A REFERRAL-REWARD EMBEDDED, BI-PHASE INFORMATION DIFFUSION TECHNIQUE

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M.Sc. (Engg.) Thesis Defense
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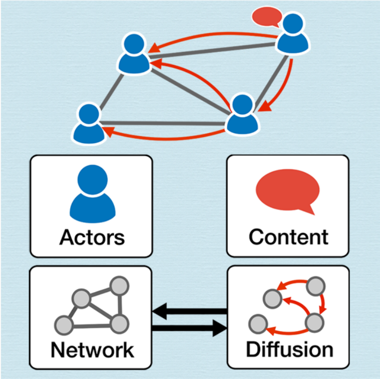
Indian Institute of Science, Bangalore

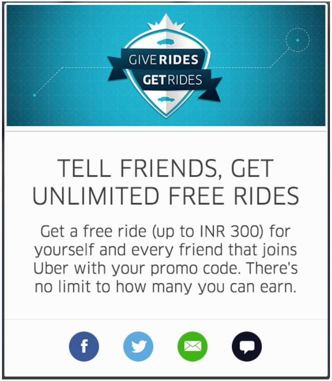
OUTLINE

INTRODUCTION

- **Diffusion via word-of-mouth**
  Identify initial adopters.
  Word-of-mouth influence propagation



- Diffusion via word-of-mouth
- Referral Rewards
  Refer product to friends and acquaintances. Get incentives for successful referral



PROBLEM SETTING

- Given
  - Target consumer base
  - Estimates for “influence” between individuals
  - Budget K as initial endowment

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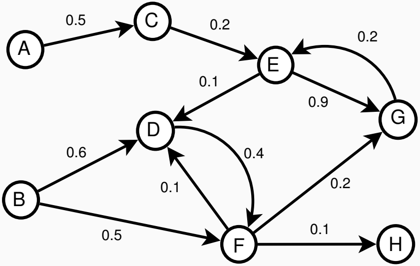
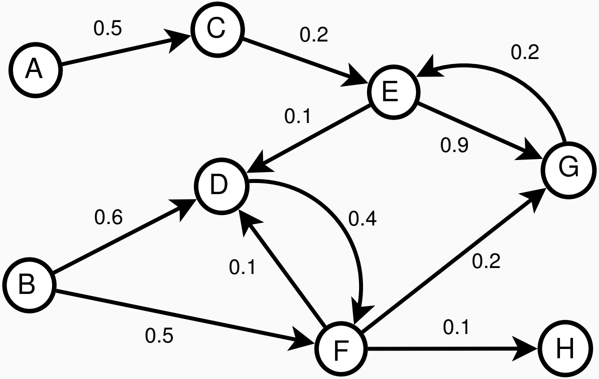
- Design Problems
  - Which advertising channels are most effective?
  - How to spread initial budget across advertising channels?

DIFFUSION MODEL - INDEPENDENT CASCADE (IC)



- Social network graph G
- When node u becomes active, it has a single chance of activating each currently inactive neighbour v
- Activation attempt succeeds with probability p_{uv}
- Process terminates when no further nodes can be activated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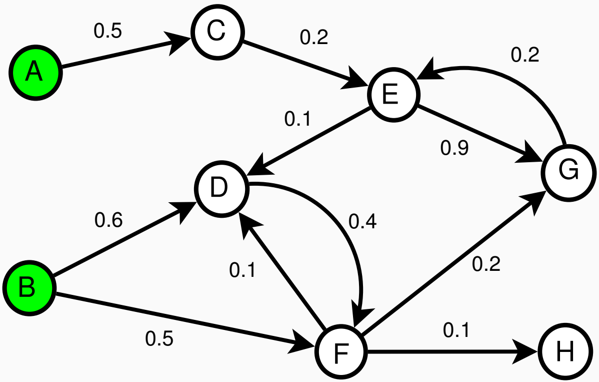

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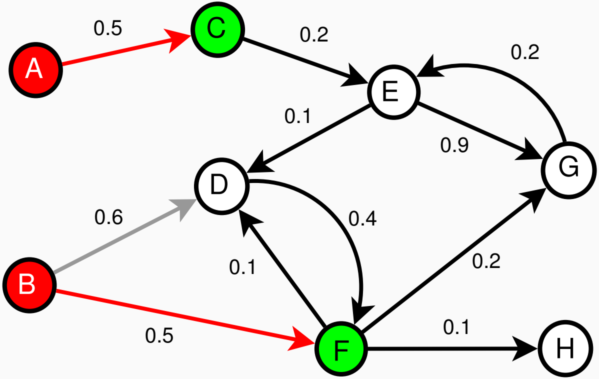
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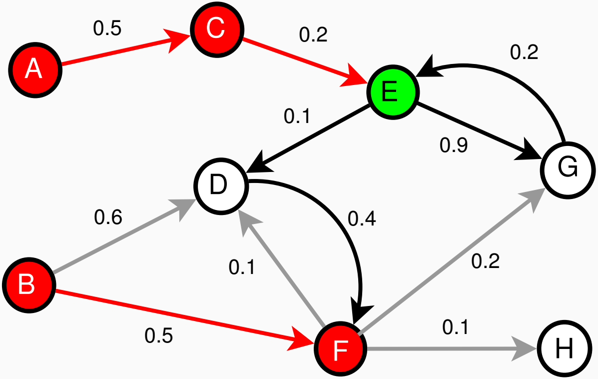
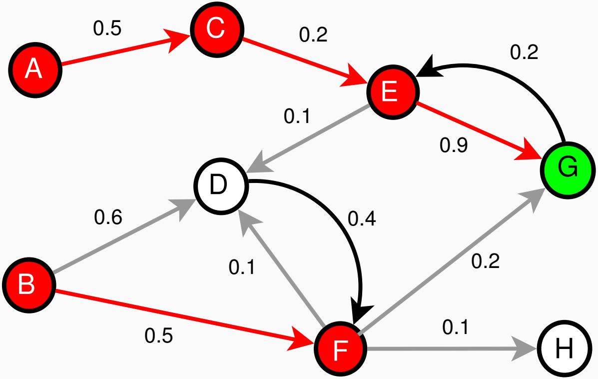
INDEPENDENT CASCADE - EXAMPLE

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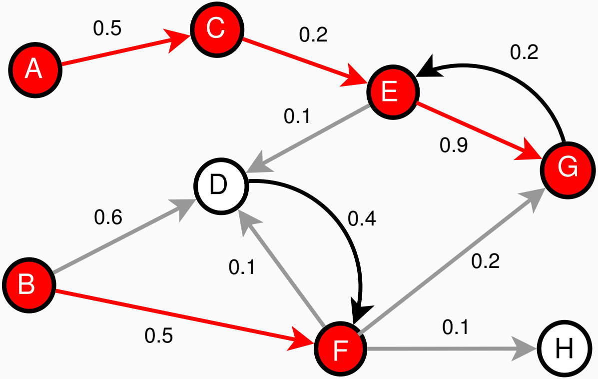

COMPUTING INFLUENCE – LIVE GRAPH

$$A \xrightarrow{0.5} B \xrightarrow{0.5} C$$

Live Graph ($\mathcal{X}$)	$f_{\mathcal{X}}(\{A\})$	$P(\mathcal{X})$
	1	0.25
	2	0.25
	1	0.25
	3	0.25

$$
\begin{aligned}
f(\{A\}) &= \sum_{\mathcal{X}} \mathrm{P}(\mathcal{X}) f_{\mathcal{X}}(\{A\}) \\
&= 0.25 * (1 + 2 + 1 + 3) \\
&= 1.75
\end{aligned}
$$

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$$\mathrm{P}(\mathcal{X}) = \prod_{e \in \mathcal{X}} p_e \prod_{e \notin \mathcal{X}} (1 - p_e)$$

$$f(S) = \sum_{\mathcal{X}} \mathrm{P}(\mathcal{X}) f_{\mathcal{X}}(S)$$

RELEVANT LITERATURE AND RESEARCH GAP

- Influence maximization in a network in a single phase using seed nodes [1]
  $\max_{|S|\leq K} f(S)$
  sub-modular optimization, greedy algorithm

[1] D. Kempe, J. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. In ACM SIGKDD, pages 137–146, 2003.

[2] P. Dayama, A. Karnik, and Y. Narahari. Optimal incentive timing strategies for product marketing on social networks. In AAMAS, pages 703–710, 2012.

[3] S. Dhamal, K. J. Prabuchandran, and Y. Narahari. Information diffusion in social networks in two phases. IEEE TNSE, 3(4):197–210, 2016.

RELEVANT LITERATURE

- Influence maximization in a network in a single phase using seed nodes [1]
- Influence maximization using referral incentives [2]
  Optimal referral pricing, maximize profit to company

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- Influence maximization in a network in a single phase using seed nodes [1]
- Influence maximization using referral incentives [2]
- Influence maximization in a network in two phases using seed nodes [3]
  Given $K \rightarrow$ select $k1$ seeds for phase 1 $\rightarrow$ observe spread $\rightarrow$ select remaining $K - k1$ seeds

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Influence maximization with budget-split in two phases, using seed nodes, followed by referral incentives

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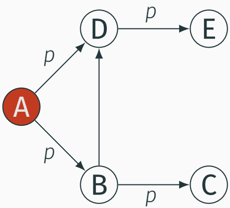
MODEL AND PROBLEM FORMULATION

EXAMPLE (K=2, k= 1, α = 0.5) PHASE 1



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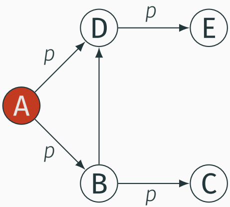

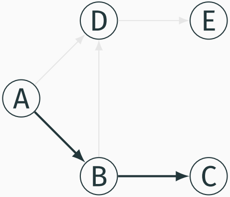

Live Graph

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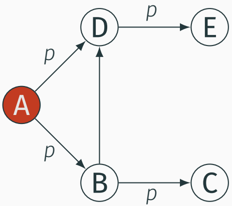

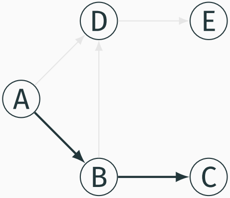

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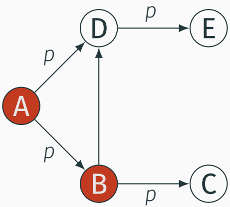

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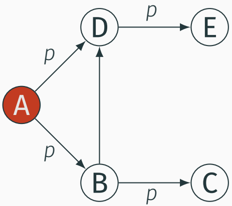

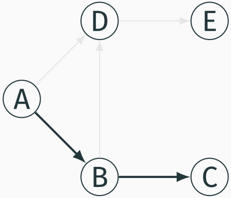

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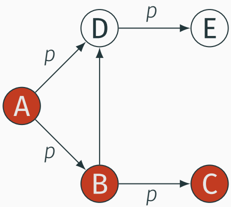

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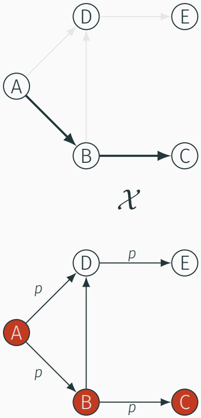
Phase 1 (k $= 1$)



Phase 2 ($\alpha = \frac{1}{2}$)

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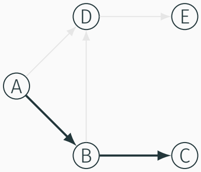
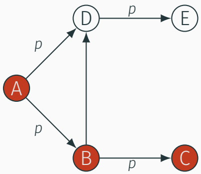


$\mathcal{X}$



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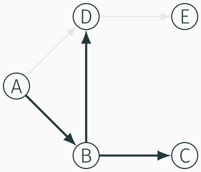

$\mathcal{Y}_1 | \mathcal{X}$

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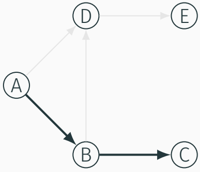

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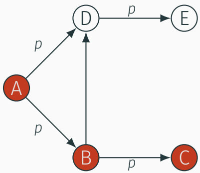

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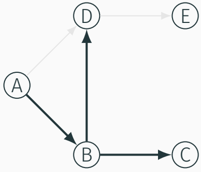

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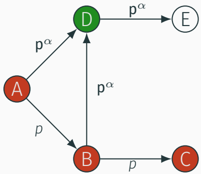

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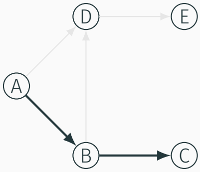

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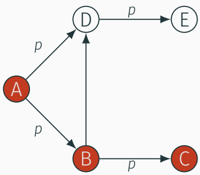

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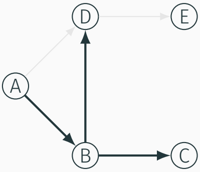

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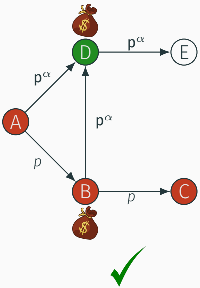

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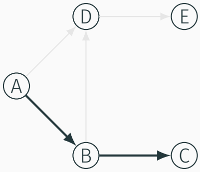

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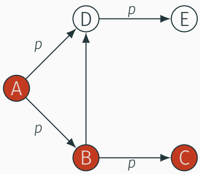

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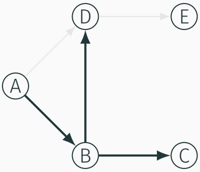

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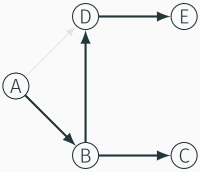

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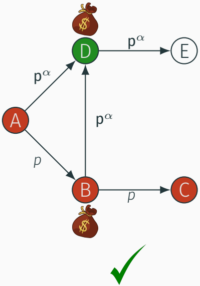

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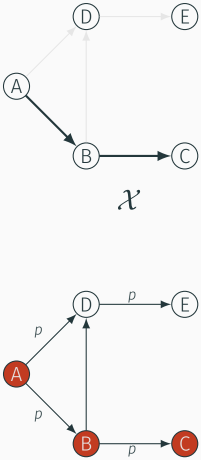

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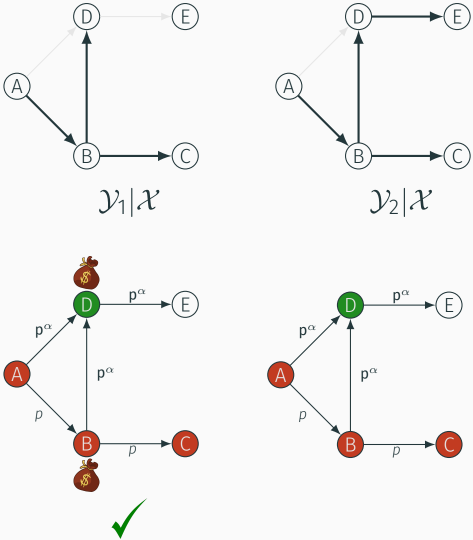

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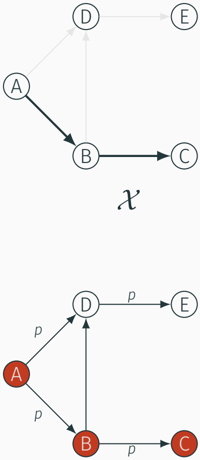

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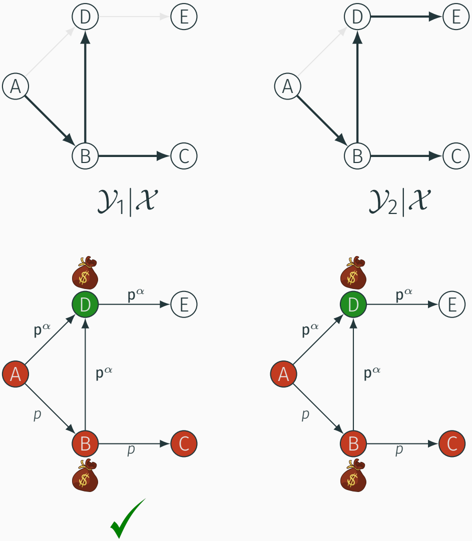

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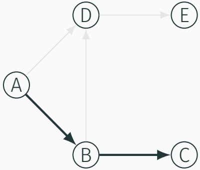

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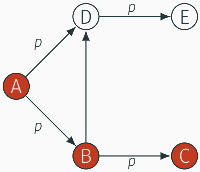

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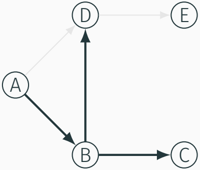

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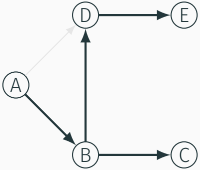
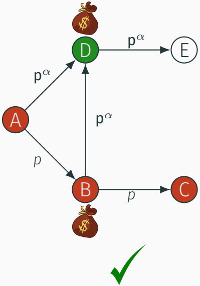
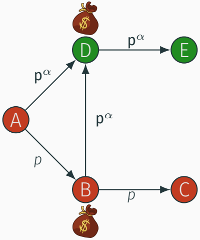


$\mathcal{Y}_2 | \mathcal{X}$





PHASE 2

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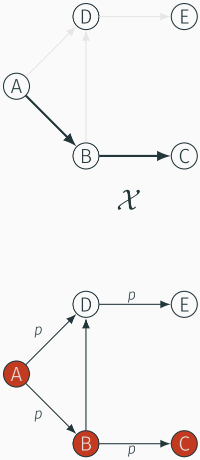

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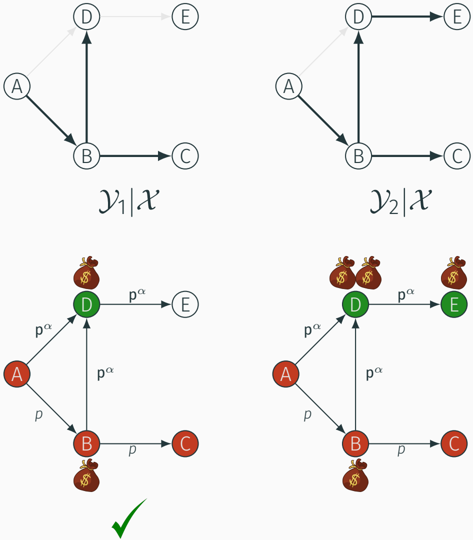

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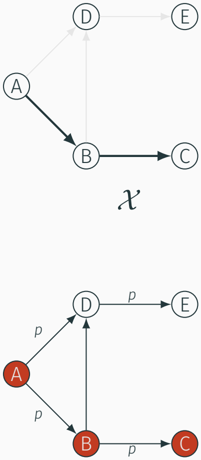

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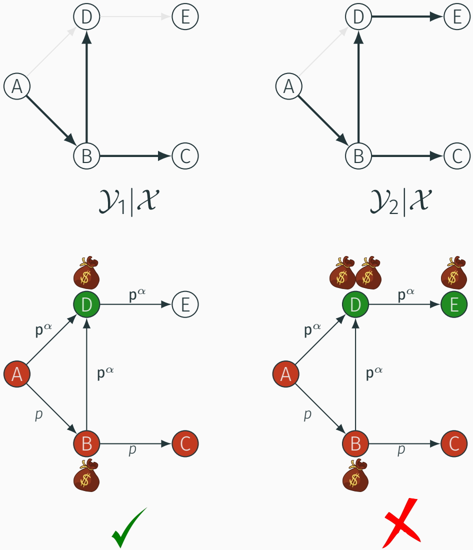

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$$A_{diff}^{\mathcal{X}} = \{v |\ v \text{ is reachable from } S^k \text{ in } \mathcal{X}\}$$

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- For a fixed α, $f(S, \alpha)$ is non-negative, monotone and sub-modular in S.

Select (S_k, α) to give

$$\max_{\substack{k \leq K, \alpha \in [0,1] \\ S_k \subset V}} f(S_k, \alpha) = \underbrace{\mathbb{E}\left[|A_{diff}(S_k)|\right]}_{\text{depends on } k} + \underbrace{\mathbb{E}\left[|A_{ref}(S_k; \alpha)|\right]}_{\text{depends on } k,\, h(\alpha)}$$

subject to

$$\mathbb{E}\left[|A_{ref}(S^k; \alpha)|\right] \leq \frac{K-k}{2\alpha}$$

Algorithm: A modified greedy algorithm for seed selection

Input: Graph G, budget K, split (k, α)
Output: Optimal seed set S_k such that $|S_k| \leq k$

$S_k \leftarrow \phi$
for $t \leftarrow 1$ ***to*** k **do**
  for $v \notin S_k$ **do**
    Compute $f(S_k \cup \{v\})$
  $V_{valid} \leftarrow \left\{v \in V \setminus S_k : \mathbb{E}|A_{ref}(S_k \cup \{v\})| \leq \frac{K-k}{2\alpha}\right\}$
  $v_t \leftarrow \arg\max_{v \in V_{valid}} f(S_k \cup \{v\}) - f(S_k)$
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EXPERIMENTAL EVALUATION

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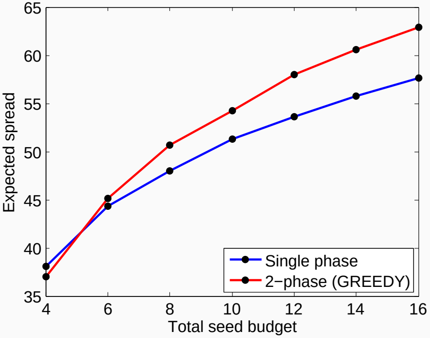
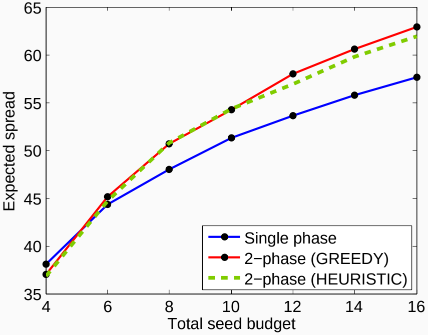
  - ◇ Suitable for running time and memory intensive algorithms
- NetHEPT
  - ◇ 15233 nodes, 31398 undirected edges
  - ◇ Exhibits most structural properties of "social-network" graphs



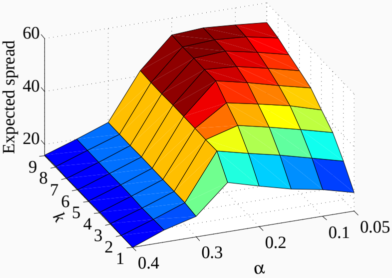
- Budget-split detrimental for small K, yields significant gains for moderate-high K, relative gain increases with K

PERFORMANCE OF 2-PHASE VS. SINGLE-PHASE (LES MISERABLES)

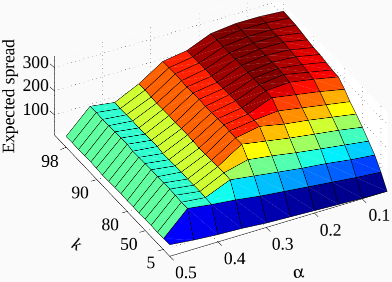


- Budget-split detrimental for small *K*, yields significant gains for moderate-high *K*
- PMIA heuristic performs nearly as well as 2-phase greedy

EFFECT OF k,α

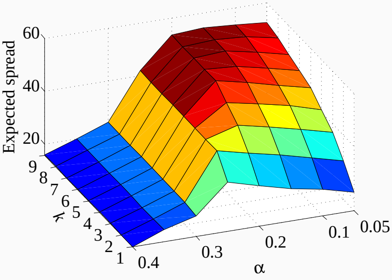


LM ($K = 10$)



NetHEPT ($K = 100$)

EFFECT OF k,α



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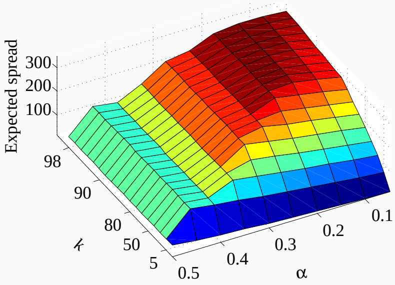

NetHEPT ($K = 100$)

- Maximum spread observed at high k, low α pairs
  Optimal split for LM : (7, 0.15). Gain $\approx$ 6%
  Optimal split for NetHEPT : (82, 0.15). Gain $\approx$ 7%

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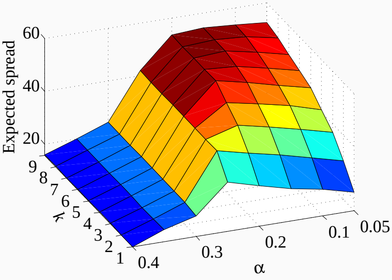

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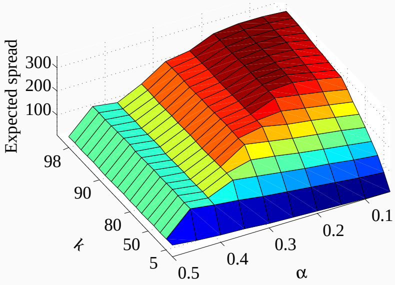

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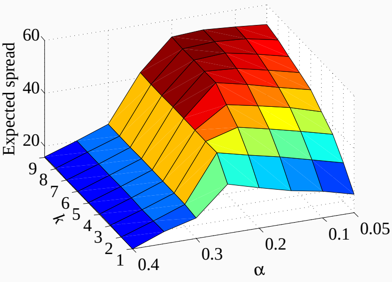

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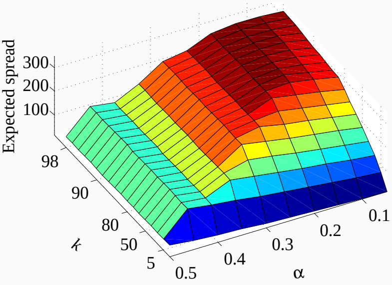

NetHEPT ($K = 100$)

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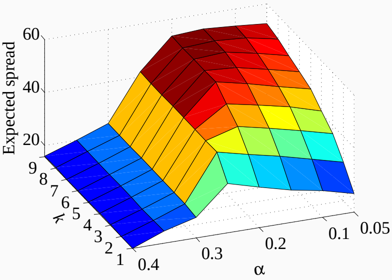
  Optimal split for NetHEPT : (82, 0.15). Gain ≈ 7%
  WHY?
  Need enough active nodes after phase 1 to act as referring agents for phase 2!



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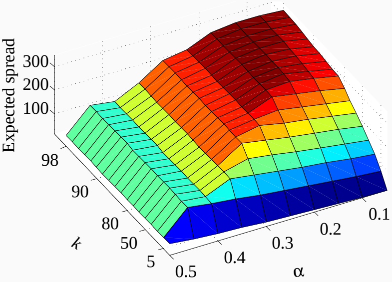

NetHEPT ($K = 100$)

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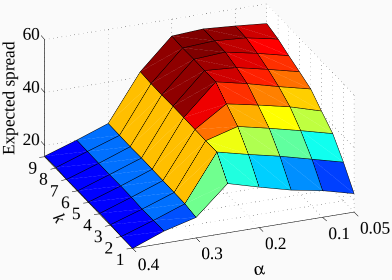
- Improved spread never attained at very high α

EFFECT OF k,α



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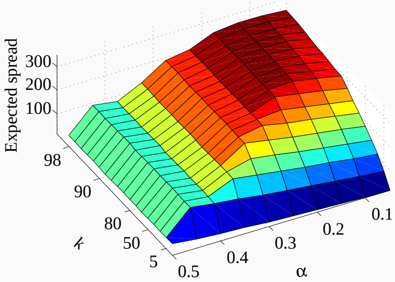

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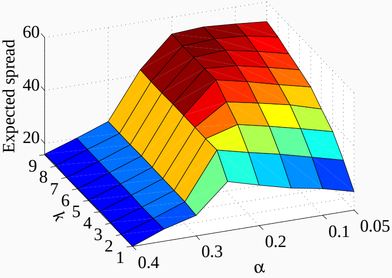

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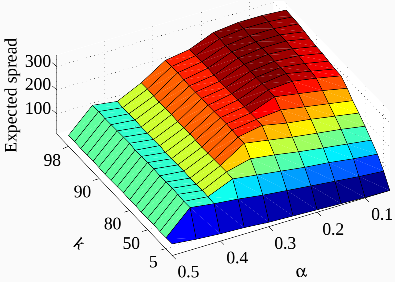

NetHEPT ($K = 100$)

- Maximum spread observed at high k, low α pairs
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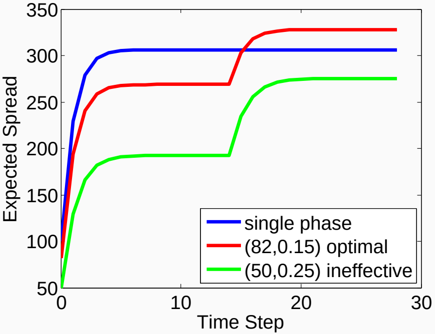
  WHY?
  Higher $\alpha \implies$ fewer permissible active nodes in phase 2!

TEMPORAL PROGRESSION OF 2-PHASE MODEL



NetHEPT ($K = 100$)

- Single phase saturates earliest
- Two-phase saturates after phase 1, shoots up on initiating phase 2
- Allocating sufficient budget for phase 1 is crucial!

SUMMARY AND FUTURE WORK

- In conclusion, we have:
    - Proposed a referral incentive based model
    - Analysed the mathematical properties of said model
    - Studied efficacy of the model on real-life datasets

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  - Proposed a referral incentive based model
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- Future Work
  - Use real cascade data to infer appropriate $h(\alpha)$
  - Analyse the modified-greedy algorithm, and establish provable guarantee for constrained optimization problem

- Sneha Mondal, Swapnil Dhamal, and Y. Narahari.
  Two-Phase Influence Maximization in Social Networks with Seed Nodes and Referral Incentives.

  Proceedings of the 11th International AAAI Conference on Web and Social Media (ICWSM-17)