CS335: Top-down Parsing

Swarnendu Biswas

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CSE, IIT Kanpur

Content influenced by many excellent references, see References slide for acknowledgements.

Example Expression Grammar

$Start \rightarrow Expr$

$Expr \rightarrow Expr + Term \mid Expr - Term \mid Term$

$Term \rightarrow Term \times Factor \mid Term \div Factor \mid Factor$

$Factor \rightarrow (Expr) \mid \mathrm{num} \mid \mathrm{name}$

priority

Derivation of $\text{name} + \text{name} \times \text{name}$

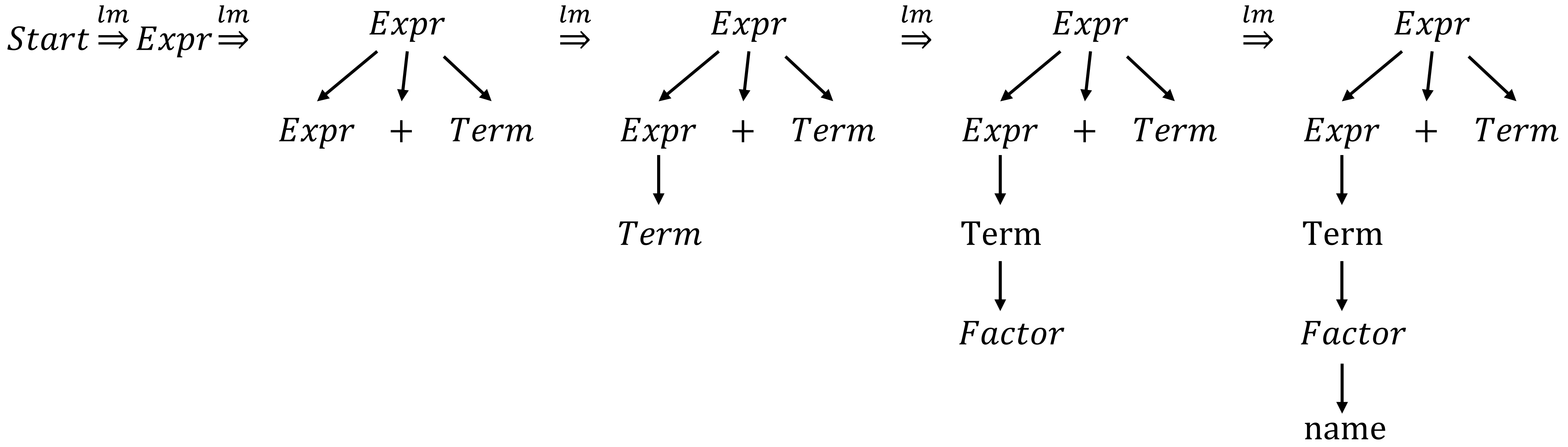
Sentential Form	Input
$Expr$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Expr + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Term + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Factor + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$\text{name} + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$\text{name} + Term$	$\text{name} \uparrow + \text{name} \times \text{name}$
$\text{name} + Term$	$\text{name} + \uparrow \text{name} \times \text{name}$
$\text{name} + Term \times Factor$	$\text{name} + \uparrow \text{name} \times \text{name}$
$\text{name} + Factor \times Factor$	$\text{name} + \uparrow \text{name} \times \text{name}$
$\text{name} + \text{name} \times Factor$	$\text{name} + \uparrow \text{name} \times \text{name}$
$\text{name} + \text{name} \times Factor$	$\text{name} + \text{name} \uparrow \times \text{name}$
$\text{name} + \text{name} \times Factor$	$\text{name} + \text{name} \times \uparrow \text{name}$
$\text{name} + \text{name} \times \text{name}$	$\text{name} + \text{name} \times \uparrow \text{name}$
$\text{name} + \text{name} \times \text{name}$	$\text{name} + \text{name} \times \text{name} \uparrow$

Derivation of $\mathbf{name + name \times name}$

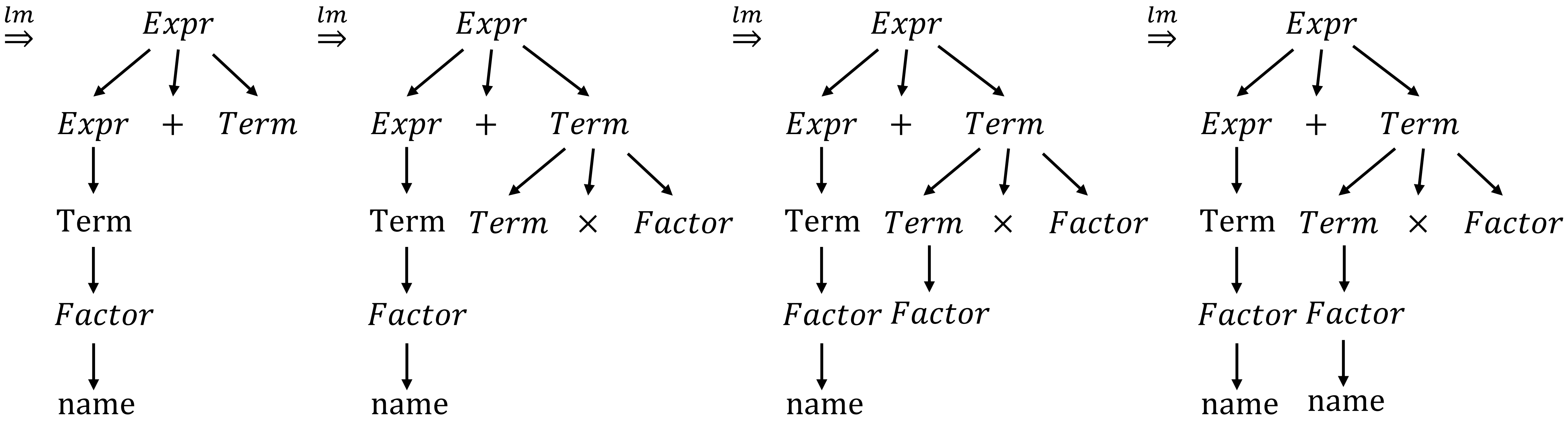
Sentential Form	Input
$Expr$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Expr + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Term + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$Factor + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$\text{name} + Term$	$\uparrow \text{name} + \text{name} \times \text{name}$
$\text{name} + Term$	$\text{name} \uparrow + \text{name} \times \text{name}$
$\text{name} + Term$	$\text{name} + \uparrow \text{name} \times \text{name}$
$\text{name} + \text{name} \times Factor$	$\text{name} + \text{name} \uparrow \times \text{name}$
$\text{name} + \text{name} \times Factor$	$\text{name} + \text{name} \times \uparrow \text{name}$
$\text{name} + \text{name} \times \text{name}$	$\text{name} + \text{name} \times \uparrow \text{name}$
$\text{name} + \text{name} \times \text{name}$	$\text{name} + \text{name} \times \text{name} \uparrow$

The current input terminal being scanned is called the lookahead symbol

Derivation of $\mathbf{name} + \mathbf{name} \times \mathbf{name}$

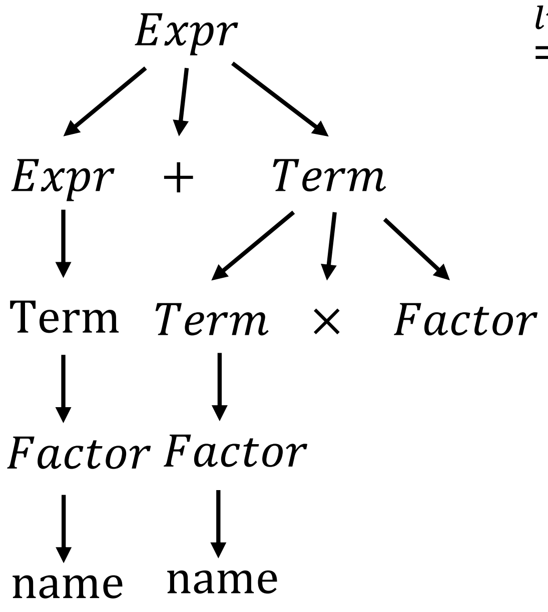


Derivation of $\mathbf{name} + \mathbf{name} \times \mathbf{name}$

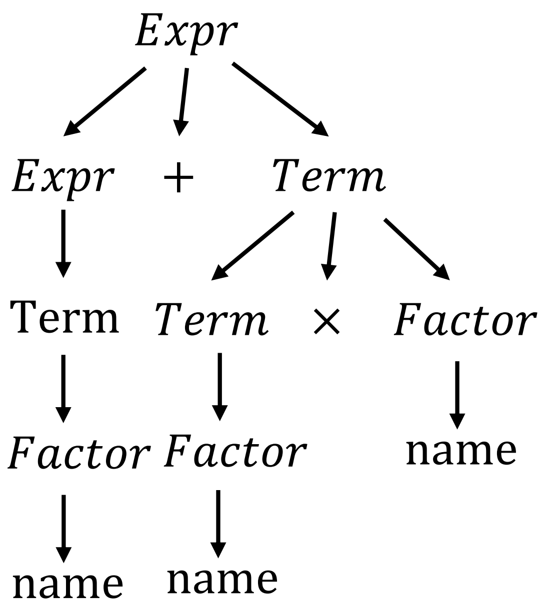


Derivation of $\mathbf{name} + \mathbf{name} \times \mathbf{name}$

$\overset{lm}{\Rightarrow}$



$\overset{lm}{\Rightarrow}$



General Idea of Top-down Parsing

Start with the root (start symbol) of the parse tree

Grow the tree downwards by expanding productions at the lower levels of the tree

- Select a nonterminal and extend it by adding children corresponding to the right side of some production for the nonterminal

Repeat till

- Lower fringe consists only terminals and the input is consumed

Top-down parsing basically finds a leftmost derivation for an input string

General Idea of Top-down Parsing

Start with the root of the parse tree

Grow the tree by expanding productions at the lower levels of the tree

- Extend a nonterminal by adding children corresponding to the right side of some production for the nonterminal

Repeat till

- Lower fringe consists only terminals and the input is consumed
- Mismatch in the lower fringe and the remaining input stream
  - Selection of a production may involve trial-and-error
  - Wrong choice of productions while expanding nonterminals
  - Input character stream is not part of the language

Leftmost Top-down Parsing Algorithm

```
root = node for Start symbol
curr = root
push(null) // Stack
word = nextWord()

while (true):
  if curr ∈ Nonterminal:
    pick next rule A ⟶ β1β2 …βn to
expand curr
    create nodes for β1, β2, …, βn as
children of curr
    push(βn, βn−1,β1)
    curr = β1
```

```
if curr == word:
  word = nextWord()
  curr = pop()
if word == eof and curr == null:
  accept input
else
  backtrack
```

Implementing Backtracking

- Extend the previous algorithm to backtrack
  - Set `curr` to parent and delete the children
  - Expand the node `curr` with untried rules if any
    - Create child nodes for each symbol in the right hand of the production
    - Push those symbols onto the stack in reverse order
    - Set `curr` to the first child node
  - Move `curr` up the tree if there are no untried rules
  - Report a syntax error when there are no more moves

Example of Top-down Parsing

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Expr + Term$
2	$Expr \rightarrow Expr - Term$
3	$Expr \rightarrow Term$
4	$Term \rightarrow Term \times Factor$
5	$Term \rightarrow Term \div Factor$
6	$Term \rightarrow Factor$
7	$Factor \rightarrow (Expr)$
8	$Factor \rightarrow$ num
9	$Factor \rightarrow$ name

Rule #	Sentential Form	Input
	$Expr$	↑ name + name × name
1	$Expr + Term$	↑ name + name × name
3	$Term + Term$	↑ name + name × name
6	$Factor + Term$	↑ name + name × name
9	name + $Term$	↑ name + name × name
	name + $Term$	name ↑ +name × name
	name + $Term$	name +↑ name × name
4	name + $Term \times Factor$	name +↑ name × name
6	name + $Factor \times Factor$	name +↑ name × name
9	name + name × $Factor$	name +↑ name × name
	name + name × $Factor$	name + name ↑× name
	name + name × $Factor$	name + name ×↑ name
9	name + name × name	name + name ×↑ name
	name + name × name	name + name × name ↑

Example of Top-down Parsing

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Expr + Term$
2	$Expr \rightarrow Expr - Term$
3	$Expr \rightarrow Term$
4	$Term \rightarrow Term \times Factor$
5	$Term \rightarrow Term \div Factor$
6	
7	
8	$Factor \rightarrow$ num
9	$Factor \rightarrow$ name

Rule #	Sentential Form	Input
	$Expr$	↑ name + name × name
1	$Expr + Term$	↑ name + name × name
3	$Term + Term$	↑ name + name × name
6	$Factor + Term$	↑ name + name × name
9	name + $Term$	↑ name + name × name
	name + $Term$	name ↑ +name × name
		× name
		× name
6	name + $Factor \times Factor$	name +↑ name × name
9	name + name × $Factor$	name +↑ name × name
	name + name × $Factor$	name + name ↑× name
	name + name × $Factor$	name + name ×↑ name
9	name + name × name	name + name ×↑ name
	name + name × name	name + name × name ↑

How does a top-down parser choose which rule to apply?

Example of Top-down Parsing

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Expr + Term$
2	$Expr \rightarrow Expr - Term$
3	$Expr \rightarrow Term$
4	$Term \rightarrow Term \times Factor$
5	$Term \rightarrow Term \div Factor$
6	$Term \rightarrow Factor$
7	$Factor \rightarrow (Expr)$
8	$Factor \rightarrow$ num
9	$Factor \rightarrow$ name

Rule #	Sentential Form	Input
	$Expr$	↑ name + name × name
1	$Expr + Term$	↑ name + name × name
1	$Expr + Term + Term$	↑ name + name × name
1	$Expr + Term + Term + \cdots$	↑ name + name × name
1	…	↑ name + name × name
1	…	↑ name + name × name

Example of Top-Down Parsing

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Expr + Term$
2	$Expr \rightarrow Expr - Term$
3	$Expr \rightarrow Term$
4	$Term \rightarrow Term \times Factor$
5	
6	
7	$Factor \rightarrow (Expr)$
8	$Factor \rightarrow$ num
9	$Factor \rightarrow$ name

Rule #	Sentential Form	Input
	$Expr$	↑ name + name × name
1	$Expr + Term$	↑ name + name × name
1	$Expr + Term + Term$	↑ name + name × name
1	$Expr + Term + Term + \cdots$	↑ name + name × name
1	$\cdots$	↑ name + name × name
		× name

A top-down parser can loop indefinitely with left-recursive grammar

Left Recursion

- A grammar is left-recursive if it has a nonterminal A such that there is a derivation $A \overset{+}{\Rightarrow} A\alpha$ for some string α
  - **Direct** left recursion: There is a production of the form $A \rightarrow A\alpha$
  - **Indirect** left recursion: First symbol on the right-hand side of a rule can derive the symbol on the left

We can often reformulate a grammar to avoid left recursion

Remove Left Recursion

$$A \rightarrow A\alpha_1 | A\alpha_2 | \dots | A\alpha_m | \beta_1 | \dots | \beta_n$$

$$\Downarrow$$

$$A \rightarrow \beta_1 A' | \beta_2 A' | \dots | \beta_n A'$$
$$A' \rightarrow \alpha_1 A' | \alpha_2 A' | \dots | \alpha_m A' | \epsilon$$

Remove Left Recursion

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \textbf{id}$$

$$E \rightarrow TE'$$
$$E' \rightarrow +TE'$$
$$T \rightarrow FT'$$
$$T' \rightarrow * FT'$$
$$F \rightarrow (E)|\textbf{id}$$

Non-Left-Recursive Expression Grammar

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Expr + Term$
2	$Expr \rightarrow Expr - Term$
3	$Expr \rightarrow Term$
4	$Term \rightarrow Term \times Factor$
5	$Term \rightarrow Term \div Factor$
6	$Term \rightarrow Factor$
7	$Factor \rightarrow (Expr)$
8	$Factor \rightarrow$ num
9	$Factor \rightarrow$ name

Rule #	Production
0	$Start \rightarrow Expr$
1	$Expr \rightarrow Term\ Expr'$
2	$Expr' \rightarrow + Term\ Expr'$
3	$Expr' \rightarrow - Term\ Expr'$
4	$Expr' \rightarrow \epsilon$
5	$Term \rightarrow Factor\ Term'$
6	$Term' \rightarrow \times Factor\ Term'$
7	$Term' \rightarrow \div Factor\ Term'$
8	$Term' \rightarrow \epsilon$
9	$Factor \rightarrow (Expr)$
10	$Factor \rightarrow$ num
11	$Factor \rightarrow$ name

Indirect Left Recursion

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

- There is a left recursion because $S \rightarrow Aa \rightarrow Sda$

Eliminating Left Recursion

- **Input**: Grammar G with no cycles or $\epsilon-$productions
- **Algorithm**

Arrange nonterminals in some order $A_1, A_2, \dots, A_n$
for $i \leftarrow 1 \dots n$
    for $j \leftarrow 1$ to $i - 1$
        If ∃ a production $A_i \rightarrow A_j\gamma$
            Replace $A_i \rightarrow A_j\gamma$ with one or more productions that expand A_j
    Eliminate the immediate left recursion among the A_i productions

Eliminating Left Recursion

- **Input**: Grammar G with no cycles or $\epsilon-$productions
- **Algorithm**

Arrange nonterminals in some order $A_1, A_2, \dots, A_n$
for $i \ \leftarrow 1 \dots n$
    for $j \leftarrow 1$ to $i - 1$
        If ∃ a production $A_i \ \rightarrow A_j\gamma$
            Replace $A_i \ \rightarrow A_j\gamma$ with one or more productions that expand A_j
    Eliminate the immediate left recursion among the A_i productions

Loop invariant at the start of outer iteration i

$$\forall k < i, \text{no production expanding } A_k \text{ has } A_l \text{ in its righthand side for all } l < k$$

Eliminating Indirect Left Recursion

$$S \rightarrow Aa \mid b$$
$$A \rightarrow Ac \mid Sd \mid \epsilon$$

$\Rightarrow$

$$S \rightarrow Aa \mid b$$
$$A \rightarrow bdA' \mid A'$$
$$A' \rightarrow cA' \mid adA' \mid \epsilon$$

Cost of Backtracking

Backtracking is expensive

- Parser expands a nonterminal with the wrong rule
- Mismatch between the lower fringe of the parse tree and the input is detected
- Parser undoes the last few actions
- Parser tries other productions if any

Avoid Backtracking

- Parser is to select the next rule
  - Compare the `curr` symbol and the next input symbol called the lookahead
  - Use the lookahead to disambiguate the possible production rules

- Backtrack-free grammar is a CFG for which the leftmost, top-down parser can always predict the correct rule with one word lookahead
  - Also called a predictive grammar

FIRST Set

- **Intuition**
  - Each alternative for the leftmost nonterminal leads to a distinct terminal symbol
  - Which rule to choose becomes obvious by comparing the next word in the input stream

- Given a string γ of terminal and nonterminal symbols, FIRST*(γ)* is the set of all terminal symbols that can begin any string derived from γ
  - We also need to keep track of which symbols can produce the empty string
  - $\mathrm{FIRST}: (NT \cup T \cup \{\epsilon, \mathrm{EOF}\}) \rightarrow (T \cup \{\epsilon, \mathrm{EOF}\})$

Steps to Compute FIRST Set

1. If X is a terminal, then $\mathrm{FIRST}(X) = \{X\}$
2. If $X \rightarrow \epsilon$ is a production, then $\epsilon \in \mathrm{FIRST}(X)$
3. If X is a nonterminal and $X \rightarrow Y_1 Y_2 \dots\ Y_k$ is a production
   I. Everything in $\mathrm{FIRST}(Y_1)$ is in $\mathrm{FIRST}(X)$
   II. If for some i, $a\ \in \mathrm{FIRST}(Y_i)$ and $\forall 1 \leq j < i,\ \epsilon \in \mathrm{FIRST}(Y_j)$, then $a\ \in \mathrm{FIRST}(X)$
   III. If $\epsilon \in \mathrm{FIRST}(Y_1 \dots Y_k)$, then $\epsilon \in \mathrm{FIRST}(X)$

FIRST Set

- Generalize FIRST relation to string of symbols

$\text{FIRST}(X\gamma) \rightarrow \text{FIRST}(X)$ if $X \nrightarrow \epsilon$

$\text{FIRST}(X\gamma) \rightarrow \text{FIRST}(X) \cup \text{FIRST}(\gamma)$ if $X \rightarrow \epsilon$

Compute FIRST Set

$Start \rightarrow Expr$

$Expr \rightarrow Term\ Expr'$

$Expr' \rightarrow +Term\ Expr'$

$\quad\quad |-Term\ Expr' \mid \epsilon$

$Term \rightarrow Factor\ Term'$

$Term' \rightarrow \times\ Factor\ Term'$

$\quad\quad | \div\ Factor\ Term' \mid \epsilon$

$Factor \rightarrow (Expr) \mid \text{num} \mid \text{name}$

Compute FIRST Set

$Start \rightarrow Expr$

$Expr \rightarrow Term\ Expr'$

$Expr' \rightarrow +Term\ Expr'$

$\quad\quad |-Term\ Expr' \mid \epsilon$

$Term \rightarrow Factor\ Term'$

$Term' \rightarrow \times\ Factor\ Term'$

$\quad\quad |\div\ Factor\ Term' \mid \epsilon$

$Factor \rightarrow (Expr) \mid \mathrm{num} \mid \mathrm{name}$

$\mathrm{FIRST}(Expr) = \{(, \mathrm{name}, \mathrm{num}\}$

$\mathrm{FIRST}(Expr') = \{+, -, \epsilon\}$

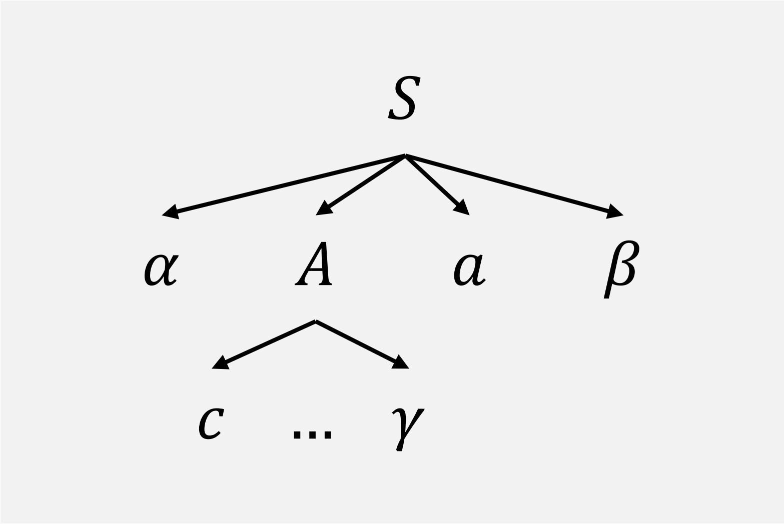
$\mathrm{FIRST}(Term) = \{(, \mathrm{name}, \mathrm{num}\}$

$\mathrm{FIRST}(Term') = \{\epsilon\ \times, \div\}$

$\mathrm{FIRST}(Factor) = \{(, \mathrm{name}, \mathrm{num}\}$

FOLLOW Set

- $\text{FOLLOW}(X)$ is the set of terminals that can immediately follow X
  - That is, $t \in \text{FOLLOW}(X)$ if there is any derivation containing Xt



Terminal c is in $\text{FIRST}(A)$ and a is in $\text{FOLLOW}(A)$

Steps to Compute FOLLOW Set

1. Place \$ in $\mathrm{FOLLOW}(S)$ where S is the start symbol and \$ is the end marker
2. If there is a production $A \rightarrow \alpha B \beta$, then everything in $\mathrm{FIRST}(\beta)$ except ϵ is in $\mathrm{FOLLOW}(B)$
3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where $\mathrm{FIRST}(\beta)$ contains ϵ, then everything in $\mathrm{FOLLOW}(A)$ is in $\mathrm{FOLLOW}(B)$

Compute FOLLOW Set

$Start \rightarrow Expr$

$Expr \rightarrow Term\ Expr'$

$Expr' \rightarrow +Term\ Expr'$

$\quad |-Term\ Expr' \mid \epsilon$

$Term \rightarrow Factor\ Term'$

$Term' \rightarrow \times\ Factor\ Term'$

$\quad |\div\ Factor\ Term' \mid \epsilon$

$Factor \rightarrow (Expr) \mid \text{num} \mid \text{name}$

Compute FOLLOW Set

$Start \rightarrow Expr$

$Expr \rightarrow Term\ Expr'$

$Expr' \rightarrow +Term\ Expr'$

$\quad |-Term\ Expr' \mid \epsilon$

$Term \rightarrow Factor\ Term'$

$Term' \rightarrow \times\ Factor\ Term'$

$\quad |\div\ Factor\ Term' \mid \epsilon$

$Factor \rightarrow (Expr) \mid \text{num} \mid \text{name}$

$\mathrm{FOLLOW}(Expr) = \{\$,)\}$

$\mathrm{FOLLOW}(Expr') = \{\$,)\}$

$\mathrm{FOLLOW}(Term) = \{\$, +, -,)\}$

$\mathrm{FOLLOW}(Term') = \{\$,+, -,)\}$

$\mathrm{FOLLOW}(Factor) = \{\$, +, -,\times,\div,)\}$

Conditions for Backtrack-Free Grammar

- Consider a production $A \rightarrow \beta$

$$\text{FIRST}^{+} = \begin{cases} \text{FIRST}(\beta) & \text{if } \epsilon \notin \text{FIRST}(\beta) \\ \text{FIRST}(\beta) \cup \text{FOLLOW}(A) & \text{otherwise} \end{cases}$$

- For any nonterminal A where $A \rightarrow \beta_1 | \beta_2 | \ldots | \beta_n$, a backtrack-free grammar has the property

$$\text{FIRST}^{+}(A \rightarrow \beta_i) \cap \text{FIRST}^{+}(A \rightarrow \beta_j) = \phi, \qquad \forall 1 \leq i, j \leq n, i \neq j$$

Backtracking

$Start \rightarrow Expr$

$Expr \rightarrow Term Expr'$

$Expr' \rightarrow +Term Expr'$

$\quad |-Term Expr' \mid \epsilon$

$Term \rightarrow Factor Term'$

$Term' \rightarrow \times Factor Term'$

$\quad |\div Factor Term' \mid \epsilon$

$Factor \rightarrow$ name

$\quad |$ name [$Arglist$]

$\quad |$ name ($Arglist$)

$Arglist \rightarrow Expr \; MoreArgs$

$MoreArgs \rightarrow , Expr \; MoreArgs$

$\quad \mid \epsilon$

Backtracking

$Start \rightarrow Expr$

$Expr \rightarrow Term Expr'$

$Expr' \rightarrow +Term Expr'$

$\quad | -Term Expr' \mid \epsilon$

$Term \rightarrow Factor Term'$

$Term' \rightarrow \times Factor Term'$

$\quad | \div Factor Term' \mid \epsilon$

$Factor \rightarrow$ name

| name [$Arglist$]

| name ($Arglist$)

$Arglist \rightarrow Expr \; MoreArgs$

$MoreArgs \rightarrow , Expr \; MoreArgs$

$\quad | \; \epsilon$

Not all grammars are backtrack free

Left Factoring

- Left factoring is the process of extracting and isolating common prefixes in a set of productions

$$Factor \rightarrow name\ Arguments$$
$$Arguments \rightarrow [\ ArgList\ ]\ |\ (\ ArgList\ )\ |\ \epsilon$$

- Algorithm

$$A \rightarrow \alpha\beta_1|\alpha\beta_2|\ \dots\ |\alpha\beta_n|\gamma_1|\gamma_2|\ \dots\ |\gamma_j$$

$$\Downarrow$$

$$A \rightarrow \alpha B|\gamma_1|\gamma_2|\ \dots\ |\gamma_j$$
$$B \rightarrow \beta_1|\beta_2|\ \dots\ |\beta_n$$

Key Insight in Using Top-Down Parsing

- Efficiency depends on the accuracy of selecting the correct production for expanding a nonterminal
  - Parser may not terminate in the worst case
- A large subset of the context-free grammars can be parsed without backtracking

Recursive-Descent Parsing

Recursive-Descent Parsing

- Recursive-descent parsing is a form of top-down parsing that **may** require backtracking
- Consists of a set of procedures, one for each nonterminal

```
void A() {
    Choose an A-production A → X₁X₂ … Xₖ
    for i ← 1 … k
        if Xᵢ is a nonterminal
            call procedure Xᵢ()
        else if Xᵢ equals the current input symbol a
            advance the input to the next symbol
        else
            // error
}
```

Limitations with Recursive-Descent Parsing

- Consider a grammar with two productions $X \rightarrow \gamma_1$ and $X \rightarrow \gamma_2$
- Suppose $\mathrm{FIRST}(\gamma_1) \cap \mathrm{FIRST}(\gamma_2) \neq \phi$
  - Say a is the common terminal symbol
- Function corresponding to X will not know which production to use on input token a

Recursive-Descent Parsing with Backtracking

- To support backtracking
  - All productions should be tried in some order
  - Failure for some production implies we need to try remaining productions
  - Report an error only when there are no other rules

Predictive Parsing

- Special case of recursive-descent parsing that does not require backtracking
  - Lookahead symbol unambiguously determines which production rule to use
  - Advantage is that the algorithm is simple and the parser can be constructed by hand

$$
\begin{aligned}
stmt \rightarrow\ & \textbf{expr}\ ; \\
\mid\ & \textbf{if}\ (\ expr\ )\ stmt \\
\mid\ & \textbf{for}\ (\ optexpr\ ;\ optexpr\ ;\ optexpr\ )\ stmt \\
\mid\ & \textbf{other} \\
optexpr \rightarrow\ & \epsilon \mid \textbf{expr}
\end{aligned}
$$

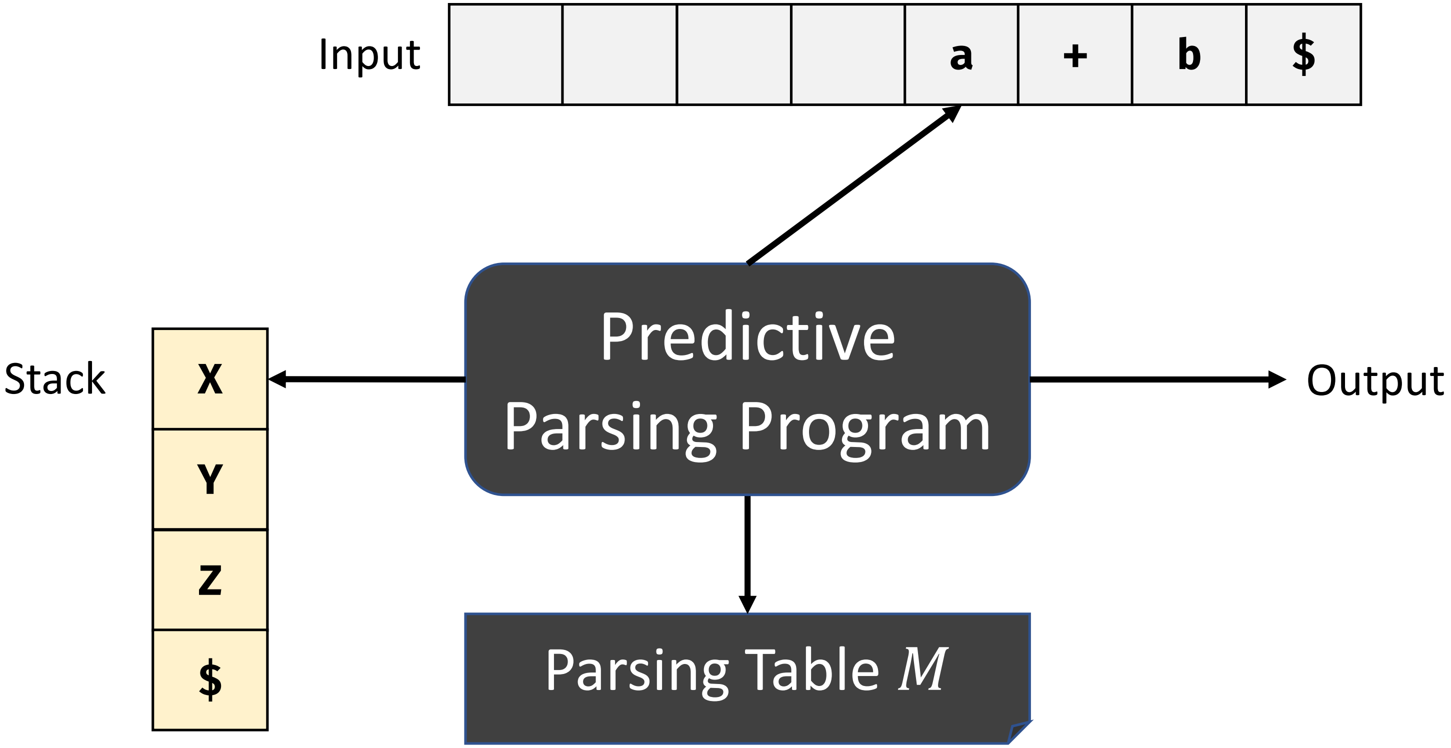
Pseudocode for a Predictive Parser

```
void stmt() {
  switch(lookahead) {
    case expr:
      match(expr); match(‘;’); break;
    case if:
      match(if); match(‘(‘); match(expr); match(‘)’); stmt(); break;
    case for:
      match(for); match(‘(‘); optexpr(); match(‘;’); optexpr();
      match(‘;’); optexpr(); match(‘)’); stmt(); break;
    case other:
      match(other); break;
    default:
      report(“syntax error”);
  }
}
```

LL(1) Grammars

- Class of grammars for which no backtracking is required
  - First L stands for left-to-right scan, second L stands for leftmost derivation
  - There is one lookahead token
- No left-recursive or ambiguous grammar can be LL(1)
- In LL(k), k stands for k lookahead tokens
  - Predictive parsers accept LL(k) grammars
  - Every LL(1) grammar is a LL(2) grammar

Nonrecursive Table-Driven Predictive Parser



Predictive Parsing Algorithm

- **Input**: String w and parsing table M for grammar G
- **Algorithm**:

Let a be the first symbol in w
Let X be the symbol at the top of the stack
while $X \neq \$$:
    if $X == a$:
        pop the stack and advance the input
    else if X is a terminal or $M[X, a]$ is an error entry:
        error
    else if $M[X, a] == X \rightarrow Y_1 Y_2 \ldots Y_k$:
        output the production
        pop the stack
        push $Y_k Y_{k-1} \ldots Y_1$ onto the stack
    $X \leftarrow$ top stack symbol

Predictive Parsing Table

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \epsilon$
$T \rightarrow FT'$
$T' \rightarrow * FT' \mid \epsilon$
$F \rightarrow (E) \mid \textbf{id}$

Nonterminal	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow$ id			$F \rightarrow (E)$		

Construction of a Predictive Parsing Table

- **Input**: Grammar G

- **Algorithm**:
  - For each production $A \rightarrow \alpha$ in G,
    - For each terminal a in $\mathrm{FIRST}(\alpha)$, add $A \rightarrow \alpha$ to $M[A, a]$
    - If ϵ is in $\mathrm{FIRST}(\alpha)$, then for each terminal b in $\mathrm{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, b]$
    - If ϵ is in $\mathrm{FIRST}(\alpha)$ and \$ is in $\mathrm{FOLLOW}(A)$, add $A \rightarrow \alpha$ to $M[A, a]$
    - No production in $M[A, a]$ indicates error

Working of Predictive Parser

Matched	Stack	Input	Action
	$E\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	
	$TE'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	Output $E \rightarrow TE'$
	$FT'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	Output $T \rightarrow FT'$
	$\textbf{id}T'E'\$$	$\textbf{id} + \textbf{id} * \textbf{id}\$$	Output $F \rightarrow \textbf{id}$
id	$T'E'\$$	$+\textbf{id} * \textbf{id}\$$	Match **id**
id	$E'\$$	$+\textbf{id} * \textbf{id}\$$	Output $T' \rightarrow \epsilon$
id	$+TE'\$$	$+\textbf{id} * \textbf{id}\$$	Output $E' \rightarrow +TE'$
id $+$	$TE'\$$	$\textbf{id} * \textbf{id}\$$	Match $+$
id $+$	$FT'E'\$$	$\textbf{id} * \textbf{id}\$$	Output $T \rightarrow FT'$
id $+$	$\textbf{idT}'E'\$$	$\textbf{id} * \textbf{id}\$$	Output $F \rightarrow \textbf{id}$

Working of Predictive Parser

Matched	Stack	Input	Action
…			
id $+$	**id**$T'E'\$$	**id** $*$ **id**$\$$	Output $F \rightarrow$ **id**
id $+$ **id**	$T'E'\$$	$*$ **id**$\$$	Match **id**
id $+$ **id**	$* FT'E'\$$	$*$ **id**$\$$	Output $T' \rightarrow * FT'$
id $+$ **id**$*$	$FT'E'\$$	**id**$\$$	Match $*$
id $+$ **id**$*$	**id**$T'E'\$$	**id**$\$$	Output $F \rightarrow$ **id**
id $+$ **id**$*$**id**	$T'E'\$$	$\$$	Match **id**
id $+$ **id**$*$**id**	$E'\$$	$\$$	Output $T' \rightarrow \epsilon$
id $+$ **id**$*$**id**	$\$$	$\$$	Output $E' \rightarrow \epsilon$

Predictive Parsing

- Grammars whose predictive parsing tables contain no duplicate entries are called LL(1)

- If grammar G is left-recursive or is ambiguous, then parsing table M will have at least one multiply-defined cell
- Some grammars cannot be transformed into LL(1)
  - The adjacent grammar is ambiguous

$$S \rightarrow iEtSS' \mid a$$
$$S' \rightarrow eS \mid \epsilon$$
$$E \rightarrow b$$

Predictive Parsing Table

$S \rightarrow iEtSS' \mid a$
$S' \rightarrow eS \mid \epsilon$
$E \rightarrow b$

Nonterminal	a	b	e	i	t	$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow \epsilon$ $S' \rightarrow eS$			$S' \rightarrow \epsilon$
E		$E \rightarrow b$		$T \rightarrow FT'$		

Error Recovery in Predictive Parsing

- Error conditions
  - Terminal on top of the stack does not match the next input symbol
  - Nonterminal A is on top of the stack, a is the next input symbol, and $M[A, a]$ is error
- Choices
  - Raise an error and quit parsing
  - Print an error message, try to recover from the error, and continue with compilation

Error Recovery in Predictive Parsing

- Panic mode – skip over symbols until a token in a set of synchronizing (synch) tokens appears
  - Add all tokens in $\text{FOLLOW}(A)$ to the synch set for A
  - Add symbols in $\text{FIRST}(A)$ to the synch set for A
  - Add keywords that can begin sentences
  - …

Predictive Parsing Table with Synchronizing Tokens

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \epsilon$
$T \rightarrow FT'$
$T' \rightarrow * FT' \mid \epsilon$
$F \rightarrow (E) \mid \textbf{id}$

Nonterminal	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$	synch	synch
E'		$E' \rightarrow +TE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	synch		$T \rightarrow FT'$	synch	synch
T'		$T' \rightarrow \epsilon$	$T' \rightarrow * FT'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow$ id	synch	synch	$F \rightarrow (E)$	synch	synch

Error Recover Moves by Predictive Parser

Stack	Input	Remark
$E\$$	$)\mathbf{id} * +\mathbf{id}\$$	Error, skip)
$E\$$	$\mathbf{id} * +\mathbf{id}\$$	**id** is in $\mathrm{FIRST}(E)$
$TE'\$$	$\mathbf{id} * +\mathbf{id}\$$	
$FTE'\$$	$\mathbf{id} * +\mathbf{id}\$$	
$\mathbf{id}TE'\$$	$\mathbf{id} * +\mathbf{id}\$$	
$T'E'\$$	$* +\mathbf{id}\$$	
$* FT'E'\$$	$* +\mathbf{id}\$$	
$FT'E'\$$	$+\mathbf{id}\$$	Error, $M[F, +] = \mathrm{synch}$
$T'E'\$$	$+\mathbf{id}\$$	F has been popped
$E'\$$	$+\mathbf{id}\$$	

Error Recover Moves by Predictive Parser

Stack	Input	Remark
$+TE'\$$	$+\mathbf{id}\$$	
$TE'\$$	$\mathbf{id}\$$	
$FT'E'\$$	$\mathbf{id}\$$	
$\mathbf{id}T'E'\$$	$\mathbf{id}\$$	
$T'E'\$$	$\$$	
$E'\$$	$\$$	
$\$$	$\$$	

References

- A. Aho et al. Compilers: Principles, Techniques, and Tools, 2nd edition, Chapter 4.4.
- K. Cooper and L. Torczon. Engineering a Compiler, 2nd edition, Chapter 3.3.