) = (2,0)$

$T(0) + T(0) = (2,0) \neq (1,0) = T(0+0)$

b) $T(x_1, x_2) = (x_2, x_1)$ let $v, w \in \mathbb{R}^2$ $c \in F$

$T(cv + w) = cTv + Tw$ for linear transformation

$T(cv + w) = T\left(c(v_1, v_2) + (w_1, w_2)\right) = T\left((cv_1 + w_1, cv_2 + w_2)\right)$

$= (cv_2 + w_2, cv_1 + w_1) = c(v_2, v_1) + (w_2, w_1)$

$= cT(v) + T(w)$

$\Rightarrow$ T is a valid linear transformation

c) $T(x_1, x_2) = (x_1^2, x_2)$ $\quad v, w \in \mathbb{R}^2$

$$T(v+w) = T((v_1+w_1, v_2+w_2)) = ((v_1+w_1)^2, v_2+w_2)$$

$$= (v_1^2 + 2v_1w_1 + w_1^2, v_2+w_2) = (v_1^2, v_2) + (w_1^2 + w_2) + (2v_1w_1, 0)$$

$$Tv + Tw = (v_1^2, v_2) + (w_1^2, w_2)$$

$\Rightarrow$ T is not a valid linear transformation.

d) $T(x_1, x_2) = (\sin x_1, x_2)$

$$T(cv+w) = T((cv_1+w_1, v_2+w_2)) = (\sin(cv_1+w_1), cv_2+w_2)$$

$$cTv + Tw = c(\sin v_1, v_2) + (\sin w_1, w_2)$$

$$= (c\sin v_1 + \sin w_1, cv_2 + w_2)$$

$$\sin(cv_1+w_1) = \sin(cv_1)\cos(w_1) + \cos(cv_1)\sin(w_1)$$

$$\neq c\sin v_1 + \sin w_1$$

$\Rightarrow$ not a valid linear transformation.

Tyler Olivieri HW 4

1b) #1 e)

$$T(x_1, x_2) = (x_1 - x_2, 0)$$

$$T(cv + w) = T((cv_1 + w_1, cv_2 + w_2))$$

$$= (cv_1 + w_1 - (cv_2 + w_2), 0)$$

$$= (cv_1 - cv_2, 0) + (w_1 - w_2, 0)$$

$$= c(v_1 - v_2, 0) + (w_1 - w_2, 0)$$

$$= cT(v) + T(w)$$

$\Rightarrow$ T is a valid linear transformation.

1) b) 5)

If $\alpha_1 = (1,-1)$ $\beta_1 = (1,0)$

$\alpha_2 = (2,-1)$ $\beta_2 = (0,1)$

$\alpha_3 = (-3,2)$ $\beta_3 = (1,1)$

is there a linear transformation T from $\mathbb{R}^2$ into $\mathbb{R}^2$

s.t. $T\alpha_i = \beta_i$ for $i = 1, 2, 3$

for a linear transformation to be valid,

$T(0) = 0$

Assume T is a valid linear transformation.

then, $T(\alpha_1 + \alpha_2 + \alpha_3) = T\alpha_1 + T\alpha_2 + T\alpha_3 = \beta_1 + \beta_2 + \beta_3$

$T(\alpha_1 + \alpha_2 + \alpha_3) = T((1,-1) + (2,-1) + (-3,2)) = T((0,0)) = 0$

$\beta_1 + \beta_2 + \beta_3 = (1,0) + (0,1) + (1,1) = (2,2) \neq 0$

Contradiction.

There is no valid linear transformation T from $\mathbb{R}^2$ into $\mathbb{R}^2$

1b) a)

Let V be the v.s of all n×n matrices over the field F

and let B be a fixed n×n matrix.

If $T(A) = AB - BA$, verify that T is a linear transformation

from V into V. $T: V \longrightarrow V$

let $A_1, A_2 \in$ V.s of all n×n matrices

$c \in F$

$T(cA_1 + A_2) = cTA_1 + TA_2$

if T is valid linear transformation.

$$T(cA_1 + A_2) = (cA_1 + A_2)B - B(cA_1 + A_2)$$

$$= cA_1B + A_2B - BcA_1 + BA_2$$

$$= cA_1B - cBA_1 + A_2B - BA_2$$

$$= c(A_1B - BA_1) + A_2B - BA_2$$

$$= cT(A_1) + T(A_2)$$

$\Rightarrow$ T is valid linear transformation.

Tyler Olivieri HW#4

1) b) 8)

Describe a linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^3$ which has its range the subspace spanned by $(1,0,-1)$ and $(1,2,2)$

from theorem 1 in section 3.1

$\exists!\ T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ s.t $T\alpha_j = \beta_j$ where α_j is a basis vector of $\mathbb{R}^3$ and β_j is a vector in $\mathbb{R}^3$.

Let $\{e_1, e_2, e_3\}$ be a basis for $\mathbb{R}^3$

let

$$Te_1 = (1,0,-1)$$

$$Te_2 = (1,2,2)$$

$$Te_3 = (0,0,0)$$

Then the linear transformation T has a range of the subspace spanned by $(1,0,-1)$ and $(1,2,2)$.

such a linear transformation is

$$T(x_1, x_2, x_3) = T(x_1 + x_2,\ 2x_2 - x_1,\ -x_1 + 2x_2)$$

1b) #10

Tyler Olivieri HW4

Let V be the set of all complex numbers regarded as a vector space over the field of real numbers (usual operations). Find a function from V into V which is a linear transformation on the above v.s but is not a linear transformation on C^1, which is not complex linear

Let $T: V \to V$ sending $a+bi \mapsto a$

let $v, w \in$ Complex # / $\mathbb{R}$ $\quad v, w$ have form $a+bi$
$v_1 + v_2 i$
$w_1 + w_2 i$

$c \in \mathbb{R}$

Prop Then $T(cv+w) = cTv + Tw$ is valid transformation

$$T(cV+w) = T(c(v_1+v_2 i) + (w_1 + w_2 i))$$

$$= T((cv_1 + cv_2 i) + (w_1 + w_2 i))$$

$$= T((cv_1 + w_1 + (cv_2 + w_2)i))$$

$$= cv_1 + w_1$$

$$= cT(V) + T(w)$$

Now consider the same vectors in v.s / $\mathbb{C}'$ $\qquad c \in \mathbb{C}'$

$T(0+i) = 0$, $T(i) = T(1\cdot i) = i\,T(1) = i\cdot 1 = i$

$0 \neq i$ forms contradiction

Tyler Olivieri HW4

2) Which of the following are linear transformations?

a) $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ sending $(x,y,z) \longmapsto (x-y, y-z, z-x, 0)$

let $a_1 = (x_1, y_1, z_1) \in \mathbb{R}^3$

$a_2 = (x_2, y_2, z_2) \in \mathbb{R}^3$

$T(ca_1 + a_2) = cTa_1 + Ta_2$ iff T is linear transform

$$T(ca_1 + a_2) = T(c(x_1, y_1, z_1) + (x_2, y_2, z_2))$$

$$= T((cx_1 + x_2, cy_1 + y_2, cz_1 + z_2))$$

$$= ((cx_1 + x_2) - (cy_1 + y_2), (cy_1 + y_2) - (cz_1 + z_2), (cz_1 + z_2) - (cx_1 + x_2), 0)$$

$$= (cx_1 - cy_1 + x_2 - y_2, cy_1 - cz_1 + y_2 - z_2, cz_1 - cx_1 + z_2 - x_2, 0)$$

$$= (cx_1 - cy_1, cy_1 - cz_1, cz_1 - cx_1, 0) + (x_2 - y_2, y_2 - z_2, z_2 - x_2, 0)$$

$$= (c(x_1 - y_1), c(y_1 - z_1), c(z_1 - x_1), 0) + \text{" "}$$

$$= c(x_1 - y_1, y_1 - z_1, z_1 - x_1, 0) + \text{" "}$$

$$= cT((x_1, y_1, z_1)) + T((x_2, y_2, z_2))$$

$$= cTa_1 + Ta_2 \implies T \text{ is valid linear transform}$$

2b)

$M: \mathbb{R}^2 \longrightarrow \mathbb{R}$ sending $(x,y) \mapsto xy$ (Here we view $\mathbb{R} = \mathbb{R}^1$)

let

$a_1 = (x_1, y_1) \in \mathbb{R}^2$

$a_2 = (x_2, y_2) \in \mathbb{R}^2$

for M to be linear transformation

$$M(ca_1 + a_2) = cMa_1 + Ma_2$$

$$M(ca_1 + a_2) = M((cx_1 + x_2, cy_1 + y_2))$$

$$= (cx_1 + x_2)(cy_1 + y_2) = c^2 x_1 y_1 + cx_1 y_2 + x_2 c y_1 + x_2 y_2$$

$$= c^2 x_1 y_1 + cx_1 y_2 + cx_1 x_2 + x_2 y_2$$

$$\neq$$

$$cMa_1 + Ma_2 = cx_1 y_1 + x_2 y_2$$

$\Rightarrow$ not a valid linear transformation.

Tyler Olivieri HW4

2) c)

$R: Z \to Z$ where Z is v.s of sequences of real numbers

sending $(a_1, a_2, a_3, \dots) \mapsto (0, a_1, a_2, a_3, \dots)$

for R to be a valid linear transformation

$$R(cA_1 + B_1) = cRA_1 + RB_1$$

let

$A_1 = a_1, a_2, a_3, \dots \in Z$

$B_1 = b_1, b_2, b_3, \dots \in Z$

$$R(cA_1 + B_1) = R(c(a_1, a_2, a_3, \dots) + (b_1, b_2, b_3, \dots))$$

$$= R((ca_1 + b_1, ca_2 + b_2, \dots))$$

$$= (0, ca_1 + b_1, ca_2 + b_2, ca_3 + b_3, \dots)$$

$$= (0, ca_1, ca_2, ca_3, \dots) + (0, b_1, b_2, b_3, \dots)$$

$$= c(0, a_1, a_2, a_3, \dots) + (0, b_1, b_2, b_3, \dots)$$

$$= cRA_1 + RB_1$$

$\Rightarrow$ R is a valid linear transformation.

2) d)

$L: Z \to Z$ (where Z is an in part (c)) sending

$$(a_1, a_2, a_3, \ldots) \mapsto (a_2, a_3, a_4, \ldots)$$

let $A_1, B_1 \in Z$

$$L(cA_1 + B_1) = L((ca_1 + b_1, ca_2 + b_2, ca_3 + b_3, \ldots))$$

$$= ((ca_2 + b_2), ca_3 + b_3, ca_4 + b_4, \ldots)$$

$$cLA_1 + LB_1 = c(a_2, a_3, a_4, \ldots) + (b_2, b_3, b_4 \ldots)$$

$$= (ca_2 + b_2, ca_3 + b_3, ca_4 + b_4, \ldots)$$

$$\Rightarrow L(cA_1 + B_1) = cLA_1 + LB_1$$

$\Rightarrow L$ is a valid linear transformation.

Tyler Olivieri HW#4

2 e) $I : W \to \mathbb{R}$ (where W is the vector space of cont. real-valued functions on the closed interval $[0,1]$) sending $f \mapsto \int_0^1 f(x)\,dx$

let $f_1, f_2 \in W$ $c \in F$

$$I(cf_1 + f_2) \overset{?}{=} cIf_1 + If_2$$

$$I(cf_1 + f_2) = \int_0^1 (cf_1(x) + f_2(x))\,dx = \int_0^1 cf_1(x)\,dx + \int_0^1 f_2(x)\,dx$$

$$= c\int_0^1 f_1(x)\,dx + \int_0^1 f_2(x)\,dx = cIf_1 + If_2$$

$\Rightarrow I$ is a valid linear transformation

2) f)

$D: V \to V$ (where V is the vector space of infinitely differentiable functions on $\mathbb{R}$)

sending $f \mapsto f'$

let $f_1, f_2 \in V$ $c \in F$

$D(cf_1 + f_2) \stackrel{?}{=} cDf_1 + Df_2$

$D(cf_1+f_2) = (cf_1+f_2)' = (cf_1)' + f_2' = cDf_1 + Df_2$

$\Rightarrow$ valid linear transformation.

Tyler Ollivieri HW4

#2 g) $E: V \to \mathbb{R}$ (where V is as in part (f))

sending $f \mapsto f(0)$

let $f_1, f_2 \in V$ $c \in F$

$$E(cf_1 + f_2) = (cf_1 + f_2)(0) = cf_1(0) + f_2(0)$$

$$= cEf_1 + Ef_2$$

$\Rightarrow$ valid linear transformation

#2 h) $Q: V \to V$ sending $f \mapsto f^2$

$$Q(cf_1 + f_2) = (cf_1 + f_2)^2 = c^2 f_1^2 + 2cf_1 f_2 + f_2^2$$

$$cQf_1 + Qf_2 = cf_1^2 + f_2^2$$

$$Q(cf_1 + f_2) \neq cQf_1 + Qf_2$$

$\Rightarrow$ Q is not a linear transformation

#2D) i) $S: V \longrightarrow V$ sending $f(x) \mapsto f(x)\sin(x)$

$$S(cf_1(x)+f_2(x)) = (cf_1(x)+f_2(x))\sin(x)$$

$$= cf_1 \sin(x) + f_2(x)\sin(x)$$

$$= cSf_1 + Sf_2$$

$\Rightarrow$ valid linear transformation

Tyler Olivieri HW #4 #3 For each map in #2 that is a linear transformation, find the following:

a) The range of the map, also known as the image $T: V \to W$ $\{w \in W \mid w = T(v)$ for some $v \in V\}$

b) the nullspace of the map, also known as the kernel. (The nullspace of a linear transformation $T: V \to W$ is by definition $\{v \in V \mid T(v) = 0\}$

3. 2a) was a valid ~~transfor~~ linear transformation.

$T: \mathbb{R}^3 \to \mathbb{R}^4$ sending $(x, y, z) \mapsto (x-y, y-z, z-x, 0)$

Range (T) $\{w \in \mathbb{R}^4 \mid w = T(v), v = (a, b, c), a \neq b$ or $b \neq c$ or $a \neq c\}$

nullspace (T) is $\{v \in \mathbb{R}^3 \mid v = (x, y, z), x = y = z\}$

a line in $\mathbb{R}^3$, passes through origin.

3. 2c) was a valid v.s

$R: Z \to Z$ sending $(a_1, a_2, a_3, \ldots) \mapsto (0, a_1, a_2, a_3, \ldots)$

Range (R) = $\{A \in Z\}$ (all sequences except 0 sequence)

nullspace (R) = $\{(0, 0, 0, \ldots)\}$

3. 2d) was a valid linear transformation

$L: Z \to Z$ sending $(a_1, a_2, a_3, \dots) \mapsto (a_2, a_3, a_4, \dots)$

$\text{Range}(L) = \{ A \in Z \}$ ← all sequences except 0 sequence

$\text{Nullspace}(L) = \{(0, 0, 0, \dots)\}$

---

3. 2c) was a valid linear transformation

$I: W \to \mathbb{R}$ sending $f \mapsto \int_0^1 f(x)\,dx$

$\text{Range}(I) = \{\mathbb{R}$ except $0\}$

$\text{nullspace}(I) = \{$ all functions that have equal negative and positive area on $(0,1)$ [e.g. $\int_0^1 f(x)\,dx = 0$], some sine, cos, $f(x) = 0$ on $(0,1)$, $\dots \}$

---

3. 2f) was a valid linear transformation

$D: V \to V$ (where V is the vector space of infinetley diff functions on $\mathbb{R}$)

$\text{nullspace}(D) = \{ \mathbb{R}$ (all real numbers)$\}$

$D(\mathbb{R}) = 0$ derivative of a real number is 0. 0 is still infinetly differentiable.

$\text{Range}(D) = \{$ all infinetley differential functions $\}$

3.2g) was a linear transformation.

$E: V \to \mathbb{R}$ sending $f \mapsto f(0)$

nullspace(E) = { any function $f \in V \mid f(0) = 0$ }

range(E) = $\{ \mathbb{R} \}$

there exists a function that is infinetley differentiable that at $f(0)$ will cover some point $\in \mathbb{R}$

3.2i) was a linear transformation.

$S: V \to V$ sending $f(x) \to f(x)\sin(x)$

nullspace$(S) = \{0\}$

Range(S) = { every function $\in V$. }

Tyler Olivieri HW4

4) Let $S: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation taking $(x,y) \in \mathbb{R}^2$ to $(a,b,c) \in \mathbb{R}^3$ whenever

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

I'm assuming $x = y$ and $a = b = c$ is possible

if $a \neq b \neq c$ there is no nullspace because $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ couldn't happen ...

Find the kernel/nullspace of S.

Find the image/Range of S.

Kernel of S

$$\text{Kernel}(S) = \{ v \in (x,y) \in \mathbb{R}^2 \mid x = -2y \}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} y = 0$$

this describes a line in $\mathbb{R}^2$ passing through origin.

$$y = -\frac{x}{2}$$

$$x = -2y$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} -2y + \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} y = 0$$

$$\begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} y + \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} y = 0$$

$$\text{image}(S) = \{ v = (a,b,c) \in \mathbb{R}^3 \mid a = \tfrac{1}{2}b = \tfrac{1}{3}c \}$$

this is a line in $\mathbb{R}^3$ passing through origin

all scalar multiples of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$c\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} y = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ will be linear combinations of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ ...

but $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ is lin dependendent.

5) Tyler Olivieri HW4

Let V be a real v.s, and suppose that $S: V \to \mathbb{R}$ and $T: V \to \mathbb{R}$ are linear transformations. Define $P: V \to \mathbb{R}^2$ by $P(v) = (S(v), T(v))$

a) show that P is a linear transformation

let $a_1, a_2 \in V$ $\quad c \in F$

P is a linear transformation iff

$$P(ca_1 + a_2) = cPa_1 + Pa_2$$

$$P(ca_1 + a_2) = (S(ca_1 + a_2), T(ca_1 + a_2))$$

Because S is a linear transformation $S: V \to \mathbb{R}$ and $a_1, a_2 \in V$

$$S(ca_1 + a_2) = cSa_1 + Sa_2$$

Similarly for P

$$P(ca_1 + a_2) = cPa_1 + Pa_2$$

$$(S(ca_1 + a_2), T(ca_1 + a_2)) = (cSa_1 + Sa_2, cTa_1 + Ta_2)$$

$$= (cSa_1, cTa_1) + (Sa_2, Ta_2) = c(S(a_1), T(a_1)) + (S(a_2), T(a_2))$$

$$= cP(a_1) + P(a_2)$$

$\Rightarrow P$ is a linear transformation.

Tyler Olivier HW4

5 b)

Find the kernel of P in terms of the kernels of S and T.

$$\text{Kernel}(P) = \{ v \in V \mid P(v) = 0 \}$$

$$\text{Kernel}(S) = \{ v \in V \mid S(v) = 0 \}$$

$$\text{Kernel}(T) = \{ v \in V \mid T(v) = 0 \}$$

$$P(v) = (S(v), T(v)) = (0, 0)$$

when $S(v) = 0$

and $T(v) = 0$

or nullsp

$$\Rightarrow \text{Kernel}(P) = \{ v \in V \mid \text{kernel}(S) \cap \text{kernel}(T) \}$$

intersection of kernel of S and kernel of T

it is the intersection of those two subspaces.