3. Matrix Chain Multiplication
1.1, 1.2, 1.3, 2, 4, 5

4. OBST
4.2.1, 4.3.1, 4.3.2

5. Telescopic Scheduling
1, 2, 5, 7, 8

6. Optimal Rod Cutting
1, 2.

8. Knapsack problem
1, 2.

9. Longest Common Sequence
1,

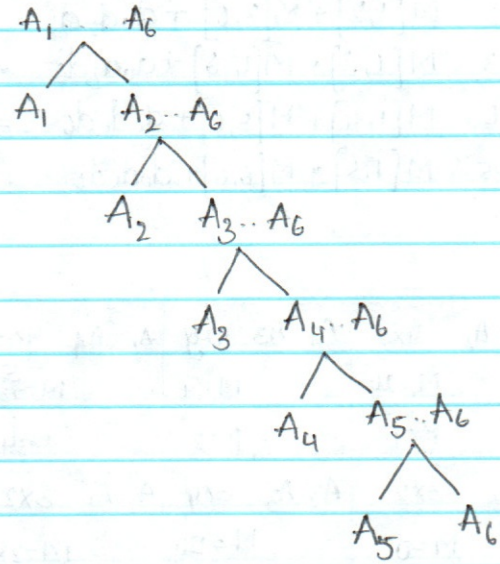
12. Maximum Independent Set of a Tree
1, 2

13. Tracking turns with a coin
Question 1

Depth First Search
2.1, 2, 5, 6.

Matrix Chain Multiplication

1\. 1. Draw execution tree for $A_1 \ldots A_6$



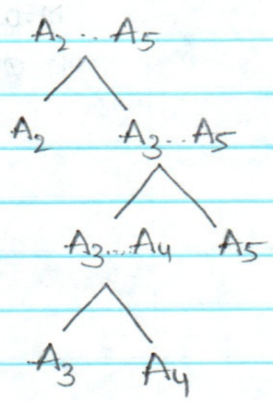
2\. $A_2 \ldots A_5$

3X3

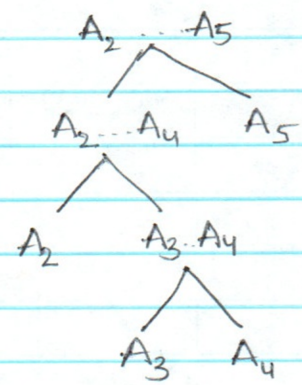
M = 46

P = 2, 4

P = 2



P = 4



3. $M[1,6] \overset{k=1}{=} M[1,1]+M[2,6]+d_0d_1d_6 = 0+64+4\times3\times3 = 100$

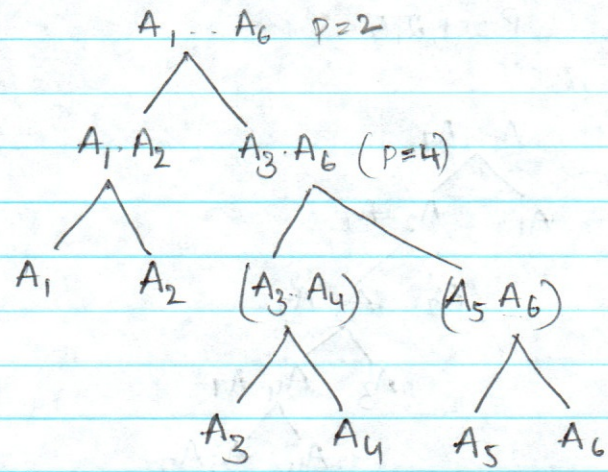
✓ $k=2 \quad M[1,2]+M[3,6]+d_0d_2d_6 = 24+46+4\times2\times3 = 94$

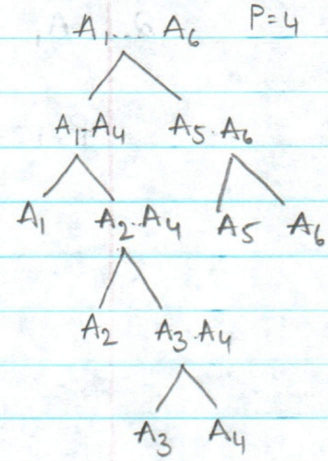
$k=3 \quad M[1,3]+M[4,6]+d_0d_3d_6 = 56+42+4\times4\times3 = 146$

✓ $k=4 \quad M[1,4]+M[5,6]+d_0d_4d_6 = 52+18+4\times\cancel{9}\,2\times3 = \cancel{108}\ 94$

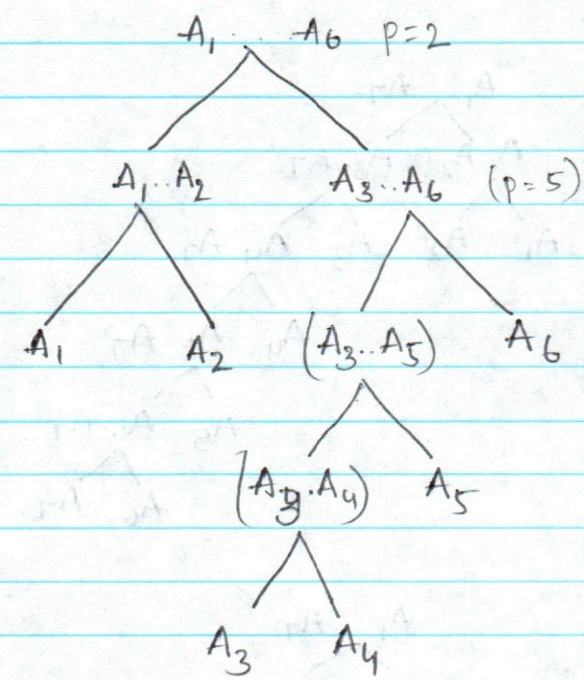
$k=5 \quad M[1,5]+M[6,6]+d_0d_5d_6 = 76+0+4\times2\times3 = 100$

A_1 4x3 M=0 P=∅	$A_1 \cdot A_2$ 4x2 M=24 P=1	$A_1 \cdots A_3$ 4x~~9~~2 M=56 P=2	$A_1 \cdots A_4$ 4x3 M=~~76~~ 52 P=~~2,4~~ 1	$A_1 \cdots A_5$ 4x3 M=76 P=2,4	$A_1 \cdots A_6$ 4x3 M=94 P=2
	A_2 3x2 M=0 P=∅	$A_2 \cdot A_3$ 3x4 M=24 P=2	$A_2 \cdots A_4$ 3x2 M=28 P=2	$A_2 \cdots A_5$ 3x3 M=46 P=2,4	$A_2 \cdots A_6$ 3x3 M=64 P=2,4
		A_3 2x4 M=0 P=∅	$A_3 A_4$ 2x2 M=16 P=3	$A_3 \cdots A_5$ 2x3 M=28 P=4	$A_3 \cdots A_6$ 2x3 M=46 P=4,5
			A_4 4x2 M=0 P=∅	$A_4 A_5$ 4x3 M=24 P=4	$A_4 \cdots A_6$ 4x3 M=42 P=4
				A_5 2x3 M=0 P=∅	$A_5 A_6$ 2x3 M=18 P=5
					A_6 3x3 M=0 P=∅


A1 .. A6 P=2
A1 A2
A3 A6 (P=4)
A1
A2
(A3 A4)
(A5 A6)
A3
A4
A5
A6

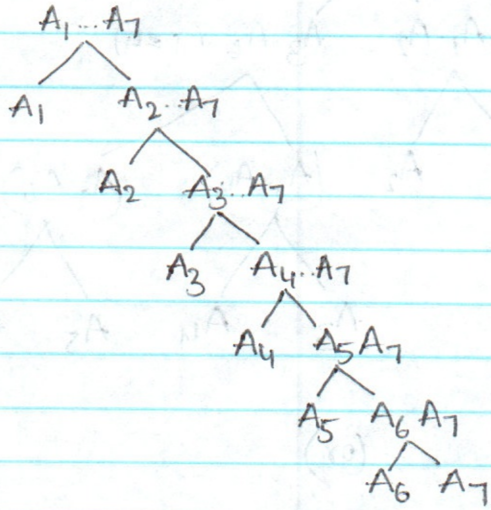

A1 ... A6 P=4
A1 A4
A5 A6
A1
A2 A4
A5
A6
A2
A3 A4
A3
A4

(or)

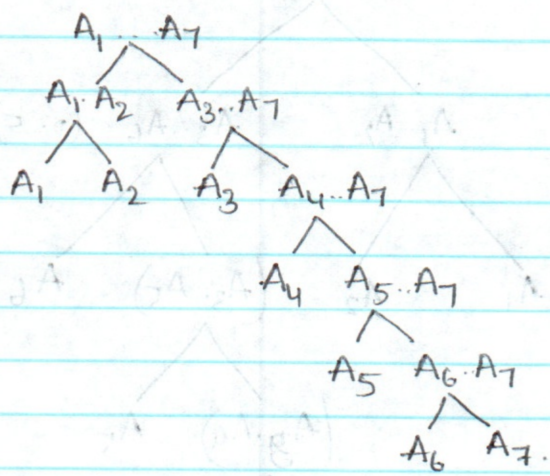

A1 .. A6 P=2
A1 .. A2
A3 .. A6 (P=5)
A1
A2
(A3 .. A5)
A6
(A3 A4)
A5
A3
A4

2. $A_1 \ldots A_7$ $P = 1, 2, 5$

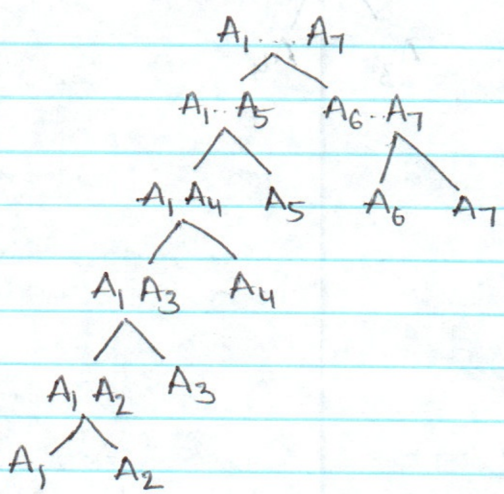
$P = 1$



$P = 2$



$P = 5$



4. Remove Largest

A_1 A_2 A_3

2x2 2x4 4x2

Cost $= 2\times4\times2 = 16$

A_1 $A_2 \cdot A_3$

2x2 2x2

Cost $= 2\times2\times2 = 8$

total Cost $= 16+8 = 24$

Remove Smallest

A_1 A_2 A_3

2x2 2x4 4x2

Cost $= 2\times2\times4 = 16$

$A_1 \cdot A_2$ A_3

2x4 4x2

Cost $= 2\times4\times2 = 16$

total cost $16+16 = 32$

Remove Largest

A_1 A_2 A_3

6x7 7x5 5x2

Cost $= 6\times7\times5$
$= 210$

$A_1 \cdot A_2$ A_3

6x5 5x2

Cost $= 6\times5\times2$
$= 60$

total cost $= 210+60 = 270$

Remove Smallest

A_1 A_2 A_3

6x7 7x5 5x2

Cost $= 7\times5\times2$
$= 70$

A_1 $A_2 \cdot A_3$

6x7 7x2

Cost $= 6\times7\times2$
$= 84$

total cost $= 84+70 = 154$

Hence removing small, large dimensions lead to best case.

5. $M[i,j] = \min_{k, \text{ with } i \le k < j} \{ M[i,k] + M[k+1,j] + d_{i-1} d_k d_j \}$ for $j = i+2 \ldots n$

if we change min to max in the above equation the resulting number $M[1,n]$ represent the most costly way to multiply a matrix chain.

Example

A_1 4×3 $M=0$ $P=0$	$A_1 \cdot A_2$ 4×2 $M=24$ $P=1$	$A_1 \cdots A_3$ 4×4 $M=72$ $P=1$
	A_2 3×2 $M=0$ $P=\phi$	$A_2 \cdot A_3$ 3×4 $M=24$ $P=2$
		A_3 2×4 $M=0$ $P=\phi$

$$M[1,3] = \overset{k=1}{M[1,1]} + M[2,3] + d_0 d_1 d_3 = 0 + 24 + 48 = 72 \checkmark$$
$$k=2 \quad M[1,2] + M[3,3] + d_0 d_2 d_3 = 24 + 0 + 32 = 54$$

OBST

4.2.1 W = 32

T = 75

R = 4,5

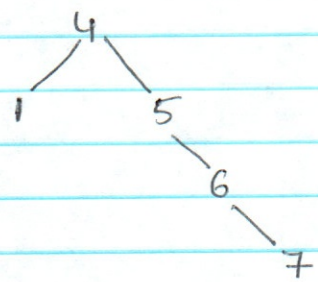
4.3.1 W = 68

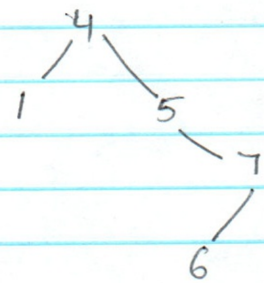
T = 185

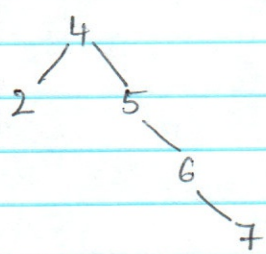
R = 4.

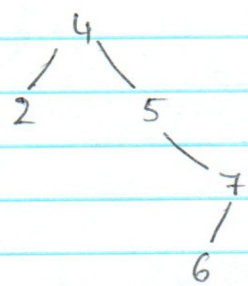
4.3.2

1. R = 4 doesn't always have nodes 1,2,3 present.

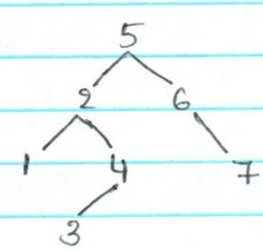


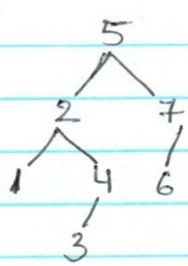






R = 5





Weighted Interval Scheduling - Telescope

5.2

Q.1

i	s_i	f_i	b_i	Pred(i)	$b_i + B_{pred(i)}$		$0 + B_{i-1}$	B_i	Track
0									
1	2	3	3	0	3+0	>	0	3	Y
2	4	5	3	1	3+3=6	>	3	6	Y
3	2	7	5	0	5+0=5	<	6	6	N
4	6	9	4	2	4+6=10	>	6	10	Y
5	10	11	3	4	3+10=13	>	10	13	Y
6	10	13	3	4	3+10=13	=	13	13	T
7	8	15	5	3	5+6=11	<	13	13	N
8	14	19	4	6	4+13=17	>	13	17	Y
9	16	23	1	7	1+13=14	<	17	17	N
10	22	25	1	8	1+17=18	>	17	18	Y
11	14	29	2	6	2+13=15	<	18	18	N

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8	i=9	i=10	i=11	Check
Include	Y	Y	N	Y	Y	T/N	N	Y	N	Y	N	
	3	3		4	3			4		1		$B_i=18$

2.

i	s_i	f_i	b_i	pred(i)	$b_i + B_{pred(i)}$	$0 + B_{i-1}$	
12	1	29	46	0	46+0	45	Yes

So, Yes it is possible $B_{12} = 46 > B_{11} = 45$

5.

i	s_i	f_i	b_i	pred(i)	$b_i + B_{pred(i)}$		$0 + B_{i-1}$	B_i	Track
0									
1	2	3	3	0	3+0=3	>	0	3	Y
2	4	5	3	1	3+3=6	>	3	6	Y
3	2	7	15	0	15+0=15	>	6	15	Y
4	6	9	9	2	9+6=15	=	15	15	T
5	10	11	3	4	3+15=18	>	15	18	Y

$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	Check if
Y	Y	Y	T	Y	
3	3	15	9	3	$B_5 = 18$

Schedule 1

$i=1$ $i=2$ $i=4$ $i=5$

$3 + 3 + 9 + 3 = 18$

(or) Schedule 2

$i=3$ $i=5$

$15 + 3 = 18$.

7

i	s_i	f_i	b_i	Pred(i)	$b_i + B_{pred(i)}$	$0 + B_{i-1}$	B_i	Track
9	16	23	15	7	15+25	35	40	Yes

Yes it is possible.

8. As $B_{10} = B_{11}$ and $i = 11$ is not included.
Optimal Schedule remains the same.

Optimal Rod Cutting.

1. $n = 13$

$R[n] = 38$

$l = 3, 10, 13$

$n = 14$

$R[14] = 40$

$l = 2, 10, 14$

$n = 15$

$R[15] = 43$

$l = 2, 3, 10, 13$

$n = 16$

$R[16] = 47$

$l = 6, 10.$

2. $n=3$ maximum revenue $R[3] = 8$ is at $l = 3$

Sell it entirely with out making cuts to make max. profit

$n=4$ maximum revenue $R[4] = 10$ is at $l = 2$

Cut the rod to make length $= 2$ (2 pieces of length 2)

$n=10$ maximum revenue $R[10] = 30$ at $l = 10$

Sell it entirely with out making cuts

$n = 11$ maximum revenue $R[11] = 31$ at $l = 10, 1$

Cut the rod to make length $= 10$ and length $= 1$ rods.

Knapsack Problems

1. $W = 17$

	V-table	MyTrack
$i=1$	18	Y
$i=2$	40	Y
$i=3$	40	N
$i=4$	50	Y
$i=5$	53	Y

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	Check
Included?	N	Y	N	Y	N	
v_i		22		28	3	$V[n,W] = 53$
w_i		6		8	3	$6+8+3 \leq W$? Yes.

2.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$v_6=20$	0	0	0	~~0~~ 3	20	20	22	23	28	28	42	42	48	48	~~48~~ 50	51	51
$w_6=4$	N	N	N	N	N	N	N	Y	N	N	Y	Y	Y	Y	N	Y	Y

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	Check
Included?	N	N	N	Y	Y	Y	$V[n,W] = 51$
v_i	18	22	5	28	3	20	
w_i	8	6	7	8	3	4	$8+3+4 \leq W=16$ Yes

Longest Common Subsequence

1.

	j=0 $y_j=0$	j=1 y_j A	j=2 y_j=A	j=3 y_j=A	j=4 y_j=B	j=5 y_j=B	j=6 y_j=B	j=7 y_j=B	j=8 y_j=A
i=0	nil	nil	nil	nil	nil	nil	nil	nil	nil
$x_0=0$	0	0	0	0	0	0	0	0	0
i=1	nil	↖	↖	↖					↖
x_i=A	0	1	1	1	← 1	← 1	← 1	← 1	1
i=2	nil	↑	↑	↑	↖	↖	↖	↖	
x_i=B	0	1	1	1	2	2	2	2	← 2
i=3	nil	↑	↑	↑	↖	↖	↖	↖	
x_i=B	0	1	← 1	← 1	2	3	3	3	← 3
i=4	nil	↖	↖	↖	↑	↑	↑	↑	↖
x_i=A	0	1	2	2	← 2	3	← 3	← 3	4
i=5	nil	↖	↖	↖		↑	↑	↑	↖
x_i=A	0	1	2	3	← 3	← 3	← 3	← 3	4
i=6	nil	↑	↑	↑	↖	↖	↖	↖	↑
x_i=B	0	1	2	3	4	4	4	4	← 4

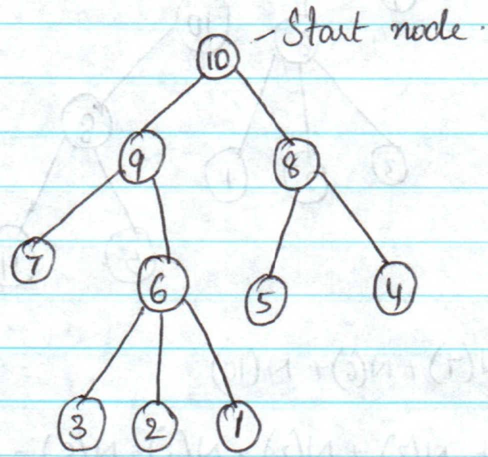
Length of LCS is 4.

LCS − ABBB
ABBA
AAAB

Not Unique.

Maximum Independent Set of a Tree.

1. 10 nodes



$$N(10) = \max \begin{cases} N(9) + N(8) & = 4+2=6 \\ 1 + N(7) + N(6) + N(5) + N(4) & = 1+1+3+1+1 = 7 \checkmark \end{cases}$$

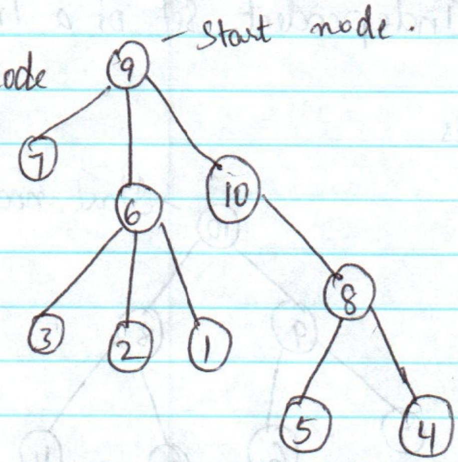
$$N(9) = \max \begin{cases} N(7) + N(6) \\ 1 + N(3) + N(2) + N(1) \end{cases} = 4$$

$$N(6) = \max \begin{cases} N(3) + N(2) + N(1) \\ 1+0 \end{cases} = 3$$

$$N(8) = \max \begin{cases} N(5) + N(4) \\ 1+0 \end{cases} = 2.$$

MIST $\{10, 7, 5, 4, 3, 2, 1\}$

2. Changing the start node



$$N(9) = \max \begin{cases} N(7) + N(6) + N(10) & = 1 + 3 + 3 = 7 \checkmark \\ 1 + N(3) + N(2) + N(1) + N(8) & = 1 + 1 + 1 + 1 + 2 = 6 \end{cases}$$

$$N(6) = \max \begin{cases} N(3) + N(2) + N(1) & = 3 \\ 1 + 0 \end{cases}$$

$$N(10) = \max \begin{cases} 1 & = 3 \\ 1 + N(5) + N(4) \end{cases}$$

$$N(8) = \max \begin{cases} N(5) + N(4) & = 2 \\ 1 + 0 \end{cases}$$

MIST = $\{7, 3, 2, 1, 10, 5, 4\}$

This is same as before.

Taking turns with a coin

	1	2	3	4	5	6
1	20	25	55	55	86	88
2		25	35	53	68	76
3			35	35	63	68
4				28	33	51
5					33	33
6						23