Q3)

Given,

Training data i.e input vectors and its labels :

$x_1 = [1 \ 1]$ $\quad y_1 = 1$

$x_2 = [1 \ -1]$ $\quad y_2 = -1$

$$\boxed{\text{Mean squared error} = \frac{1}{2} \sum_i [y_i - w^T x_i]^2}$$

where,

y_i = actual label

$w^T x_i$ = predicted label

$w^T \rightarrow$ weight vector

a)

$$\text{Error} = \frac{1}{2}[1 - (w^T x_1)]^2 + \frac{1}{2}[-1 - (w^T x_2)]^2$$

$$w^T x_1 = [w_1 \ w_2] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = w_1 + w_2$$

$$w^T x_2 = [w_1 \ w_2] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = w_1 - w_2$$

So,

$$\text{Error} = \frac{1}{2}\left[(1 - (w_1 + w_2))^2 + (-1 - (w_1 - w)^2)^2 \right]$$

$$= \frac{1}{2}\left[(1 + w_1^2 + w_2^2 - \cancel{2w_1} - 2w_2 + \cancel{2w_1 w_2}) + (1 + w_1^2 + w_2^2 + \cancel{2w_1} - 2w_2 - \cancel{2w_1 w_2}) \right]$$

$$E = \frac{1}{2}[2 + 2w_1^2 + 2w_2^2 - 4w_2]$$

c)

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by

So

$$E = 1 + w_1^2 + w_2^2 - 2w_2$$

$$E = w_1^2 + (w_2 - 1)^2$$

clearly this function is an equation of <u>ellyptical paraboloid</u>.

curvature :- Ellyptic paraboloid has guassian and mean curvature

→ curvature of ellyptic paraboloid decreases as point on surface moves further away from origin.

Minimum value of E is when :

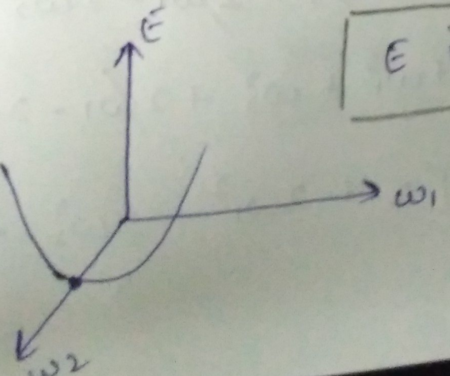
Differentiate the Error function w.r.t w_1 and w_2,

$$E = w_1^2 + (w_2 - 1)^2$$

$$\frac{\delta E}{\delta w_1} = 2w_1 = 0 \Rightarrow w_1 = 0$$

$$\frac{\delta E}{\delta w_2} = 2(w_2 - 1) = 0 \Rightarrow w_2 = 1$$

So, error function will take its minimum value at $w_1 = 0$ and $w_2 = 1$



$$\boxed{\begin{array}{c} E \text{ is minimum at} \\ w_1 = 0 \\ w_2 = 1 \end{array}}$$

b) Hessian of the Error function :-

Hessian matrix is a second-order partial derivatives of a scalar valued function.

$$H = \begin{bmatrix} \frac{\partial^2 E}{\partial w_1^2} & \frac{\partial^2 E}{\partial w_1 \partial w_2} \\ \frac{\partial^2 E}{\partial w_2 w_1} & \frac{\partial^2 E}{\partial w_2^2} \end{bmatrix}$$

So, let's find its value

$$\frac{\partial E}{\partial w_1} = 2w_1 \quad \text{So,} \quad \boxed{\frac{\partial^2 E}{\partial w_1^2} = 2}$$

$$\boxed{\frac{\partial \left(\frac{\partial E}{\partial w_1}\right)}{\partial w_2} = 0}$$

and also

$$\frac{\partial E}{\partial w_2} = 2(w_2 - 1)$$

So, $\boxed{\frac{\partial^2 E}{\partial w_2^2} = 2}$

$$\boxed{\frac{\partial \left(\frac{\partial E}{\partial w_2}\right)}{\partial w_1} = 0}$$

So, our Hessian matrix will be

$$H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

As it is a diagonal matrix so, its diagonal elements will be the eigen values

So, this matrix has eigen values 2,2

as both the values are ≥ 0 So,

$\lambda_1 = 2$, $\lambda_2 = 2$

Since H is a positive semi-definite matrix.