Half Wave Diode Rectifier using Fourier Series Analysis

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Abstract: This Manual gives a quick overview of half wave diode rectifiers and also analysing the output with Fourier series.

Half Wave Diode Rectifier

Problem 1. Refer to the diagram given below and find the output across the load resistor R. The figure 1.1 shows a half wave diode rectifier.

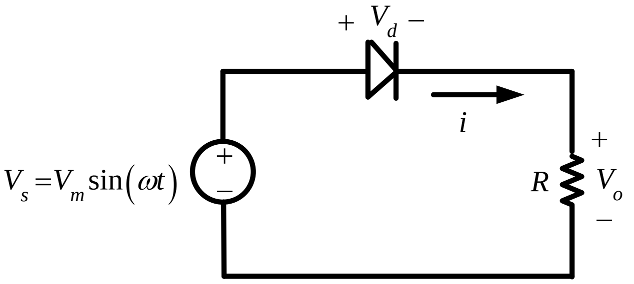

Fig. 1.1: Circuit diagram of half wave diode rectifier

Solution:

Here input source voltage (V_s) is $V_m\sin(\omega t)$. At positive half cycle of supply, the diode becomes forward biased and behaves like a short circuit connection, while in negative half cycle; it becomes reverse biased and hence open circuit. The output voltage across the load resistance R can be represented mathematically as:

$$V(t) = \begin{cases} V sin\omega t & 0 < \omega t < \pi \\ 0 & \pi < \omega t < 2\pi \end{cases} \quad (1)$$

The below program is executed in **Octave** to plot the output voltage of half wave diode rectifier.

```
l=linspace(0,10,100);
sig=sin(2*pi*50*l);
subplot(211)
plot(sig);
xlabel("Input AC Voltage");ylabel(“Amplitude”);
grid
% u=1:9;
% t=(1:)
for t=1:100
if sin(2*pi*50*l(t))<=0
   sig(t)=0;
else
   sig(t) = sin(2*pi*50*l(t));
end
end
subplot(212)
plot(sig);
xlabel("Rectified output
Voltage");ylabel(“Amplitude”);
grid
```

The Current output relation is as follows:

$$i(t) = \begin{cases} V sin\omega t/R & 0 < \omega t < \pi \\ 0 & \pi < \omega t < 2\pi \end{cases} \quad (2)$$

The Input voltage (V_s) and output voltage (V_o) waveforms are shown in the figure 1.2. The input current (I_s) and output current (I_o) waveforms are presented in the figure 1.3.

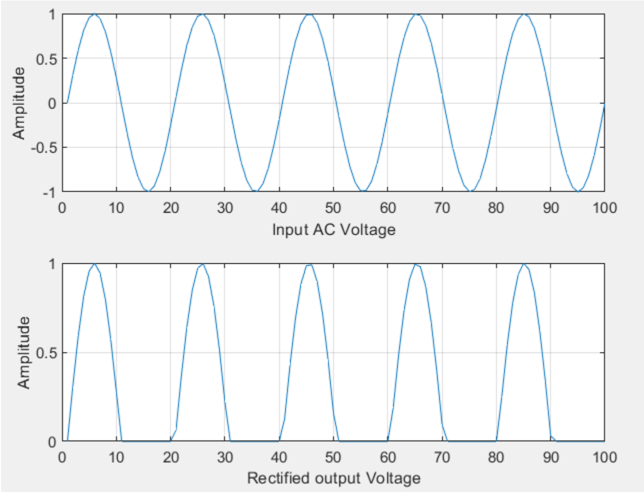

Figure 1.2. Input and output voltage waveforms

Similarly, with minor changes to the program, gives the input and output current waveforms.

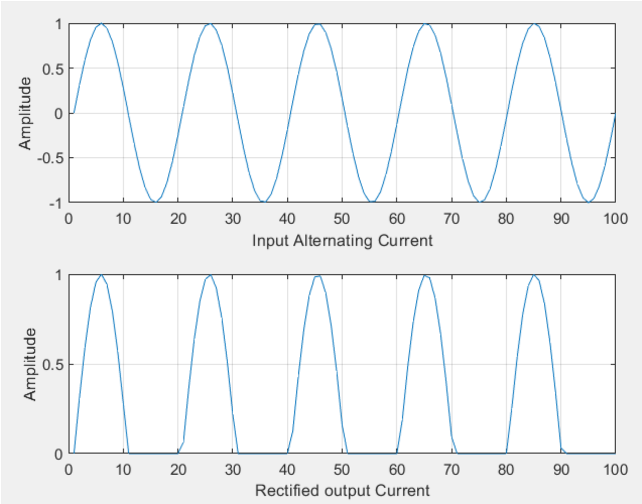

Figure 1.3. Input and output current waveforms

Problem 2. Do the Fourier series analysis on half wave rectified output in problem 1.

Solution:

History of Fourier Series: Joseph Fourier (1768–1830), a French mathematician solved a problem in heat conduction using a function as an infinite series of sine and cosine functions.

Consider ωt = x

It is known that the output of half wave rectifier can be represented as:

$$f(x) = \begin{cases} V sinx & 0 < x < \pi \\ 0 & x < x < 2\pi \end{cases}$$

General Formula of Fourier series:

$f(x) =$ $a_0 + \sum_{n=1}^{\infty}(a_n\, cosnx + b_n\, sinnx)$ (3)

Here $a_0 = 1/2\pi \int_0^{2\pi} f(x)\, dx$ (4)

$a_0 = 1/2\pi\, [\, \int_0^{\pi} f(x)\, dx + \int_{\pi}^{2\pi} f(x)\, dx\,]$

$= 1/2\pi \int_0^{\pi} V sinx\, dx = V/\pi$ (5)

$a_n = 1/\pi \int_0^{\pi} f(x)\, cosnx\, dx$ (6)

$= V/\pi \int_0^{\pi} sinx.cosnx\, dx$

$= V/2\pi \int_0^{\pi} (sin(n+1)x - sin(n-1)x)\, dx$

Further simplifying,

$$= V/\pi \left\{\frac{((-1)^{n-1}-1)}{(n^2-1)}\right\}$$

$$= \begin{cases} 0 & n = odd \\ \frac{2V}{\pi(n^2-1)} & n = even \end{cases} \qquad (7)$$

$b_n = 1/\pi \int_0^{\pi} f(x)\, sinnx\, dx$ (8)

$= V/\pi \int_0^{\pi} sinx.sinnx\, dx$ (9)

From (7), $b_n = 0$, except for b_1

$b_1 = V/\pi \int_0^{\pi} sin^2x\, dx = V/2\pi \int_0^{\pi} (1 - cos2x)dx$

$= V/2$ (10)

Substituting (5), (7), (10) in (3)

$f(x) = V/\pi + V/2 (\sin t) + \sum_{n=1}^{\infty}(\frac{2V}{\pi(n^2-1)}\, cosnt)$ (11)

Equation 11 shows the Fourier series analysis on half wave rectified output shown in figure 1.2.

Problem 3. Determine Fourier series coefficients of a Half wave rectified square wave by considering the values Time period T = 2 and the number of cycles of the n^{th} harmonic in the analysis interval N = 7.

Solution: Fourier series analysis is done using the given Octave program. The graph for the same is shown in Figure 1.4.

```
clc;
syms t n
n=1:7;
f=heaviside(t)-heaviside(t-1);
ezplot(f,[0,2]);
xlabel("Rectified output signal");
ylabel("Amplitude");
T=2;
w0=2*pi/T;
a0=1/T*int(f,t,0,T)
an=2/T*int(f*cos(n*w0*t),t,0,T)
bn=2/T*int(f*sin(n*w0*t),t,0,T)
```

Output:

a0 = 1/2
an =[0, 0, 0, 0, 0, 0, 0]
bn =[2/pi, 0, 2/(3*pi), 0, 2/(5*pi), 0, 2/(7*pi)]

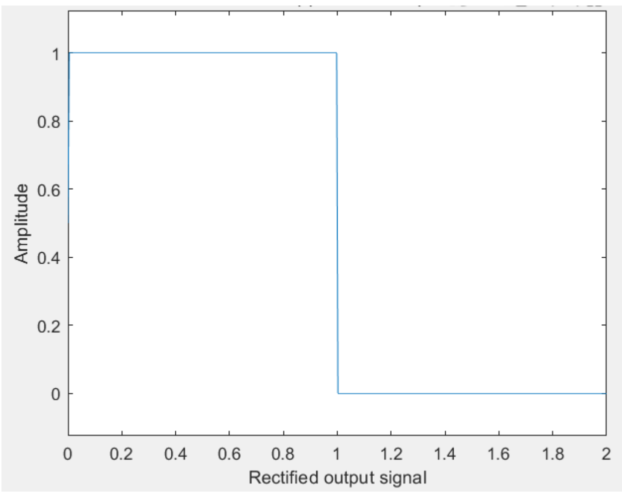

Figure 1.4. Rectified Square wave.

Important Equations:

Average current is

$I_{DC} = 1/2\pi \int_0^{\pi} V_m \sin\Theta \, d\Theta / R = V_m/\pi R_L = I_m/\pi$ (12)

Similarly, average voltage equation is

$V_{DC} = V_m/\pi$ (13)

RMS value of output current,

$I_{RMS} = 1/2\pi \int_0^{\pi} I_m^2 \sin^2\Theta \, d\Theta = I_m/2$ (14)

Similarly, RMS voltage across the load is

$V_{RMS} = V_m/2$ (15)

Output frequency,

Fout =Fin, Fin=Input frequency= 50 Hz (16)

Ripple factor,

$r = I_{AC}/I_{DC} = \sqrt{\mathrm{IRMS}^2 + \mathrm{Idc}^2} \; / I_{DC} = \sqrt{(\pi^2/4) - 1} = 1.21$ (17)

DC output power,

$P_{DC} = (I_m/\pi)^2 . R_L$ (18)

Total AC input power,

$P_{AC} = (I_m/2)^2 . R$, R is the sum of resistances (19)

Rectifier efficiency,

$\eta = P_{DC} / P_{AC} = 40.6\ \%$ (20)

DESIGNING OF HALF WAVE RECTIFIER:

A. Here, the supply voltage is fixed and known since it is deriving from the utility supply. The RMS value or effective value of AC supply voltage in India is 230V (+ or-5%) at 50Hz.
B. Resistance of load depends on the application model like lighting / heating /charging.
C. The power diode will be selected based on following Important specifications:
   - Power handling capacity
   - Frequency handling of the diode
   - I_{diode} (forward current) should be minimum 20% higher than I_{load}
   - PIV ratings (Peak reverse Voltage): Best practice is selecting diode with PIV rating minimum 20% higher than expected reverse voltage.
   - Maximum surge current
   - Forward voltage
   - Reverse current

Using the above specifications, the user may search for suitable diode in vendor / datasheet web sources and select a desired one for the experiment.

Example 1: Find Voltage drop (V_{dc}) and current (I_{dc}) through R=100 Ω connected to 230 V_{rms} single phase half wave rectifier. Also find average DC power consumed by load. Suggest the power diode that can be used.

Results: $V_{dc} = V_{max}/\pi = 0.318 * V_{max} = 0.45 * V_{rms}$
$= 0.45*230 = 103.5V$
$I_{dc} = V_{dc}/R = 103.5/100 = 1.035A$
$P = I^2R = 1.035^2 * 100 = 107.12W$

Here the suitable diode with required specifications is of 1N400x series, for example: 1N4003 diode.

Most commonly used power diode for basic electronic application is 1N400x series Glass Passivated type diode, with rating of I=1A, PIV rating of 1N4003 testing up to PIV rating of 200V.

EXERCISES:

Problem 4. Theoretically solve and verify the results of Fourier analysis of Square wave. (Hint: refer problem 3)

Problem 5. A crystal diode having internal resistance r_f = 20Ω is used for half-wave rectification. If the applied voltage v = 50 sin ω t and load resistance R_L= 800 Ω, find: (i) I_m, I_{dc}, I_{rms} (ii) a.c. power input and d.c. power output (iii) d.c. output voltage (iv) efficiency of rectification.
(Hint: refer equations 12 to 20)

Problem 6. A half-wave rectifier is used to supply 50 V d.c. to a resistive load of 800 Ω. The diode has a resistance of 25 Ω. Calculate a.c. voltage required.