F.E - I | App Maths - I | 05-12-14

(REVISED COURSE) **QP Code :11932**

(3 Hours) **Total Marks : 80**

N. B. : (1) **Q. No.1** is **compulsory.**
(2) **Attempt** any **three** questions from question **no.2** to question **no.6**.
(3) **Figures** to the right indicate **full** marks.

1. (a) If $\tanh x = \frac{2}{3}$, find the value of x and then cosh 2x

(b) If $u = \tan^{-1}\left(\frac{y}{x}\right)$, Find these value of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ **3**

(c) if $x = r\cos\theta,\ y = r\sin\theta$ **3**

Find $\frac{\partial(x,y)}{\partial(r,\theta)}$

(d) Prove that $\log\sec x = \frac{1}{2}x^2 + \frac{1}{1z}x^4 + \frac{1}{45}x^6$ **3**

(e) Show that every square matrix can be uniquely expressed as the sum of Hermitian martix and a skew Hermitian matrix. **4**

(f) Find the n^{th} derivative of **4**
$y = \sin x \sin 2x \sin 3x$

2. (a) Solve the equation $x^6+1=0$ **6**

(b) Reduce the matrix to normal form and find its rank, where **6**

$$A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

(c) State and prove Eulers theorem for a homogeneous function in two variables: Hence verify the Eulers theorem for **8**

$$u = \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$$

3. (a) Test the consistency of the following equations and solve them if they are consistent. **6**
$2x - y + z = 8, \quad 3x - y + z = 6$
$4x - y + 2z = 7, \quad -x + y - z = 4$

(b) Find the stationary values **6**
$x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

(c) Separate into real and imaginary parts of $\sin^{-1}(e^{i\theta})$ **8**

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4. (a) If $x = uv,\ y = \frac{u}{v}$ prove that $J.J' = 1$ — 6

(b) Show that for real values of a and b, $e^{2ai\cot^{-1}b}\left[\frac{bi-1}{bi+1}\right]^{-a} = 1$ — 6

(c) Solve the following equations by Gauss-seidel method — 8

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

5. (a) Expond $\cos^7\theta$ in a series of cosines of multiple of θ — 6

(b) If $\lim_{x \to 0} \frac{a\sinh x + b\sin x}{x^3} = \frac{5}{3}$, find a and b — 6

(c) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ then prove that $(1 - x^2)\, y_{n+1} - (2n + 1)\, xy_n - n^2 y_{n-1} = 0$ — 8

6. (a) Examine whether the vectors — 6

$x_1 = [3, 1, 1]$ $\quad x_2 = [2, 0, -1]$

$x_3 = [4, 2, 1]$ are linearly independent.

(b) If $u = f(x-y,\ y-z,\ z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ — 6

(c) Fit a straight line for the following data — 8

x	1	2	3	4	5	6
y	49	54	60	73	80	86
