PH 160 LAB 9

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- **Aim:**

  To observe the transmitted and reflected waves through a potential barrier.

- **Theory:**

When we talk about the tunneling effect in quantum mechanics we need to know about the whole story, i.e., from where did we thought about something like tunneling.
So before coming onto the concept of tunnel effect we will first understand the concept of particle trapped in a box and about the finite potential well problem.

Particle trapped in a box:

Here we solved for the allowable energy levels and found some wave-functions for that particular particle. But while solving we took an assumption that the wall of the box is infinite in height and are infinitely thick so that all the possibilities of particle passing through the walls were avoided and energy loss was taken as 0 and constant and hence, we calculated the following things.

$$\psi = A \sin n\pi x/L,$$

This was the wave function we got by those assumptions we made above and also, we got an expression for the energy levels allowed i.e.,

$$E_n = n^2h^2/ 8mL^2$$

Now after this we came onto the concept of finite potential well i.e., the walls of the barrier were now not infinite rather they were finite.

Finite potential well:

Here we took the height of the potential well finite, and we got the following plot as shown in the below figure:

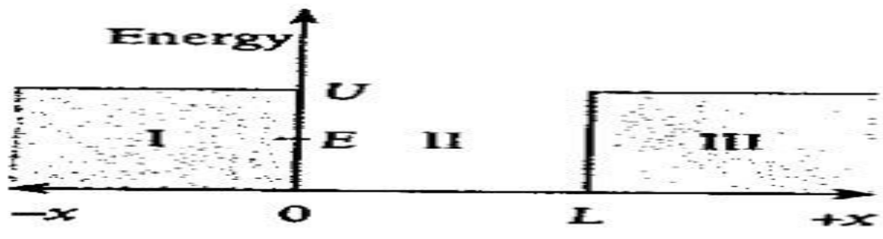


Figure 5.7 A square potential well with finite barriers. The energy E of the trapped particle is less than the height U of the barriers.

Now here the particle can have the possibility of passing regions 1 and 3 lowering the energy levels it had before. So, region 1 and 3 follow the steady state equation of Schrodinger i.e.,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

Now we have wave functions as shown for all the three regions as shown below.

In region 2 the wave function will be same as that we had inside the box when particle was trapped but outside in region 1 and 3 it is,

The solutions to Eq. (5.53) are real exponentials:

$$\psi_{\mathrm{I}} = Ce^{ax} + De^{-ax} \tag{5.55}$$

$$\psi_{\mathrm{III}} = Fe^{ax} + Ge^{-ax} \tag{5.56}$$

Both ψ_{I} and ψ_{III} must be finite everywhere. Since $e^{-ax} \to \infty$ as $x \to -\infty$ and $e^{ax} \to \infty$ as $x \to \infty$, the coefficients D and F must therefore be 0. Hence we have

$$\psi_{\mathrm{I}} = Ce^{ax} \tag{5.57}$$

$$\psi_{\mathrm{III}} = Ge^{-ax} \tag{5.58}$$

Where, ψ_2 and ψ_3 are the wave functions in region 1 and 3.

And finally, after these calculations we got some graphs and we came onto the conclusion that in real world the energy levels are lesser than that we found in Particle Trapped in a box topic because on passing it loses some energy.

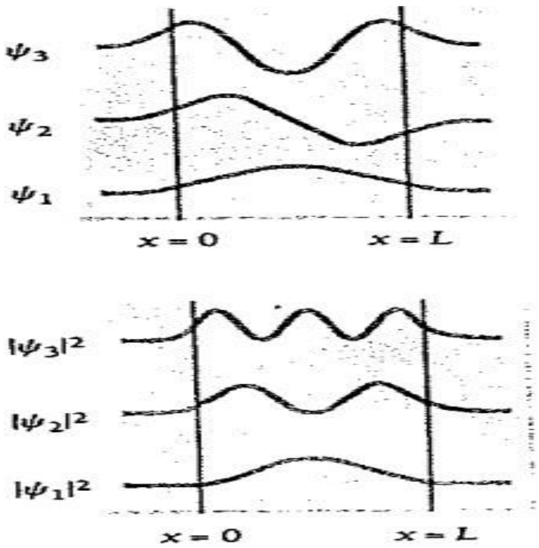


Figure 5.8 Wave functions and probability densities of a particle in a finite potential well. The particle has a certain probability of being found outside the wall.

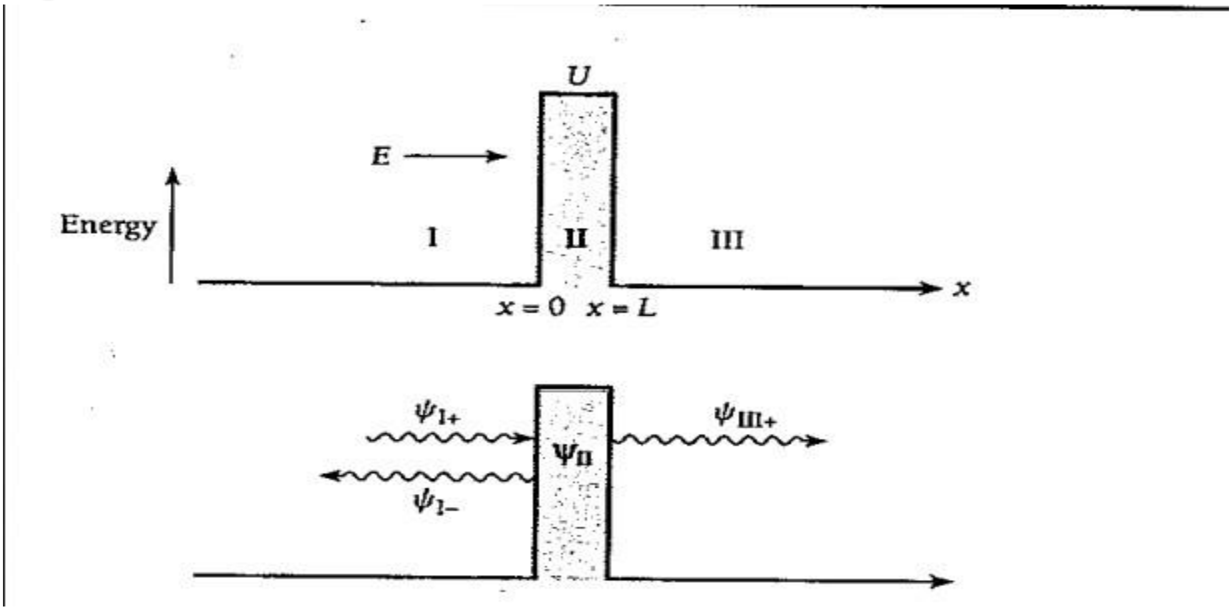
Now we finally come on to the concept of Tunnel Effect

Tunneling effect:

Now we saw that when the particle strikes at the wall of the barrier, it has a tendency to reflect but also a small amount of it passes through their, although the probability of transmission is very less still some of them do pass.

We can relate this with the experiment of Rutherford alpha particle scattering, in that there was one observation that some particle did transmit

through the potential barrier of the nucleus, the amount was 1 particle is 10^{38} particles.
So, we can see that although the probability is so less still some of the particles do transmit.
The main concept that we try to understand here is that,
A particle without the energy to pass over a potential barrier ma still tunnel through it as shown in the figure.



Ψ_1 is the wavefunction of the coming wave and ψ_1 is the wavefunction of the reflected wave and ψ_3 is the wavefunction of transmitted wave.
The transmission probability is very less.
We must also note that the reflection probability (R) and transmission probability (T) depends on the:

1.) Energy of the wave

2.) Energy of the barrier

3.) Width of the barrier

The transmission probability is given by $T = e^{-2 k_2 L}$

Where $k_2 = (\sqrt{2m(U-E)})/(h/2\pi)$

Now let us make some observations on it.

• Formula used:

$T = e^{-2 k_2 L}$

Where, $k_2 = (\sqrt{2m(U-E)})/(h/2\pi)$

$R = 1 - T$

• Observations:

1.) Barrier height Constant

For only wave packet.

S.No.	Barrier width (nm)	Barrier Height (nm)	Reflectance	Transmittance
1	0.8	0.79	0.84	0.16
2	1.1	0.79	0.91	0.09
3	1.4	0.79	0.94	0.06
4	1.7	0.79	0.95	0.05
5	2.0	0.79	0.96	0.04

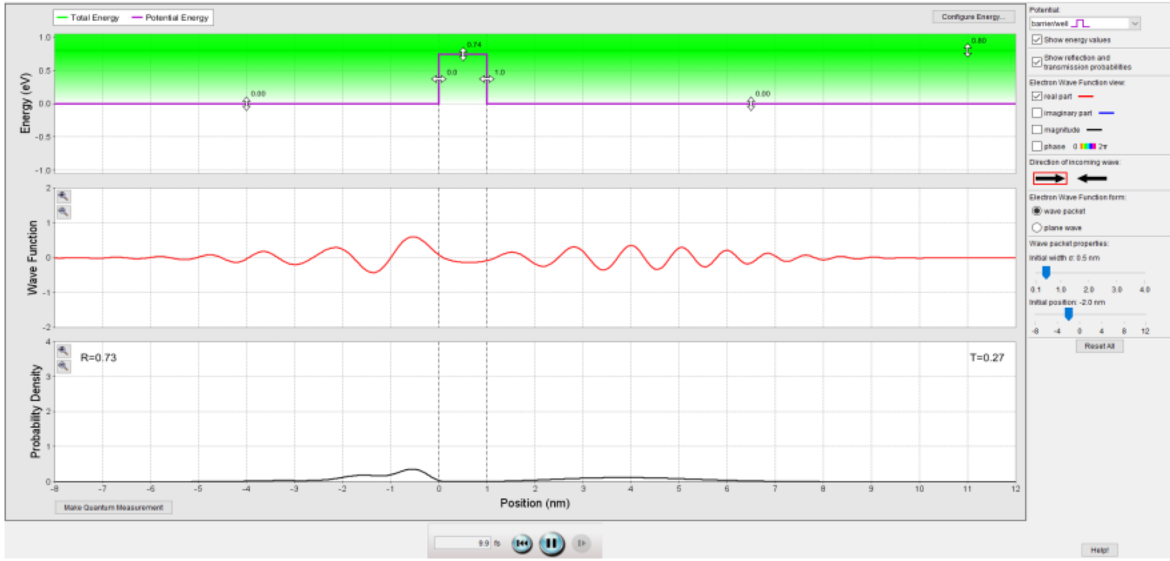
2.) Barrier Width Constant

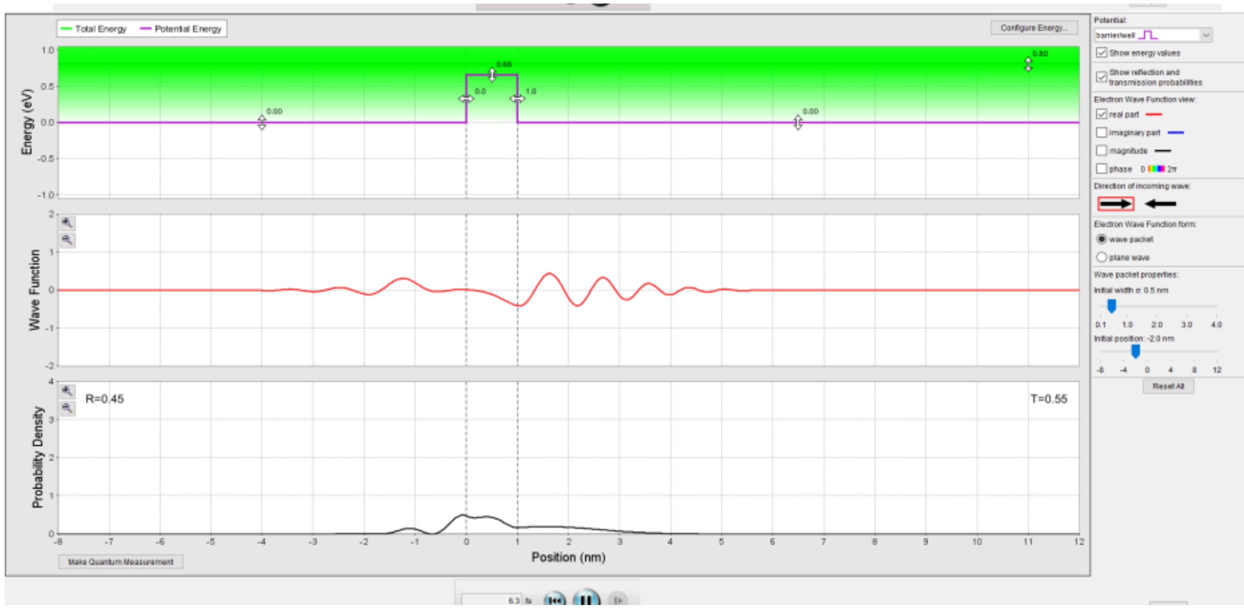
For only wave packet height.

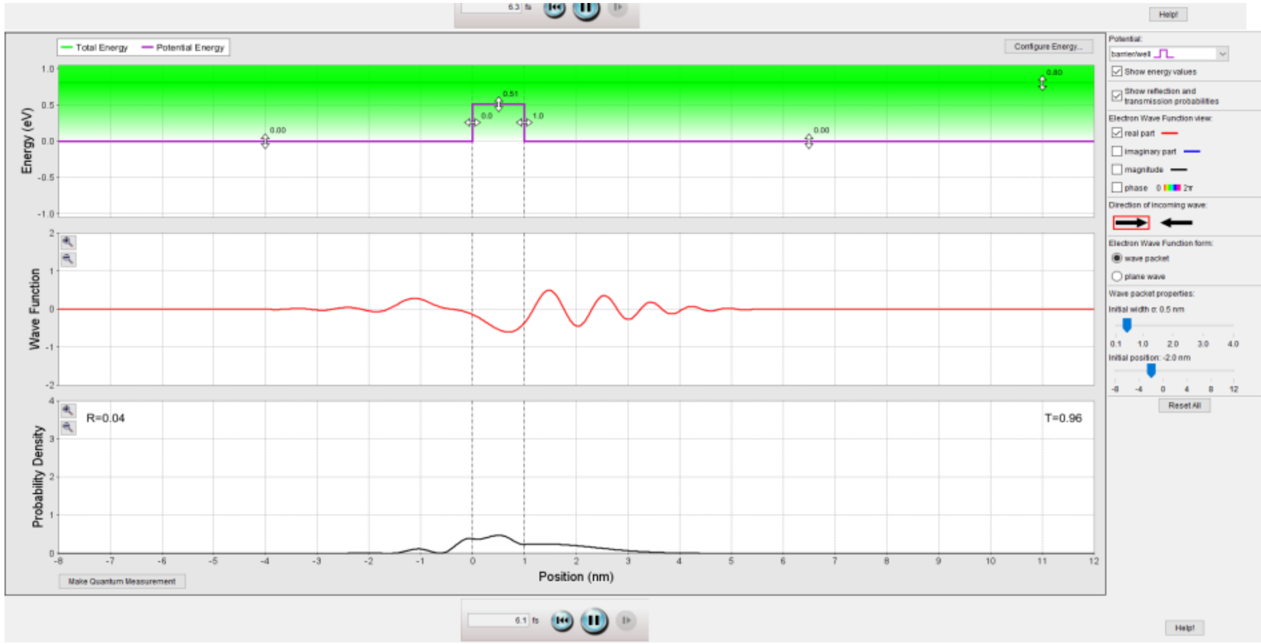
S.No.	Barrier width (nm)	Barrier Height	Reflectance	Transmittance
1	1.9	0.02	0.05	0.95
2	1.9	0.08	0.10	0.90
3	1.9	0.13	0.14	0.86
4	1.9	0.16	0.15	0.85
5	1.9	0.22	0.16	0.84

At around 0.8 eV we get complete reflection and no transmittance.
Let us look at some images of the observations that we get at varying barrier width and height

A.) Varying Barrier Height for a particular wave packet.

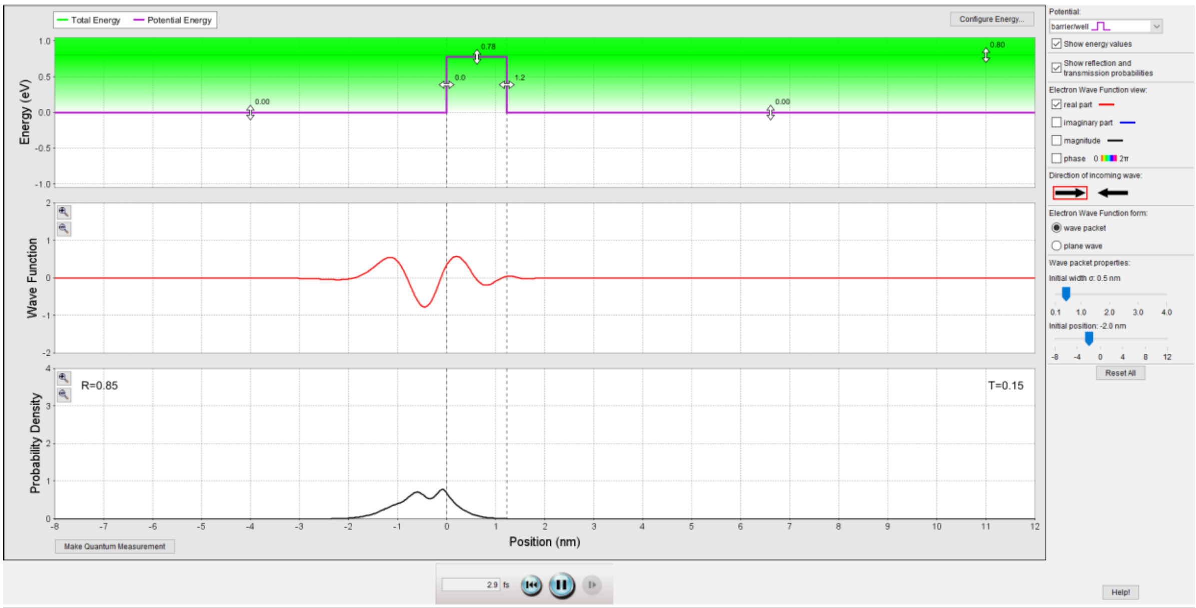

Total Energy
Potential Energy
Configure Energy...
Energy (eV)
Wave Function
Probability Density
R=0.73
T=0.27
Position (nm)
Make Quantum Measurement
Potential:
Show energy values
Show reflection and transmission probabilities
Electron Wave Function view:
real part
imaginary part
magnitude
phase
Direction of incoming wave:
Electron Wave Function form:
wave packet
plane wave
Wave packet properties:
Reset All
Help!

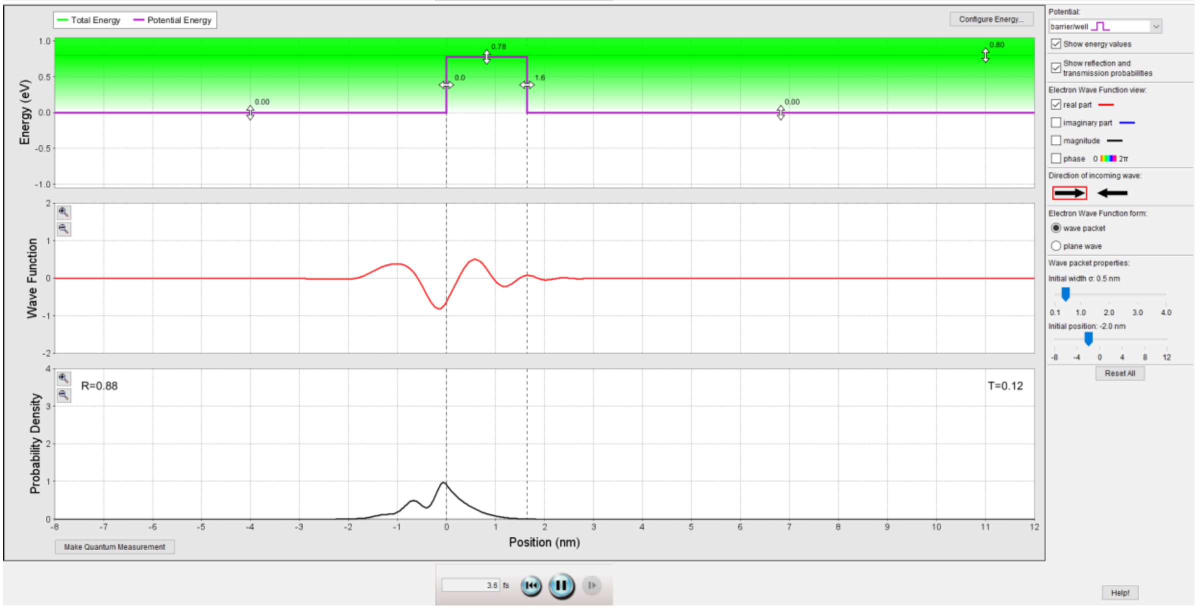

Total Energy
Potential Energy
Configure Energy...
Energy (eV)
Wave Function
Probability Density
R=0.45
T=0.55
Position (nm)
Make Quantum Measurement
Potential:
Show energy values
Show reflection and transmission probabilities
Electron Wave Function view:
real part
imaginary part
magnitude
phase
Direction of incoming wave:
Electron Wave Function form:
wave packet
plane wave
Wave packet properties:
Reset All

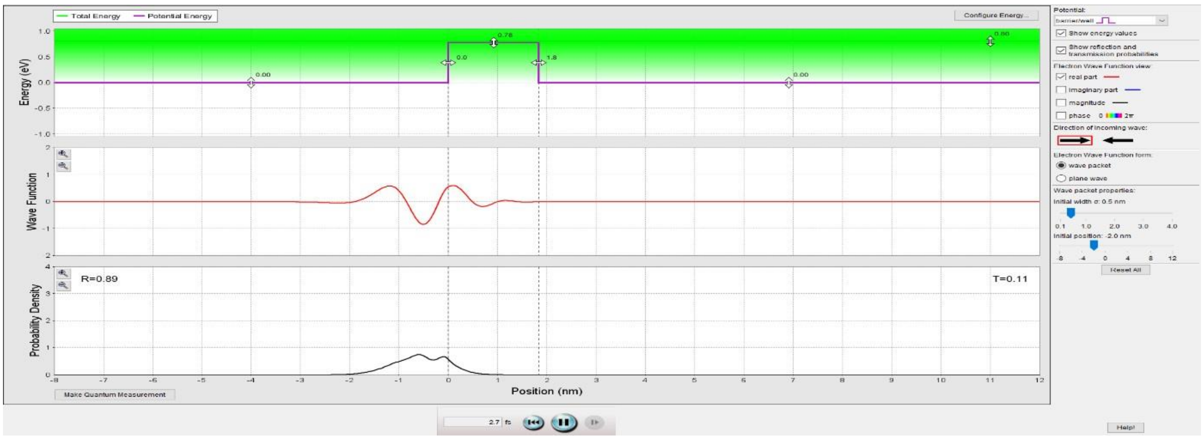


Total Energy
Potential Energy
Configure Energy...
Energy (eV)
Wave Function
Probability Density
R=0.04
T=0.96
Position (nm)
Make Quantum Measurement
Help!
Reset All

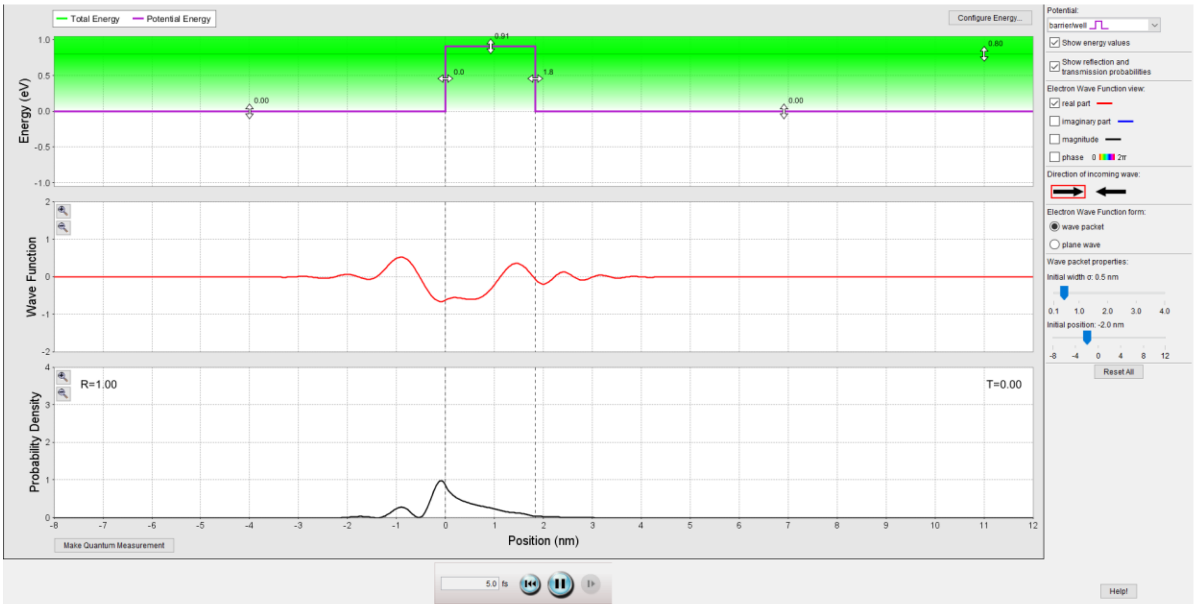
B.) Varying barrier width for a particular wave packet width



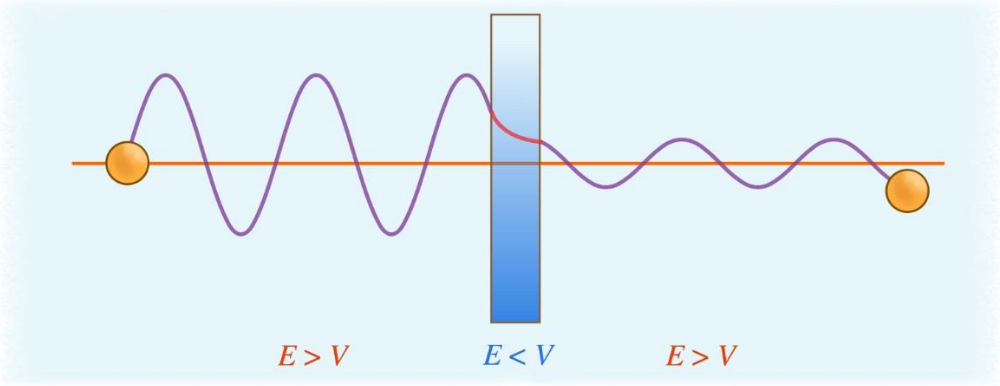




Also, we can see that the transmission probability becomes 0 and reflection probability becomes 1 at a particular height of barrier in a particular packet as shown below.



• Calculations:



Using the formula:

$T = e^{-2 k_2 L}$

Where, $k_2 = (\sqrt{2m(U-E)})/(h/2\pi)$

$R = 1 - T$

We can calculate the value of transmittance probability and the reflection probability using this formula by keeping the given values in it.

• Error in result:

Common sources of error can be instrumental, environmental, procedural, and human. They are very random in nature. Instrumental error occurs when the instrument we are using is not perfect, environmental error occurs when there is some environmental parameters affecting the experiment in bad way, human error can be that the human can do wrong calculations, procedural error can be due to not following the proper procedure for the experiment.

To calculate that error, we can use:
$\% \text{ error} = (|\textit{expected value} - \textit{experimental value}|) / (\textit{expected value})$

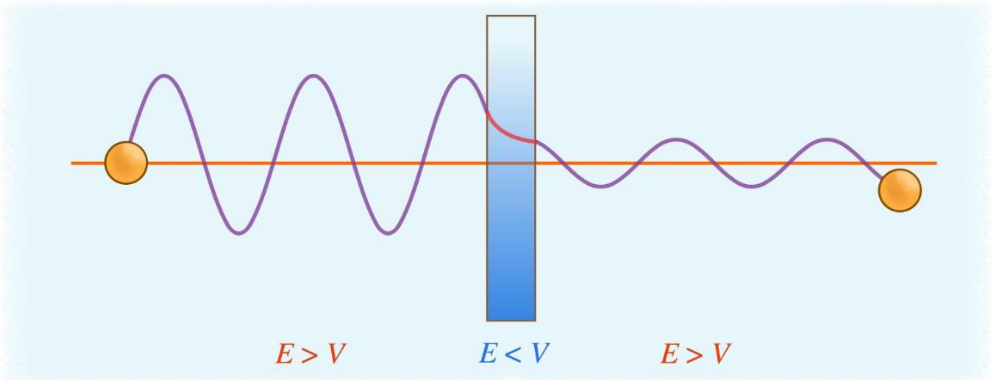
Result analysis:

1.) On increasing the barrier height with constant barrier width for a particular wave packet width, we observed that the transmission probability was decreasing and reflection probability was increasing.

    Maximum transmission probability was when the barrier height was minimum.

2.) On increasing the barrier width with constant barrier height for a particular wave packet width, we observed that the transmission probability was decreasing and the reflection probability was increasing.

3.) Also, we saw that the energy of the wave decreased upon transmission as shown in below figure.



4.) Also, we can say that the transmission probability is maximum when both the barrier height and width are minimum for a given width of wave packet.

5.) The transmission probability is very much sensitive to the change in width and height of the barrier.

6.) Also, we can conclude that the transmission in tunneling effect depends on the width, height of the barrier basically height of the barrier is the energy of the barrier and also on the energy of the wave

Hence all our theories match the simulation results and observations and now we can understand the effect of quantum tunneling more effectively.

Thank you