Q1. Coordinating advertising and production

1a) Formulate this decision question as a Linear Programming Model. Define the variables, write down the constraints and the objective function in mathematical (algebraic) terms (variables, inequalities, etc.)

Answer 1a

Decision variables:

- Treg = number of table lamps produced during regular time (in thousands)
- Tovr = number of table lamps produced during overtime (in thousands)
- Freg = number of floor lamps produced during regular time (in thousands)
- Fovr = number of floor lamps produced during overtime (in thousands)
- Creg = number of ceiling lamps produced during regular time (in thousands)
- Covr = number of ceiling lamps produced during overtime (in thousands)
- Preg = number of pendant lamps produced during regular time (in thousands)
- Povr = number of pendant lamps produced during overtime (in thousands)
- AT = advertising spend for table lamps (in thousands of dollars)
- AF = advertising spend for floor lamps (in thousands of dollars)
- AC = advertising spend for ceiling lamps (in thousands of dollars)
- AP = advertising spend for pendant lamps (in thousands of dollars)

Objective function:

The goal is to maximize total profit. We can express this as:

Profit = (120 - 66 - 16)TReg + (120 - 66 - 18)TOvr + (150 - 85 - 16)FReg + (150 - 85 - 18)FOvr + (100 - 50 - 12)CReg + (100 - 50 - 15)COvr + (160 - 80 - 12)PReg + (160 - 80 - 15)POvr - AT - AF - AC - AP

This represents the revenue minus material costs and production costs for each lamp type, minus the advertising costs.

Constraints:

- **Production capacity constraints:**
  Treg + Freg ≤ 100 (Department 1 regular time)
  Tovr + Fovr ≤ 25 (Department 1 overtime)
  Creg + Preg ≤ 90 (Department 2 regular time)
  Covr + Povr ≤ 24 (Department 2 overtime)

  These ensure that production doesn't exceed available capacity in each department.

- **Demand constraints:**
  Treg + Tovr ≤ 60(1 + 0.12AT/10)
  Freg + Fovr ≤ 20(1 + 0.10AF/10)

Creg + Covr ≤ 100(1 + 0.08AC/10)
Preg + Povr ≤ 35(1 + 0.15AP/10)

These link production to demand, incorporating the effect of advertising.

- **Total advertising budget constraint:**
  AT + AF + AC + AP ≤ 18

  This ensures the total advertising spend doesn't exceed the budget.

- **Individual advertising spend limits:**
  AT ≤ 10
  AF ≤ 10
  AC ≤ 10
  AP ≤ 10

  This ensures the maximum amount spent on advertising for each of the products doesn't exceed the specified limit.

- **Non-negativity constraints:**
  All variables ≥ 0

  This ensures we don't produce negative quantities or spend negative amounts on advertising.

1b) Find the optimal solution using SOLVER and generate the sensitivity report. What is an optimal output for the company?

Answer 1b

Based on the Excel Solver results here's the optimal solution for the Future Bright Company:

Production Quantities (000 units):

1. Table lamps:
   - Regular time (TReg): 67.2
   - Overtime (TOvr): 0
2. Floor lamps:
   - Regular time (FReg): 20
   - Overtime (FOvr): 0
3. Ceiling lamps:
   - Regular time (CReg): 50.8
   - Overtime (COvr): 24
4. Pendant lamps:
   - Regular time (PReg): 39.2
   - Overtime (POvr): 0

Advertising Spend (000 USD):

1. Table lamps (AT): 10
2. Floor lamps (AF): 0
3. Ceiling lamps (AC): 0
4. Pendant lamps (AP): 8

Total Profit: The optimal solution yields a total profit of $8951.6 (000 USD).

This solution maximizes the company's profit while respecting all constraints, including production capacities, demand influenced by advertising, and the advertising budget limit. The company focuses its production on table lamps, floor lamps, and ceiling lamps, with some overtime production for ceiling and pendant lamps. The advertising budget is primarily allocated to table lamps and pendant lamps, which likely have the highest impact on demand and profitability.

1c) For each department, what is the marginal value of additional overtime capacity?

Answer 1c

To answer this, we need to look at the shadow prices for the overtime capacity constraints in the sensitivity report:

- **Department 1 Overtime:** The shadow price is $0. This means each additional unit of overtime capacity in Department 1 would increase the overall profit by $0.
- **Department 2 Overtime:** The shadow price is $35. This means each additional unit of overtime capacity in Department 2 would increase the overall profit by $35.

1d) If the company cut the total budget of advertising by $6,000, how would you expect this to impact the overall profit, assuming all other conditions remain the same?

Answer 1d

To answer this, we need to look at the shadow price for the total advertising budget constraint:

The shadow price for the advertising budget constraint is $14.75. This means that for each dollar increase in the advertising budget, the profit would increase by $14.75.

Therefore, if we decrease the advertising budget by $6,000:

Expected impact on profit = $14.75 * (-6,000) = -$88,500 (approximately)

We would expect the overall profit to decrease by about $88,500 if the advertising budget was cut by $6,000, assuming all other conditions remain the same.

Q2. Optimize purchase plan and cost schedule

2a) Formulate this model algebraically.

Answer 2a

Decision variables:

- A = quantity of fertilizer A
- B = quantity of fertilizer B
- C = quantity of fertilizer C

Objective function:

The goal is to minimize the total cost of using each of the fertilizers while also satisfying the mineral requirement. This can be expressed as:

Total Cost (z) = 0.01*A + 0.008*B + 0.007*C

Constraints:

We have constraints on the total quantity of each mineral that needs to be sprayed and this can be expressed as follows:

- **Nitrogen:** 0.02*A + 0.01*B + 0.015*C >= 1.2
- **Phosphorus:** 0.01*A + 0.005*B + 0.01*C >= 1.4
- **Potash:** 0.005*A + 0.015*B + 0.005*C >= 1.8

2b) How much of each fertilizer should be purchased to satisfy the above requirements at minimum cost? Formulate and solve this problem using Solver on a spreadsheet.

Answer 2b

Problem solved using Solver in Excel (refer to Excel file)

Upon formulating the above minimization problem, following quantities of fertilizers should be purchased to satisfy the requirements at minimum cost:

- Quantity of fertilizer A = 0lb
- Quantity of fertilizer B = 88lb
- Quantity of fertilizer C = 96lb

This leads to a total minimum cost of purchasing fertilizers as $1.38 while satisfying the given mineral content requirements.

2c) Solve this problem using Python and either the Gurobipy or Pyomo package.

Answer 2c

Colab link: BA885-HW1-2c-SnehaEkka.ipynb