1) $m_A = 1\,kg$ $\quad m_B = 1\,kg$

$V_A = 50\,m/s$ $\quad V_B = 125\,m/s$

$$\therefore V_A' = \frac{m_A V_A - m_B V_A + 2 m_B V_B}{m_A + m_B}$$

$$= \frac{50 - 50 + 50}{1+1} = \frac{50}{2} = 25\,m/s$$

$$\therefore V_B' = V_A - V_B + V_A' \quad (\text{from } ③)$$

$$= 50 - 25 + 25 = 50\,m/s$$

3) $m_A = 1\,kg$ $\quad m_B = 5\,kg$

$V_A = 100\,m/s$ $\quad V_B = 10\,m/s$

$$V_A' = \frac{1(100) - 5(100) + 2(5)(10)}{1+5}$$

$$= \frac{-300}{6} = -50\,m/s$$

$$V_B' = 100 - 10 + (-50)$$

$$= 40\,m/s$$

5) $m_A = 1\,kg$ $\quad m_B = 5kg$

$V_A = 50\,m/s$ $\quad V_B = -50\,m/s$

$$V_A' = \frac{1(50) - 5(50) + 2(5)(-50)}{1+5}$$

$$= \frac{-700}{6} = -116.67\,m/s$$

$$V_B' = 50 - (-50) + (-116.67)$$

$$= -16.67\,m/s$$

2) $m_A = 2\,kg$ $\quad m_B = 4\,kg$

$V_A = 40\,m/s$ $\quad V_B = 0\,m/s$

$$V_A' = \frac{2(40) - 4(40) + 2(4)(0)}{2+4}$$

$$= -\frac{80}{6} = -13.34\,m/s$$

$$V_B' = 40 - 0 + (-13.34)$$

$$= 26.67\,m/s$$

4) $m_A = 5\,kg$ $\quad m_B = 1\,kg$

$V_A = 100\,m/s$ $\quad V_B = 10\,m/s$

$$V_B' = \frac{1(10) - 5(10) + 2(5)(100)}{1+5}$$

$$= \frac{960}{6} = 160\,m/s$$

$$V_A' = 10 - 100 + 160$$

$$= 70\,m/s$$

6) $m_A = 3\,kg$ $\quad m_B = 3\,kg$

$V_A = 80\,m/s$ $\quad V_B = -40\,m/s$

$$V_A' = \frac{3(80) - 3(80) + 2(3)(-40)}{3+3}$$

$$= -\frac{240}{6} = -40\,m/s$$

$$V_B' = 80 - (-40) + (-40)$$

$$= 80\,m/s$$

1) $p_A = m_A v_A$ $\quad p_B = m_B v_B$

$= 50 \text{ kg m/s}$ $\quad = 25 \text{ kg m/s}$

$p_A' = m_A v_A'$ $\quad p_B' = m_B v_B'$

$= 25 \text{ kg m/s}$ $\quad = 50 \text{ kg m/s}$

$KE_A = \frac{1}{2} m_A v_A^2$ $\quad KE_B = \frac{1}{2} m_B v_B^2$

$= 1250 \text{ J}$ $\quad = 312.5 \text{ J}$

$KE_A' = \frac{1}{2} m_A v_A^2$ $\quad KE_B' = \frac{1}{2} m_B v_B^2$

$= 312.5 \text{ J}$ $\quad = 1250 \text{ J}$

3) $p_A = 100 \text{ kg m/s}$ $\quad p_B = 5(10)$

$= 50 \text{ kg m/s}$

$p_A' = 1(-50)$ $\quad p_B' = 5(40)$

$= -50 \text{ kg m/s}$ $\quad = 200 \text{ kg m/s}$

$KE_A = \frac{1}{2}(1)(10000)$ $\quad KE_B = \frac{1}{2}(5)(100)$

$= 5000 \text{ J}$ $\quad = 250 \text{ J}$

$KE_A' = \frac{1}{2}(1)(2500)$ $\quad KE_B' = \frac{1}{2}(5)(1600)$

$= 1250 \text{ J}$ $\quad = 4000 \text{ J}$

5) $p_A = 1(50)$ $\quad p_B = 5(-50)$

$= 50 \text{ kg m/s}$ $\quad = -250 \text{ kg m/s}$

$p_A' = 1(-116.67)$ $\quad p_B' = 5(-16.67)$

$= -116.67 \text{ kg m/s}$ $\quad = -83.35 \text{ kg m/s}$

2) $P_A = 2(40)$
$= 80 \text{ kg m/s}$

$P_A' = 2(-13.34)$
$= -26.68 \text{ kg m/s}$

$P_B = 4(0)$
$= 0 \text{ kg m/s}$

$P_B' = 4(-26.67)$
$= -106.68 \text{ kg m/s}$

$KE_A = \frac{1}{2}(2)(1600)$
$= 1600 \text{ J}$

$KE_A' = \frac{1}{2}(2)\left(\frac{1600}{9}\right)$
$= 177.7 \text{ J}$

$KE_B = \frac{1}{2}(4)(0)$
$= 0 \text{ J}$

$KE_B' = \frac{1}{2}(4)\left(\frac{6400}{9}\right)$
$= 1422.2 \text{ J}$

4) $P_A = 5(100)$
$= 500 \text{ kg m/s}$

$P_A' = 5(70)$
$= 350 \text{ kg m/s}$

$P_B = 1(10)$
$= 10 \text{ kg m/s}$

$P_B' = 1(160)$
$= 160 \text{ kg m/s}$

$KE_A = \frac{1}{2}(5)(10000)$
$= 25000 \text{ J}$

$KE_A' = \frac{1}{2}(5)(4900)$
$= 12250 \text{ J}$

$KE_B = \frac{1}{2}(1)(100)$
$= 50 \text{ J}$

$KE_B' = \frac{1}{2}(1)(25600)$
$= 12800 \text{ J}$

$KE_A = \frac{1}{2}(1)(2500)$
$= 1250 \text{ J}$

$KE_A' = \frac{1}{2}(1)\left(\frac{490000}{36}\right)$
$= 6805.5 \text{ J}$

$KE_B = \frac{1}{2}(5)(2500)$
$= 6250 \text{ J}$

$KE_B' = \frac{1}{2}(5)\left(\frac{2500}{9}\right)$
$= 694.4 \text{ J}$

↗

6) $P_A = 3(80)$

$= 240 \text{ kgm/s}$

$P_B = 3(-40)$

$= -120 \text{ kgm/s}$

$P'_A = 3(-40)$

$= -120 \text{ kgm/s}$

$P'_B = 3(80)$

$= 240 \text{ kgm/s}$

$KE_A = \frac{1}{2}(3)(6400)$

$= 9600 \text{ J}$

$KE_B = \frac{1}{2}(3)(1600)$

$= 2400 \text{ J}$

$KE'_A = \frac{1}{2}(3)(1600)$

$= 2400 \text{ J}$

$KE'_B = \frac{1}{2}(3)(6400)$

$= 9600 \text{ J}$

1) $m_A = 1\ kg$ $\quad m_B = 1\ kg$

$V_A = 40\ m/s$ $\quad V_B = 10\ m/s$

$e = 0.5$

$$V_B' = \frac{(m_B - e m_A) V_B + (1+e) m_A V_A}{(m_A + m_B)}$$

$$= \frac{(1 - (0.5)1)\,10 + (1+0.5)(1)(40)}{(1+1)}$$

$$= \frac{5 + 60}{2} = 32.5\ m/s$$

$$V_A' = e(V_A - V_B) + V_A'$$

$$= 0.5(10 - 40) + 32.5$$

$$= 17.5\ m/s$$

3) $m_A = 2\ kg$ $\quad m_B = 4\ kg$

$V_A = 60\ m/s$ $\quad V_B = 20\ m/s$

$e = 0.8$

$$V_A' = \frac{(2 - 0.8(4))\,60 + 1.8(4)(20)}{2+4}$$

$$= \frac{-72 + 144}{6} = \frac{72}{6} = 12\ m/s$$

$$V_B' = 0.8(60 - 20) + 12$$

$$= 32 + 12 = 44\ m/s$$

2) $m_A = 1\ kg$ $\quad m_B = 5\ kg$

$v_A = 50\ m/s$ $\quad v_B = -50\ m/s$

$e = 0.2$

$$v_A' = \frac{(1 - 0.2(5))50 + (1.2)(5)(-50)}{1+5}$$

$$= \frac{-300}{6} = -50\ m/s$$

$$v_B' = 0.2(50 - (-50)) + (-50)$$

$$= 20 - 50$$

$$= -30\ m/s$$

4) $m_A = 2\ kg$ $\quad m_B = 4\ kg$

$v_A = 80\ m/s$ $\quad v_B = -40\ m/s$

$e = 0$

$$v_A' = \frac{(2 - 0(4))80 + 1(4)(-40)}{2+4}$$

$$= \frac{160 - 160}{6} = 0\ m/s$$

$$v_B' = 0(80 - (-40)) + 0$$

$$= 0\ m/s$$

5) $m_A = 3\,kg$ $\quad m_B = 5\,kg$

$v_A = 30\,m/s$ $\quad v_B = -30\,m/s$

$e = 0.4$

$$v_A' = \frac{(3 - 0.4(5))\,30 + 1.4\,(5)(-30)}{3+5}$$

$$= -\frac{180}{8} = -22.5\,m/s$$

$$v_B' = 0.4(30 - (-30)) + (-22.5)$$

$$= 24 - 22.5 = 1.5\,m/s$$

6) $m_A = 1\ kg$ $\quad m_B = 1\ kg$

$V_A = 50\ m/s$ $\quad V_B = 0\ m/s$

$e = 0.6$

$$V_A' = \frac{(1 - 0.6(1))\,50 + 1.6\,(1)\,(0)}{1+1}$$

$$= \frac{20}{2} = 10\ m/s$$

$$V_B' = 0.6(50-0) + 10$$

$$= 40\ m/s$$

1) $P_A = 1(40)$
$= 40 \text{ kg m/s}$

$P_A' = 1(17.5)$
$= 17.5 \text{ kg m/s}$

$KE_A = \frac{1}{2}(1)(1600)$
$= 800 \text{ J}$

$KE_A' = \frac{1}{2}(1)(289.25)$
$= 144.625 \text{ J}$

$P_B = 1(10)$
$= 10 \text{ kg m/s}$

$P_B' = 1(32.5)$
$= 32.5 \text{ kg m/s}$

$KE_B = \frac{1}{2}(1)(100)$
$= 50 \text{ J}$

$KE_B' = \frac{1}{2}(1)(1024.25)$
$= 512.125 \text{ J}$

3) $P_A = 2(60)$
$= 120 \text{ kg m/s}$

$P_A' = 2(12)$
$= 24 \text{ kg m/s}$

$KE_A = \frac{1}{2}(2)(3600)$
$= 3600 \text{ J}$

$KE_A' = \frac{1}{2}(2)(144)$
$= 144 \text{ J}$

$P_B = 4(20)$
$= 80 \text{ kg m/s}$

$P_B' = 4(44)$
$= 176 \text{ kg m/s}$

$KE_B = \frac{1}{2}(4)(400)$
$= 800 \text{ J}$

$KE_B' = \frac{1}{2}(4)(1936)$
$= 3872 \text{ J}$

2) $P_A = 1(50)$ $P_B = 5(-50)$

$= 50 \text{ kg m/s}$ $= -250 \text{ kg m/s}$

$P_A' = 1(-50)$ $P_B' = 5(-30)$

$= -50 \text{ kg m/s}$ $= -150 \text{ kg m/s}$

$KE_A = \frac{1}{2}(1)(2500)$ $KE_B = \frac{1}{2}(5)(2500)$

$= 1250 \text{ J}$ $= 6250 \text{ J}$

$KE_A' = \frac{1}{2}(1)(2500)$ $KE_B' = \frac{1}{2}(5)(900)$

$= 1250 \text{ J}$ $= 2250 \text{ J}$

4) $P_A = 2(80)$ $P_B = 4(-40)$

$= 160 \text{ kg m/s}$ $= -160 \text{ kg m/s}$

$P_A' = 2(0)$ $P_B' = 4(0)$

$= 0 \text{ kg m/s}$ $= 0 \text{ kg m/s}$

$KE_A' = \frac{1}{2}(2)(6400)$ $KE_B = \frac{1}{2}(4)(1600)$

$= 6400 \text{ J}$ $= 3200 \text{ J}$

$KE_A' = \frac{1}{2}(2)(0)$ $KE_B' = \frac{1}{2}(4)(0)$

$= 0 \text{ J}$ $= 0 \text{ J}$

5) $P_A = 3(30)$

$= 90 \text{ kg m/s}$

$P_B = 5(-30)$

$= -150 \text{ kg m/s}$

$P_A' = 3(-22.5)$

$= -67.5 \text{ kg m/s}$

$P_B' = 5(1.5)$

$= 7.5 \text{ kg m/s}$

$KE_A = \frac{1}{2}(3)(900)$

$= 1350 \text{ J}$

$KE_B = \frac{1}{2}(5)(900)$

$= 2250 \text{ J}$

$KE_A' = \frac{1}{2}(3)\left(\frac{2025}{4}\right)$

$= 759.375 \text{ J}$

$KE_B' = \frac{1}{2}(5)(2.25)$

$= 5.625 \text{ J}$

6) $P_A = 1(50)$

$= 50 \text{ kgm/s}$

$P_A' = 1(10)$

$= 10 \text{ kgm/s}$

$KE_A = \frac{1}{2}(1)(2500)$

$= 1250 \text{ J}$

$KE_A' = \frac{1}{2}(1)(100)$

$= 50 \text{ J}$

$P_B = 1(0)$

$= 0 \text{ kgm/s}$

$P_B' = 1(40)$

$= 40 \text{ kgm/s}$

$KE_B = \frac{1}{2}(1)(0)$

$= 0 \text{ J}$

$KE_B' = \frac{1}{2}(1)(1600)$

$= 800 \text{ J}$

Elastic Collisions :- Momentum and kinetic energy, both are Conserved
∴ e = 1 (Coeff of restitution)

let mass of A = m_A — mass of B = m_B
velocity of A before Collision = V_A — velocity of B before Collision = V_B
velocity of A after Collision = V_A' — velocity of B after Collision = V_B'

∴ During elastic Collision, momentum (P) is Conserved;
∴ $(P_A + P_B)$ before Collision = $(P_A' + P_B')$ after Collision
$\Rightarrow m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$ — ①

Also, Kinetic Energy is Conserved in elastic Collision,
∴ $(KE_A + KE_B)$ before Collision = $(KE_A' + KE_B')$ after Collision
$\Rightarrow \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = \frac{1}{2} m_A V_A'^2 + \frac{1}{2} m_B V_B'^2$

$\Rightarrow m_A V_A^2 + m_B V_B^2 = m_A V_A'^2 + m_B V_B'^2$ — ②

But, $e = 1 = \frac{V_B' - V_A'}{V_A - V_B} \Rightarrow V_B' - V_A' = V_A - V_B$
$\Rightarrow V_B' = V_A - V_B + V_A'$ — ③

putting ③ in ①,
$m_A V_A + m_B V_B = m_A V_A' + m_B (V_A - V_B + V_A')$
$\Rightarrow m_A V_A + m_B V_B - m_B V_A + m_B V_B = m_A V_A' + m_B V_A'$
$\Rightarrow m_A V_A - m_B V_A + 2 m_B V_B = (m_A + m_B) V_A'$
$\Rightarrow \frac{(m_A V_A - m_B V_A + 2 m_B V_B)}{(m_A + m_B)} = V_A'$ — (A)

∴ $V_B' = V_A - V_B + V_A'$
$= V_A - V_B + \frac{m_A V_A - m_B V_A + 2 m_B V_B}{(m_A + m_B)} = \frac{2 m_A V_A - m_A V_B + m_B V_B}{(m_A + m_B)}$

$V_B' = \frac{m_B V_B - m_A V_B + 2 m_A V_A}{(m_A + m_B)}$ — (B)

Inelastic Collision :- Momentum Conserved, but not Kinetic Energy

:- $0 \le e < 1$

let mass of body A = m_A — mass of body B = m_B

vel of A before Collision = v_A — vel. of B before Collision = v_B

vel of A after Collision = v_A' — vel. of B after Collision = v_B'

Coeff. of restitution of the System = e

In this type of Collision, there will be Conservation of Momentum

$\therefore$ $(P_A + P_B)$ before Coll. = $(P_A' + P_B')$ after Coll.

$\Rightarrow$ $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$ — (1)

But, the Coeff. of restitution = $e = \dfrac{v_B' - v_A'}{v_A - v_B}$

$\Rightarrow$ $e v_A - e v_B = v_B' - v_A'$

$\Rightarrow$ $e v_A - e v_B + v_A' = v_B'$ — (2)

$\therefore$ In (1), $m_A v_A + m_B v_B = m_A v_A' + m_B (e v_A - e v_B + v_A')$

$\Rightarrow$ $m_A v_A + m_B v_B - e m_B v_A + e m_B v_B = m_A v_A' + m_B v_A'$

$\Rightarrow$ $(m_A - e m_B) v_A + (m_B + e m_B) v_B = (m_A + m_B) v_A'$

$\Rightarrow$ $\dfrac{(m_A - e m_B) v_A + (1+e) m_B v_B}{(m_A + m_B)} = v_A'$ — (A)

$\therefore$ $v_B' = e v_A - e v_B + \dfrac{(m_A - e m_B) v_A + (1+e) m_B v_B}{(m_A + m_B)}$ (from (2))

$= \dfrac{e m_A v_A - e m_A v_B + e m_B v_A - e m_B v_B + m_A v_A - e m_B v_A + (1+e) m_B v_B}{(m_A + m_B)}$

$= \dfrac{(1+e) m_A v_A + (1-e) m_B v_B - e m_A v_B}{(m_A + m_B)}$

$\Rightarrow$ $v_B' = \dfrac{(m_B - e m_A) v_B + (1+e) m_A v_A}{(m_A + m_B)}$ — (B)