Sem I / CBSGS / Applied Maths - I / 07-12-2016

**QP CODE : 529302**

**(3 hours)** **[Total Marks : 80]**

**Note: i) Q.No 1 is compulsory**

**ii) Attempt any three from remaining.**

**ii) All questions carry equal marks.**

Q.No.1)a)If tanhx = $\frac{1}{2}$, find sinh2x, cosh2x. (3)

b) If z = $xyf(\frac{y}{x})$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial y}{\partial y} = 2z$ (3)

c) If x = u(1-v), y = uv – uvw, z = uvw find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ (3)

d) Using Maclaurins expansion , Prove that (3)

$$e^x secx = 1 + x + \frac{2x^2}{2!} + \frac{4x^3}{3!} + \cdots$$

e) Show that every square matrix A can be uniquely expressed as P+iQ, where P & Q are Hermitian Matrices. (4)

f) Find nth derivative of $e^x cosxcos2x$ (4)

Q.No.2)a) If x =cos $\theta$+isin $\theta$, y = cos$\emptyset$ + i sin$\emptyset$, show that

$$\frac{x-y}{x+y} = itan\frac{\theta-\emptyset}{2}$$ (6)

b) For the following matrix A , find non singular matrices P and Q such that PAQ is in normal form and hence find the rank of A, A = $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ (6)

c) If u = $cosec^{-1}\sqrt{(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}})}$ , show that (8)

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial xy} + y^2\frac{\partial^2 u}{\partial x^2} = \frac{tanu}{12}(\frac{13}{12} + \frac{tan^2 u}{12})$$

Q.No 3)a) For what values of $\lambda$, the system of equations 3x-y+4z=3,x+2y-3z=-2, 6x+5y+λz=-3 has a unique solution. Determine the solution in each case. (6)

b) Find the maxima and minima of the function (6)

f(x,y) = $x^3 + y^3$- 3x - 12y +20

c) Show that $tan^{-1}i\left(\frac{x-a}{x+a}\right) = \frac{i}{2}\log(\frac{x}{a})$ (8)

Q.No.4)a) Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$, using partial derivatives for (6)

$$xe^y + ye^z + logx - 2 - 3log2 = 0$$ at P(1,log2,log3)

**[TURN OVER**

b) Find the principal value of $(1+i)^{1-i}$. (6)

c) Solve the following system of equation by crouts method (8)

x + y + z = 3, 2x – y + 3z = 16, 3x + y - z = -3

Q.No.5)a) Show that $\frac{sin6\theta}{sin2\theta} = 16cos^4\theta - 16cos^2\theta + 3$ (6)

b) Find a and b such that $\lim_{x\to 0} \frac{x(1-acosx)+bsinx}{x^4} = \frac{1}{3}$ (6)

c) If $y = (1-x)^{-\alpha}e^{-\alpha x}$, show that (8)

(i) (1- x) $y_1 = \alpha xy$

(ii) $(1-x)y_{n+1} - (n+\alpha x)y_n - n\alpha y_{n-1} = 0$

Q.No.6) a) Show that the rows of the following matrix are linearly dependent and find the relationship between them $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 1 \\ 4 & 6 & 2 & -4 \\ -6 & 0 & -3 & -4 \end{bmatrix}$ (6)

b) If $\emptyset\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$, prove that px + qy = 3z (6)

c) Fit a second degree parabola to the following data (8)

| x : | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y: | -3.150 | -1.390 | 620 | 2.880 | 5.378 |

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Q. 6 → C  Read as 0.620

1 → B ⇒ $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$  Read as $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$

2 – C ⇒ $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{xx}$ Read as

$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{xy}$

6 → B ~~as~~ add the sentence where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.