

Q.P. Code : 5001

(3 Hours)

[Total Marks : 100]

N.B.:-i) Q.No.1) is compulsory

ii) Attempt any THREE from remaining

iii) All questions carry equal marks

Q.No.1) a) If $\log \tan x = y$ then prove that $\sinh(n+1)y + \sinh(n-1)y = 2 \sinh ny \cdot \operatorname{cosec} 2x$

$$b) \text{ If } z = \log(\tan x + \tan y) \text{ then prove that } \sin 2x \frac{\partial z}{\partial x} + \sin 2y \frac{\partial z}{\partial y} = 2 \quad (3)$$

$$c) \text{ If } x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta \text{ then find } \frac{\partial(r, \theta, \phi)}{\partial(x, y, z)} \quad (3)$$

$$d) \text{ Prove that } \log \sec x = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \dots \quad (3)$$

$$e) \text{ Find the values of } a, b, c \text{ and } A^{-1} \text{ when } A = \frac{1}{9} \begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix} \text{ is orthogonal} \quad (4)$$

$$f) \text{ If } y = \sin \theta + \cos \theta \text{ then prove that } y_n = r^n \sqrt{1 + (-1)^n \sin 2\theta} \text{ where } \theta = rx \quad (4)$$

$$Q.No.2) a) \text{ If } z = -1 + i\sqrt{3} \text{ then prove that } \left(\frac{z}{2}\right)^n + \left(\frac{\bar{z}}{2}\right)^n = \begin{cases} 2 & \text{if } n \text{ is multiple of } 3 \\ -1 & \text{if } n \text{ is not multiple of } 3 \end{cases} \quad (6)$$

$$b) \text{ If } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \text{ then find two non-singular matrices } P \text{ \& } Q \text{ such that } PAQ \text{ is in normal form also find } \rho(A) \text{ and } A^{-1} \quad (6)$$

c) State and prove Euler's theorem for functions of two independent variable hence prove that

$$\left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right) \left(x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y}\right) = 0 \text{ if } x = e^u \tan v, y = e^u \sec v \quad (8)$$

$$Q.No.3) a) \text{ Determine the values of } a \text{ and } b \text{ such that system } \begin{cases} 3x - 2y + z = b \\ 5x - 8y + 9z = 3 \\ 2x + y + az = -1 \end{cases}$$

has i) no solution, ii) a unique solution, iii) infinite number of solutions (5)

$$b) \text{ Discuss the maximum and minimum of } f(x, y) = x^3 + 3xy^2 - 15(x^2 + y^2) + 72x \quad (6)$$

$$c) \text{ Show that } \tan^{-1} \left(\frac{x+y}{x-y} \right) = \frac{\pi}{4} + \frac{1}{2} \log \left(\frac{x+y}{x-y} \right) \quad (8)$$

[TURN OVER]

Q.No.4) a) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ then prove that $\frac{\partial x}{\partial u} = \frac{1}{(x-y)(x-z)}$ (6)

b) If $\sqrt{1 - \frac{v^2}{c^2}} = \alpha + i\beta$ then prove that i) $\alpha^2 + \beta^2 = e^{-\frac{\pi\beta}{\alpha}}$ ii) $\tan\left(\frac{\beta}{\alpha}\right) = \frac{\pi\alpha}{\beta}$ (6)

c) Apply Croust's method to solve $\begin{cases} x - y + 2z = 2 \\ 3x + 2y - 3z = 2 \\ 4x - 4y + 2z = 2 \end{cases}$ (8)

Q.No.5) a) If $\cos^6 \theta + \sin^6 \theta = \alpha \cos 4\theta + \beta$ then prove that $\alpha + \beta = 1$ (6)

b) Find the values of a , b & c such that $\lim_{x \rightarrow 0} \frac{ae^{ax} - be^{-x} + cx}{x - \sin x} = 4$ (6)

c) If $x = \cos\left[\log\left(y^{1/m}\right)\right]$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (m^2 + n^2)y_n = 0$ (8)

Q.No.6) a) Define linear dependence and independence of vectors, Examine for linear dependence of following set of vectors and find the relation between them if dependent

$$X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} \quad (6)$$

b) If $z = f(u, v)$, $u = x^2 - y^2$, $v = 2xy$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4\sqrt{u^2 + v^2} \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$ (6)

c) Fit a straight line passing through points $(0,1)$, $(1,2)$, $(2,3)$, $(3,4.5)$, $(4,6)$, $(5,7.5)$ (8)