

REVISED COURSE

(3 Hours)

[Total Marks : 100

N.B. : (1) Question No. 1 is compulsory.

(2) Attempt any three questions from question No. 2 to question no. 6.

(3) Figures to the right indicate full marks.

1. (a) Solve the equation $7\cosh x + 8\sinh x = 1$ for real values of x 3(b) If $z(x+y) = (x-y)$ find $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2$ 3(c) If $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$ 3(d) Prove that $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \dots$ 3

(e) Find the rank of the Matrix by reducing it to normal form. 4

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

(f) Find n^{th} derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$ 42. (a) If α, β are the roots of the equation $x^2 - 2\sqrt{3} \cdot x + 4 = 0$ find the value of $\alpha^3 + \beta^3$ 6

(b) Examine whether the vectors 6

$$X_1 = \begin{bmatrix} 3 & 1 & 1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 & 0 & -1 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix}$$

are linearly independent.

(c) (i) State and prove Euler's theorem for a Homogeneous function in two variables. 4

(ii) If $y = x \cos u$ find the value of $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$ 4

[TURN OVER

3. (a) Is the following system has trivial or non trivial solution ? Obtain the non trivial solution if exist. 6

$$3x_1 + 4x_2 - x_3 - 9x_4 = 0$$

$$2x_1 + 3x_2 + 2x_3 - 3x_4 = 0$$

$$2x_1 + x_2 - 14x_3 - 12x_4 = 0$$

$$x_1 + 3x_2 + 13x_3 + 3x_4 = 0$$

- (u) Discuss the stationary points for Maxima and Mininima of 6
 $x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 10$

- (c) (i) If $\tan(x+iy) = a + ib$ prove that $\tanh 2y = \frac{2b}{1+a^2+b^2}$ 4
 (ii) Separate into real and imaginary parts of $\text{Log}(3+4i)$ 4

4. (a) If $x = u \cos v$, $y = u \sin v$ 6

Prove that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$

- (b) Show that $\log[e^{i\alpha} + e^{i\beta}] = \log\left[2 \cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$ 6

- (c) (i) Solve the system of equation by Gauss Jordan Method 4
 $x + 2y + 6z = 22$, $3x + 4y + z = 26$, $6x - y - z = 19$

- (ii) Solve the system of equation by Gauss Siedel Method. 4
 Correct upto three decimal.
 $2x - 4y + 49z = 49$
 $43x + 2y + 25z = 23$
 $3x + 53y + 3z = 91$

5. (a) Prove that $\cos^6 \theta + \sin^6 \theta = \frac{1}{8} [3 \cos 4\theta + 5]$ 6

- (b) Find the value of a and b 6

if $\lim_{x \rightarrow 0} \frac{x(1 + b \cos x) - b \sin x}{x^3} = 1$

- (c) (i) If $y = e^x \cos 2x \cos x$ find y_n 4
 (ii) If $y = e^{\tan^{-1} x}$ prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$ 4

[TURN OVER]

- (a) Find non-Singular Matrices P & Q such that,

6

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix} \text{ is reduced to normal form. Also find rank.}$$

- (b) If $u = f(e^{x^2}, e^{y^2}, e^{z^2})$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

6

- (c) (i) Fit a straight line to the following data :

4

Year x :	1951	1961	1971	1981	1991
Production y :	10	12	8	10	15

- (ii) Fit a second degree parabolic curve to the following data :

4

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9