

REINFORCEMENT LEARNING

Spring 2023

CS6431 Natural Language Processing

Credits

1. B2: *Machine learning: an algorithmic perspective*. 2nd Edition, Marsland, Stephen. CRC press, 2015
2. <https://www.samyzaf.com/ML/rl/qmaze.html>
3. <http://www.gwydir.demon.co.uk/jo/maze/makemaze/index.htm>
4. <https://www.freecodecamp.org/news/an-introduction-to-q-learning-reinforcement-learning-14ac0b4493cc/>

Assignment

Read:

B2: Chapter 11

Problems:

B2: 11.1, 11.2, 11.4

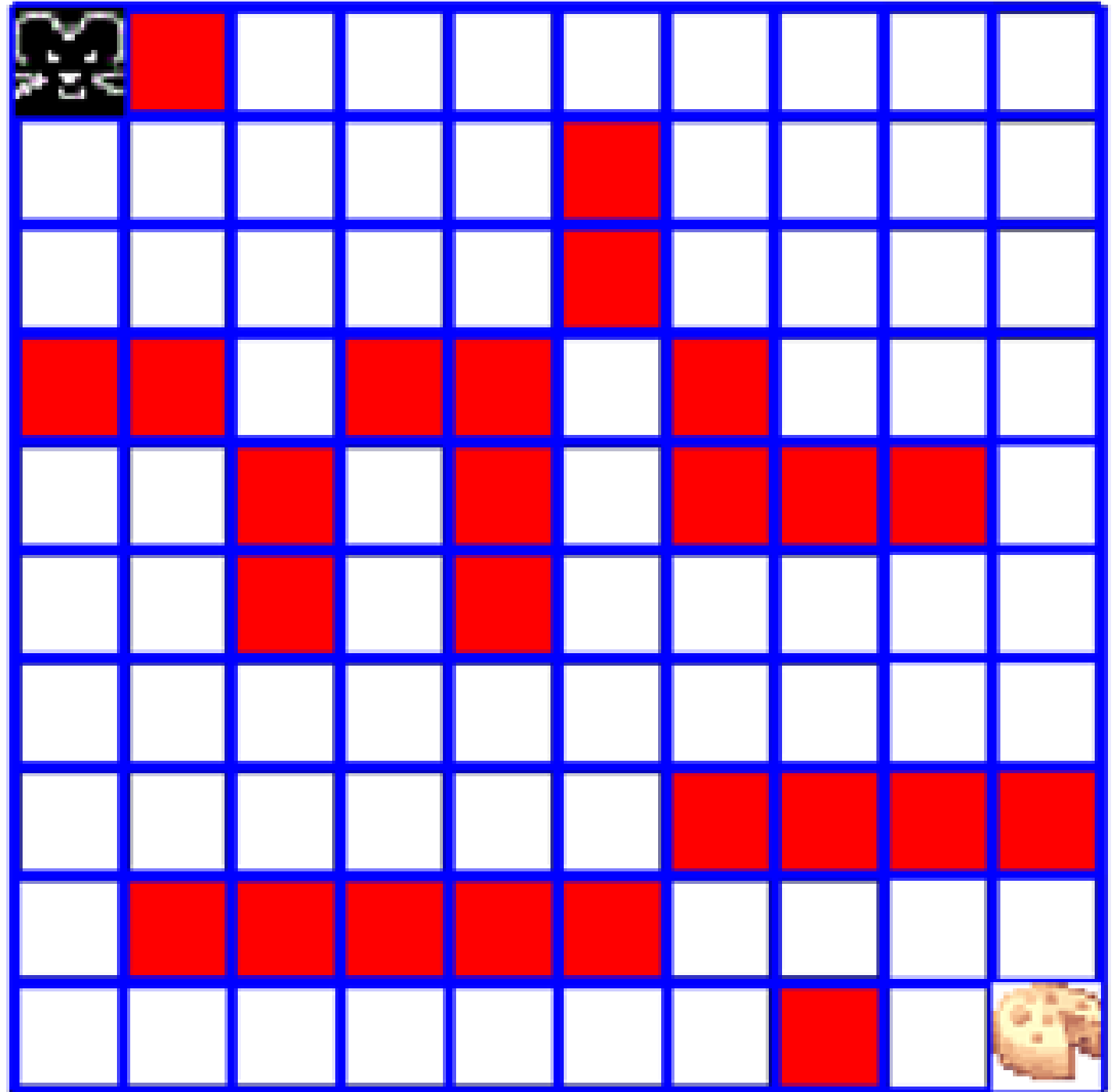
Reinforcement Learning

- Information is provided about whether or not the answer is correct, but not how to improve it.
- Reinforcement learner has to try out different strategies and see which work best.
 - ▣ Trying out \equiv searching, is a fundamental part of any reinforcement learner
 - ▣ Searches over the state space of possible inputs and outputs in order to try to maximize a **reward**.

- Reinforcement Learning: Comes from Animal intelligence
 - ▣ You repeat an action that gives you more satisfaction (or reward). Such an action gets reinforced with the situation that caused it.

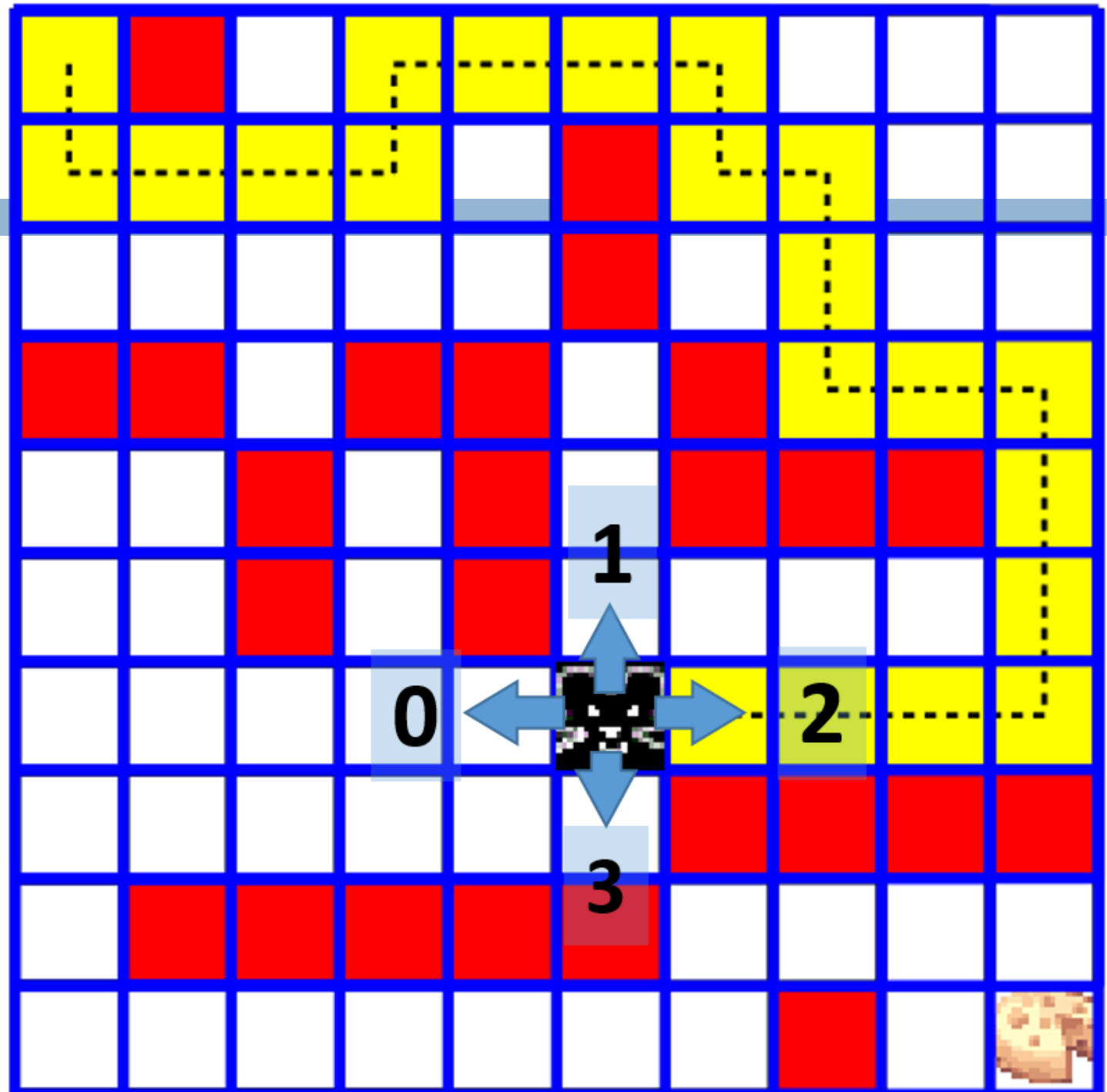
An Example

- Robot mouse has no idea of the room layout – obstructions and ways
- Robot mouse only knows how to recognize block with cheese – the end goal.

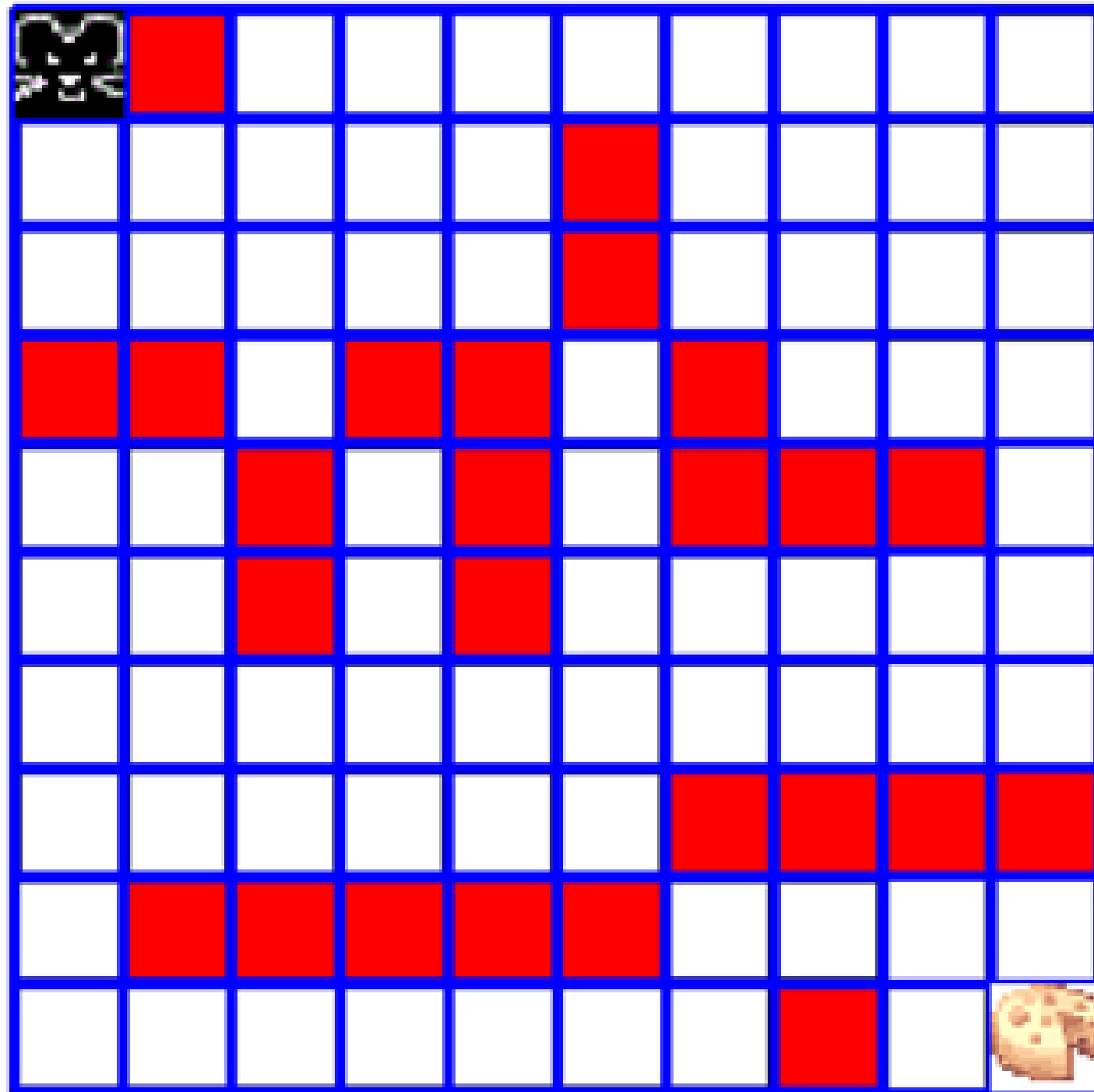


Restricted Actions

- Can move only in one of the four ways

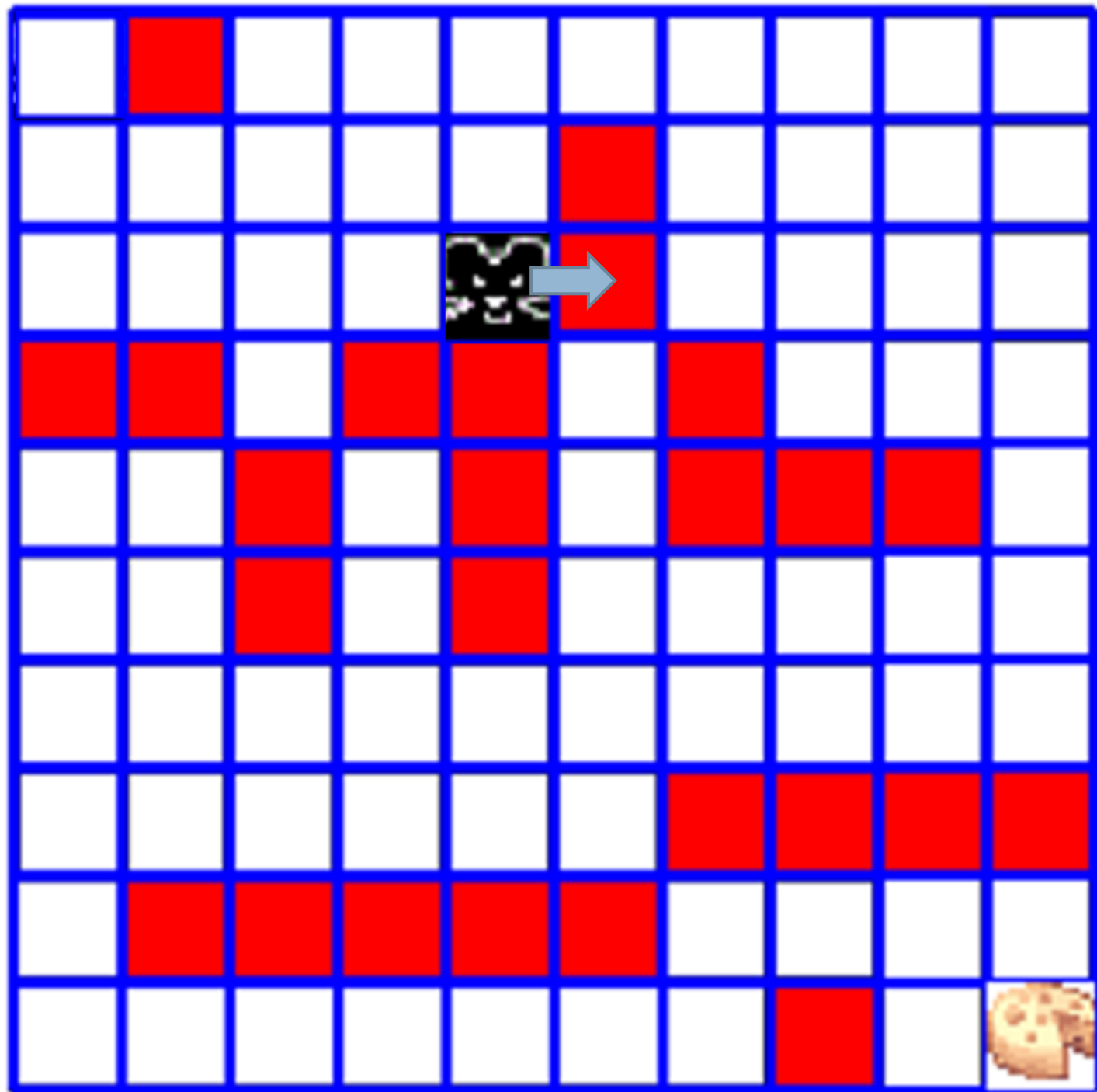


- Approach: explore and learn

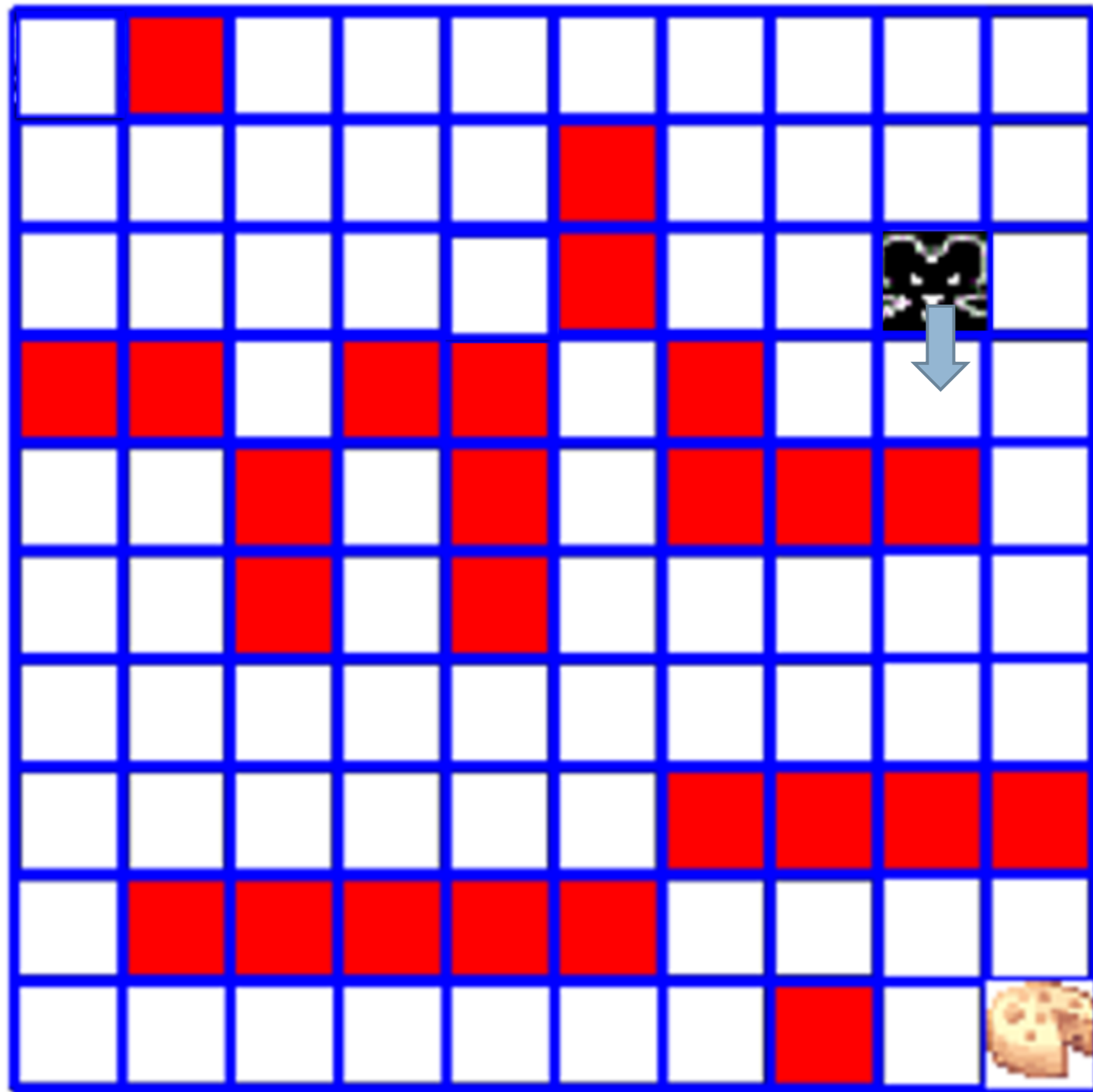


Learning Part

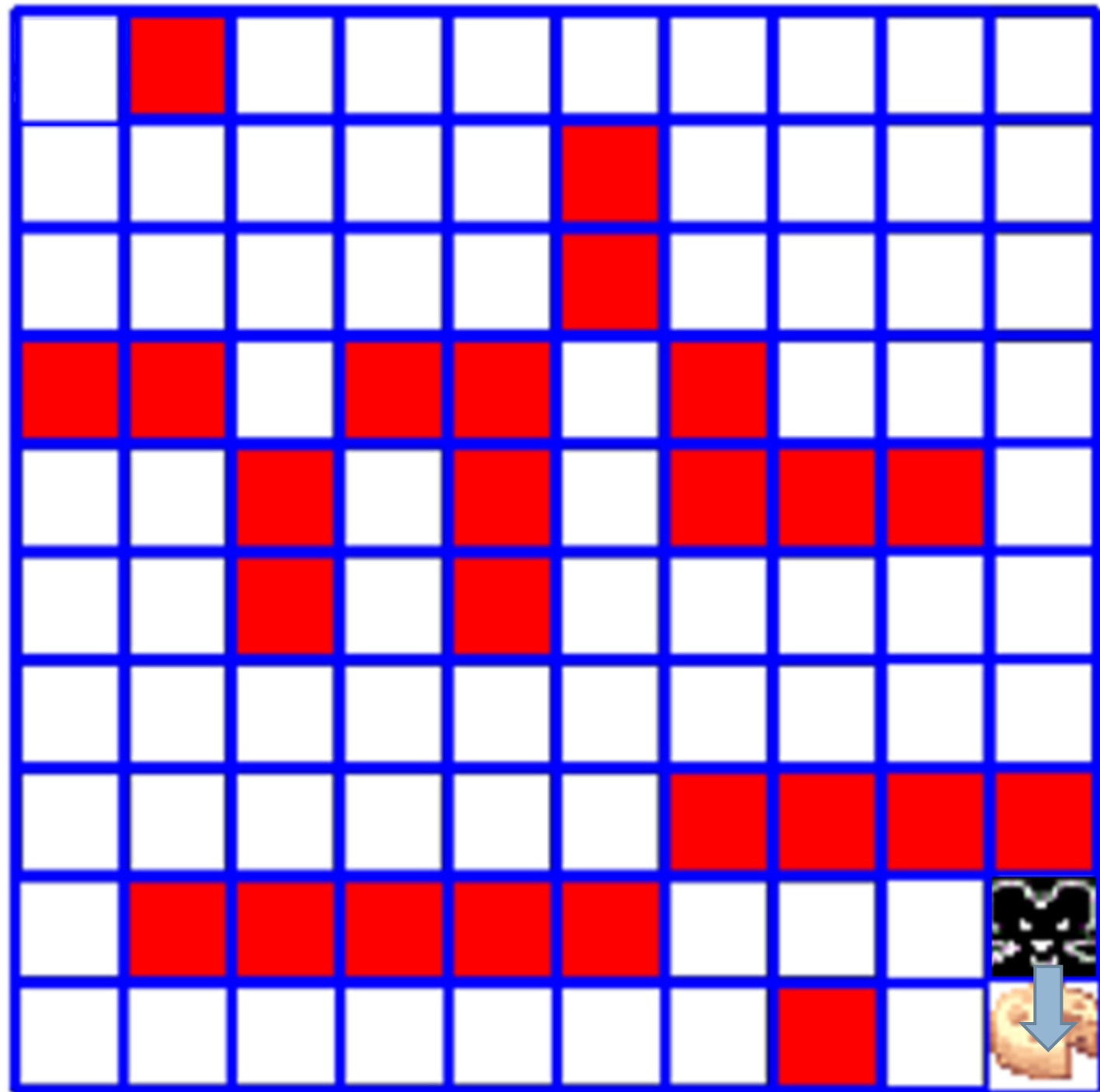
- Cell (3,5) and action 2 (going right) results in punishment
- ▣ Punishment can be denoted by some -ve value.



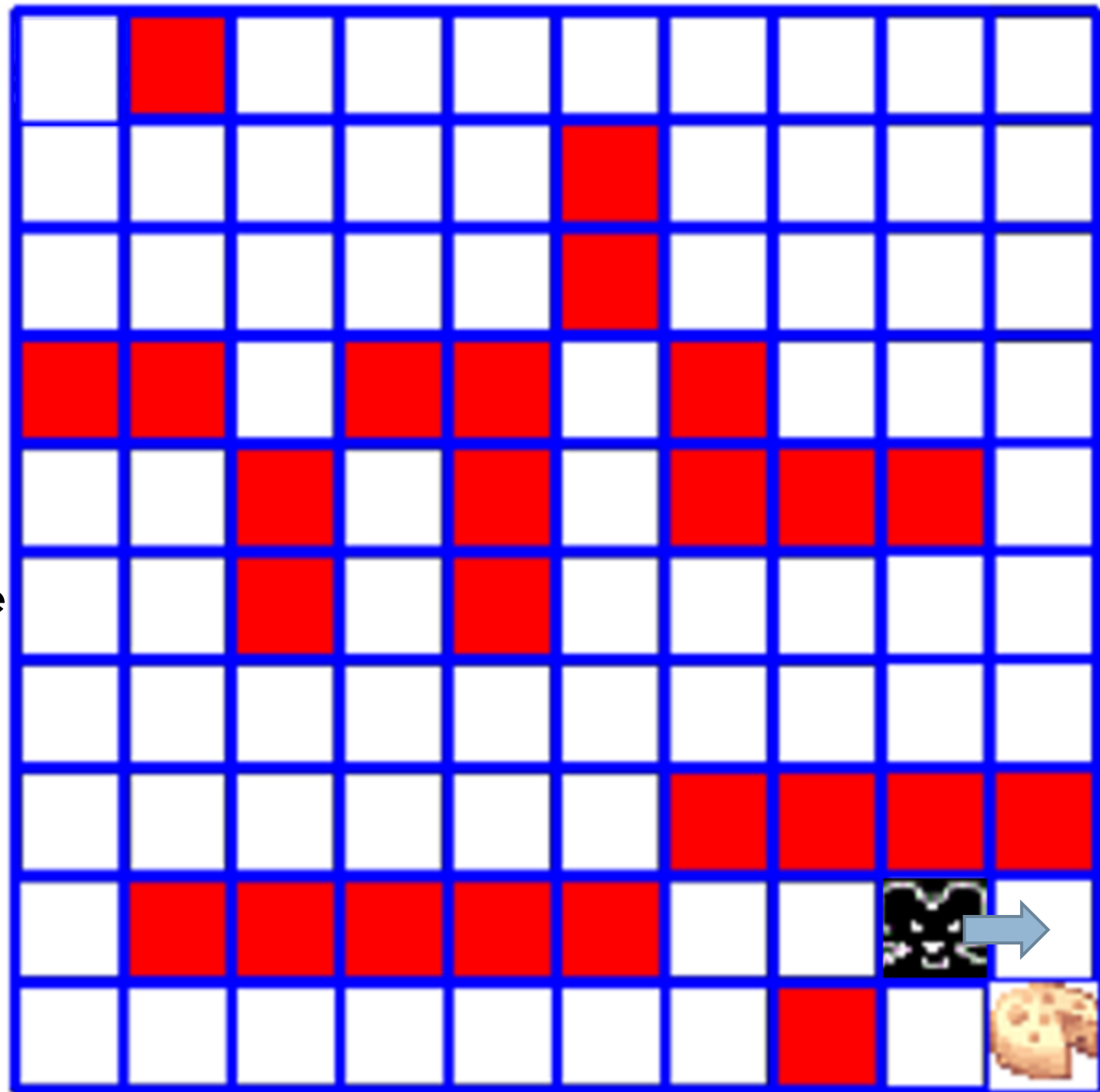
- Cell (3,9) and action 3 (going down) results in just another block
- ▣ That is, neither hitting an obstruction nor reaching the goal
- ▣ Can be represented by a zero value



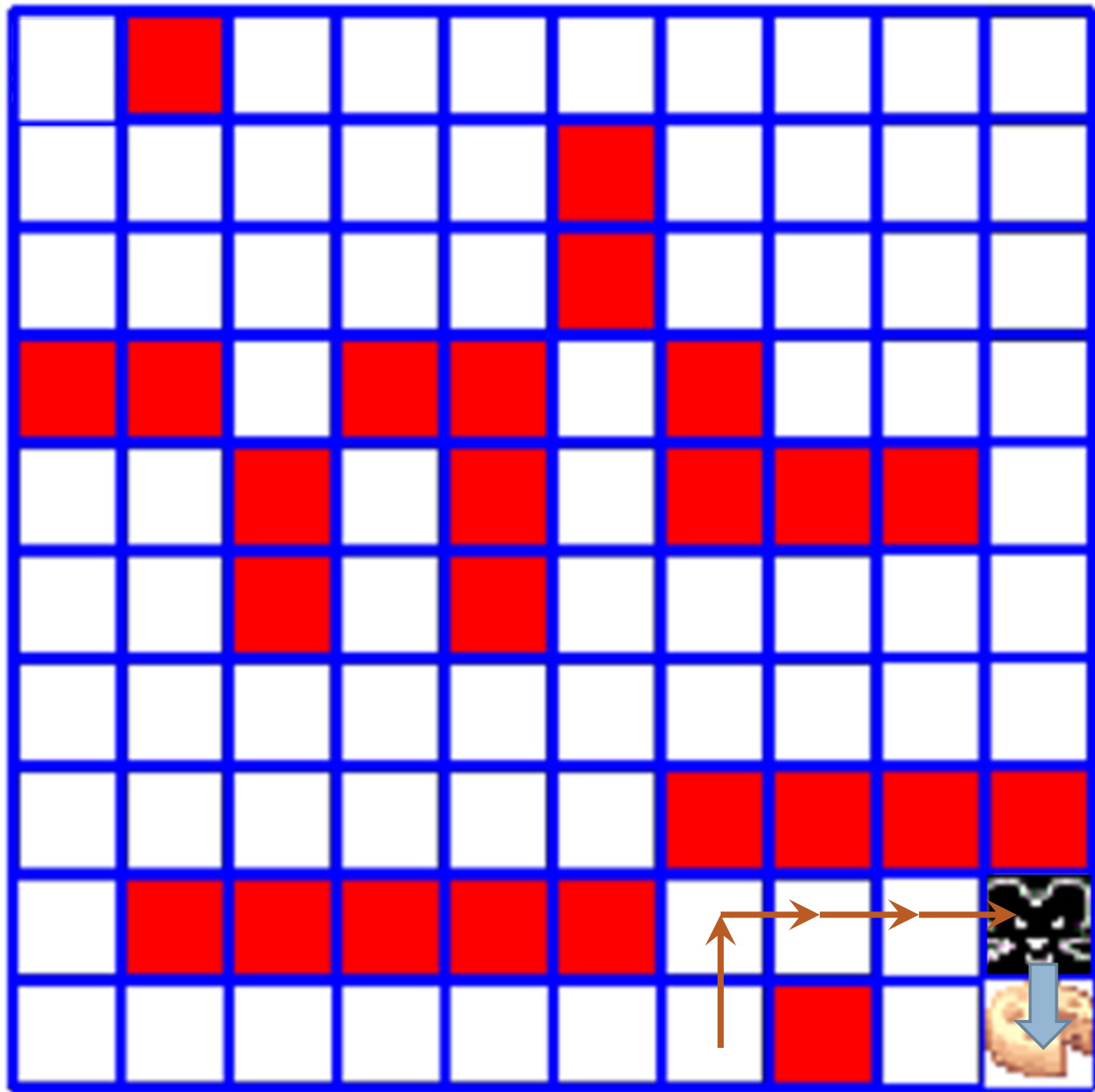
- Cell (9,10) and action 3 (going down) results in reaching the goal
 - ▣ Can be represented by a high positive value
- This also makes the cell (9,10) very valuable because it has a way to reach the goal



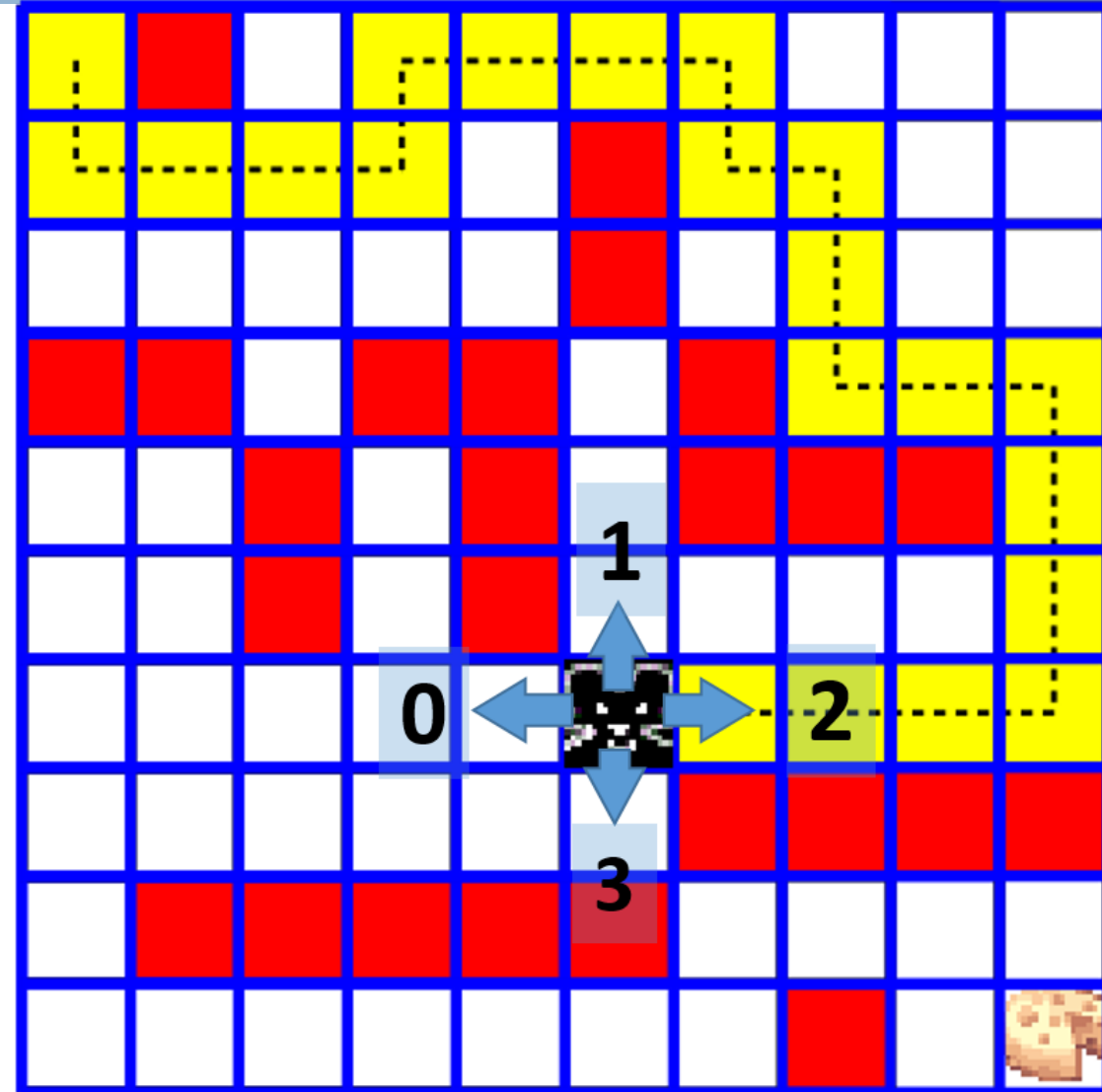
- Cell (9,9) and action 2 (going right) results in reaching the valuable cell (9,10)
 - ▣ Can be given a high positive value
- Hence, cell (9,9) also becomes valuable.



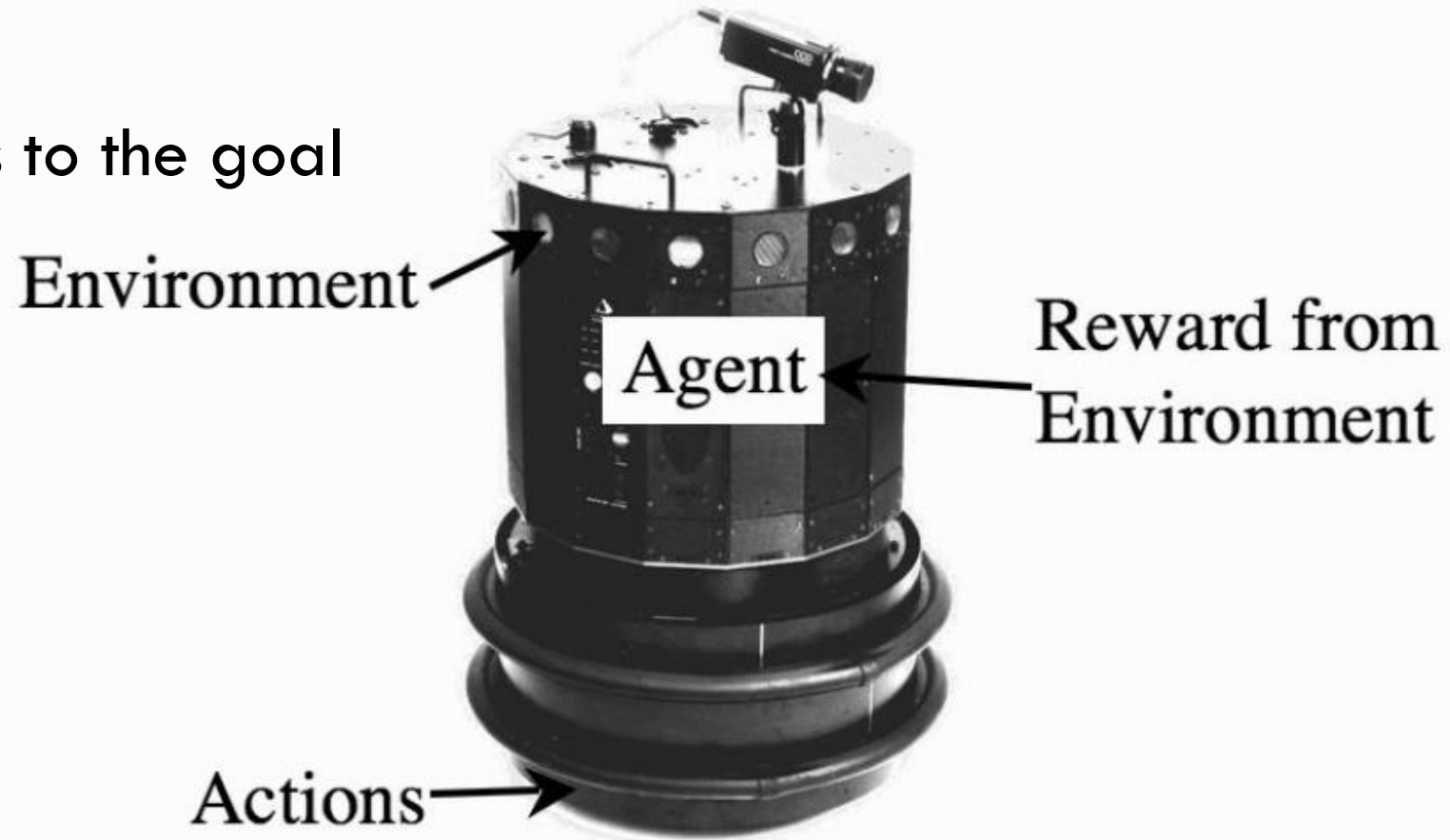
- How much “history” we want to maintain is also a choice.

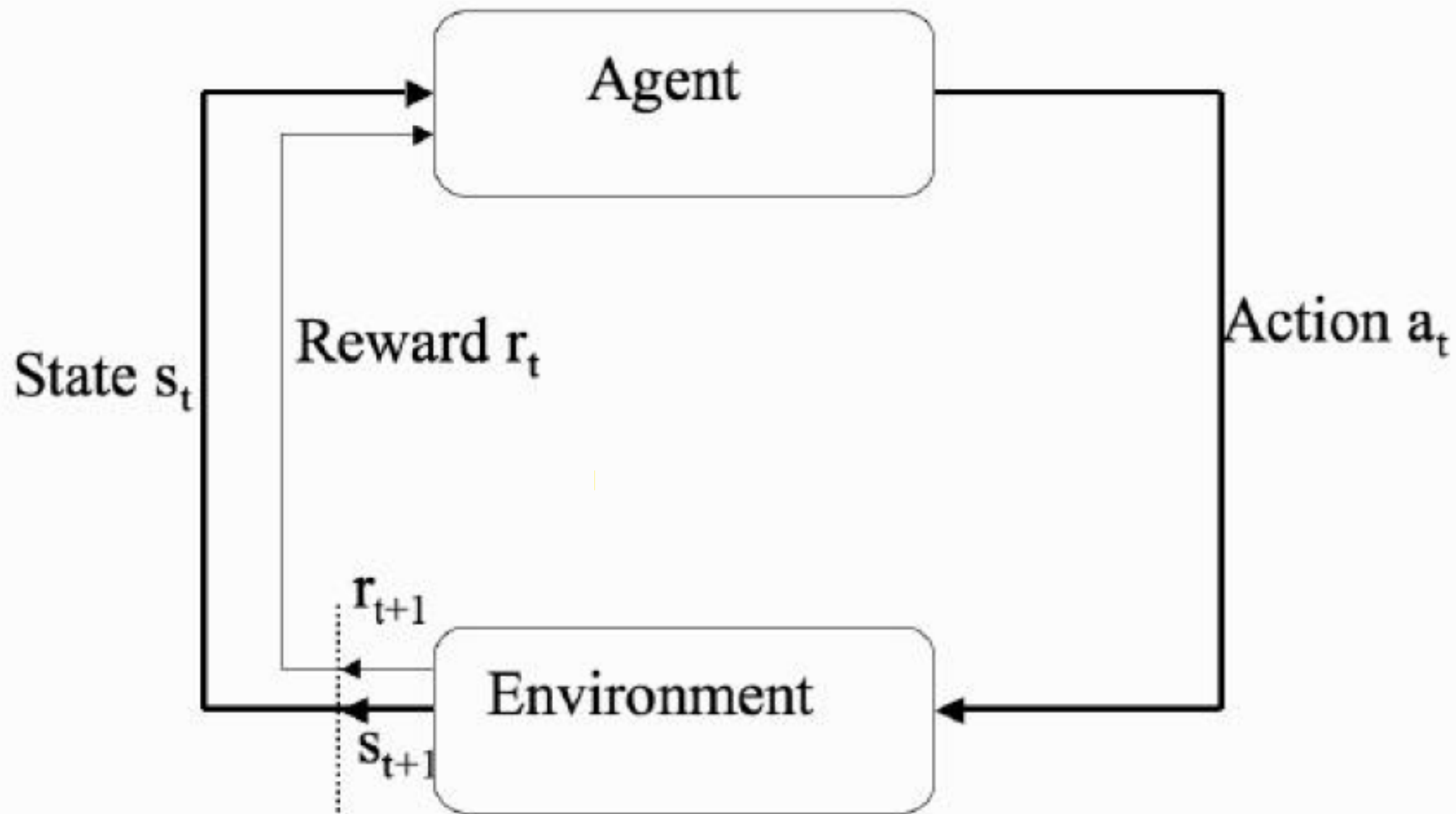


- Three things
 - ▣ Current input or the *state*
 - ▣ Possible things that can be done or the *actions*
 - ▣ Aim is to maximize *reward*
- **Agent:** that is doing or learning
- **Environment:** where the agent acts



- State: sensor readings
 - May not tell everything to the robot
 - There may be noise/inaccuracies
- Actions: possible ways in which the robot can drive its motors
 - Actions on the environment
- Reward: how well it navigates to the goal with minimal crashing



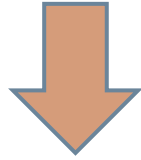


- Reward can be delayed or it can be instant
 - ▣ E.g. a robot travelling a maze vs. a robot driving a car
- **Policy:** Choice of action that should be taken
 - ▣ Should be a combination of exploitation and exploration

Policy: exploitation vs. exploration

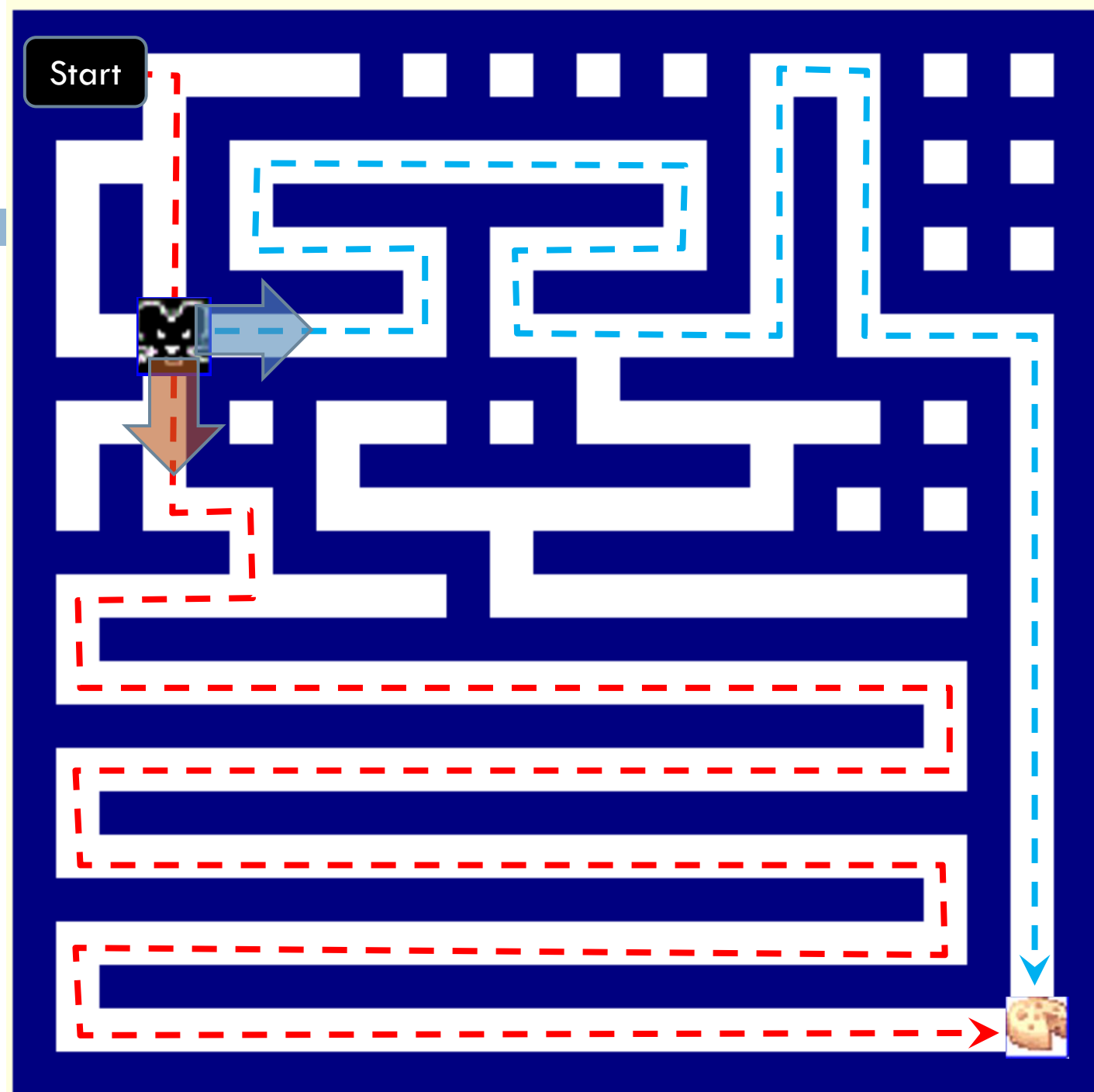
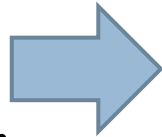
Exploitation

- Take the best action learnt so far



Exploration

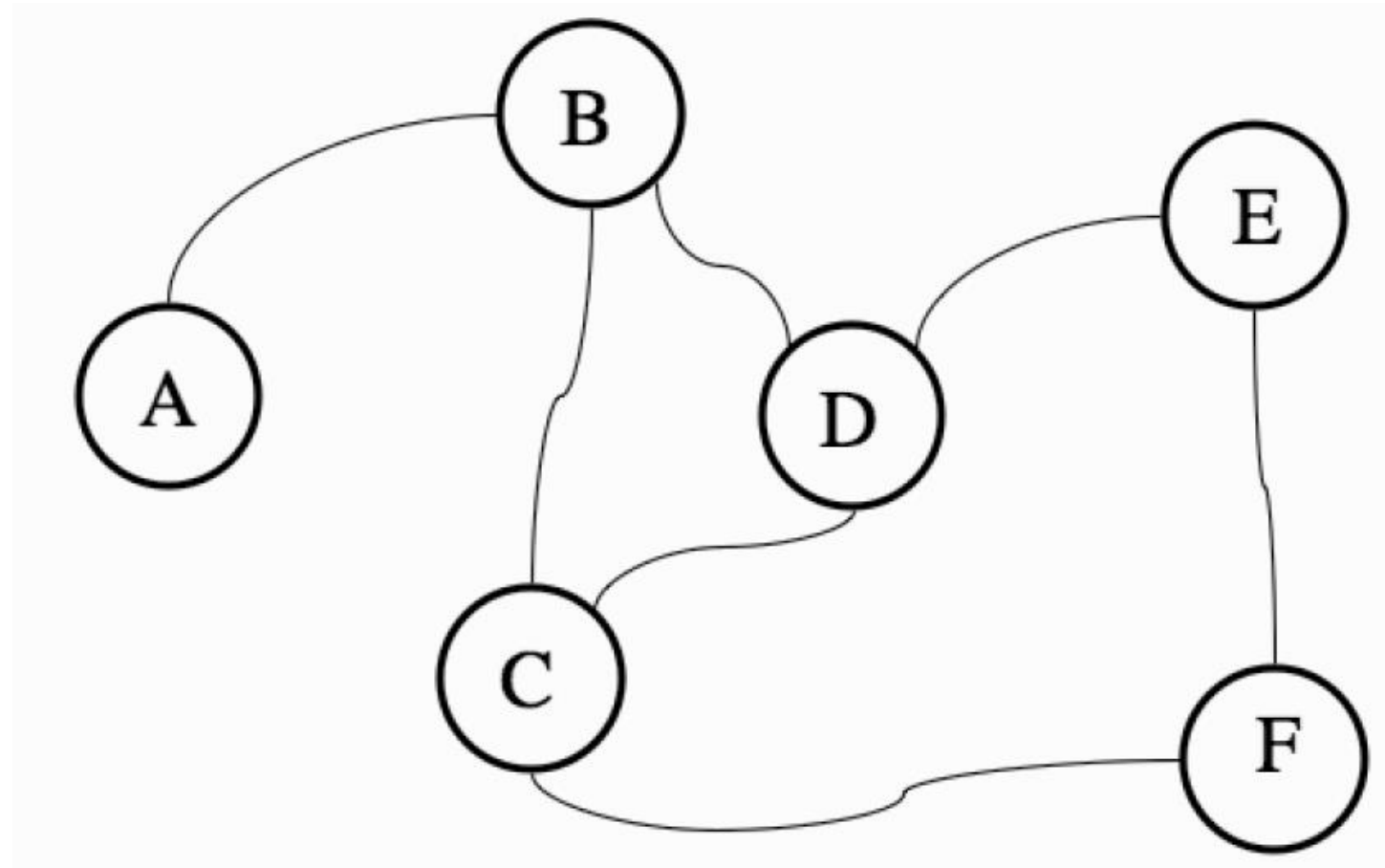
- Give sub-optimal actions a chance



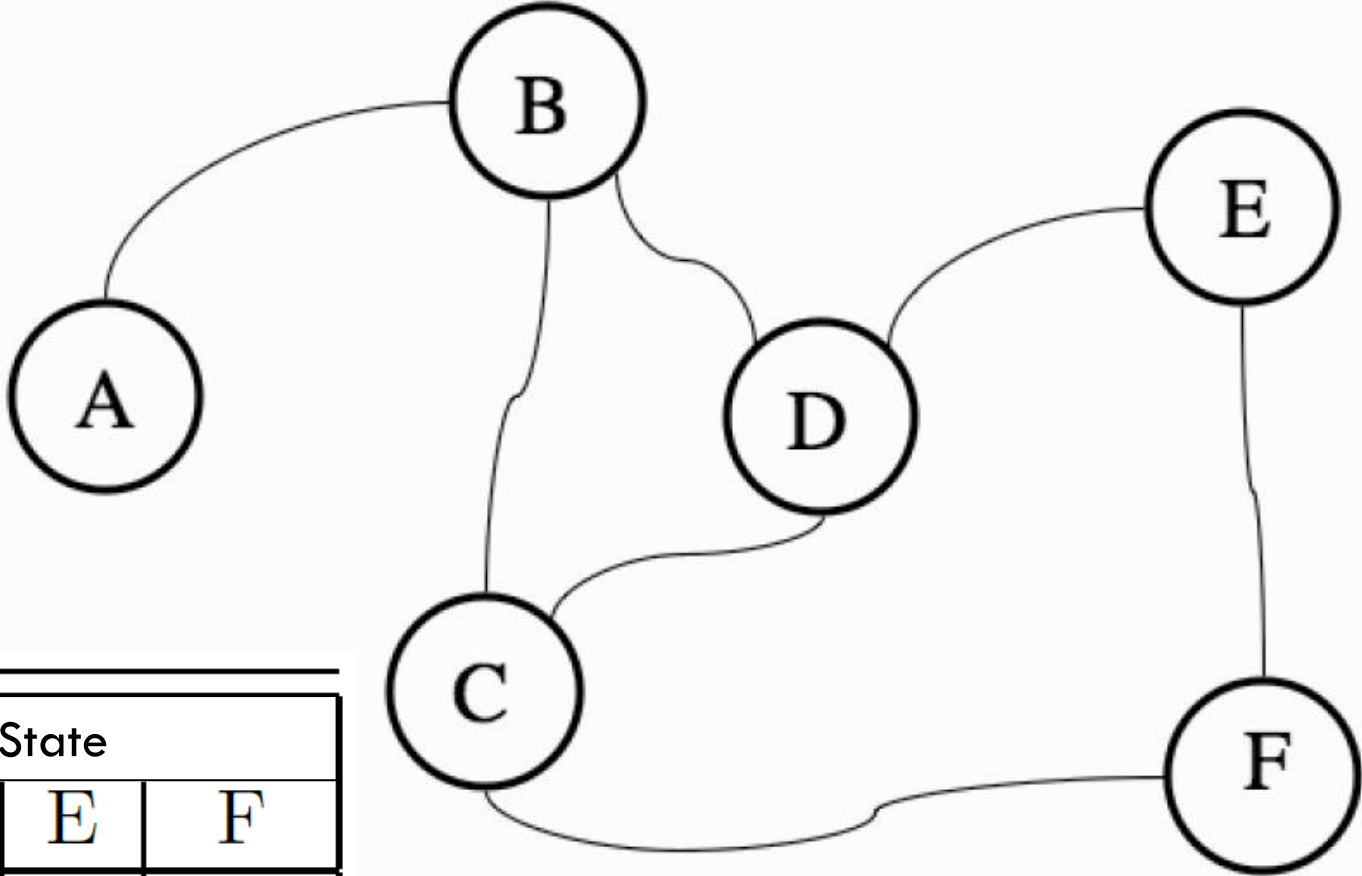
- Reward can be delayed or it can be instant
 - ▣ E.g. a robot travelling a maze vs. a robot driving a car
- **Policy:** Choice of action that should be taken
 - ▣ Should be a combination of exploration and exploitation
- **Absorbing state:** your goal state, when the solution you were searching for is found
 - ▣ Also called **Terminal** or **Accepting** state.
- **State space:** set of all states that the learner can experience
- **Action space:** set of all actions that the learner can take

EXAMPLE: GETTING LOST

- You know that F is the absorbing state ,because you are vaguely familiar with it
- You decide to reward yourself with chips only if you reach F



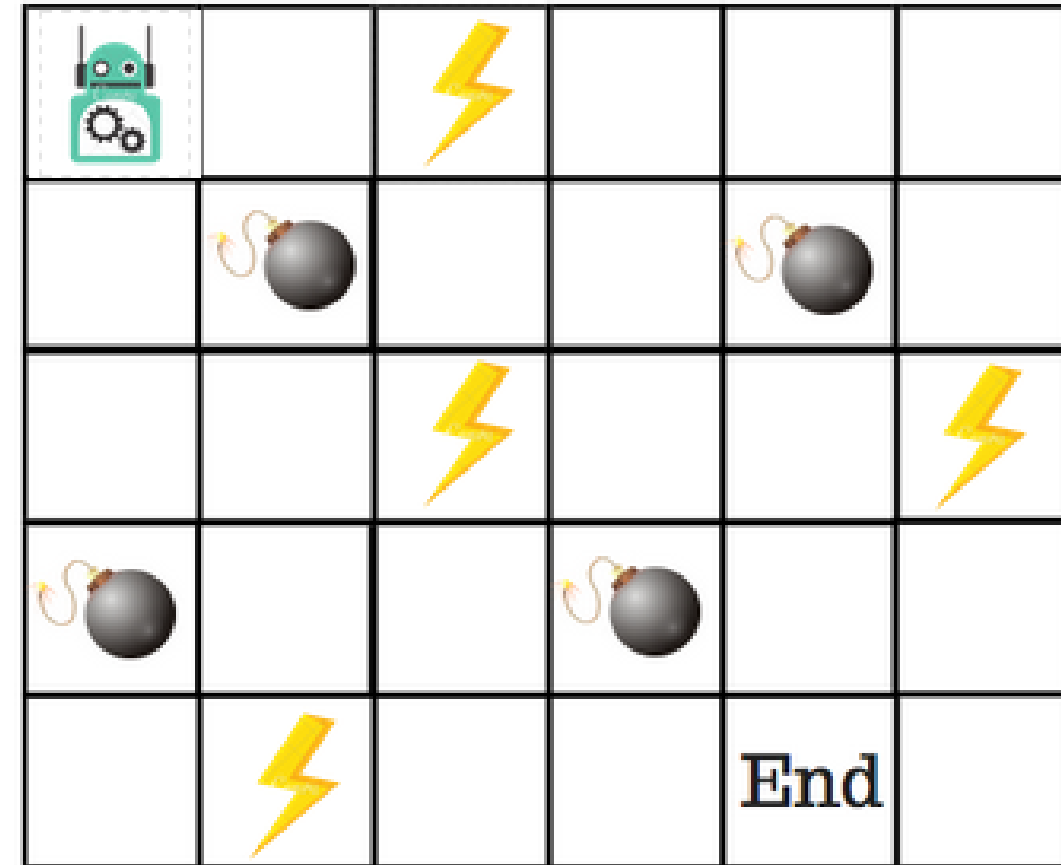
□ You don't have the city map with you, hence, you are not aware of the following **reward matrix**



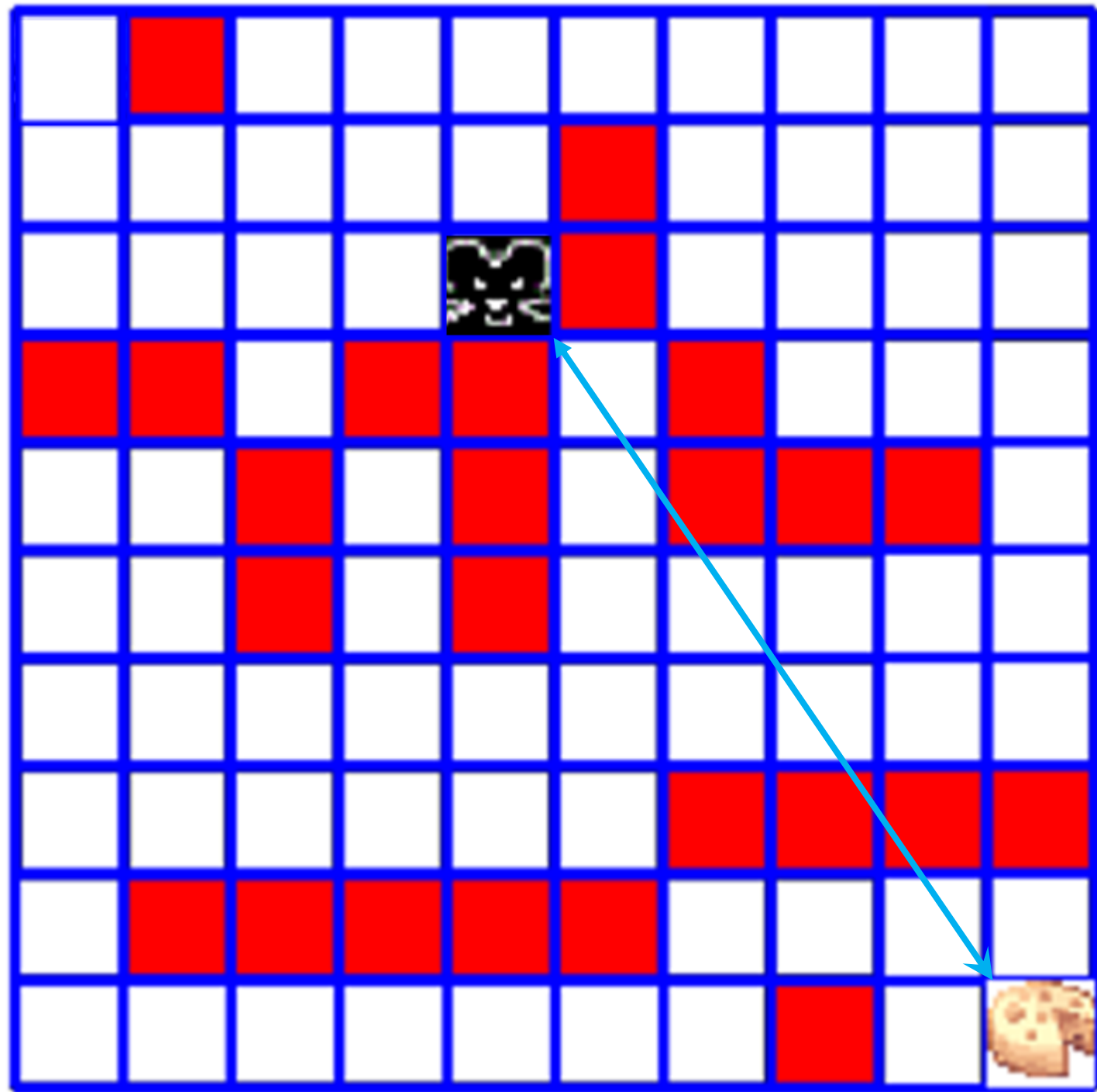
| Current State | Action or Next State | | | | | |
|---------------|----------------------|----|----|----|----|-----|
| | A | B | C | D | E | F |
| A | -5 | 0 | - | - | - | - |
| B | 0 | -5 | 0 | 0 | - | - |
| C | - | 0 | -5 | 0 | - | 100 |
| D | - | 0 | 0 | -5 | 0 | - |
| E | - | - | - | 0 | -5 | 100 |
| F | - | - | 0 | - | 0 | - |

Carrots and Sticks: The Reward Function

- Reward Function
 - ▣ Input: Current State and Chosen Action
 - ▣ Output: Numerical reward based on them
- It is generated by the environment around the learner
 - ▣ It is not internal to the learner
- Two parts
 - ▣ An intermediate part (at every step or so)
 - ▣ A pay-off in the end



- The reward tells the learner what the goal is, not how the goal should be achieved, which would be supervised learning.
- ▣ Usually a bad idea to include sub-goals like speed up of learning
- ▣ Learner can find methods of achieving sub-goals without actually achieving the real goal



- For continual tasks (e.g., learning to walk): there is no terminal state
- In general, we want to predict the reward into the infinite future
- Solution: **Discounting**:
 - ▣ We discount future rewards depending upon how “far” they are in the future

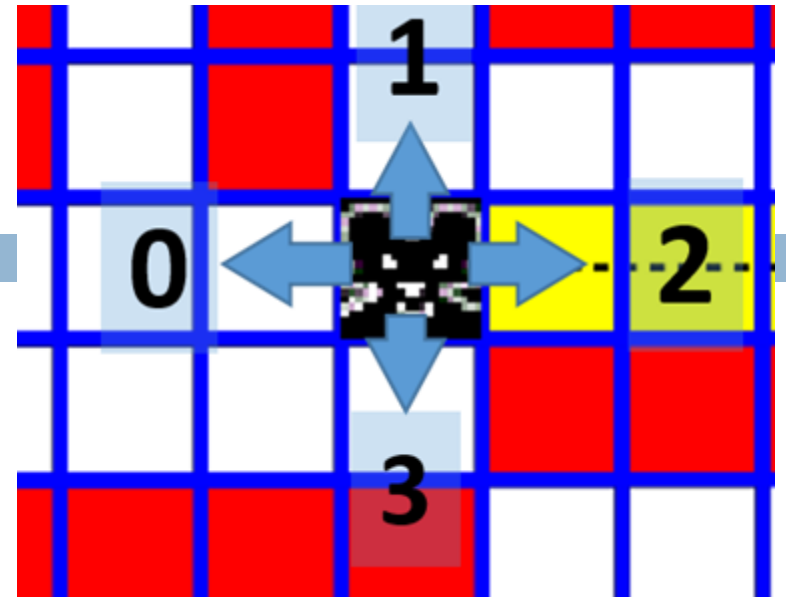
$$R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \dots + \gamma^{k-1} r_k + \dots = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

Where, $0 < \gamma < 1$ is the discounting factor

write on desk

Action Selection

- Each action is assigned a value based on
 - ▣ The current state
 - ▣ How it has been rewarded in the past
- A reward action a has received on being taken t times from the state s : $Q_{s,t}(a)$
 - ▣ Will eventually converge to true predictions



Based on prediction of $Q_{s,t}(a)$, there are three methods of choosing a

- **Greedy**: pick action with highest $Q_{s,t}(a)$
- **ϵ -Greedy**: similar to greedy, but small probability where we pick some other action at random
 - ▣ Works better than greedy in practice.
- **Soft-max**: refinement of ϵ -Greedy

$$P(Q_{s,t}(a)) = \frac{\exp(Q_{s,t}(a)/\tau)}{\sum_b \exp(Q_{s,t}(b)/\tau)} \quad \text{write on desk}$$

- **τ (temperature)**:
 - ▣ Large τ : all actions have similar probability
 - ▣ Small τ : individual probabilities matter more

Policy

- Choice of which action to take in each state in order to get optimal results: π
 - ▣ It is a mapping from states to actions.
- Goal: learn better π for each state s_t as we proceed.
- Two things:
 - ▣ How much past information we need to know while getting into the current state (use of Markov Decision Process)
 - ▣ How we assign a value to the current state

MARKOV DECISION PROCESSES

- Is the information of current state sufficient to compute reward for the next move (e.g., chess)
 - ▣ Such a state is called **Markov state**

$$Pr(r_t = r', s_{t+1} = s' | s_t, a_t)$$

write on desk

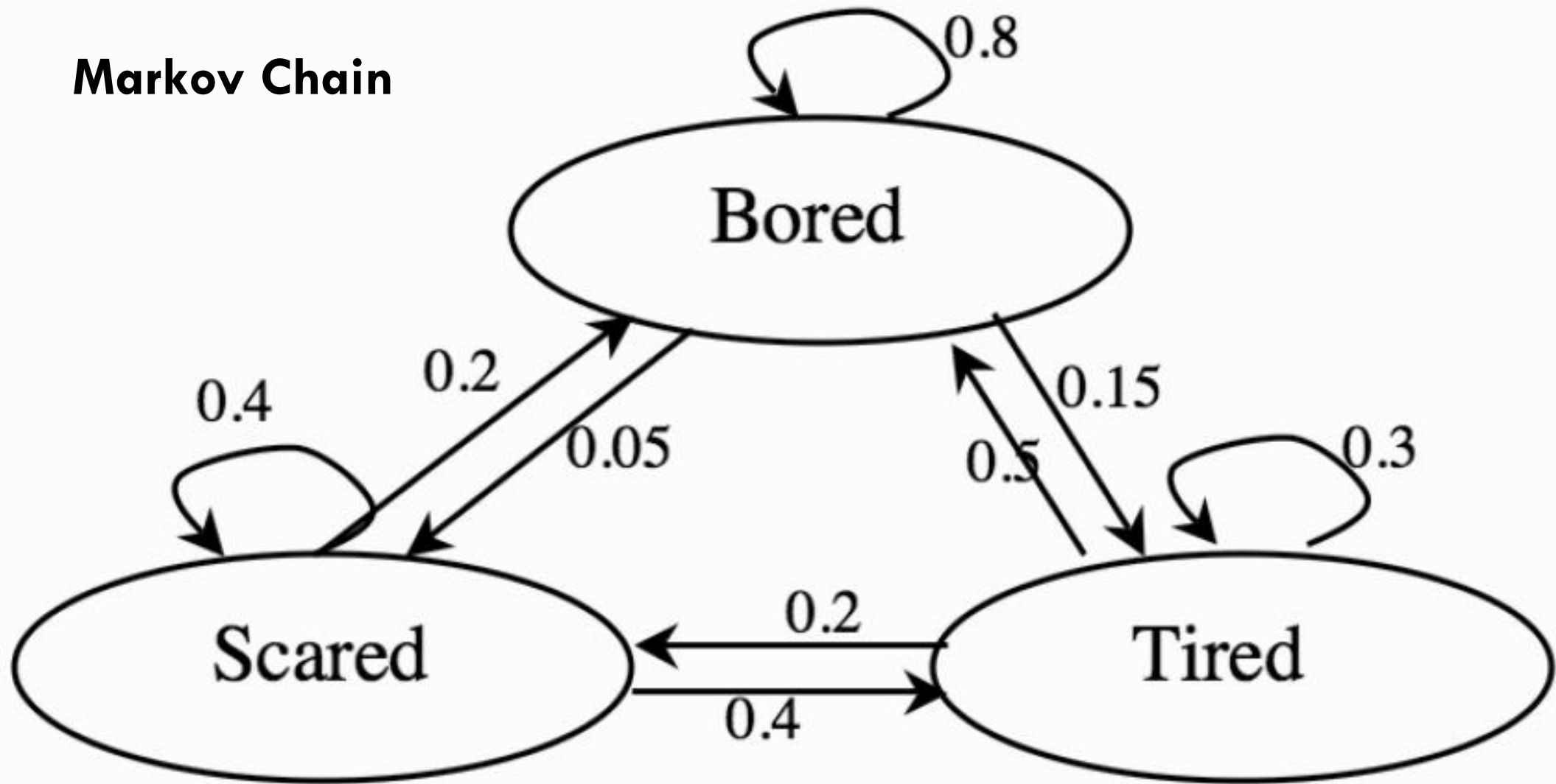
OR

- We need to consider how we got to the current state, i.e., a sense of history needs to be stored.

$$Pr(r_t = r', s_{t+1} = s' | s_t, a_t, r_{t-1}, s_{t-1}, a_{t-1}, \dots, r_1, s_1, a_1, r_0, s_0, a_0).$$

- A reinforcement learning problem that follows Markov state property is known as a **Markov Decision Process (MDP)**
- The number of possible states and actions are assumed to be finite.
- Markov Chain: States + transition probabilities
- Markov Decision Process: Extension of Markov Chain with actions and rewards

Markov Chain

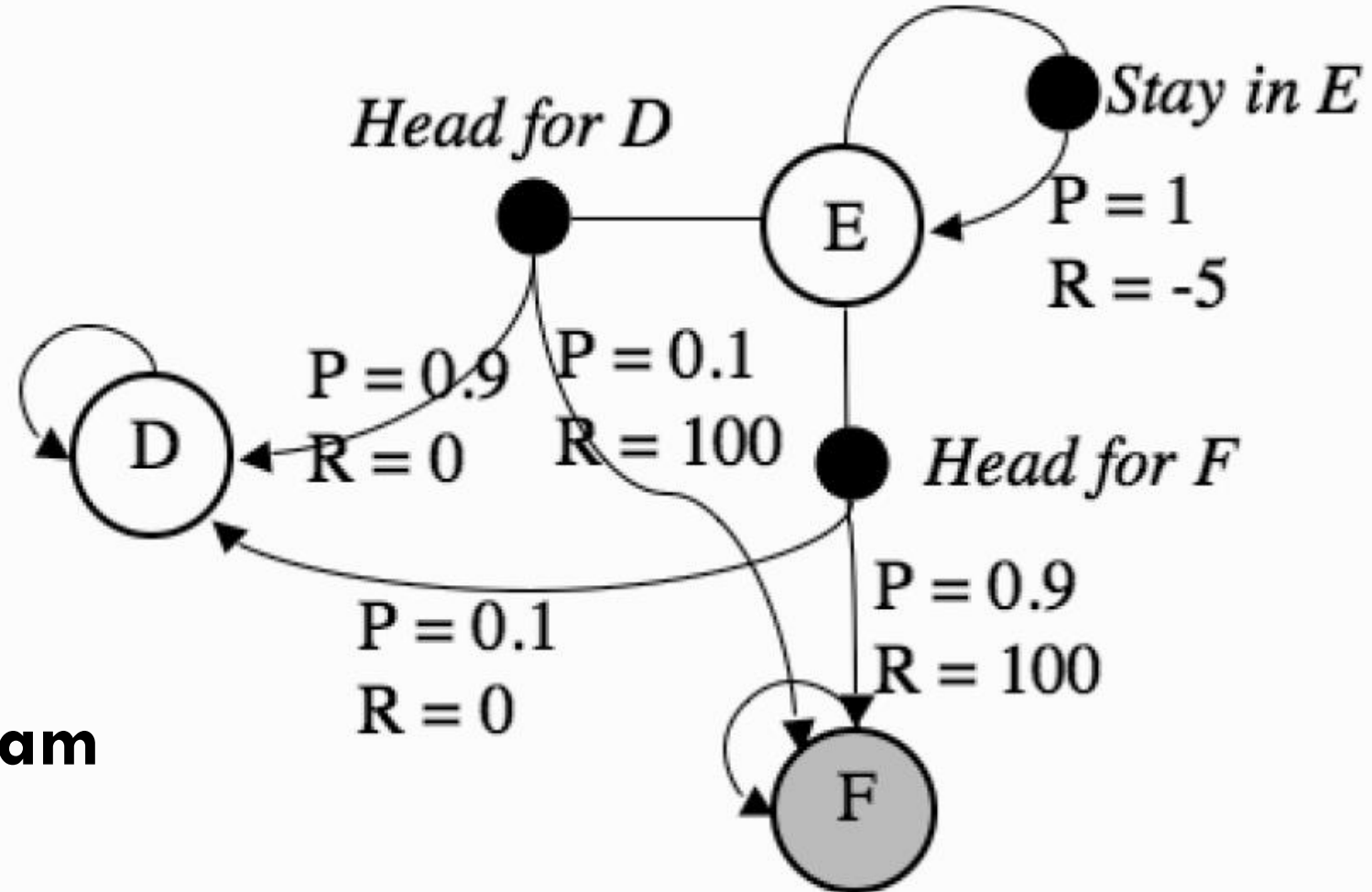


A simple example of a Markov decision process to decide on the state of your mind tomorrow given your state of mind today.

Markov Decision Process

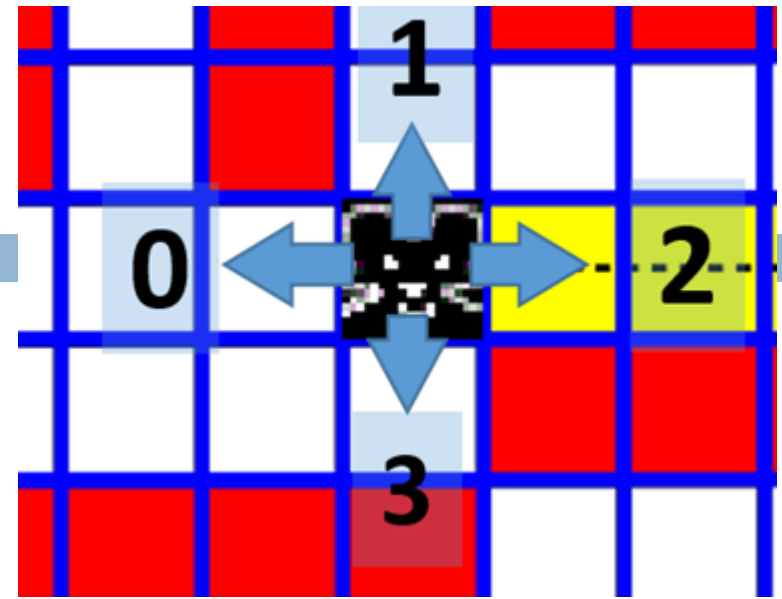
- Addition: Selecting an action does not guarantee a state.
- We add action nodes coming out of state nodes.
- Multiple state nodes can be connected to an action node.

Transition Diagram



MDP for getting lost example

VALUES



- **Value** denotes the expected future reward the reinforcement learner is trying to maximize

□ Two ways:

- ▣ **State-value function** $V(s)$: average the reward across all of the actions that can be taken

$$V(s) = E(r_t | s_t = s) = E \left\{ \sum_{i=0}^{\infty} \gamma^i r_{t+i+1} | s_t = s \right\}$$

write on desk

- ▣ **Action-value function** $Q(s, a)$: consider each possible action that can be taken from the current state separately.

$$Q(s, a) = E(r_t | s_t = s, a_t = a) = E \left\{ \sum_{i=0}^{\infty} \gamma^i r_{t+i+1} | s_t = s, a_t = a \right\}$$

□ Two ways:

- ▣ **State-value function** $V(s)$: average the reward across all of the actions that can be taken

$$V(s) = E(r_t | s_t = s) = E \left\{ \sum_{i=0}^{\infty} \gamma^i r_{t+i+1} | s_t = s \right\}$$

- ▣ **Action-value function** $Q(s, a)$: consider each possible action that can be taken from the current state separately.

$$Q(s, a) = E(r_t | s_t = s, a_t = a) = E \left\{ \sum_{i=0}^{\infty} \gamma^i r_{t+i+1} | s_t = s, a_t = a \right\}$$

- ▣ State-value function is less accurate but easier to compute

- ▣ Action-value function is more accurate but requires more information to compute

- Now, we have two problems
 - ▣ Deciding which action to take, i.e., the policy
 - ▣ Predicting the value function
- Optimal policy, π^* , is the one (not necessarily unique) in which the value function is the greatest over all possible states.
- Optimal value functions can be of two types: $V^*(s) = \max_a(Q^*(s, a))$
 - ▣ Optimal state value function: **write on desk**

$$V^*(s) = \max_{\pi} V^{\pi}(s) \text{ for all possible states } s$$

- ▣ Optimal action value function:

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a) \text{ for all possible states } s \text{ and actions } a$$

□ Optimal state value function:

$$V^*(s) = \max_{\pi} V^{\pi}(s) \text{ for all possible states } s$$

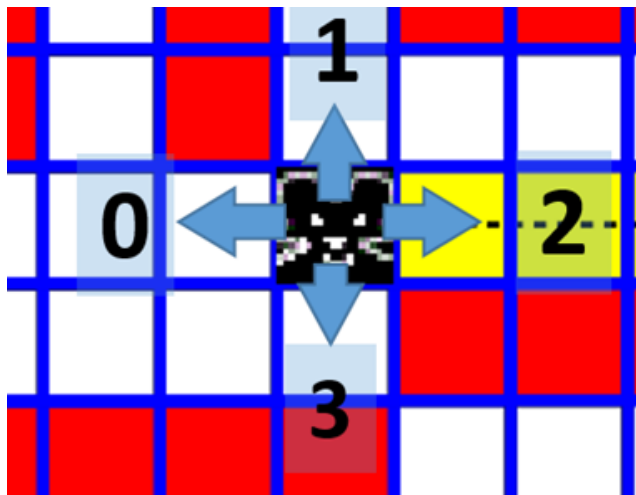
▣ Assumes taking the optimal action in each case

□ Optimal action value function:

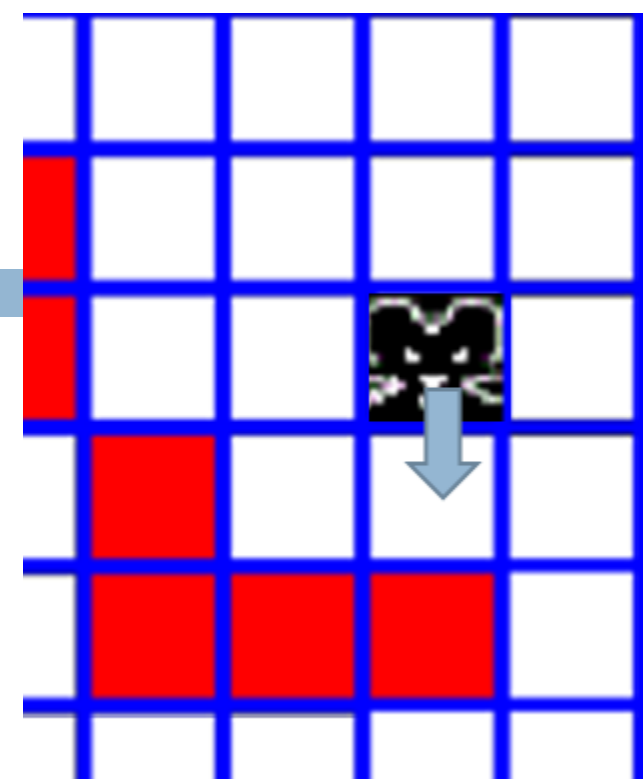
$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a) \text{ for all possible states } s \text{ and actions } a$$

▣ Considers taking action a this time, and then following the optimal policy from then on

□ Hence, optimal action value = current reward + discounted estimate of the future reward



$$\begin{aligned} Q^*(s, a) &= E(r_{t+1}) + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) \\ &= E(r_{t+1}) + \gamma V^*(s_{t+1} | s_t = s, a_t = a) \end{aligned}$$



- How do we update values?

Updated Estimate \leftarrow Current Estimate +
 $\mu(\text{New Estimate} - \text{Current Estimate})$

Temporal Difference (TD) method

$$V(s_t) \leftarrow V(s_t) + \mu(r_{t+1} + \gamma V(s_{t+1}) - V(s_t))$$

$$Q(s, a) \leftarrow Q(s, a) + \mu(r + \gamma \max_{a'} Q(s', a') - Q(s, a))$$

Write on desk

- If we are remembering previous λ states and also updated their value functions, then the above algorithm is called $TD(\lambda)$.

- Initialisation

- set $Q(s, a)$ to small random values for all s and a

- Repeat:

- initialise s

- repeat:

- * select action a using ϵ -greedy or another policy

- * take action a and receive reward r

- * sample new state s'

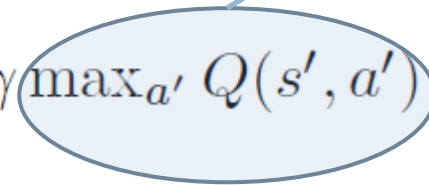
- * update $Q(s, a) \leftarrow Q(s, a) + \mu(r + \gamma \max_{a'} Q(s', a') - Q(s, a))$

- * set $s \leftarrow s'$

- For each step of the current episode

- Until there are no more episodes

Off-policy decision



The Sarsa Algorithm

- Initialisation

- set $Q(s, a)$ to small random values for all s and a

- Repeat:

- initialise s
- choose action a using the current policy
- repeat:

- * take action a and receive reward r

- * sample new state s'

- * choose action a' using the current policy

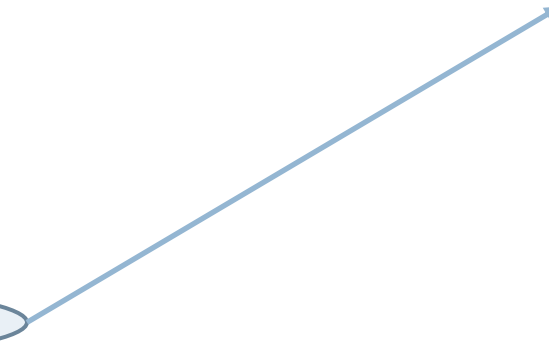
- * update $Q(s, a) \leftarrow Q(s, a) + \mu(r + \gamma Q(s', a') - Q(s, a))$

- * $s \leftarrow s', a \leftarrow a'$

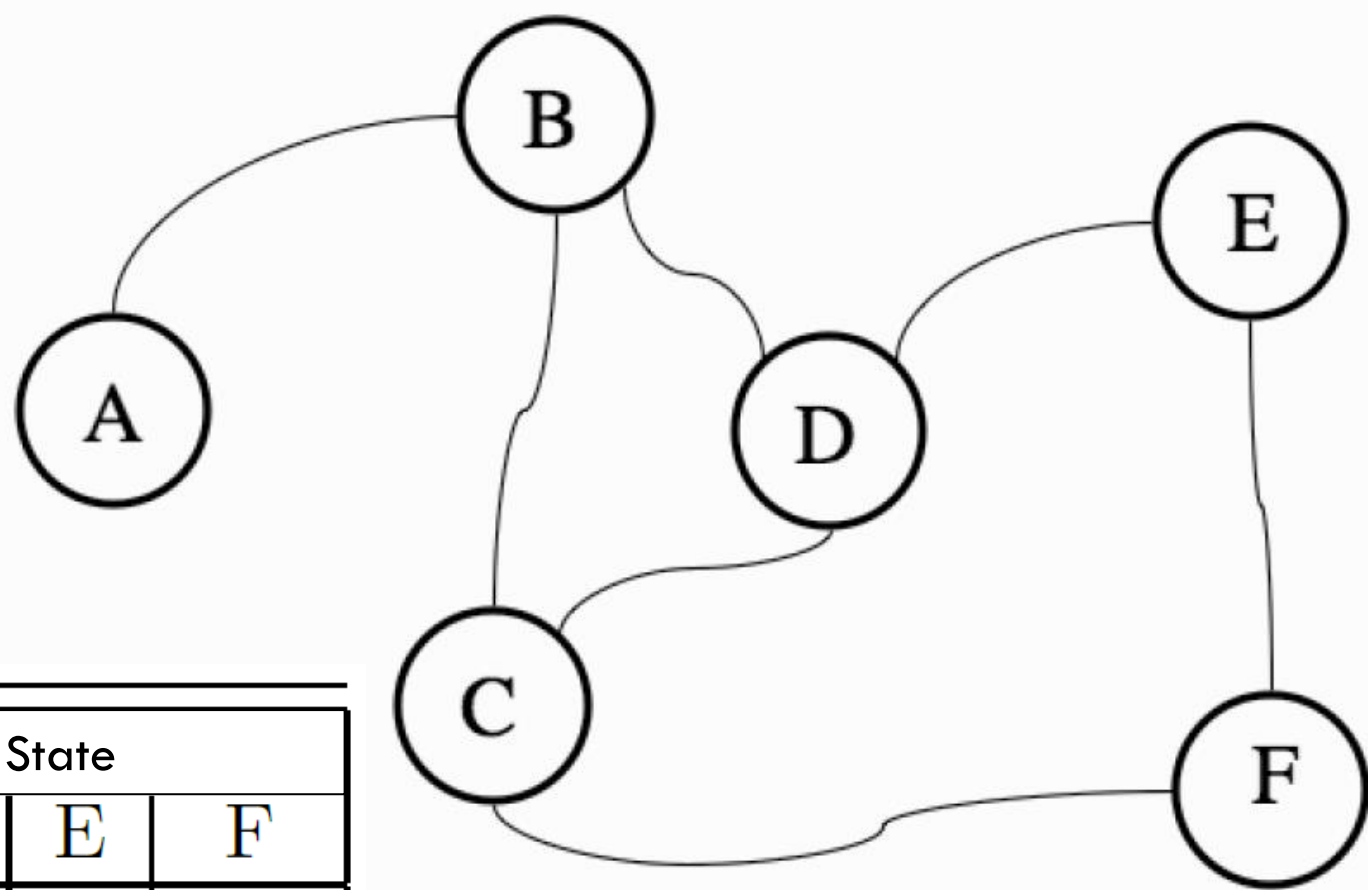
- for each step of the current episode

- Until there are no more episodes

On-policy decision



Recall Getting Lost Example



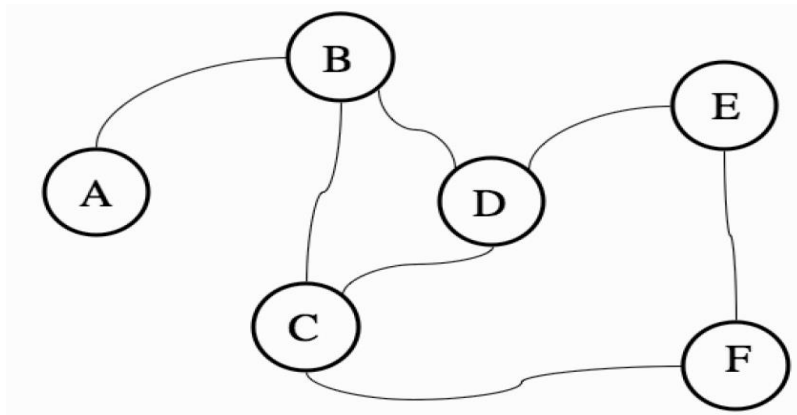
Reward Matrix

| Current State | Action or Next State | | | | | |
|---------------|----------------------|----|----|----|----|-----|
| | A | B | C | D | E | F |
| A | -5 | 0 | - | - | - | - |
| B | 0 | -5 | 0 | 0 | - | - |
| C | - | 0 | -5 | 0 | - | 100 |
| D | - | 0 | 0 | -5 | 0 | - |
| E | - | - | - | 0 | -5 | 100 |
| F | - | - | 0 | - | 0 | - |

Q-learning Example

□ For the getting lost example, consider the following trail of steps for an iteration: **A,B,A,B,D,D,B,C,F**

And the following **initial Q matrix**:



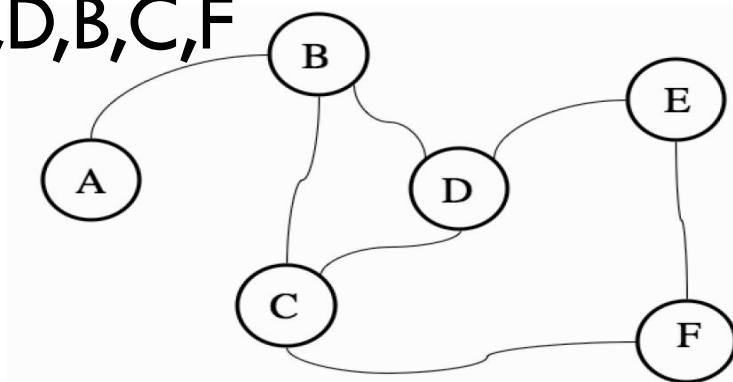
| States | Action or Next State | | | | | |
|--------|----------------------|--------|--------|--------|--------|--------|
| | A | B | C | D | E | F |
| A | -0.039 | -0.028 | -0.026 | -0.018 | 0.013 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | 0.013 | -0.026 |
| C | -0.018 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | 0.013 | 0.025 | -0.004 | 0.043 | -0.015 | 0.011 |
| E | -0.018 | 0.011 | -0.018 | -0.009 | -0.038 | -0.033 |
| F | 0.013 | -0.018 | 0.039 | -0.026 | 0.011 | -0.002 |

Compute matrix updates as during the iteration. Use μ (learning rate) = .6 and γ (discounting factor) = .3

Trail: A,B,A,B,D,D,B,C,F

$$\mu = .6$$

$$\gamma = .3$$



For $A \rightarrow B$: $s = A, a = B, s' = B, r = 0$

$$\begin{aligned}
 Q(A, B) &= Q(A, B) \\
 &+ .6 \left(0 + .3 \times \max_{a'} Q(B, a') - Q(A, B) \right) \\
 &= -0.028 + .6(.3 \times 0.044 + 0.028) \\
 &= -0.003
 \end{aligned}$$

For $B \rightarrow A$: $s = B, a = A, s' = A, r = 0$

$$Q(s, a) \leftarrow Q(s, a) + \mu(r + \gamma \max_{a'} Q(s', a') - Q(s, a))$$

| States | Action or Next State | | | | | |
|---------|----------------------|--------|--------|--------|--------|--------|
| | A | B | C | D | E | F |
| A | -0.039 | -0.028 | -0.026 | -0.018 | 0.013 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | 0.013 | -0.026 |
| C | -0.018 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | 0.013 | 0.025 | -0.004 | 0.043 | -0.015 | 0.011 |
| E | -0.018 | 0.011 | -0.018 | -0.009 | -0.038 | -0.033 |
| F | 0.013 | -0.018 | 0.039 | -0.026 | 0.011 | -0.002 |
| Updated | A | B | C | D | E | F |
| A | -0.039 | -0.003 | -0.026 | -0.018 | 0.013 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | 0.013 | -0.026 |
| C | -0.018 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | 0.013 | 0.025 | -0.004 | 0.043 | -0.015 | 0.011 |
| E | -0.018 | 0.011 | -0.018 | -0.009 | -0.038 | -0.033 |
| F | 0.013 | -0.018 | 0.039 | -0.026 | 0.011 | -0.002 |

Q matrix after iteration # 10

```
[[ -0.0 0.0 -inf -0.0 0.0 0.0]
[ -0.0 0.0 38.2 -0.0 0.0 -inf]
[ -0.0 -0.0 -0.0 -0.0 0.0 99.8]
[ -0.0 0.0 -0.0 -0.0 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 91.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q matrix after iteration # 30

```
[[ -0.0 16.0 -inf -0.0 -inf -inf]
[ 4.3 0.0 40.0 -0.0 0.0 -inf]
[ -0.0 -0.0 24.5 4.4 0.0 100.0]
[ -0.0 15.9 -0.0 0.8 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 99.8]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q matrix after iteration # 100

```
[[ -0.0 16.0 -inf -0.0 -inf -inf]
[ 5.8 7.7 40.0 -0.0 0.0 -inf]
[ -0.0 -0.0 34.1 6.2 0.0 100.0]
[ -0.0 16.0 -0.0 0.8 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

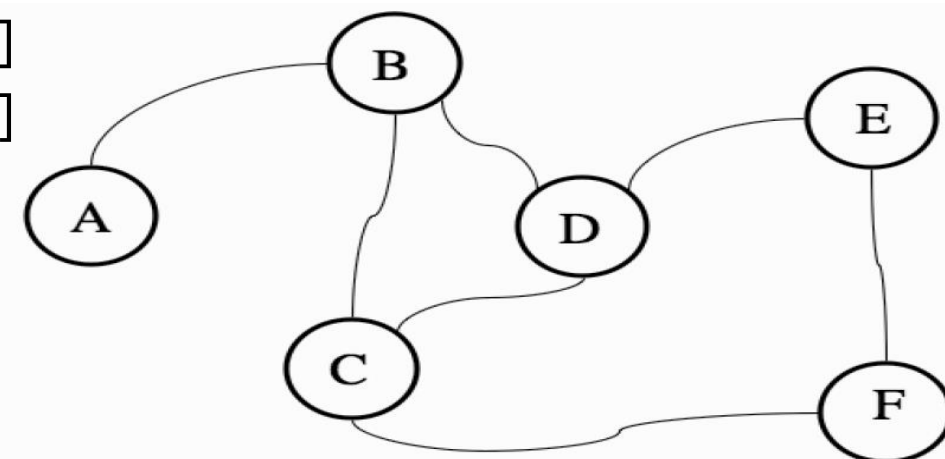
Q matrix after iteration # 500

```
[[ 1.4 16.0 -inf -0.0 -inf -inf]
[ 6.4 11.0 40.0 15.7 0.0 -inf]
[ -0.0 16.0 35.0 15.9 0.0 100.0]
[ -0.0 16.0 36.4 8.0 40.0 -inf]
[ -0.0 -0.0 -0.0 15.6 34.1 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q matrix after iteration # 1000

```
[[ 1.4 16.0 -inf -0.0 -inf -inf]
[ 6.4 11.0 40.0 16.0 0.0 -inf]
[ -0.0 16.0 35.0 16.0 0.0 100.0]
[ -0.0 16.0 39.7 10.9 40.0 -inf]
[ -0.0 -0.0 -0.0 16.0 35.0 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

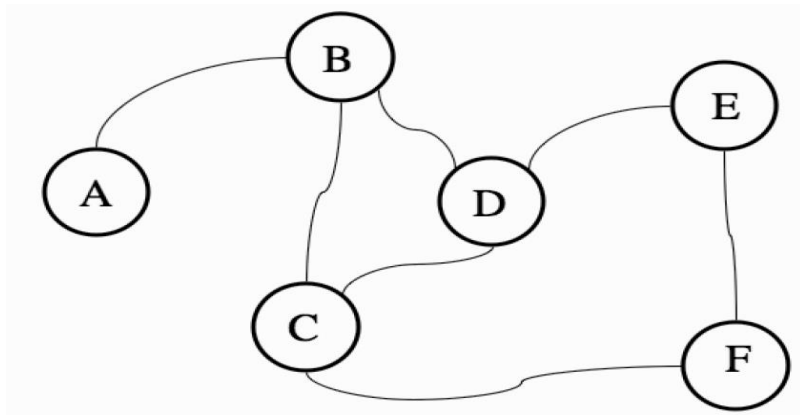
Sample run of 1000 iterations for Q-learning (TD-0) algorithm



Sarsa learning Example

- For the getting lost example, consider the following trail of steps taken by the agent: **D,B,B,B,B,C,C,C,F,C**

And the following **initial Q matrix**



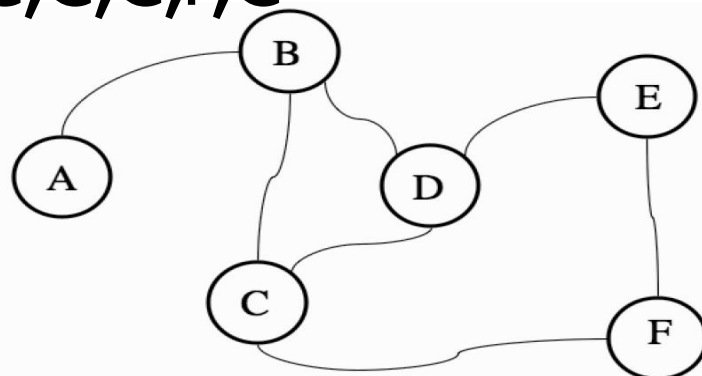
| States | Action or Next State | | | | | |
|--------|----------------------|--------|--------|--------|--------|--------|
| | A | B | C | D | E | F |
| A | -0.039 | -0.028 | -0.018 | 0.011 | -0.026 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | -0.018 | -0.026 |
| C | 0.011 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | -0.018 | 0.025 | -0.004 | 0.043 | -0.015 | -0.018 |
| E | 0.011 | -0.018 | 0.011 | -0.009 | -0.038 | -0.033 |
| F | -0.018 | -0.026 | 0.039 | -0.026 | 0.011 | -0.002 |

Compute matrix updates as during the iteration. Use μ (learning rate) = .6 and γ (discounting factor) = .3

Trail: D,B,B,B,B,C,C,C,F,C

$$\mu = .6$$

$$\gamma = .3$$



For $D \rightarrow B$: $s = D, a = B, s' = B, r = 0, a' = B$

$$Q(D, B) = Q(D, B)$$

$$\begin{aligned}
 &+.6(0 + .3 \times Q(B, B) - Q(D, B)) \\
 &= 0.025 + .6(.3 \times (-0.029) - 0.025) \\
 &= 0.005
 \end{aligned}$$

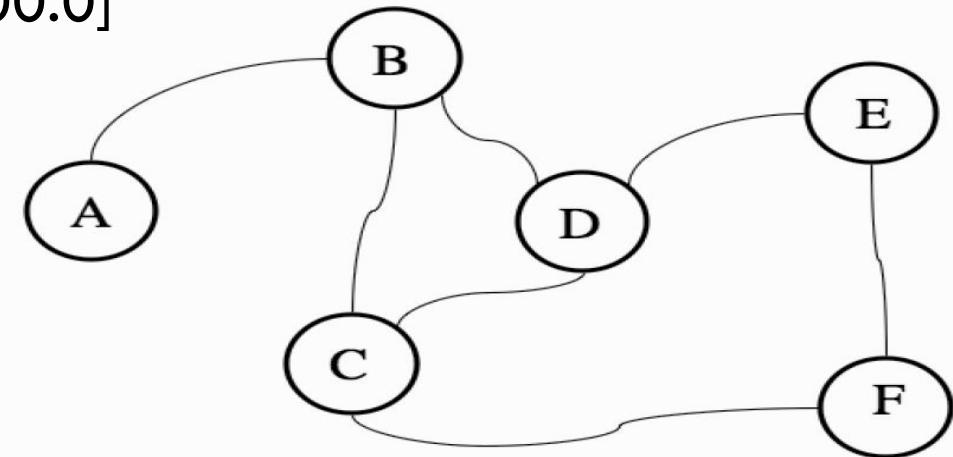
For $B \rightarrow B$: $s = B, a = B, s' = B, r = -5,$
 $a' = B$

| States | Action or Next State | | | | | |
|---------|----------------------|--------------|--------|--------|--------|--------|
| | A | B | C | D | E | F |
| A | -0.039 | -0.028 | -0.018 | 0.011 | -0.026 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | -0.018 | -0.026 |
| C | 0.011 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | -0.018 | 0.025 | -0.004 | 0.043 | -0.015 | -0.018 |
| E | 0.011 | -0.018 | 0.011 | -0.009 | -0.038 | -0.033 |
| F | -0.018 | -0.026 | 0.039 | -0.026 | 0.011 | -0.002 |
| Updated | A | B | C | D | E | F |
| A | -0.039 | -0.028 | -0.018 | 0.011 | -0.026 | 0.011 |
| B | 0.028 | -0.029 | 0.024 | 0.044 | -0.018 | -0.026 |
| C | 0.011 | -0.042 | -0.017 | -0.002 | -0.026 | 0.047 |
| D | -0.018 | 0.005 | -0.004 | 0.043 | -0.015 | -0.018 |
| E | 0.011 | -0.018 | 0.011 | -0.009 | -0.038 | -0.033 |
| F | -0.018 | -0.026 | 0.039 | -0.026 | 0.011 | -0.002 |

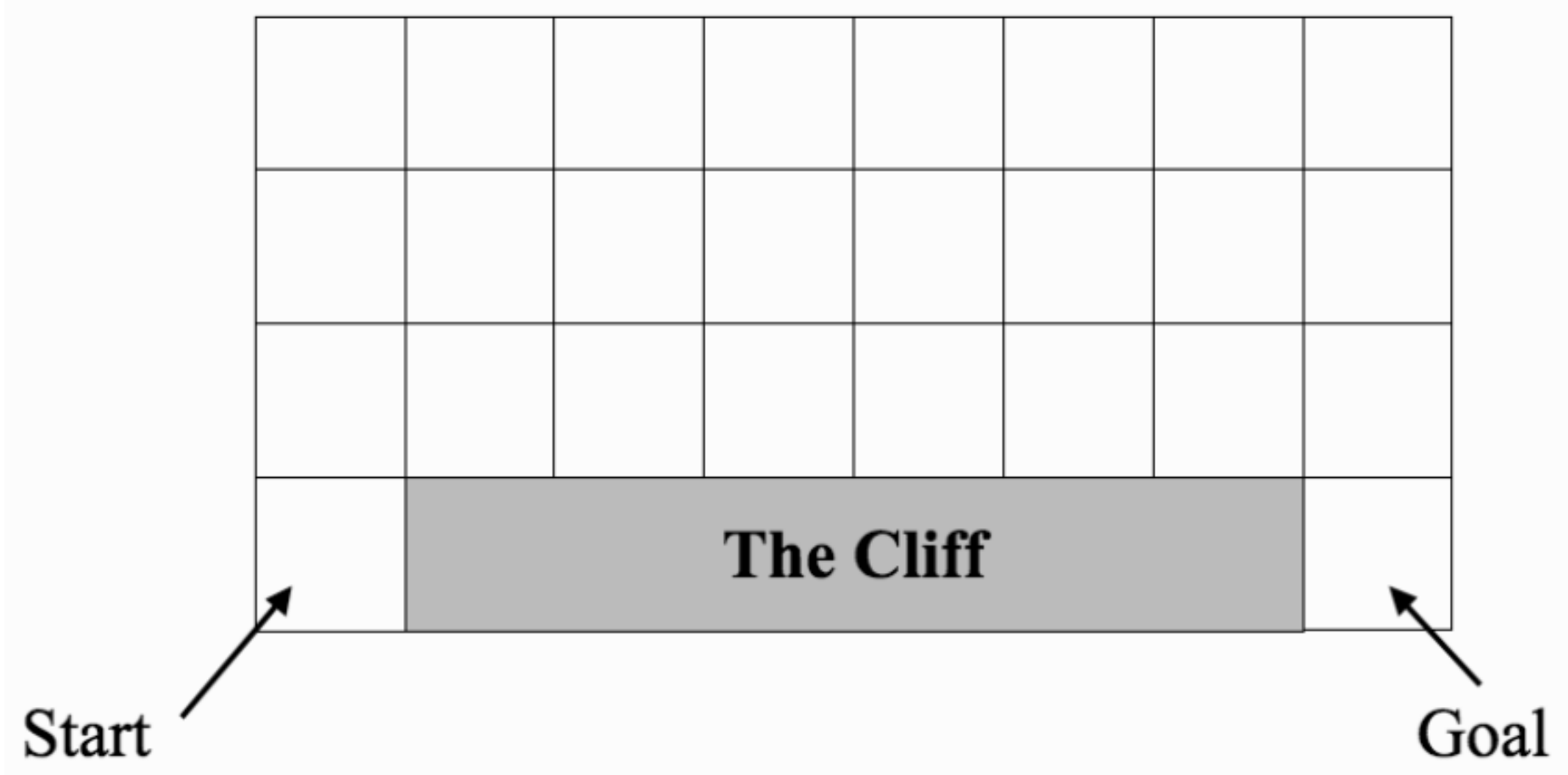
$$\text{update } Q(s, a) \leftarrow Q(s, a) + \mu(r + \gamma Q(s', a') - Q(s, a))$$

| Q matrix after iteration # 10 | Q matrix after iteration # 100 | Q matrix after iteration # 1000 |
|---|--|---|
| <code>[[-4.5 0.0 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[-4.5 16.0 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[1.4 16.0 -0.0 -0.0 -0.0 -0.0]</code> |
| <code>[-0.0 0.0 38.4 0.0 -0.0 -0.0]</code> | <code>[-0.0 7.7 40.0 0.0 -0.0 -0.0]</code> | <code>[5.4 4.4 40.0 7.4 -0.0 -0.0]</code> |
| <code>[-0.0 -0.0 -0.0 -0.0 0.0 99.9]</code> | <code>[-0.0 11.2 31.8 6.6 0.0 100.0]</code> | <code>[-0.0 9.1 35.0 11.9 0.0 100.0]</code> |
| <code>[-0.0 -0.0 -0.0 0.0 -0.0 0.0]</code> | <code>[-0.0 -0.0 40.0 -4.5 -0.0 -inf]</code> | <code>[-0.0 7.3 14.5 10.9 38.2 -inf]</code> |
| <code>[-0.0 -0.0 -0.0 -0.0 -0.0 97.3]</code> | <code>[-0.0 -0.0 -0.0 -0.0 -0.0 100.0]</code> | <code>[-0.0 -0.0 -0.0 14.0 32.8</code> |
| <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>100.0]</code> |
| Q matrix after iteration # 30 | Q matrix after iteration # 500 | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> |
| <code>[[-4.5 4.3 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[0.3 16.0 -0.0 -0.0 -0.0 -0.0]</code> | |
| <code>[-0.0 7.7 40.0 0.0 -0.0 -0.0]</code> | <code>[3.0 10.4 40.0 14.0 -0.0 -0.0]</code> | |
| <code>[-0.0 -0.0 -0.0 -0.0 0.0 100.0]</code> | <code>[-0.0 6.6 35.0 12.9 0.0 100.0]</code> | |
| <code>[-0.0 -0.0 36.4 -4.5 -0.0 -inf]</code> | <code>[-0.0 8.7 21.2 -4.5 40.0 -inf]</code> | |
| <code>[-0.0 -0.0 -0.0 -0.0 -0.0 99.9]</code> | <code>[-0.0 -0.0 -0.0 15.8 -0.0 100.0]</code> | |
| <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | |

Sample run of 1000 iterations for SARSA algorithm

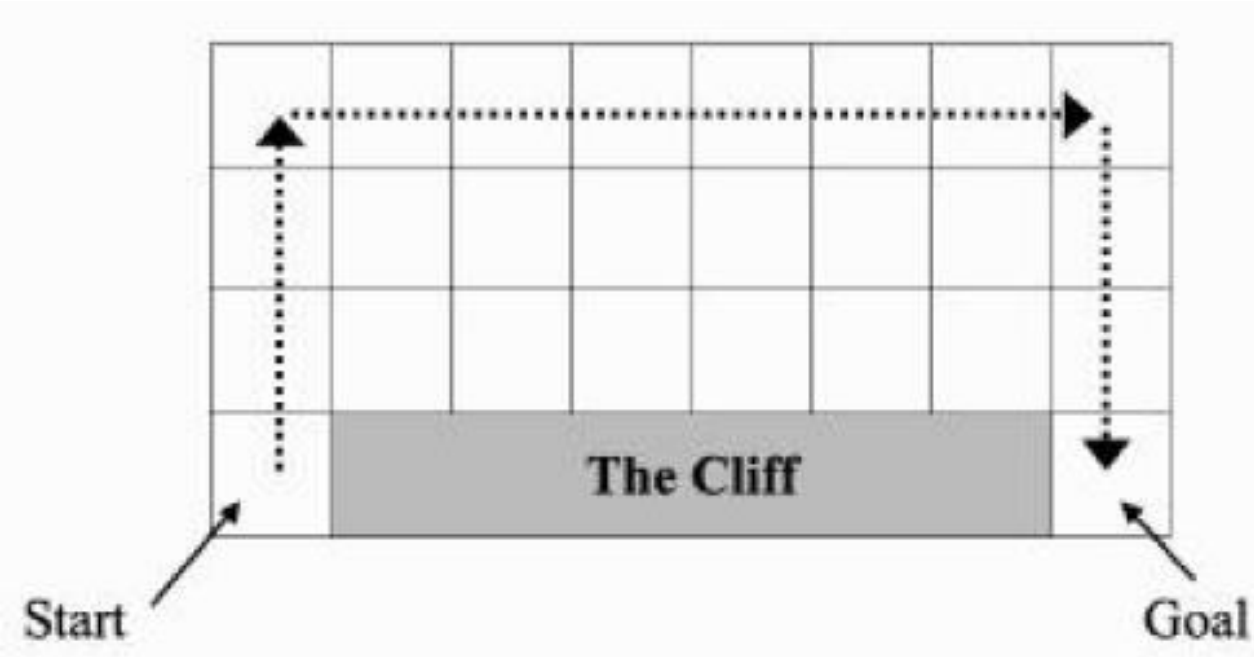


SARSA vs. Q

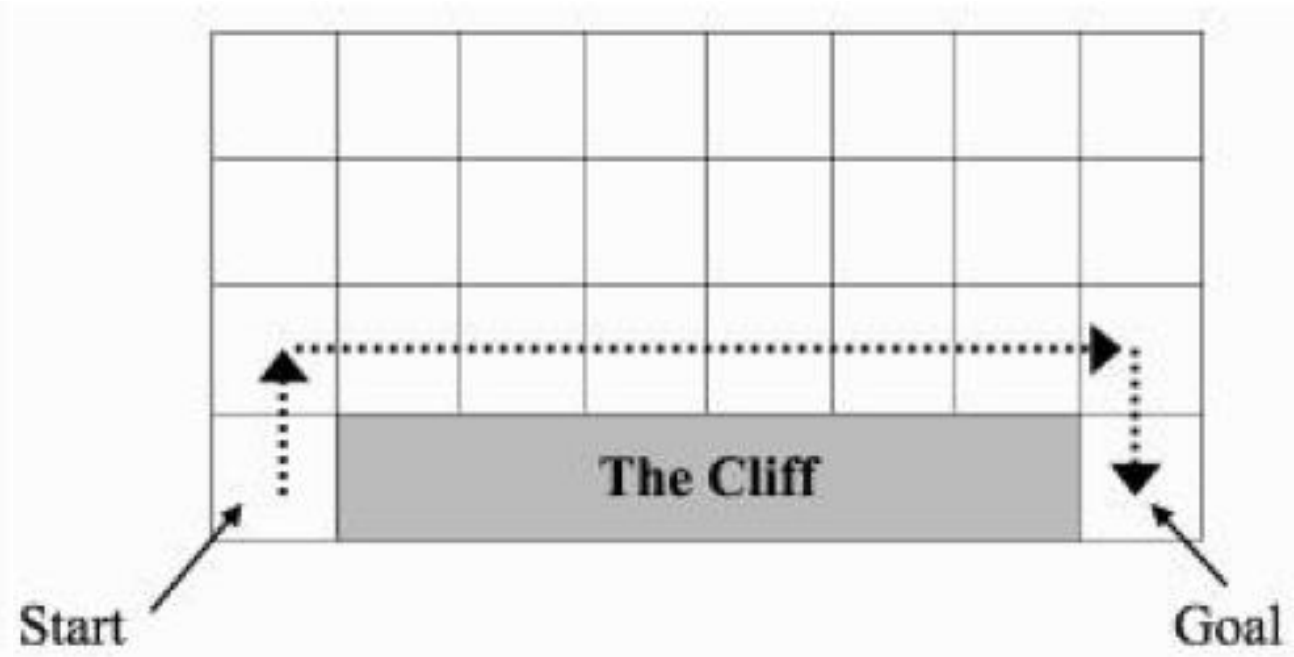


Every move gets a reward of -1

Moves that end up on cliff get a reward of -100



The SARSA solution is far from optimal, but it is safe.



The Q-learning solution is optimal, but occasionally the random search will tip it over the cliff.

□ SARSA vs. Q

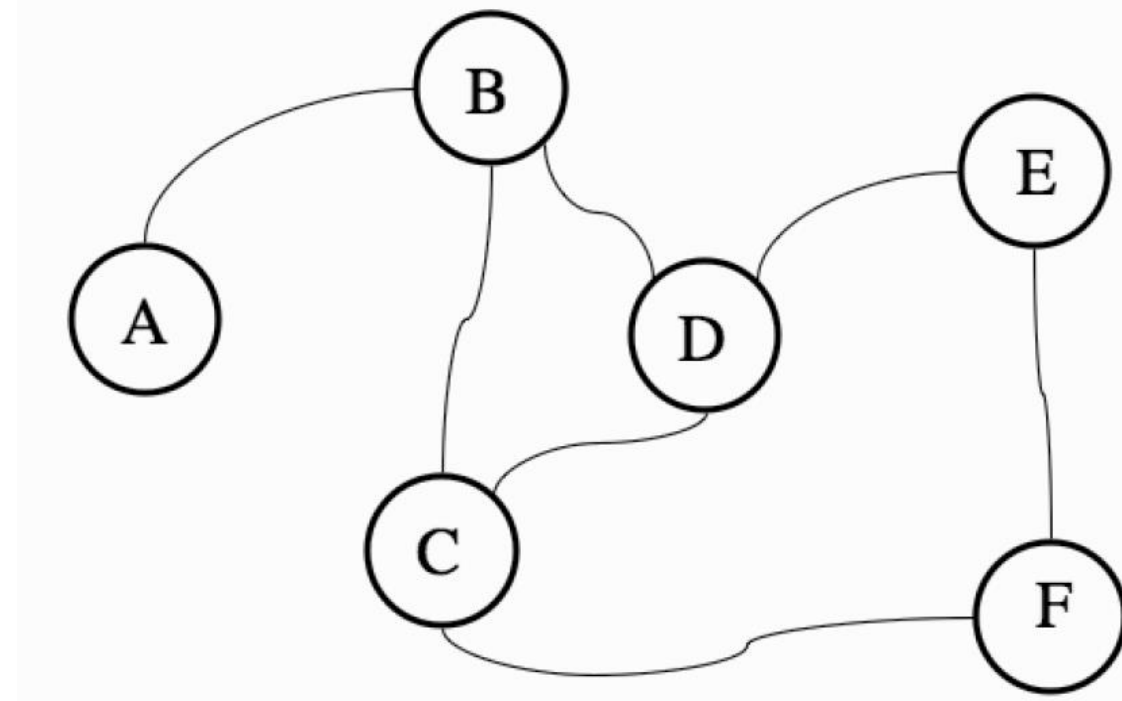
- ▣ It is our choice depending upon what we want
- ▣ Depends upon how serious a problem is falling from the cliff.

The On-Policy uses the same policy to evaluate and improve; however, the off-Policy uses behavioral policy to explore and learn and the target policy to improve.

□ Getting lost example... again!

Reward matrix used:

```
[[ -5.0  0.0 -inf -inf -inf -inf]
 [  0.0 -5.0  0.0  0.0 -inf -inf]
 [ -inf  0.0 -5.0  0.0 -inf 100.0]
 [ -inf  0.0  0.0 -5.0  0.0 -inf]
 [ -inf -inf -inf  0.0 -5.0 100.0]
 [ -inf -inf  0.0 -inf -inf  0.0]]
```



Q matrix after iteration # 10

```
[[ -0.0 0.0 -inf -0.0 0.0 0.0]
[ -0.0 0.0 38.2 -0.0 0.0 -inf]
[ -0.0 -0.0 -0.0 -0.0 0.0 99.8]
[ -0.0 0.0 -0.0 -0.0 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 91.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q matrix after iteration # 30

```
[[ -0.0 16.0 -inf -0.0 -inf -inf]
[ 4.3 0.0 40.0 -0.0 0.0 -inf]
[ -0.0 -0.0 24.5 4.4 0.0 100.0]
[ -0.0 15.9 -0.0 0.8 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 99.8]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q-learning (TD-0) algorithm – note the number of -inf

Q matrix after iteration # 100

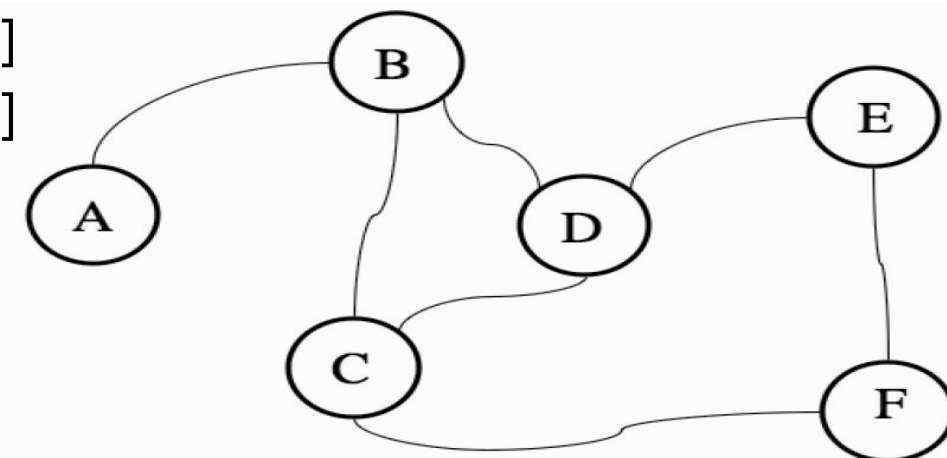
```
[[ -0.0 16.0 -inf -0.0 -inf -inf]
[ 5.8 7.7 40.0 -0.0 0.0 -inf]
[ -0.0 -0.0 34.1 6.2 0.0 100.0]
[ -0.0 16.0 -0.0 0.8 -0.0 -inf]
[ -0.0 -0.0 -0.0 0.0 -0.0 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

Q matrix after iteration # 500

```
[[ 1.4 16.0 -inf -0.0 -inf -inf]
[ 6.4 11.0 40.0 15.7 0.0 -inf]
[ -0.0 16.0 35.0 15.9 0.0 100.0]
[ -0.0 16.0 36.4 8.0 40.0 -inf]
[ -0.0 -0.0 -0.0 15.6 34.1 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```

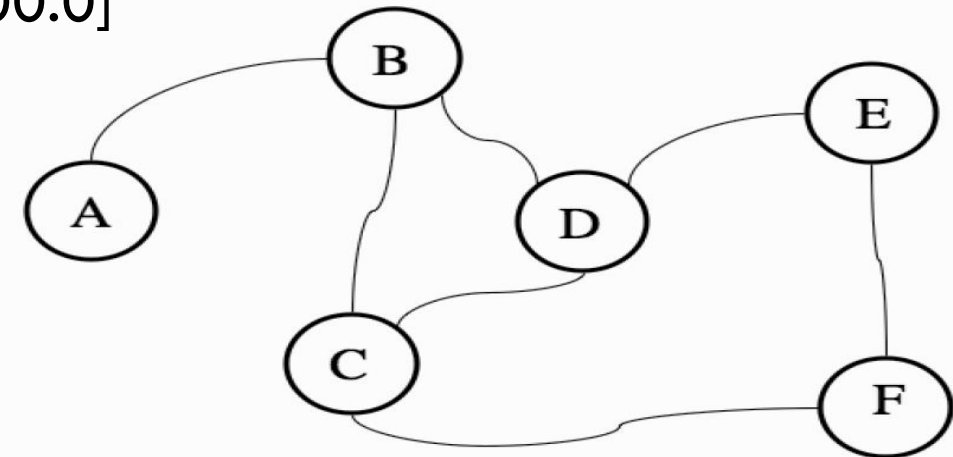
Q matrix after iteration # 1000

```
[[ 1.4 16.0 -inf -0.0 -inf -inf]
[ 6.4 11.0 40.0 16.0 0.0 -inf]
[ -0.0 16.0 35.0 16.0 0.0 100.0]
[ -0.0 16.0 39.7 10.9 40.0 -inf]
[ -0.0 -0.0 -0.0 16.0 35.0 100.0]
[ -0.0 -0.0 -0.0 -0.0 -0.0 0.0]]
```



| Q matrix after iteration # 10 | Q matrix after iteration # 100 | Q matrix after iteration # 1000 |
|---|--|---|
| <code>[[-4.5 0.0 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[-4.5 16.0 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[1.4 16.0 -0.0 -0.0 -0.0 -0.0]</code> |
| <code>[-0.0 0.0 38.4 0.0 -0.0 -0.0]</code> | <code>[-0.0 7.7 40.0 0.0 -0.0 -0.0]</code> | <code>[5.4 4.4 40.0 7.4 -0.0 -0.0]</code> |
| <code>[-0.0 -0.0 -0.0 -0.0 0.0 99.9]</code> | <code>[-0.0 11.2 31.8 6.6 0.0 100.0]</code> | <code>[-0.0 9.1 35.0 11.9 0.0 100.0]</code> |
| <code>[-0.0 -0.0 -0.0 0.0 -0.0 0.0]</code> | <code>[-0.0 -0.0 40.0 -4.5 -0.0 -inf]</code> | <code>[-0.0 7.3 14.5 10.9 38.2 -inf]</code> |
| <code>[-0.0 -0.0 -0.0 -0.0 -0.0 97.3]</code> | <code>[-0.0 -0.0 -0.0 -0.0 -0.0 100.0]</code> | <code>[-0.0 -0.0 -0.0 14.0 32.8</code> |
| <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>100.0]</code> |
| Q matrix after iteration # 30 | Q matrix after iteration # 500 | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> |
| <code>[[-4.5 4.3 -0.0 -0.0 -0.0 -0.0]</code> | <code>[[0.3 16.0 -0.0 -0.0 -0.0 -0.0]</code> | |
| <code>[-0.0 7.7 40.0 0.0 -0.0 -0.0]</code> | <code>[3.0 10.4 40.0 14.0 -0.0 -0.0]</code> | |
| <code>[-0.0 -0.0 -0.0 -0.0 0.0 100.0]</code> | <code>[-0.0 6.6 35.0 12.9 0.0 100.0]</code> | |
| <code>[-0.0 -0.0 36.4 -4.5 -0.0 -inf]</code> | <code>[-0.0 8.7 21.2 -4.5 40.0 -inf]</code> | |
| <code>[-0.0 -0.0 -0.0 -0.0 -0.0 99.9]</code> | <code>[-0.0 -0.0 -0.0 15.8 -0.0 100.0]</code> | |
| <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | <code>[-0.0 -0.0 0.0 -0.0 0.0 0.0]]</code> | |

SARSA algorithm – note the number of -inf



USES OF REINFORCEMENT LEARNING

- Most popular is in intelligent robotics
 - ▣ The robot can be left to attempt to solve the task without human intervention.
- Also popular at learning how to play games



Modelling Sample Problems

- To model sample problems, we need to model
 - ▣ Q matrix (state-action matrix)
 - ▣ Reward
- Remaining things like choice of policy, choice of algorithm, etc., are simply choices, whose options are well established.
 - ▣ They don't need to be modelled.

Wading Through Maze

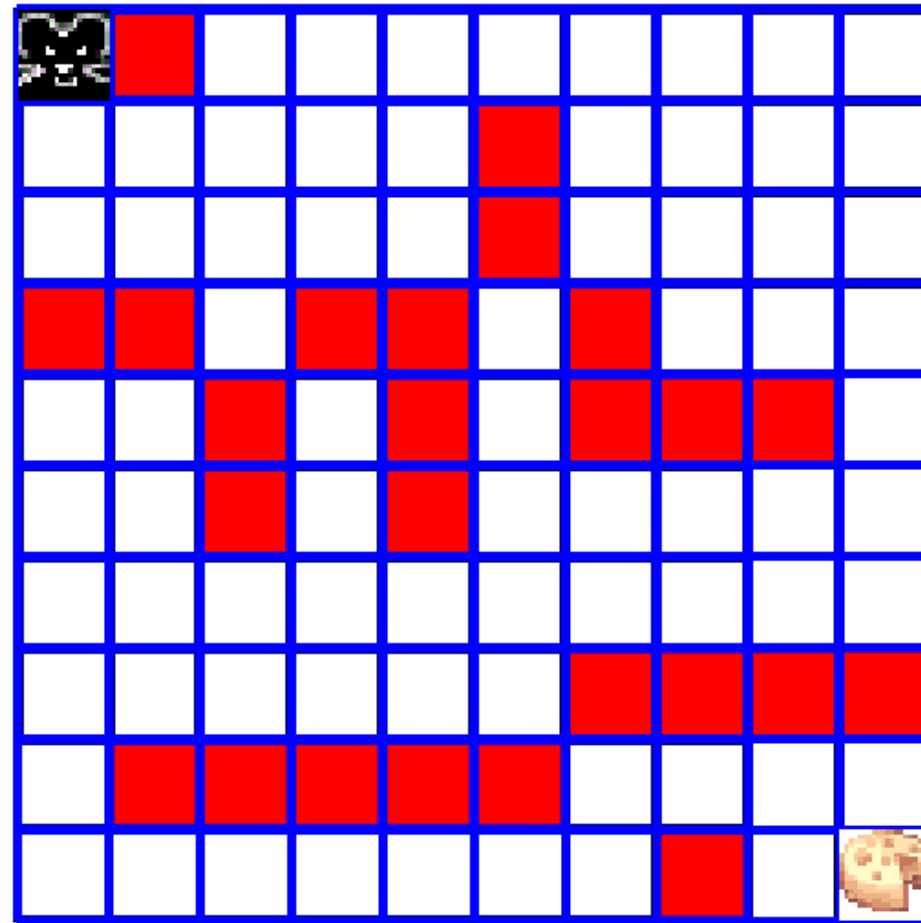
□ Taken from

<https://towardsdatascience.com/the-other-type-of-machine-learning-97ab81306ce9>

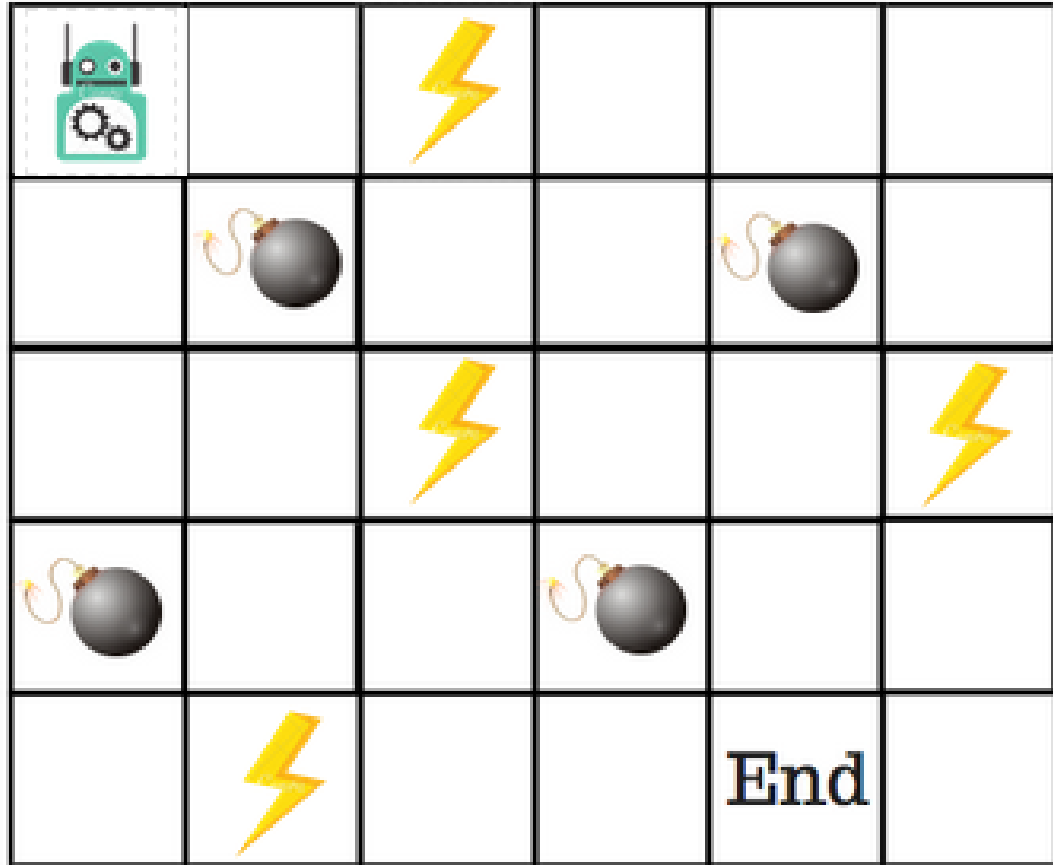
Also see:

<https://samyzaf.com/ML/rl/qmaze.html>

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | S | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | G |



Wading Through Maze - II



Actions :    

| | | | | |
|-----------------|--|--|--|--|
| Start | | | | |
| Nothing / Blank | | | | |
| Power | | | | |
| Mines | | | | |
| END | | | | |

Q-Table

Tic-Tac-Toe

