**Back Savers**

Back Savers is a company that produces backpacks primarily for students. They are  
considering offering a combination of two different models, the Collegiate and the  
Mini. Both are made from the same rip-resistant nylon fabric. Back Savers has a long-  
term contract with a supplier of the nylon and receives a 5000 square-foot shipment of  
the material each week. Each Collegiate requires 3 square feet while each Mini requires 2  
square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can  
be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates  
a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of  
$24. Back Savers has 35 laborers that each provides 40 hours of labor per week.  
Management wishes to know what quantity of each type of backpack to produce per  
week.  
a. Clearly define the decision variables  
b. What is the objective function?  
c. What are the constraints?  
d. Write down the full mathematical formulation for this LP problem.

**a. Clearly define the decision variables**?

Management wishes to know what quantity of each type of backpack to produce per  
week. The first step in LP modeling is to define the decision variables, the unknowns that the company will determine, So the first thing to do is to define the variable that we need to find.

For this problem we can define variables as:

**Xc**: Number of Collegiate backpacks produced per week

**Xm**: Number of Mini backpacks produced per week

Since the number of backpacks could not be in negative, the variables will be non-negative values.

**b**.**What is the objective function**.

The objective of the organization will be to maximize the profit without any losses of the product and the labor. Backs Savers earns profit of $32 per collegiate and $24 per Mini , the goal is to maximize the total weekly profit from production.

**Max\_profit = 32 \* Xc + 24 \* Xm**

* 32 \* Xc ​: Profit earned from selling all Collegiate backpacks produced
* 24 \* Xm ​: Profit earned from selling all Mini backpacks produced
* Max\_profit: The total profit (objective function to be maximized)

Here the company is trying to maintain balance between two products that yield the highest possible profit.

**c. What are the constraints?**

Every LP has constraints, which are real-world limitations that prevent the company from producing unlimited quantities. In this case constraints would be the shipment of material(nylon), Labor availability, sales forecast.

1. **Shipment of material:**

Each backpack requires nylon fabric:

* Collegiate: 3 sq. ft.
* Mini: 2 sq. ft.

The company receives **5,000 sq. ft. of nylon per week** under a long-term contract. Therefore:

3 \* Xc + 2 \* Xm ≤ 5000

**2. Labor (Workforce Capacity Constraint)**

Labor is another limited resource. Each worker contributes **40 hours/week**, and there are **35 workers**, giving:

35\*40\*60=84,000 minutes of labor per week

Time requirement per product:

* Collegiate: 45 minutes
* Mini: 40 minutes

Thus:

45 \* Xc + 40 \* Xm ≤ 84,000

**3. Sales Forecast (Demand Constraints)**

The marketing department forecasts maximum weekly demand for each backpack:

* At most **1,000 Collegiates** can be sold
* At most **1,200 Minis** can be sold

This results in:

Xc ​≤1000, Xm ​≤1200

These constraints prevent overproduction beyond what can realistically be sold in the market.

**4. Non-Negativity Constraints**

Since negative production levels are impossible:

Xc ≥ 0, Xm ≥ 0

**d. Write down the full mathematical formulation for this LP problem?**

Maximize: Max\_profit = 32\* Xc + 24 \* Xm (Maximize weekly profit)

Subject to: 3 \* Xc + 2 \* Xm ≤ 5000 (Fabric availability))

45 \* Xc + 40 \* Xm ≤ 84,000 (Labor capacity)

Xc ≤1000 (Max Collegiate Sales)

Xm ​≤1200 (Max Mini Sales)

Xc ≥ 0 (non-negativity of count)

Xm ≥ 0 (non-negativity of count)

This LP model provides Back Savers with a systematic way to decide the weekly production mix between Collegiate and Mini backpacks. By solving the model (using Excel Solver, Python, or other optimization software), management can determine the exact number of each backpack to produce to achieve the highest possible weekly profit while staying within fabric supply, labor availability, and market demand.

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**Weigelt Corporation**

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem

**Problem data (summary)**

| **Size** | **Profit per unit** | **Storage per unit (sq ft)** | **Sales forecast (units/day)** |
| --- | --- | --- | --- |
| Large | $420 | 20 | 900 |
| Medium | $360 | 15 | 1200 |
| Small | $300 | 12 | 750 |

| **Plant** | **Excess capacity (units/day)** | **In-process storage available (sq ft/day)** |
| --- | --- | --- |
| 1 | 750 | 13,000 |
| 2 | 900 | 12,000 |
| 3 | 450 | 5,000 |

1. **Decision variables**

Let the decision variables be the number of units of each size produced at each plant:

| **Variable** | **Definition** |
| --- | --- |
| x₁ = x₁L | Large units produced at Plant 1 |
| x₂ = x₂L | Large units produced at Plant 2 |
| x₃ = x₃L | Large units produced at Plant 3 |
| x₄ = x₁M | Medium units produced at Plant 1 |
| x₅ = x₂M | Medium units produced at Plant 2 |
| x₆ = x₃M | Medium units produced at Plant 3 |
| x₇ = x₁S | Small units produced at Plant 1 |
| x₈ = x₂S | Small units produced at Plant 2 |
| x₉ = x₃S | Small units produced at Plant 3 |
| p | Common fraction of each plant’s excess capacity used (0 ≤ p ≤ 1) |

**b. Linear programming model**

**Objective function (maximize total daily profit):**

Maximize Z =   420(x1​+x2​+x3​) + 360(x4​+x5​+x6​) + 300(x7​+x8​+x9​)

**Subject to:**

1. **Equal-percentage capacity usage at each plant** (enforces same fraction ppp of each plant’s excess capacity is used):

x1 ​+ x4 ​+ x7​ = p⋅750 (Plant 1)

x2​ + x5​ + x8​ = p⋅900 (Plant 2)

x3​ + x6 ​+ x9 ​= p⋅450 (Plant 3)

1. **Storage (in-process space) limits**:

20x1 + 15x4​ + 12x7 ​≤ 13,000 (Plant 1 storage)

20x2 ​+ 15x5 ​+ 12x8 ​≤ 12,000 (Plant 2 storage)

20x3​ + 15x6 ​+ 12x9​ ≤ 5,000 (Plant 3 storage)

1. **Market demand (sales) limits**:

x1 ​+ x2 ​+ x3​ ≤ 900 (Large)

x4 ​+ x5 ​+ x6 ​≤1200 (Medium)

x7 ​+ x8 ​+ x9 ​≤ 750 (Small)

1. **Bounds / nonnegativity**:
   1. ≤p ≤1

xi ​≥ 0 for i=1,…,9.