PH160 Laboratory session 2

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Aim : To demonstrate Elastic and Inelastic collisions.

Theory :

A) Elastic Collision :

\# Elastic Collisions :- Momentum and kinetic energy, both are Conserved

$\therefore e = 1$ (coeff of restitution)

Let mass of A = m_A

velocity of A before Collision = v_A

velocity of A after Collision = v_A'

mass of B = m_B

velocity of B before Collision = v_B

velocity of B after Collision = v_B'

$\therefore$ During elastic Collision, momentum (P) is Conserved,

$$\therefore (P_A + P_B) \text{ before Collision} = (P_A' + P_B') \text{ after Collision}$$

$$\Rightarrow m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad —①$$

Also Kinetic Energy is Conserved in elastic Collision,

$$\therefore (KE_A + KE_B) \text{ before Collision} = (KE_A' + KE_B') \text{ after Collision}$$

$$\Rightarrow \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$\Rightarrow m_A v_A^2 + m_B v_B^2 = m_A v_A'^2 + m_B v_B'^2 \quad —②$$

But, $e = 1 = \dfrac{v_B' - v_A'}{v_A - v_B} \Rightarrow v_B' - v_A' = v_A - v_B$

$$\Rightarrow v_B' = v_A - v_B + v_A' \quad —③$$

putting ③ in ①,

$$m_A v_A + m_B v_B = m_A v_A' + m_B (v_A - v_B + v_A')$$

$$\Rightarrow m_A v_A + m_B v_B - m_B v_A + m_B v_B = m_A v_A' + m_B v_A'$$

$$\Rightarrow m_A v_A - m_B v_A + 2 m_B v_B = (m_A + m_B) v_A'$$

$$\Rightarrow \frac{(m_A v_A - m_B v_A + 2 m_B v_B)}{(m_A + m_B)} = v_A' \quad —Ⓐ$$

$$\therefore v_B' = v_A - v_B + v_A'$$

$$= v_A - v_B + \frac{m_A v_A - m_B v_A + 2 m_B v_B}{(m_A + m_B)} = \frac{2 m_A v_A - m_A v_B + m_B v_B}{(m_A + m_B)}$$

$$v_B' = \frac{m_B v_B - m_A v_B + 2 m_A v_A}{(m_A + m_B)} \quad —Ⓑ$$

B) Inelastic Collision :

\# Inelastic Collision :- Momentum Conserved, but not Kinetic Energy

:- $0 \le e < 1$

let mass of body A = m_A mass of body B = m_B

vel of A before Collision = v_A vel. of B before Collision = v_B

vel of A after Collision = v_A' vel. of B after Collision = v_B'

Coeff. of restitution of the System = e

In this type of collision, there will be Conservation of Momentum

$$\therefore \quad (P_A + P_B)_{\text{before coll}} = (P_A' + P_B')_{\text{after coll}}$$

$$\Rightarrow \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \text{—— ①}$$

But, the Coeff. of restitution = $e = \dfrac{v_B' - v_A'}{v_A - v_B}$

$$\Rightarrow \quad e v_A - e v_B = v_B' - v_A'$$

$$\Rightarrow \quad e v_A - e v_B + v_A' = v_B' \quad \text{—— ②}$$

In ①, $m_A v_A + m_B v_B = m_A v_A' + m_B (e v_A - e v_B + v_A')$

$$\Rightarrow \quad m_A v_A + m_B v_B - e m_B v_A + e m_B v_B = m_A v_A' + m_B v_A'$$

$$\Rightarrow \quad (m_A - e m_B) v_A + (m_B + e m_B) v_B = (m_A + m_B) v_A'$$

$$\Rightarrow \quad \frac{(m_A - e m_B) v_A + (1 + e) m_B v_B}{(m_A + m_B)} = v_A' \quad \text{—— Ⓐ}$$

$$\therefore \quad v_B' = e v_A - e v_B + \frac{(m_A - e m_B) v_A + (1+e) m_B v_B}{(m_A + m_B)} \quad (\text{from ②})$$

$$= \frac{e m_A v_A - e m_A v_B + e m_B v_A - e m_B v_B + m_A v_A - e m_B v_A + (1+e) m_B v_B}{(m_A + m_B)}$$

$$= \frac{(1+e) m_A v_A + \cancel{(1-e)}\, m_B v_B - e m_A v_B}{(m_A + m_B)}$$

$$\Rightarrow \quad v_B' = \frac{(m_B - e m_A) v_B + (1+e) m_A v_A}{(m_A + m_B)} \quad \text{—— Ⓑ}$$

Observation :

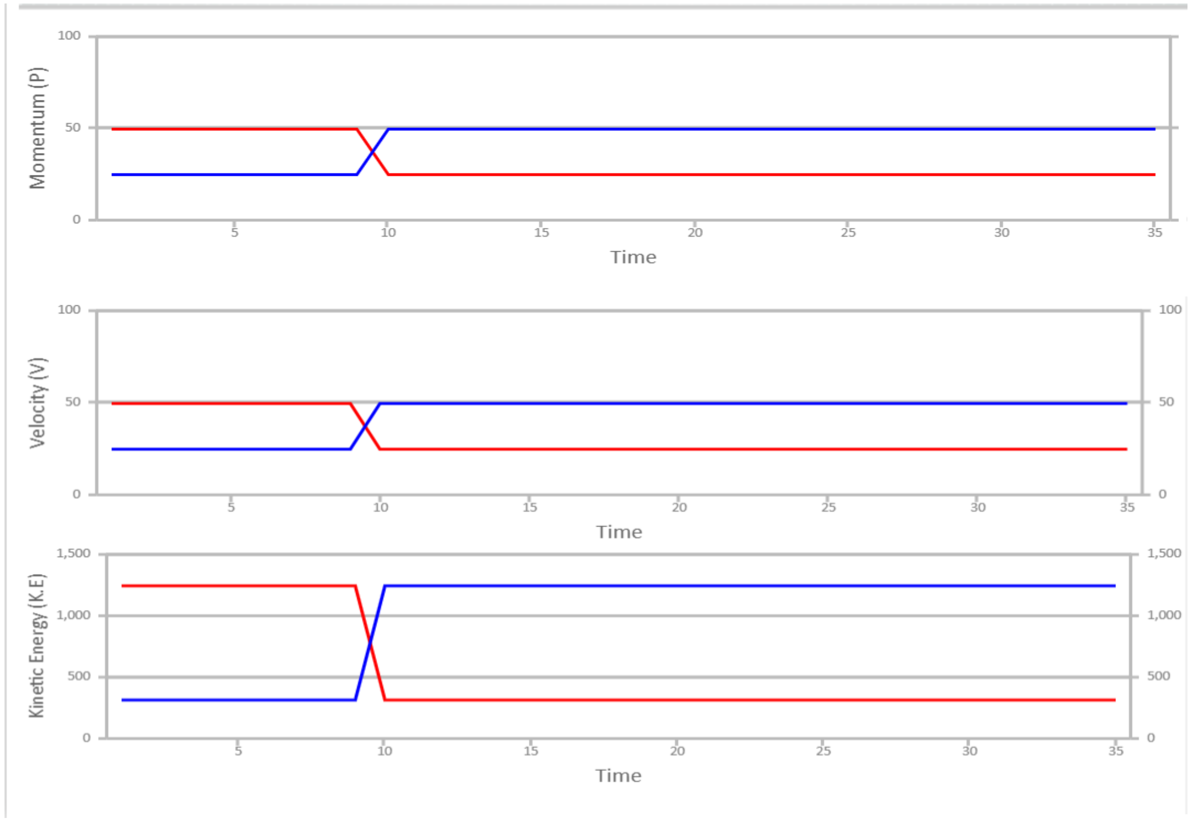
Let, mass of body A = m1
initial(before collision) velocity of A = v1
initial momentum of A = p1
initial kinetic energy of A = k1
final(after collision) velocity of A = V1
final momentum of A = P1
final kinetic energy of A = K1
Coefficient of restitution = e

mass of body B = m2
initial velocity of B = v2
initial momentum of B = p2
initial kinetic energy of B = k2
final velocity of B= V2
final momentum of B = P2
final kinetic energy of B = K2

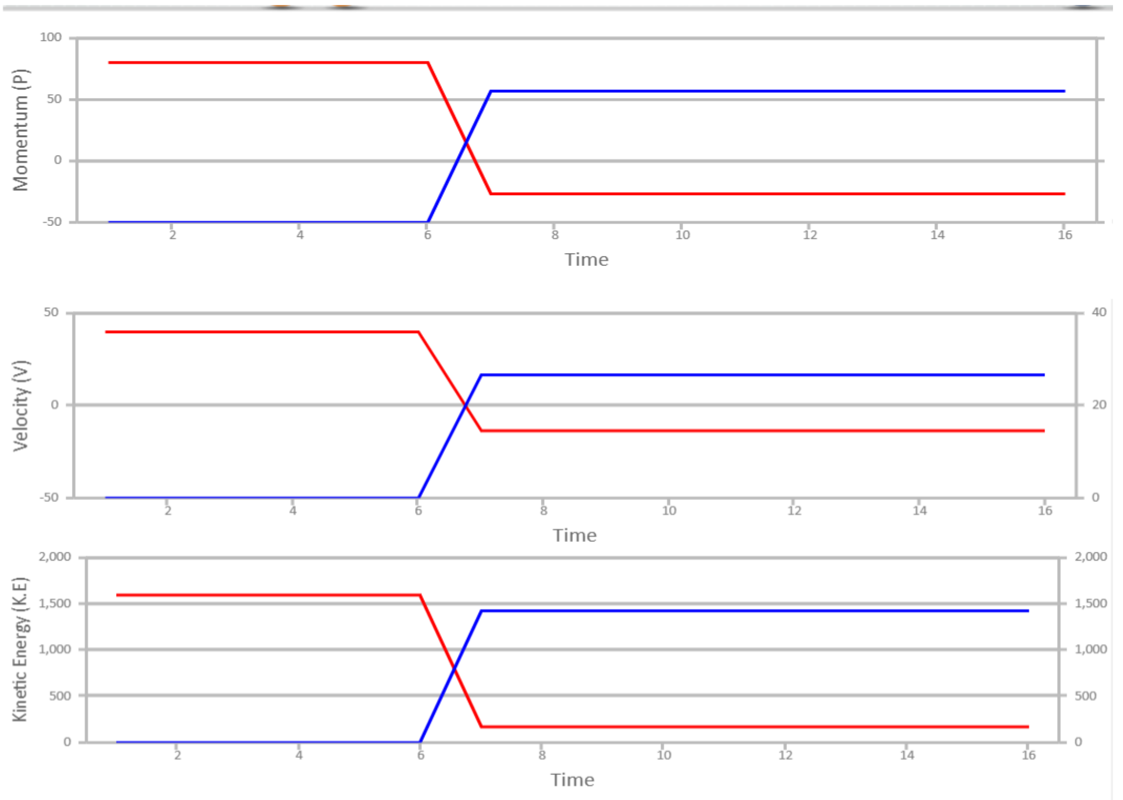
A) Elastic Collision :

	m1 (kg)	m2 (kg)	e	v1 (m/s)	v2 (m/s)	p1 (kg m/s)	p2 (kg m/s)	k1 (J)	k2 (J)	V1 (m/s)	V2 (m/s)	P1 (kg m/s)	P2 (kg m/s)	K1 (J)	K2 (J)
1	1	1	1	50	25	50	25	1250	312.5	25	50	25	50	312.5	1250
2	2	4	1	40	0	80	0	1600	0	-13.3	26.7	-26.7	106.7	177.8	1422.2
3	1	5	1	100	10	100	50	5000	250	-50	40	-50	200	1250	4000
4	1	5	1	10	100	500	10	25000	50	70	160	350	160	12250	12800
5	1	5	1	50	-50	50	-250	1250	6250	-116.7	-16.7	-116.7	-83.3	6805.6	694.4
6	3	3	1	80	-40	240	-120	9600	2400	-40	80	-120	240	2400	9600

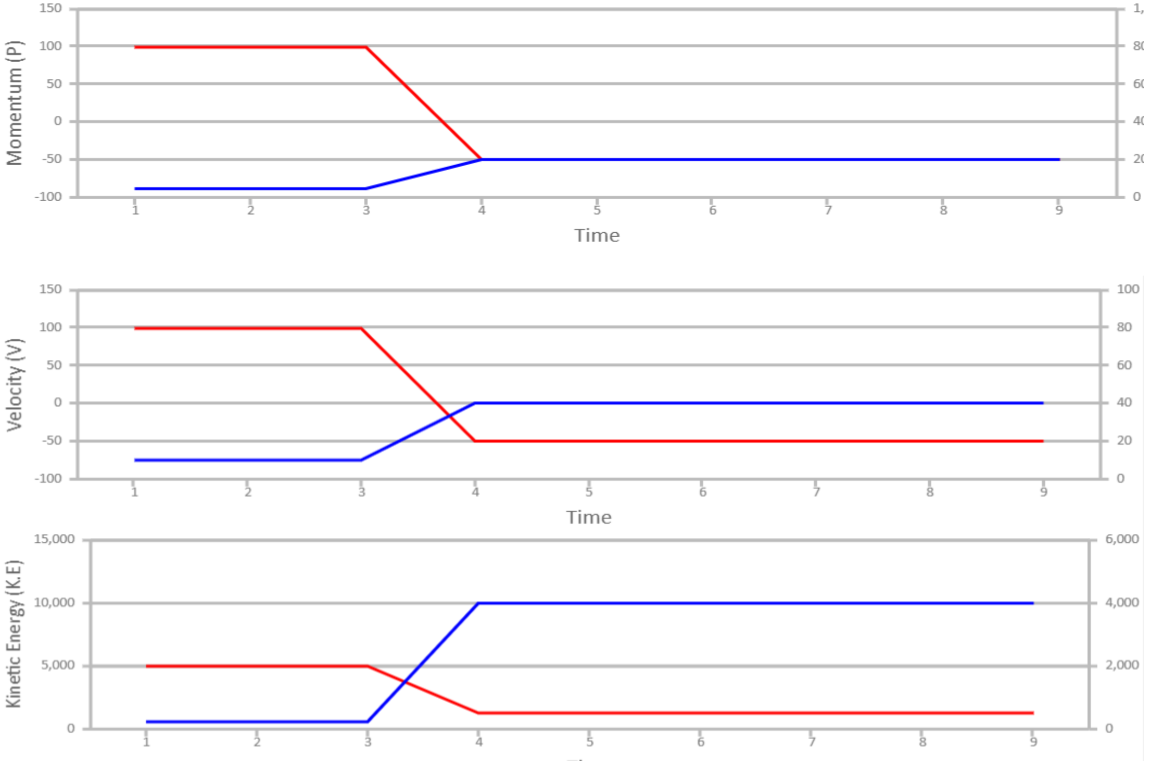
1)


Momentum (P)
Velocity (V)
Kinetic Energy (K.E)
Time

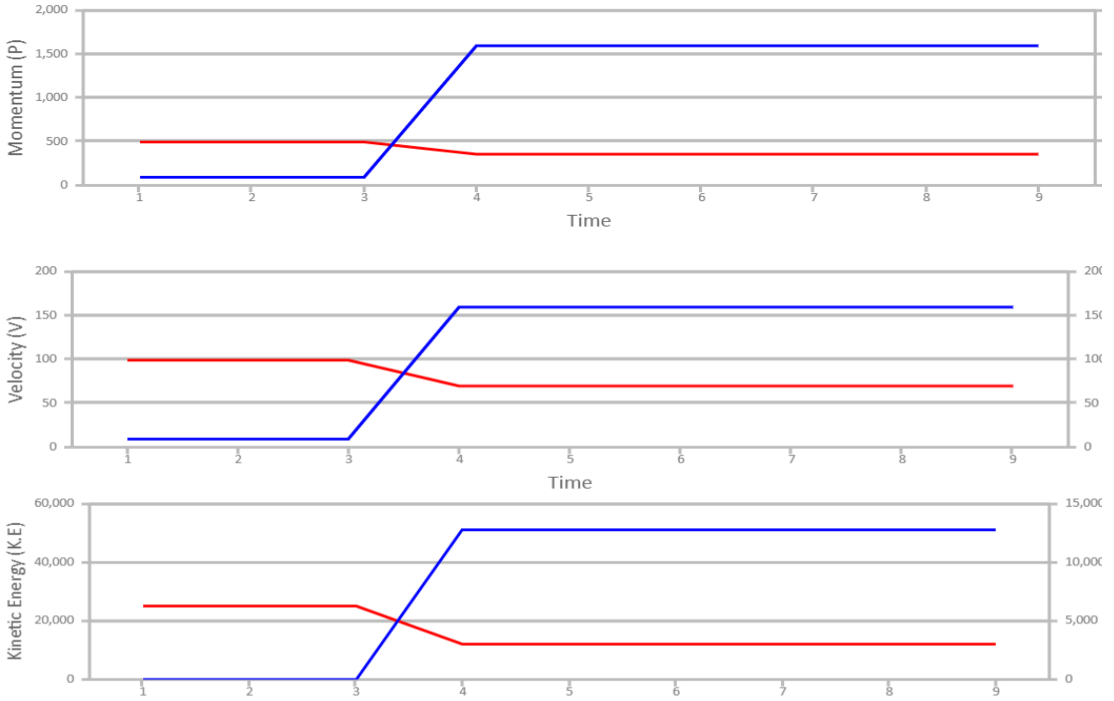
2)


Momentum (P)
Velocity (V)
Kinetic Energy (K.E)
Time

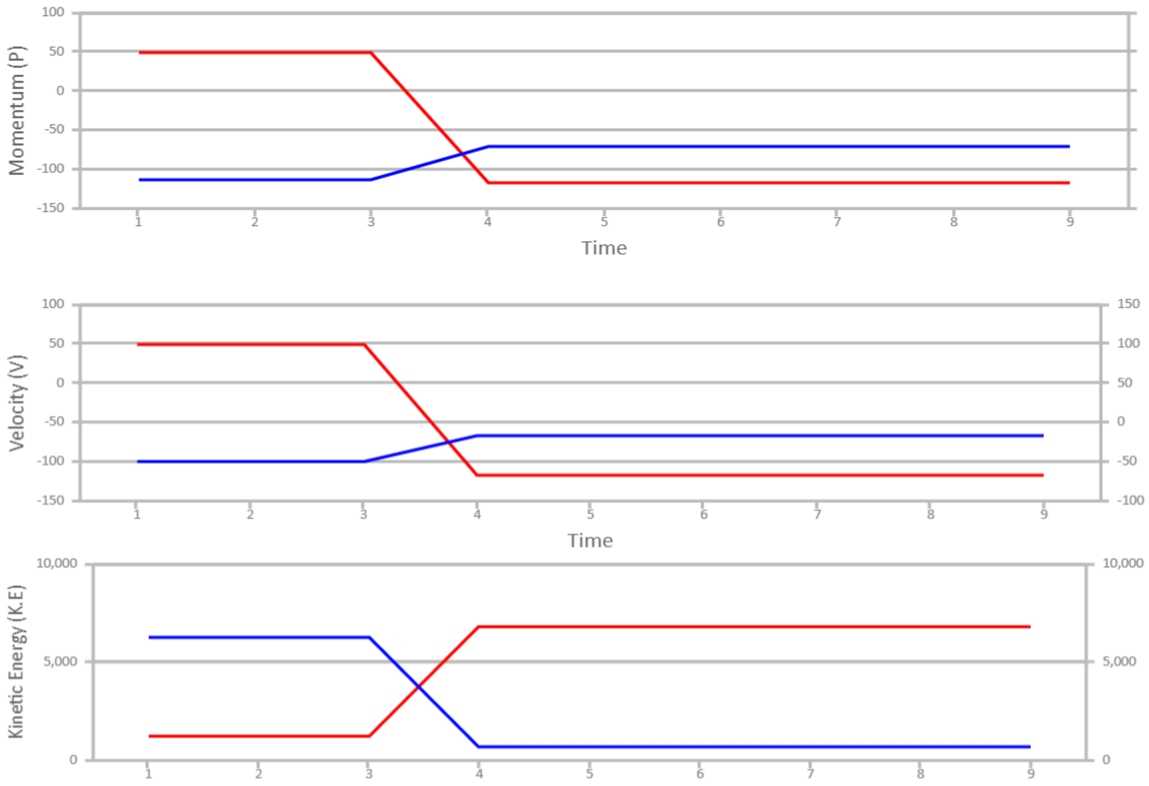
3)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)

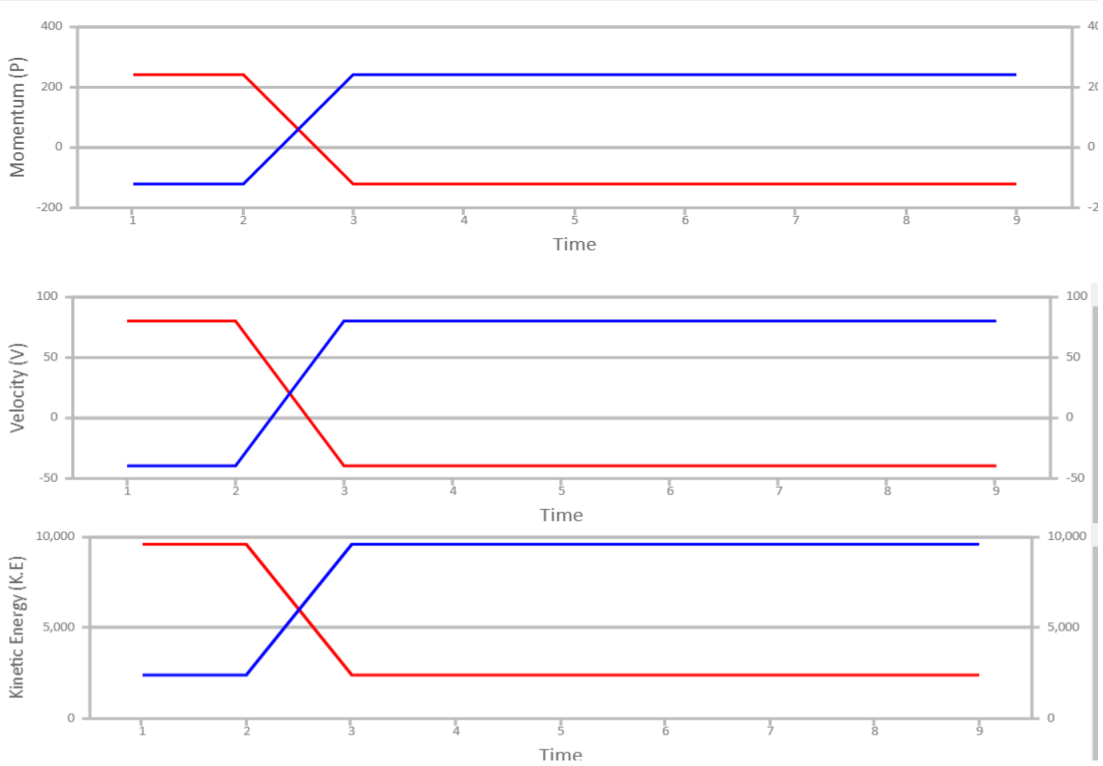
4)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)

5)


Momentum (P)
100
50
0
-50
-100
-150
1 2 3 4 5 6 7 8 9
Time
Velocity (V)
100
50
0
-50
-100
-150
150
100
50
0
-50
-100
1 2 3 4 5 6 7 8 9
Time
Kinetic Energy (K.E)
10,000
5,000
0
10,000
5,000
0
1 2 3 4 5 6 7 8 9

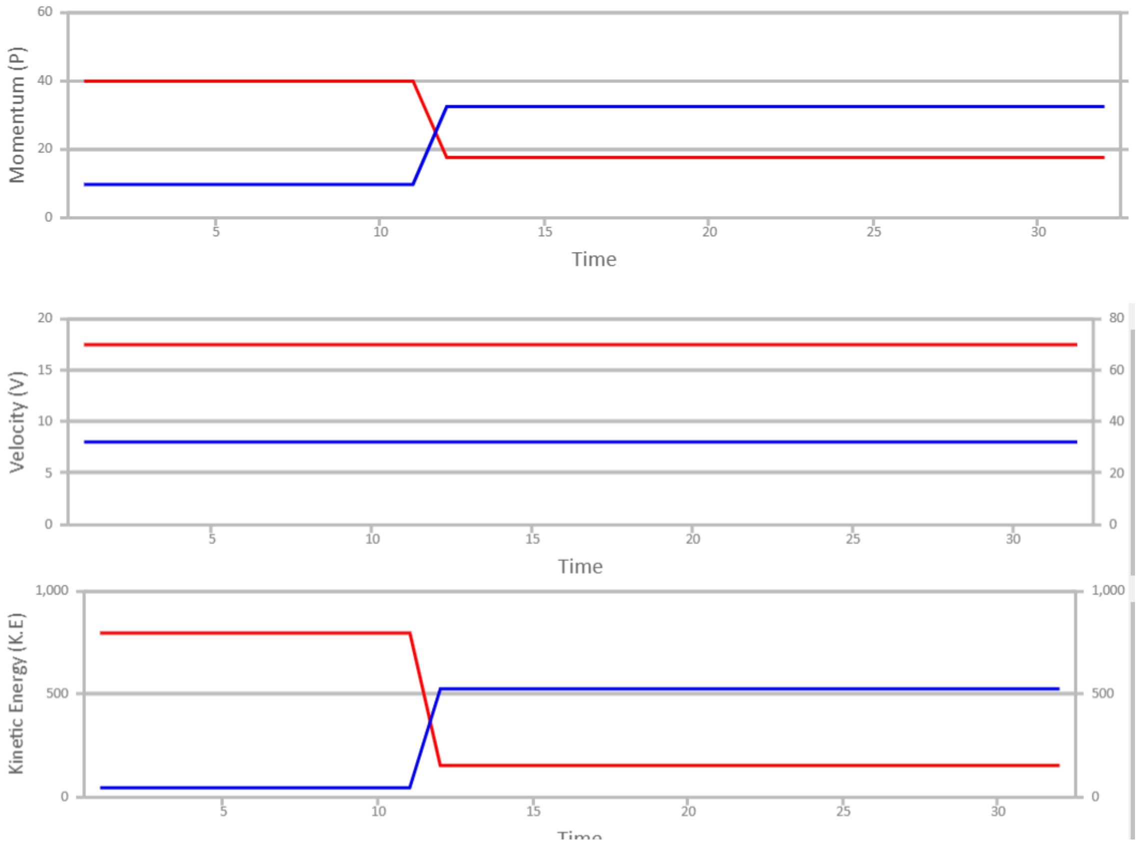
6)


Momentum (P)
400
200
0
-200
1 2 3 4 5 6 7 8 9
Time
Velocity (V)
100
50
0
-50
100
50
0
-50
1 2 3 4 5 6 7 8 9
Time
Kinetic Energy (K.E)
10,000
5,000
0
10,000
5,000
0
1 2 3 4 5 6 7 8 9
Time

B) Inelastic Collision :

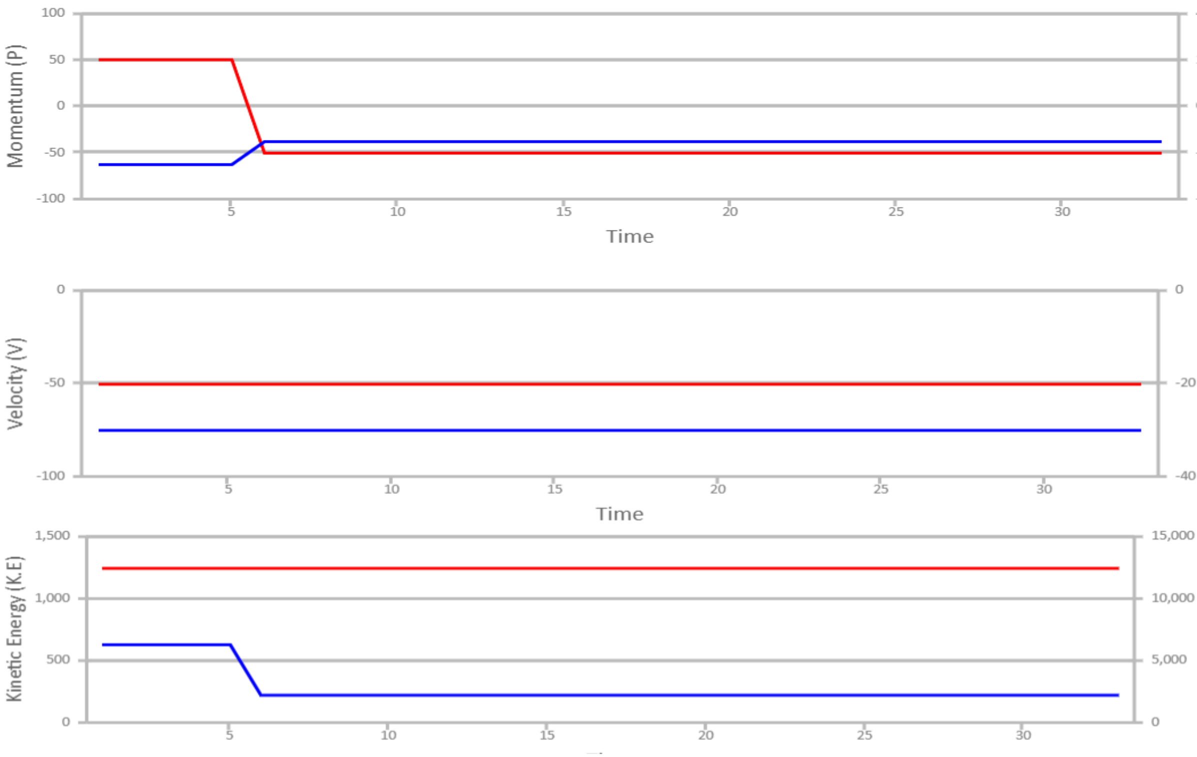
	m1 (kg)	m2 (kg)	e	v1 (m/s)	v2 (m/s)	p1 (kg m/s)	p2 (kg m/s)	k1 (J)	k2 (J)	V1 (m/s)	V2 (m/s)	P1 (kg m/s)	P2 (kg m/s)	K1 (J)	K2 (J)
1	1	1	0.5	10	40	40	10	800	50	17.5	32.5	17.5	32.5	153.1	528.1
2	1	5	0.2	50	-50	50	-250	1250	6250	-50	-30	-50	-150	1250	2250
3	2	4	0.8	60	20	120	80	3600	800	12	44	24	176	144	3872
4	2	4	0	80	-40	160	-160	6400	3200	0	0	0	0	0	0
5	3	5	0.4	30	-30	90	-150	1350	2250	-22.5	1.5	-67.5	7.5	759.4	5.6
6	1	1	0.6	50	0	50	0	1250	0	10	40	10	40	50	800

1)



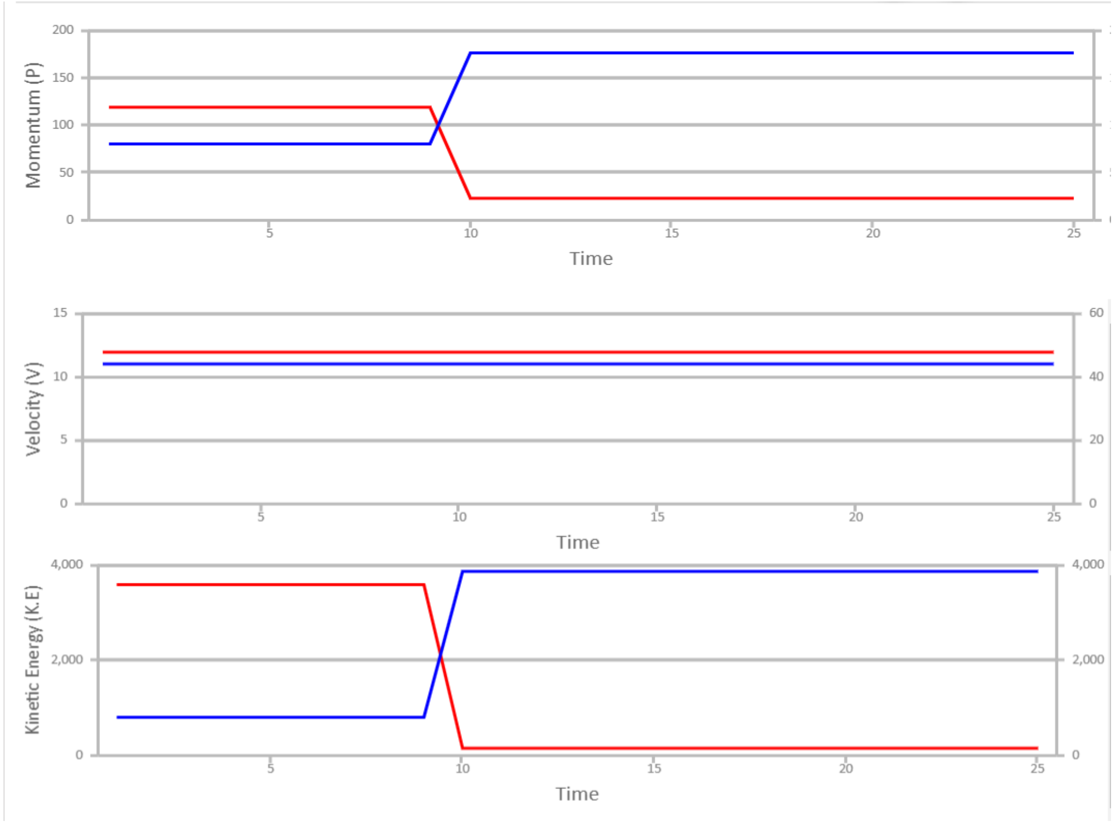
Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)
Time

2)

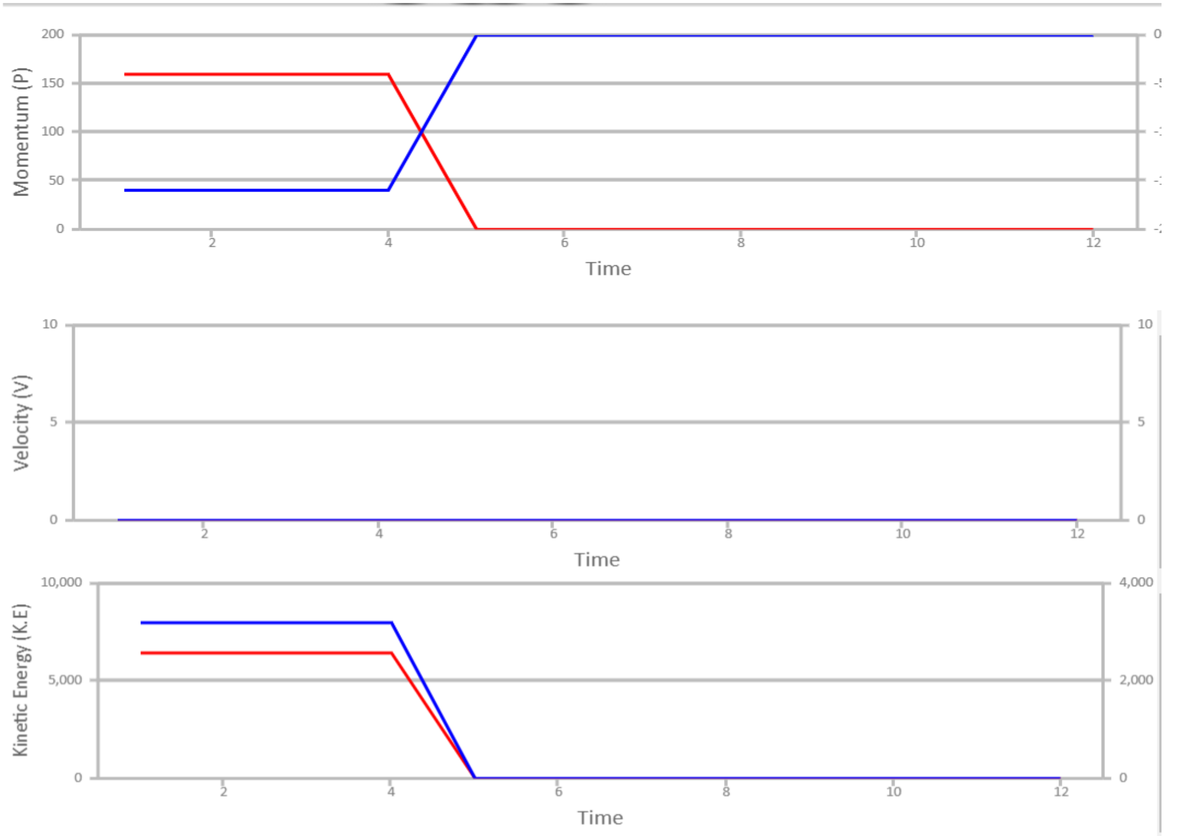


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)

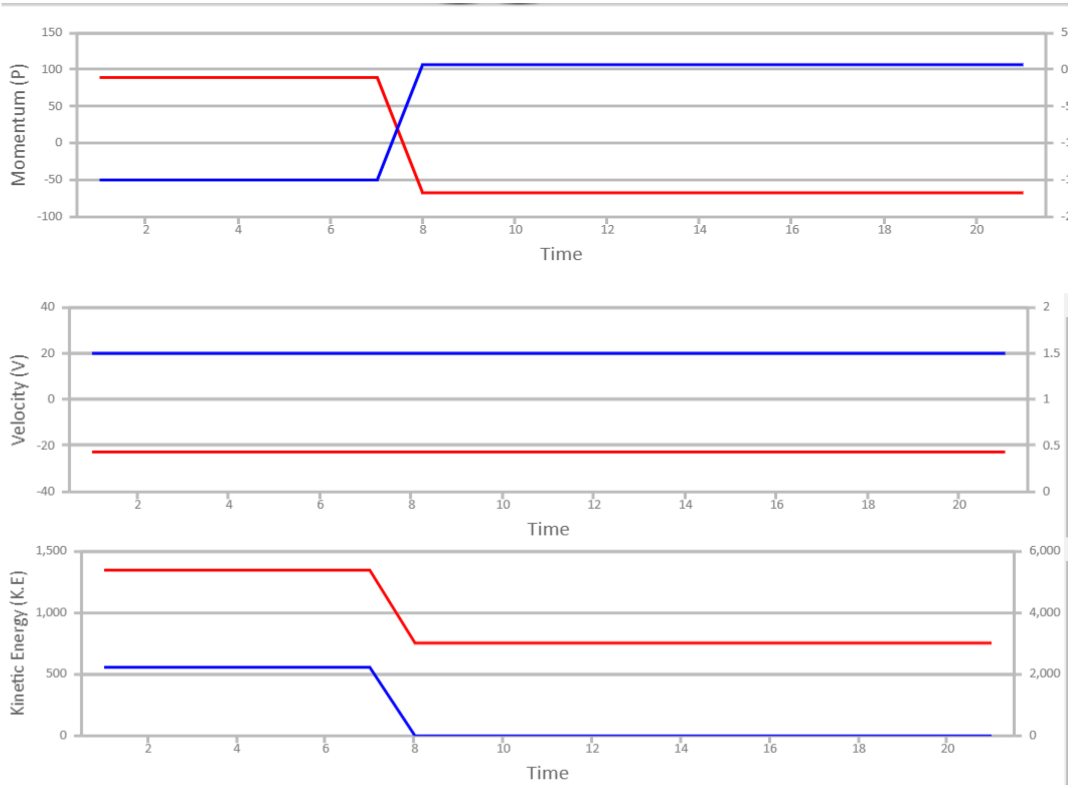
3)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)
Time

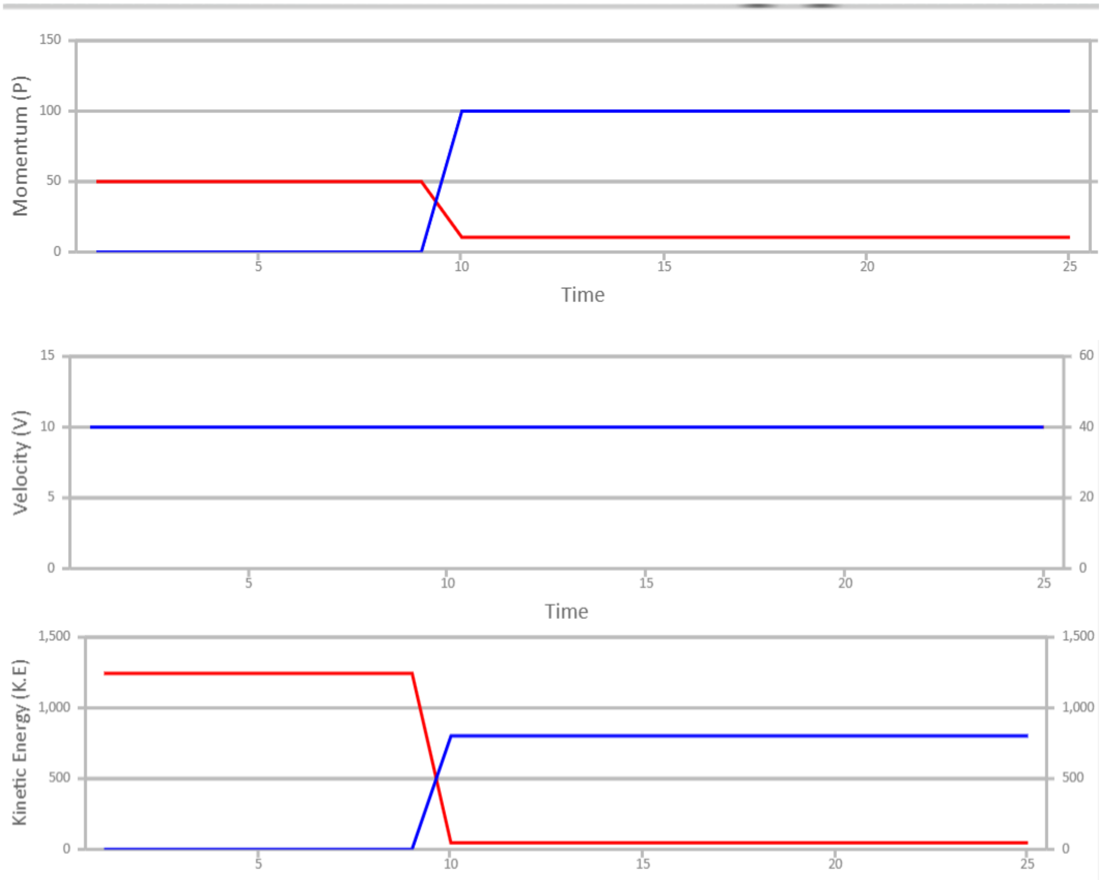
4)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)
Time

5)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)
Time

6)


Momentum (P)
Time
Velocity (V)
Time
Kinetic Energy (K.E)

Calculation :

A) Elastic Collision :

1)

1) $m_A = 1$ kg $\quad m_B = 1$ kg

$V_A = 50$ m/s $\quad V_B = 25$ m/s

$$\therefore V_A' = \frac{m_A V_A - m_B V_A + 2 m_B V_B}{m_A + m_B}$$

$$= \frac{50 - 50 + 50}{1+1} = \frac{50}{2} = 25 \text{ m/s}$$

$$\therefore V_B' = V_A - V_B + V_A' \quad (\text{from } 3)$$

$$= 50 - 25 + 25 = 50 \text{ m/s}$$

1) $P_A = m_A V_A$ $\quad P_B = m_B V_B$

$= 50$ kg m/s $\quad = 25$ kg m/s

$P_A' = m_A V_A'$ $\quad P_B' = m_B V_B'$

$= 25$ kg m/s $\quad = 50$ kg m/s

$KE_A = \frac{1}{2} m_A V_A^2$ $\quad KE_B = \frac{1}{2} m_B V_B^2$

$= 1250$ J $\quad = 312.5$ J

$KE_A' = \frac{1}{2} m_A V_A'^2$ $\quad KE_B' = \frac{1}{2} m_B V_B'^2$

$= 312.5$ J $\quad = 1250$ J

2)

2) $m_A = 2$ kg $\quad m_B = 4$ kg

$V_A = 40$ m/s $\quad V_B = 0$ m/s

$$V_A' = \frac{2(40) - 4(40) + 2(4)(0)}{2+4}$$

$$= -\frac{80}{6} = -13.34 \text{ m/s}$$

$$V_B' = 40 - 0 + (-13.34)$$

$$= 26.67 \text{ m/s}$$

2) $P_A = 2(40)$ $\quad P_B = 4(0)$

$= 80$ kg m/s $\quad = 0$ kg m/s

$P_A' = 2(-13.34)$ $\quad P_B' = 4(-26.67)$

$= -26.68$ kg m/s $\quad = -106.68$ kg m/s

$KE_A = \frac{1}{2}(2)(1600)$ $\quad KE_B = \frac{1}{2}(4)(0)$

$= 1600$ J $\quad = 0$ J

$KE_A' = \frac{1}{2}(2)\left(\frac{1600}{9}\right)$ $\quad KE_B' = \frac{1}{2}(4)\left(\frac{6400}{9}\right)$

$= 177.7$ J $\quad = 1422.2$ J

3)

3) $m_A = 1\text{ kg}$ $\quad m_B = 5\text{ kg}$

$V_A = 100\text{ m/s}$ $\quad V_B = 10\text{ m/s}$

$$V_A' = \frac{1(100) - 5(100) + 2(5)(10)}{1+5}$$

$$= -\frac{300}{6} = -50\text{ m/s}$$

$$V_B' = 100 - 10 + (-50)$$

$$= 40\text{ m/s}$$

3) $P_A = 100\text{ kg m/s}$ $\quad P_B = 5(10)$ $= 50\text{ kg m/s}$

$P_A' = 1(-50)$ $= -50\text{ kg m/s}$ $\quad P_B' = 5(40)$ $= 200\text{ kg m/s}$

$KE_A = \frac{1}{2}(1)(10000)$ $= 5000\text{ J}$ $\quad KE_B = \frac{1}{2}(5)(100)$ $= 250\text{ J}$

$KE_A' = \frac{1}{2}(1)(2500)$ $= 1250\text{ J}$ $\quad KE_B' = \frac{1}{2}(5)(1600)$ $= 4000\text{ J}$

4)

4) $m_A = 5\text{ kg}$ $\quad m_B = 1\text{ kg}$

$V_A = 100\text{ m/s}$ $\quad V_B = 10\text{ m/s}$

$$V_B' = \frac{1(10) - 5(10) + 2(5)(100)}{1+5}$$

$$= \frac{960}{6} = 160\text{ m/s}$$

$$V_A' = 10 - 100 + 160$$

$$= 70\text{ m/s}$$

4) $P_A = 5(100)$ $= 500\text{ kg m/s}$ $\quad P_B = 1(10)$ $= 10\text{ kg m/s}$

$P_A' = 5(70)$ $= 350\text{ kg m/s}$ $\quad P_B' = 1(160)$ $= 160\text{ kg m/s}$

$KE_A = \frac{1}{2}(5)(10000)$ $= 25000\text{ J}$ $\quad KE_B = \frac{1}{2}(1)(100)$ $= 50\text{ J}$

$KE_A' = \frac{1}{2}(5)(4900)$ $= 12250\text{ J}$ $\quad KE_B' = \frac{1}{2}(1)(25600)$ $= 12800\text{ J}$

5)

5) $m_A = 1\text{ kg}$ $\quad m_B = 5\text{ kg}$

$v_A = 50\text{ m/s}$ $\quad v_B = -50\text{ m/s}$

$$v_A' = \frac{1(50) - 5(50) + 2(5)(-50)}{1+5}$$

$$= -\frac{700}{6} = -116.67\text{ m/s}$$

$$v_B' = 50 - (-50) + (-116.67)$$

$$= -16.67\text{ m/s}$$

5) $P_A = 1(50)$ $\quad P_B = 5(-50)$

$= 50\text{ kgm/s}$ $\quad = -250\text{ kgm/s}$

$P_A' = 1(-116.67)$ $\quad P_B' = 5(-16.67)$

$= -116.67\text{ kgm/s}$ $\quad = -83.35\text{ kgm/s}$

$KE_A = \frac{1}{2}(1)(2500)$ $\quad KE_B = \frac{1}{2}(5)(2500)$

$= 1250\text{ J}$ $\quad = 6250\text{ J}$

$KE_A' = \frac{1}{2}(1)\left(\frac{490000}{36}\right)$ $\quad KE_B' = \frac{1}{2}(5)\left(\frac{2500}{9}\right)$

$= 6805.5\text{ J}$ $\quad = 694.4\text{ J}$

6)

6) $m_A = 3\text{ kg}$ $\quad m_B = 3\text{ kg}$

$v_A = 80\text{ m/s}$ $\quad v_B = -40\text{ m/s}$

$$v_A' = \frac{3(80) - 3(80) + 2(3)(-40)}{3+3}$$

$$= \frac{-240}{6} = -40\text{ m/s}$$

$$v_B' = 80 - (-40) + (-40)$$

$$= 80\text{ m/s}$$

6) $P_A = 3(80)$ $\quad P_B = 3(-40)$

$= 240\text{ kgm/s}$ $\quad = -120\text{ kgm/s}$

$P_A' = 3(-40)$ $\quad P_B' = 3(80)$

$= -120\text{ kgm/s}$ $\quad = 240\text{ kgm/s}$

$KE_A = \frac{1}{2}(3)(6400)$ $\quad KE_B = \frac{1}{2}(3)(1600)$

$= 9600\text{ J}$ $\quad = 2400\text{ J}$

$KE_A' = \frac{1}{2}(3)(1600)$ $\quad KE_B' = \frac{1}{2}(3)(6400)$

$= 2400\text{ J}$ $\quad = 9600\text{ J}$

B) Inelastic Collision :

1)

1) $m_A = 1\,kg$ $\quad m_B = 1\,kg$

$V_A = 40\,m/s$ $\quad V_B = 10\,m/s$

$e = 0.5$

$$V_B' = \frac{(m_B - e m_A) V_B + (1+e) m_A V_A}{(m_A + m_B)}$$

$$= \frac{(1-(0.5)1)10 + (1+0.5)(1)(40)}{(1+1)}$$

$$= \frac{5+60}{2} = 32.5\,m/s$$

$$V_A' = e(V_A - V_B) + V_A'$$

$$= 0.5(10-40) + 32.5$$

$$= 17.5\,m/s$$

1) $P_A = 1(40)$ $\quad P_B = 1(10)$

$= 40\,kg\,m/s$ $\quad = 10\,kg\,m/s$

$P_A' = 1(17.5)$ $\quad P_B' = 1(32.5)$

$= 17.5\,kg\,m/s$ $\quad = 32.5\,kg\,m/s$

$KE_A = \frac{1}{2}(1)(1600)$ $\quad KE_B = \frac{1}{2}(1)(100)$

$= 800\,J$ $\quad = 50\,J$

$KE_A' = \frac{1}{2}(1)(289.25)$ $\quad KE_B' = \frac{1}{2}(1)(1024.25)$

$= 144.625\,J$ $\quad = 512.125\,J$

2)

2) $m_A = 1\,kg$ $\quad m_B = 5\,kg$

$V_A = 50\,m/s$ $\quad V_B = -50\,m/s$

$e = 0.2$

$$V_A' = \frac{(1-0.2(5))50 + (1.2)(5)(-50)}{1+5}$$

$$= \frac{-300}{6} = -50\,m/s$$

$$V_B' = 0.2(50-(-50)) + (-50)$$

$$= 20 - 50$$

$$= -30\,m/s$$

2) $P_A = 1(50)$ $\quad P_B = 5(-50)$

$= 50\,kg\,m/s$ $\quad = -250\,kg\,m/s$

$P_A' = 1(-50)$ $\quad P_B' = 5(-30)$

$= -50\,kg\,m/s$ $\quad = -150\,kg\,m/s$

$KE_A = \frac{1}{2}(1)(2500)$ $\quad KE_B = \frac{1}{2}(5)(2500)$

$= 1250\,J$ $\quad = 6250\,J$

$KE_A' = \frac{1}{2}(1)(2500)$ $\quad KE_B' = \frac{1}{2}(5)(900)$

$= 1250\,J$ $\quad = 2250\,J$

3)

3) $m_A = 2\ kg$ $m_B = 4\ kg$

$V_A = 60\ m/s$ $V_B = 20\ m/s$

$e = 0.8$

$$V_A' = \frac{(2 - 0.8(4))60 + 1.8(4)(20)}{2+4}$$

$$= \frac{-72 + 144}{6} = \frac{72}{6} = 12\ m/s$$

$$V_B' = 0.8(60-20) + 12$$

$$= 32 + 12 = 44\ m/s$$

3) $P_A = 2(60)$ $P_B = 4(20)$

$= 120\ kgm/s$ $= 80\ kgm/s$

$P_A' = 2(12)$ $P_B' = 4(44)$

$= 24\ kgm/s$ $= 176\ kgm/s$

$KE_A = \frac{1}{2}(2)(3600)$ $KE_B = \frac{1}{2}(4)(400)$

$= 3600J$ $= 800J$

$KE_A' = \frac{1}{2}(2)(144)$ $KE_B' = \frac{1}{2}(4)(1936)$

$= 144J$ $= 3872J$

4)

4) $m_A = 2\ kg$ $m_B = 4\ kg$

$V_A = 80\ m/s$ $V_B = -40\ m/s$

$e = 0$

$$V_A' = \frac{(2 - 0(4))80 + 1(4)(-40)}{2+4}$$

$$= \frac{160 - 160}{6} = 0\ m/s$$

$$V_B' = 0(80 - (-40)) + 0$$

$$= 0\ m/s$$

4) $P_A = 2(80)$ $P_B = 4(-40)$

$= 160\ kgm/s$ $= -160\ kgm/s$

$P_A' = 2(0)$ $P_B' = 4(0)$

$= 0\ kgm/s$ $= 0\ kgm/s$

$KE_A = \frac{1}{2}(2)(6400)$ $KE_B = \frac{1}{2}(4)(1600)$

$= 6400J$ $= 3200J$

$KE_A' = \frac{1}{2}(2)(0)$ $KE_B' = \frac{1}{2}(4)(0)$

$= 0J$ $= 0J$

5)

5) $m_A = 3\,kg$ $m_B = 5\,kg$

$V_A = 30\,m/s$ $V_B = -30\,m/s$

$e = 0.4$

$$V_A' = \frac{(3 - 0.4(5))\,30 + 1.4(5)(-30)}{3+5}$$

$$= -\frac{180}{8} = -22.5\,m/s$$

$$V_B' = 0.4(30-(-30)) + (-22.5)$$

$$= 24 - 22.5 = 1.5\,m/s$$

5) $P_A = 3(30)$ $P_B = 5(-30)$

$= 90\,kg\,m/s$, $= -150\,kg\,m/s$

$P_A' = 3(-22.5)$ $P_B' = 5(1.5)$

$= -67.5\,kg\,m/s$ $= 7.5\,kg\,m/s$

$KE_A = \frac{1}{2}(3)(900)$ $KE_B = \frac{1}{2}(5)(900)$

$= 1350\,J$ $= 2250\,J$

$KE_A' = \frac{1}{2}(3)\left(\frac{2025}{4}\right)$ $KE_B' = \frac{1}{2}(5)(2.25)$

$= 759.375\,J$ $= 5.625\,J$

6)

6) $m_A = 1\,kg$ $m_B = 1\,kg$

$V_A = 50\,m/s$ $V_B = 0\,m/s$

$e = 0.6$

$$V_A' = \frac{(1 - 0.6(1))\,50 + 1.6(1)(0)}{1+1}$$

$$= \frac{20}{2} = 10\,m/s$$

$$V_B' = 0.6(50-0) + 10$$

$$= 40\,m/s$$

6) $P_A = 1(50)$ $P_B = 1(0)$

$= 50\,kg\,m/s$ $= 0\,kg\,m/s$

$P_A' = 1(10)$ $P_B' = 1(40)$

$= 10\,kg\,m/s$ $= 40\,kg\,m/s$

$KE_A = \frac{1}{2}(1)(2500)$ $KE_B = \frac{1}{2}(1)(0)$

$= 1250\,J$ $= 0\,J$

$KE_A' = \frac{1}{2}(1)(100)$ $KE_B' = \frac{1}{2}(1)(1600)$

$= 50\,J$ $= 800\,J$

Error in Result : Since the experiment is conducted virtually using a simulator, therefore no error is observed. The slight variations seen between the observed and calculated values of velocity, momentum or kinetic energy, as seen in some cases are just because the simulator has given values rounded off to lower precision.

Result Analysis : the values of final velocities, momentum and kinetic energies of the two bodies as observed using the simulator are equal to the values calculated by applying the conservation of linear momentum and using the relation of the coefficient of restitution with the initial and final velocities of both the bodies (and by applying the conservation of kinetic energy in case of elastic collisions too). This shows that all these conditions and relations considered in case of elastic and inelastic collisions hold true.

Thank you