15. Compute det(A^T A) and det(AA^T) for several random 5 × 6 matrices and several random 6 × 7 matrices. What can you say about AT A and AAT when A has more columns than rows?

```
%inputing a matix of order 5x6
a=input('Enter a matrix and a should be 5x6 matrix')

b=a'% b is transpose of a

c=a*b %c is a  matrix made by multiplcation of a martix and it
       %     transpose
d=b*a %d is a  matrix made by multiplcation of a  transpose ofmartix and it
       %   original matrix
 disp('determinant of c is'); det(c)
  disp('determinant of d is'); det(d)
```

```
%inputing matrix of order 6x7
e=input('Enter a matrix and a should be 6x7 matrix')
f=e' % f is transpose of e
g=e*f  %g is a  matrix made by multiplcation of a martix and it
      %     transpose
h=f*e  %h is a  matrix made by multiplcation of a martix and it
      %     transpose
disp('determinant of g is'); det(g)
 disp('determinant of h is'); det(h)
```

OUTPUT

```
%inputing a matix of order 5x6
a=input('Enter a matrix and a should be 5x6 matrix')
```

```
a = 5x6
     1     2     3     4     5     6
     2     5     8     6     4     5
     7     8     9     4     5     6
     4     5     6     1     2     3
     7     8     4     2     0     3
```

```
b=a'% b is transpose of a
```

```
b = 6x5
     1     2     7     4     7
     2     5     8     5     8
     3     8     9     6     4
     4     6     4     1     2
     5     4     5     2     0
     6     5     6     3     3
```

```
c=a*b %c is a  matrix made by multiplcation of a martix and it
```

```
c = 5x5
    91   110   127    64    61
   110   170   200   110   113
   127   200   271   154   175
    64   110   154    91   103
    61   113   175   103   142
```

```
        %     transpose
d=b*a %d is a  matrix made by multiplcation of a  transpose ofmartix and it
```

```
d = 6x6
   119   144   134    62    56    91
   144   182   180    91    80   124
   134   180   206   110   104   142
    62    91   110    73    66    87
    56    80   104    66    70    86
    91   124   142    87    86   115
```

```
        %   original matrix
disp('determinant of c is'); det(c)
```

```
determinant of c is
ans = 1741824
```

```
disp('determinant of d is'); det(d)
```

```
determinant of d is
ans = -1.2917e-09
```

WINDOW

OUTPUT

```
%inputing matrix of order 6x7
e=input('Enter a matrix and a should be 6x7 matrix')
```

```
e = 5x6
     1     2     3     4     5     6
     2     5     8     6     4     5
     7     8     9     4     5     6
     4     5     6     1     2     3
     7     8     4     2     0     3
```

```
f=e' % f is transpose of e
```

```
f = 6x5
     1     2     7     4     7
     2     5     8     5     8
     3     8     9     6     4
     4     6     4     1     2
     5     4     5     2     0
     6     5     6     3     3
```

```
g=e*f  %g is a  matrix made by multiplcation of a martix and it
```

```
g = 5x5
    91   110   127    64    61
   110   170   200   110   113
   127   200   271   154   175
    64   110   154    91   103
    61   113   175   103   142
```

```
      %      transpose
h=f*e  %h is a  matrix made by multiplcation of a martix and it
```

```
h = 6x6
   119   144   134    62    56    91
   144   182   180    91    80   124
   134   180   206   110   104   142
    62    91   110    73    66    87
    56    80   104    66    70    86
    91   124   142    87    86   115
```

```
      %      transpose
disp('determinant of g is'); det(g)
```

```
determinant of g is
ans = 1741824
```

```
disp('determinant of h is'); det(h)
```

```
determinant of h is
ans = -1.2917e-09
```

WINDOW

16. Solve:

0.543(10)−3 ∗ X1 + 3.21 ∗ X2 = 3.87

4.32 ∗ X1 + 2.31 ∗ X2 = 4.92

using only Gaussian elimination with back substitution to 3 significant digits.

```
% Solve:
%0.543(10)-3 * X1 + 3.21 * X2 = 3.87
%4.32 * X1 + 2.31 * X2 = 4.92

a=[0.543*10^-3 3.21 ; 4.32 2.31  ]
b=[3.87;4.92]
aug=[a b] % taking is augumented matrix
rref(aug)
```

OUTPUT

```
a = 2×2
       0.0005    3.2100
       4.3200    2.3100

b = 2×1
       3.8700
       4.9200

aug = 2×3
       0.0005    3.2100    3.8700
       4.3200    2.3100    4.9200

ans = 2×3
       1.0000         0    0.4943
            0    1.0000    1.2055
```

This is final value of x and x2

So,
x1=0.4943
and
x2=1.2055

17. Generate a random matrix A of size 6*6 and 5 vectors, bi. Check whether AX = bi has solution for each i. Write down A, bi, Xi. r1=rand(m,n): r1 is a m-by-n matrix containing real floating-point numbers drawn from a uniform distribution.

```
a=input('Enter a 6x6 random matrix ')

b1=input('Enter a vector of b1 ')
aug1=[a b1]       % finding value of x for ax=b1
rref(aug1)
b2=input('Enter a vector of b2 ')
aug2=[a b2]        % finding value of x for ax=b2
rref(aug2)
b3=input('Enter a vector of b3 ')
aug3=[a b3]               % finding value of x for ax=b3
rref(aug3)
b4=input('Enter a vector of b4 ')
aug4=[a b4]            % finding value of x for ax=b4
rref(aug4)
b5=input('Enter a vector of b5 ')
aug5=[a b5]       % finding value of x for ax=b5
rref(aug5)
b6=input('Enter a vector of b6 ')
aug6=[a b6]          % finding value of x for ax=b6
rref(aug6)
```

OUTPUT

```
a=input('Enter a 6x6 random matrix ')
```

```
a = 6x6
     1     2     3     4     5     6
     2     5     8     6     4     5
     7     8     9     4     5     6
     4     5     6     1     2     3
     7     4     5     2     6     4
     7     8     2     1     0     3
```

```
b1=input('Enter a vector of b1 ')
```

```
b1 = 6x1
     1
     2
     3
     4
     5
     6
```

```
aug1=[a b1]        % finding value of x for ax=b1
```

```
aug1 = 6x7
     1     2     3     4     5     6     1
     2     5     8     6     4     5     2
     7     8     9     4     5     6     3
     4     5     6     1     2     3     4
     7     4     5     2     6     4     5
     7     8     2     1     0     3     6
```

```
rref(aug1)
```

```
ans = 6x7
    1.0000         0         0         0         0         0  -48.0000
         0    1.0000         0         0         0         0   66.9524
         0         0    1.0000         0         0         0  -18.6190
         0         0         0    1.0000         0         0  -19.8095
         0         0         0         0    1.0000         0   64.6667
         0         0         0         0         0    1.0000  -45.5238
```

This is value of X for b1

```
b2=input('Enter a vector of b2 ')
```

```
b2 = 6x1
     2
     5
     8
     7
     6
     4
```

```
aug2=[a b2]          % finding value of x for ax=b2
```

```
aug2 = 6x7
     1     2     3     4     5     6     2
     2     5     8     6     4     5     5
     7     8     9     4     5     6     8
     4     5     6     1     2     3     7
     7     4     5     2     6     4     6
     7     8     2     1     0     3     4
```

```
rref(aug2)
```

```
ans = 6x7
    1.0000         0         0         0         0         0  -17.3333
         0    1.0000         0         0         0         0   24.1429
         0         0    1.0000         0         0         0   -5.8095
         0         0         0    1.0000         0         0   -7.9048
         0         0         0         0    1.0000         0   23.3333
         0         0         0         0         0    1.0000  -16.0952
```

This is value of X for b2

```
b3=input('Enter a vector of b3 ')
```

```
b3 = 6x1
     7
     5
     1
     2
     3
     4
```

```
aug3=[a b3]              % finding value of x for ax=b3
```

```
aug3 = 6x7
     1     2     3     4     5     6     7
     2     5     8     6     4     5     5
     7     8     9     4     5     6     1
     4     5     6     1     2     3     2
     7     4     5     2     6     4     3
     7     8     2     1     0     3     4
```

```
rref(aug3)
```

```
ans = 6x7
    1.0000         0         0         0         0         0  -66.6667
         0    1.0000         0         0         0         0   91.7143
         0         0    1.0000         0         0         0  -26.0476
         0         0         0    1.0000         0         0  -26.5238
         0         0         0         0    1.0000         0   88.6667
         0         0         0         0         0    1.0000  -61.4762
```

This is value of X for b3

```
b4=input('Enter a vector of b4 ')
```

```
b4 = 6x1
     4
     5
     0
     0
     3
     2
```

```
aug4=[a b4]            % finding value of x for ax=b4
```

```
aug4 = 6x7
     1     2     3     4     5     6     4
     2     5     8     6     4     5     5
     7     8     9     4     5     6     0
     4     5     6     1     2     3     0
     7     4     5     2     6     4     3
     7     8     2     1     0     3     2
```

```
rref(aug4)
```

```
ans = 6x7
    1.0000         0         0         0         0         0  -54.3333
         0    1.0000         0         0         0         0   75.0952
         0         0    1.0000         0         0         0  -21.4286
         0         0         0    1.0000         0         0  -20.7143
         0         0         0         0    1.0000         0   73.0000
         0         0         0         0         0    1.0000  -51.6190
```

This is value of X for b4

```
b5=input('Enter a vector of b5 ')
```

```
b5 = 6x1
     0
     0
     0
     0
     0
     0
```

```
aug5=[a b5]        % finding value of x for ax=b5
```

```
aug5 = 6x7
     1     2     3     4     5     6     0
     2     5     8     6     4     5     0
     7     8     9     4     5     6     0
     4     5     6     1     2     3     0
     7     4     5     2     6     4     0
     7     8     2     1     0     3     0
```

```
rref(aug5)
```

```
ans = 6x7
     1     0     0     0     0     0     0
     0     1     0     0     0     0     0
     0     0     1     0     0     0     0
     0     0     0     1     0     0     0
     0     0     0     0     1     0     0
     0     0     0     0     0     1     0
```

This is value of X for b5

```
b6=input('Enter a vector of b6 ')
```

```
b6 = 6x1
     7
     0
     2
     0
     4
     0
```

```
aug6=[a b6]            % finding value of x for ax=b6
```

```
aug6 = 6×7
     1     2     3     4     5     6     7
     2     5     8     6     4     5     0
     7     8     9     4     5     6     2
     4     5     6     1     2     3     0
     7     4     5     2     6     4     4
     7     8     2     1     0     3     0
```

```
rref(aug6)
```

```
ans = 6×7
    1.0000         0         0         0         0         0    5.1667
         0    1.0000         0         0         0         0   -7.4524
         0         0    1.0000         0         0         0    1.4524
         0         0         0    1.0000         0         0    1.4762
         0         0         0         0    1.0000         0   -6.3333
         0         0         0         0         0    1.0000    6.3571
```

This is value of X for b6

ND WINDOW | 2 usages of "b6" found

18 Construct a linear systems (AX = b) which has infinitely many solutions. There should be 9 equations and constant vector (b) should be formed using your student id.

```
% Giving a matrix A which have infinitly many solution to b
% matrix =[2;0;2;1;5;1;1;6;5]

a=[1 2 3 4 5 6 7 8 ;
    7 8 9 4 5 6 0 0 ;
    1 2 3 4 5 6 4 9 ;
    2 4 6 8 10 12 1 5  ;
    4 5 7 8 6 51 10 6 ;
    1 5 78 4 5 6 2  7 ;
    7 8 9 5 1 2 2 8 ;
     3 6 9 12 15 18 21 24 ;
     5/2 5 15/2 10 25/2 15 35/2 20]
b=[2 ;0 ;2; 1; 5 ;1 ;1 ;6 ;5]
aug=[a b]
rref(aug)
```

OUTPUT

```
% Giving a matrix A which have infinitly many solution to b
% matrix =[2;0;2;1;5;1;1;6;5]

a=[1 2 3 4 5 6 7 8 ;
    7 8 9 4 5 6 0 0 ;
    1 2 3 4 5 6 4 9 ;
    2 4 6 8 10 12 1 5  ;
    4 5 7 8 6 51 10 6 ;
    1 5 78 4 5 6 2  7 ;
    7 8 9 5 1 2 2 8 ;
     3 6 9 12 15 18 21 24 ;
     5/2 5 15/2 10 25/2 15 35/2 20]
```

```
a = 9×8
     1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    7.0000    8.0000
     7.0000    8.0000    9.0000    4.0000    5.0000    6.0000         0         0
     1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    4.0000    9.0000
     2.0000    4.0000    6.0000    8.0000   10.0000   12.0000    1.0000    5.0000
     4.0000    5.0000    7.0000    8.0000    6.0000   51.0000   10.0000    6.0000
     1.0000    5.0000   78.0000    4.0000    5.0000    6.0000    2.0000    7.0000
     7.0000    8.0000    9.0000    5.0000    1.0000    2.0000    2.0000    8.0000
     3.0000    6.0000    9.0000   12.0000   15.0000   18.0000   21.0000   24.0000
     2.5000    5.0000    7.5000   10.0000   12.5000   15.0000   17.5000   20.0000
```

```
b=[2 ;0 ;2; 1; 5 ;1 ;1 ;6 ;5]
```

```
b = 9×1
     2
     0
     2
     1
     5
     1
     1
     6
     5
```

```
aug=[a b]
```

```
aug = 9×9
     1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    7.0000    8.0000    2.0000
     7.0000    8.0000    9.0000    4.0000    5.0000    6.0000         0         0         0
     1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    4.0000    9.0000    2.0000
     2.0000    4.0000    6.0000    8.0000   10.0000   12.0000    1.0000    5.0000    1.0000
     4.0000    5.0000    7.0000    8.0000    6.0000   51.0000   10.0000    6.0000    5.0000
     1.0000    5.0000   78.0000    4.0000    5.0000    6.0000    2.0000    7.0000    1.0000
     7.0000    8.0000    9.0000    5.0000    1.0000    2.0000    2.0000    8.0000    1.0000
     3.0000    6.0000    9.0000   12.0000   15.0000   18.0000   21.0000   24.0000    6.0000
     2.5000    5.0000    7.5000   10.0000   12.5000   15.0000   17.5000   20.0000    5.0000
```

```
rref(aug)
```

```
ans = 9×9
     1.0000         0         0         0         0   52.8350         0         0    4.5746
          0    1.0000         0         0         0  -55.0364         0         0   -4.7547
          0         0    1.0000         0         0    2.2014         0         0    0.1838
          0         0         0    1.0000         0    9.8350         0         0    0.6652
          0         0         0         0    1.0000    3.4587         0         0    0.3402
          0         0         0         0         0         0    1.0000         0    0.0652
          0         0         0         0         0         0         0    1.0000    0.1957
          0         0         0         0         0         0         0         0         0
          0         0         0         0         0         0         0         0         0
```

Since the augmented matrix 7 pivot entry . so,one free variable which is x6. So, infinitely many solution.

19. Write codes for finding inverse of a matrix a) using Gauss-Jordan method.

```
a=input('Enter a square matrix ')

%matrix must be invertible means it is square and determinant not zer

b=length(a);
i=eye(b)% creating a indentity martrix of same order of a
aug=[a i] %Augumented a with identity matrix
rref(a) % calculatingg their reduced echelon form

rref(aug)
```

OUTPUT

```
a=input('Enter a square matrix ')
```

```
a = 2×2
       1     2
       3     8
```

```
%matrix must be invertible means it is square and determinant not zero

b=length(a);
i=eye(b)% creating a indentity martrix of same order of a
```

```
i = 2×2
       1     0
       0     1
```

```
aug=[a i] %Augumented a with identity matrix
```

```
aug = 2×4
       1     2     1     0
       3     8     0     1
```

```
rref(aug)
```

```
ans = 2×4
    1.0000         0    4.0000   -1.0000
         0    1.0000   -1.5000    0.5000
```

This is inverse of a

20. Construct a 2 × 3 matrix A, not in echelon form, such that the solution of Ax = 0 is a line in R^3

. Find equation of that line.

```
a=[1 2 3;4 5 6] %constructing a martix a
[m, n]=size(a); % taking it size of a
b=zeros(m,1) % creating a matrix b such that it order is mx1
aug=[a b] % Augumented a and b to get x to reduced to row echleon
rref(aug)
```

OUTPUT

```
a = 2×3
     1     2     3
     4     5     6

b = 2×1
     0
     0

aug = 2×4
     1     2     3     0
     4     5     6     0

ans = 2×4
     1     0    -1     0
     0     1     2     0
```

So, a argument b is

$$= \begin{bmatrix} \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 \end{bmatrix}$$

have two pivot entry so, x_3 free variable

let $x_3 = c$.

$x_2 + 2c = 0 \Rightarrow x_2 = -2c$.

$x_1 - c = 0$

$\boxed{x_1 = c}$

Now, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} c \\ -2c \\ c \end{bmatrix}$.

eqn of x in R^3 is $= c \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ $c \in \underline{R}$.