

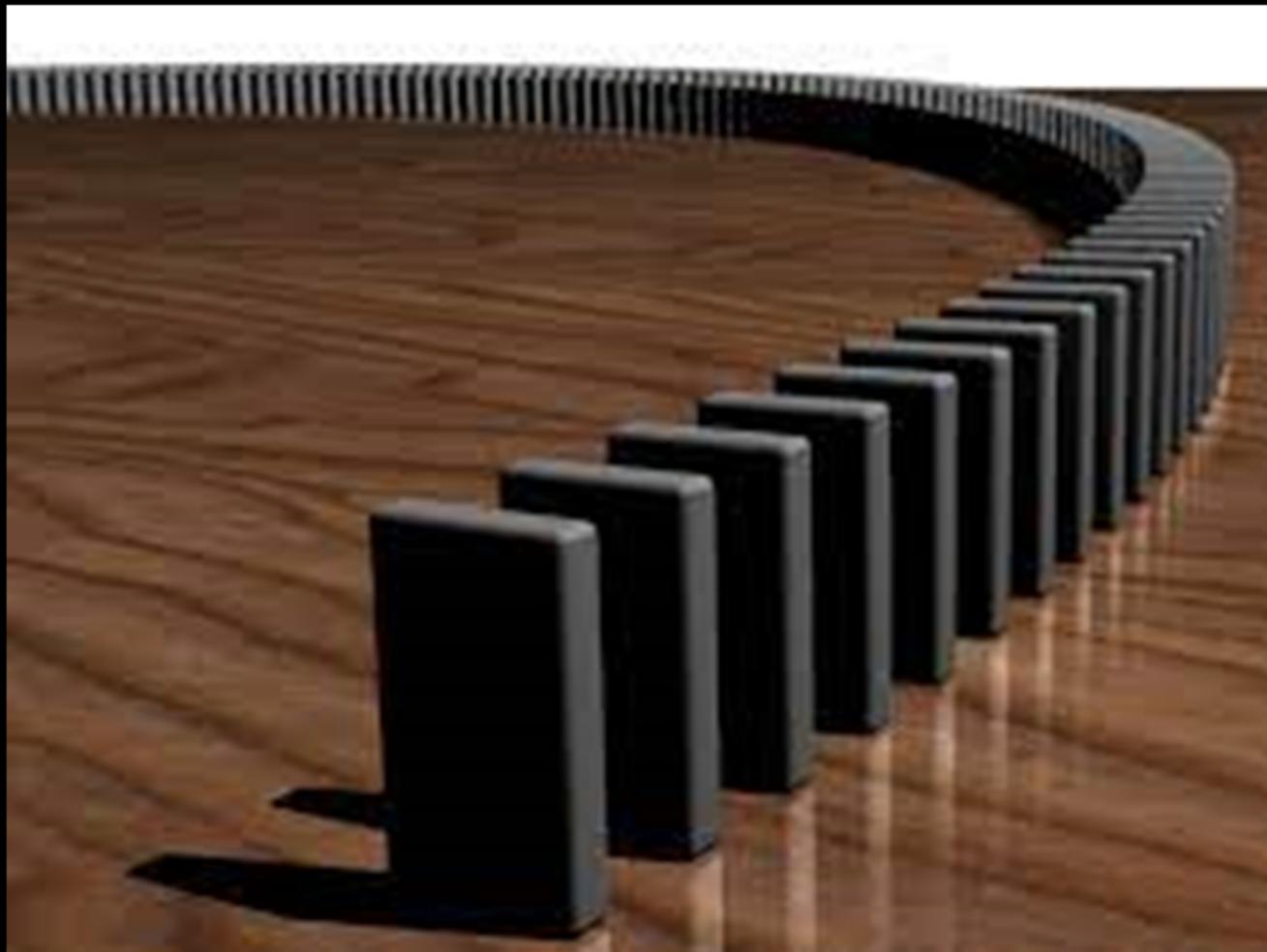


The proof of Mathematical Induction is in book(P-36)

Proof using
Mathematical Induction:

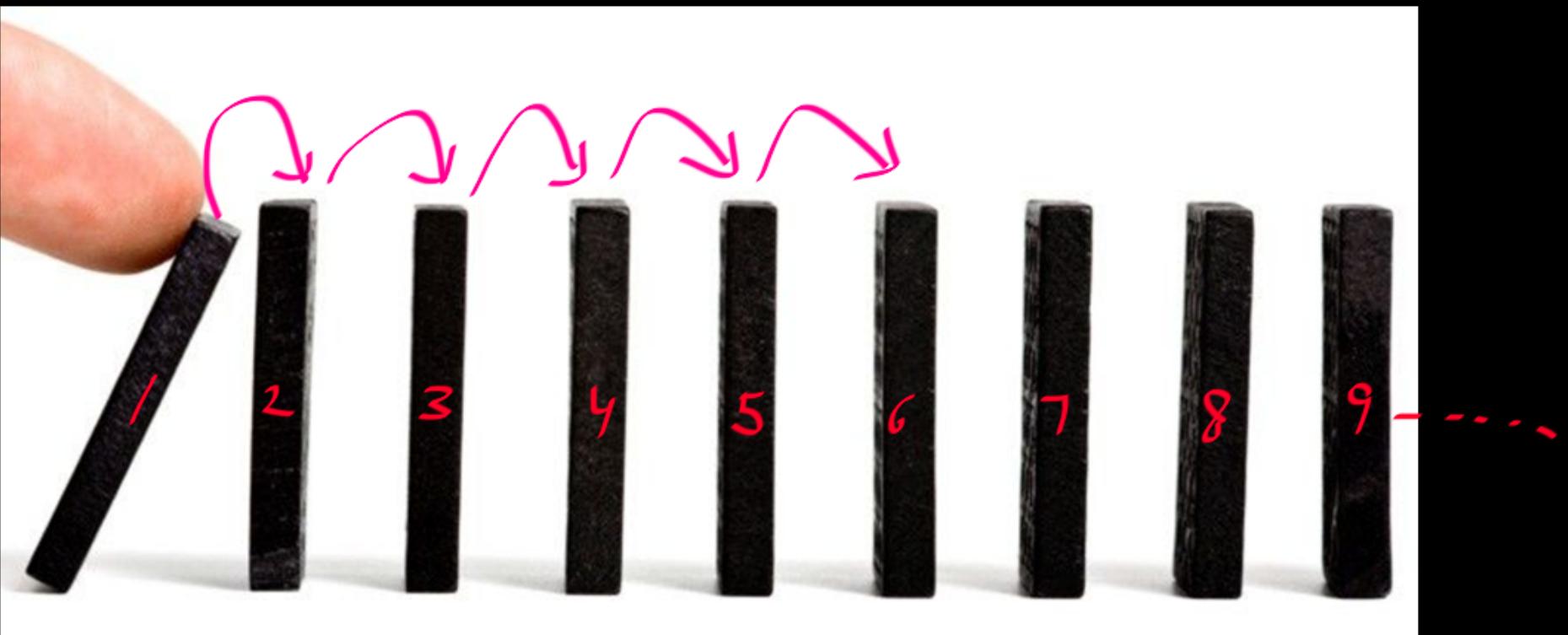


Discrete Mathematics





Discrete Mathematics

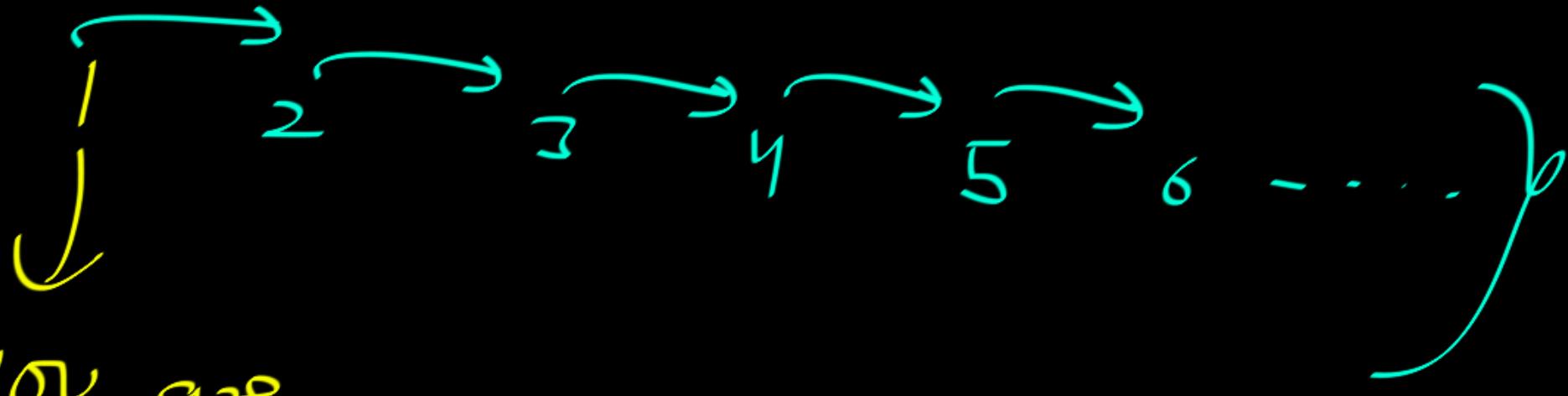




Domino Effect

Why all Dominos are falling ?

- ① Pushed first Domino } Starting Point
- ② k^{th} Domino is pushing $(k+1)^{\text{th}}$ Domino.
for every $k \geq 1$

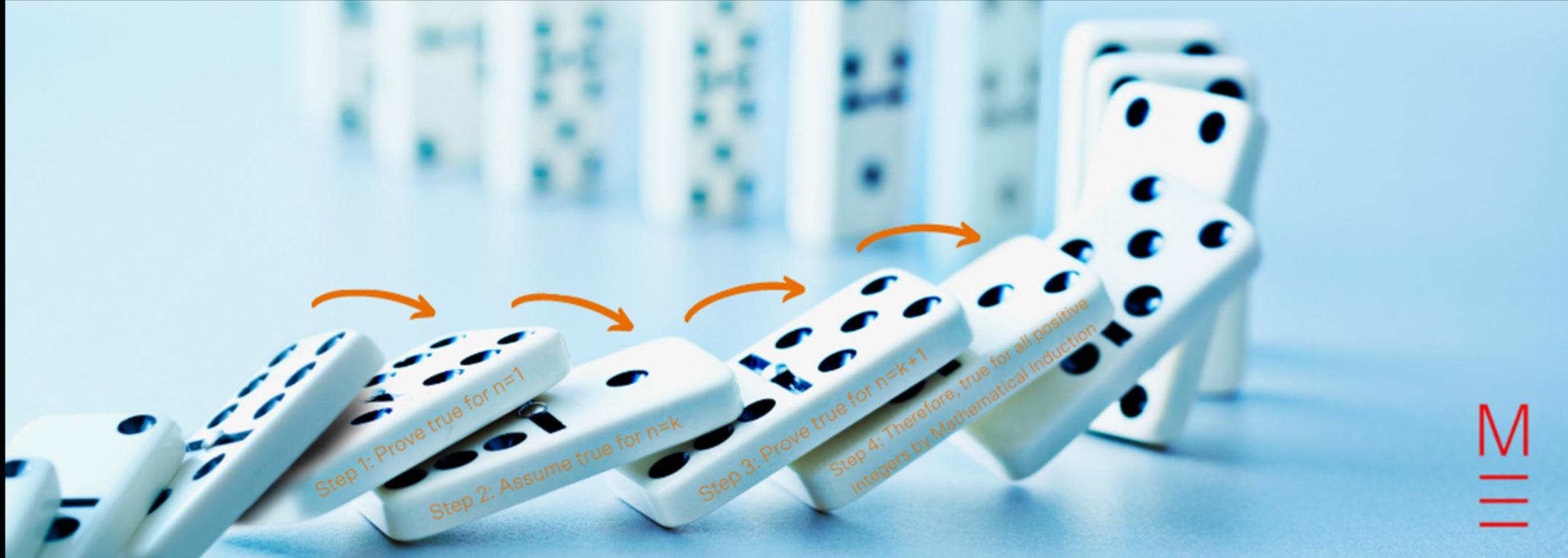


You are
pushing

So all
Dominos fall.

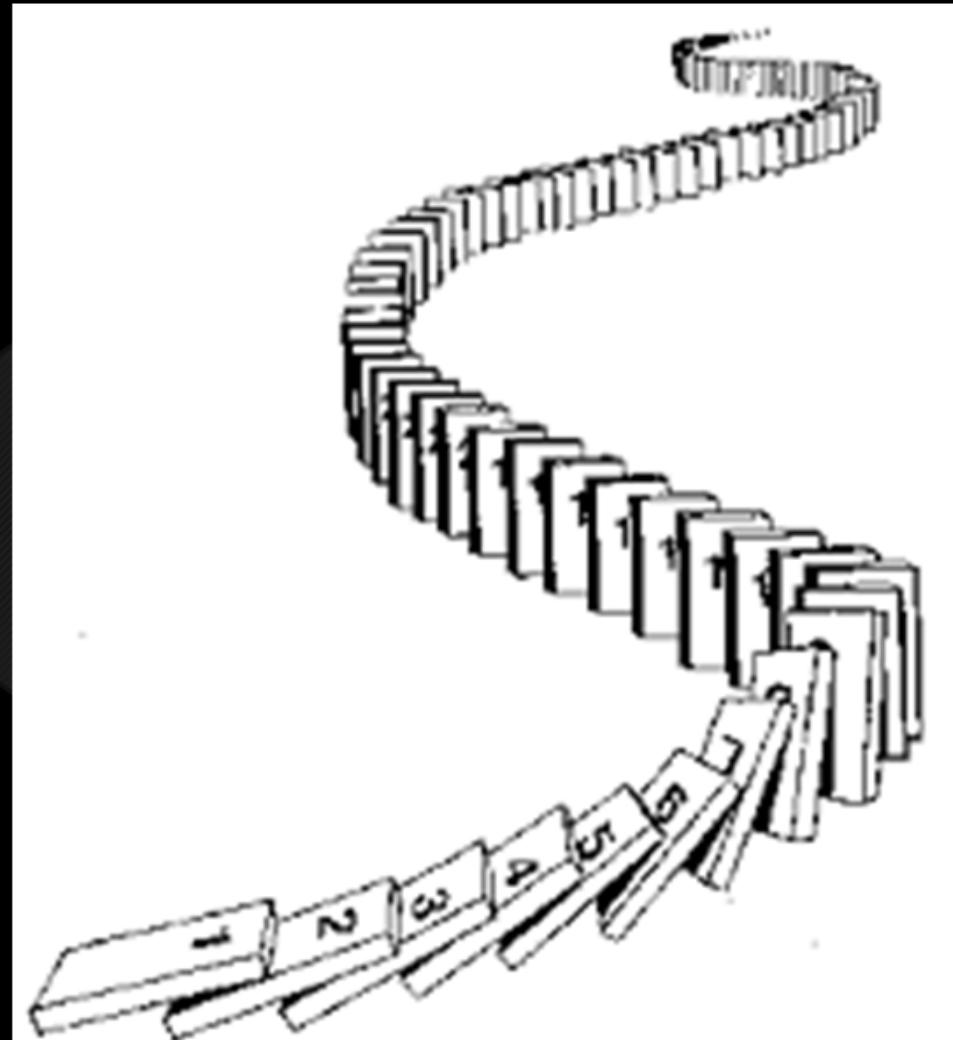


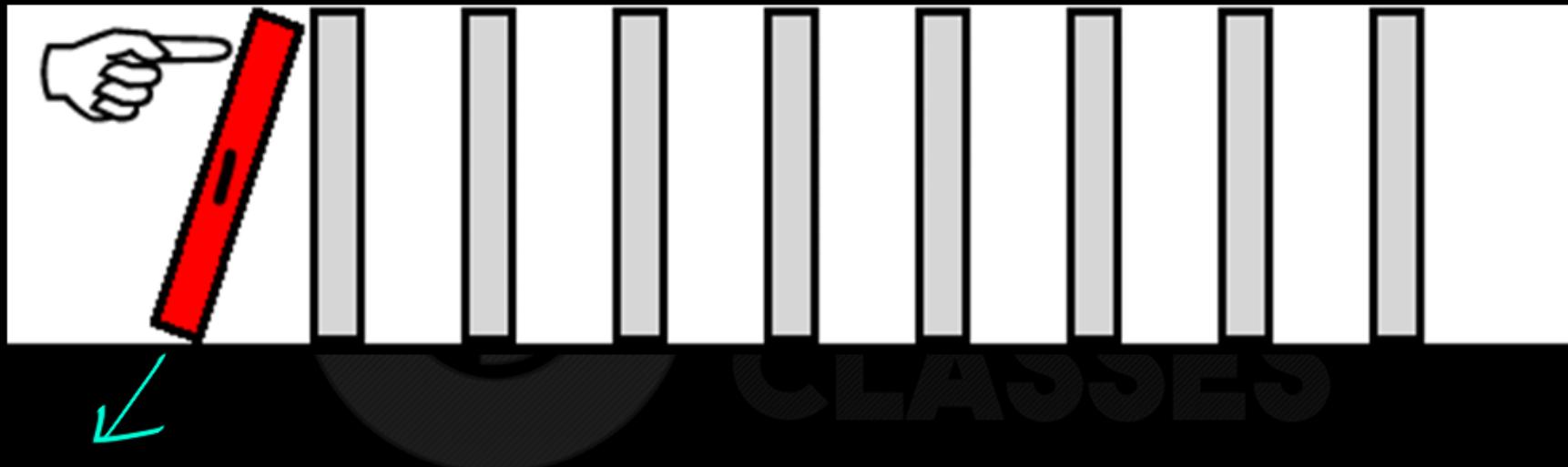
Discrete Mathematics





Discrete Mathematics

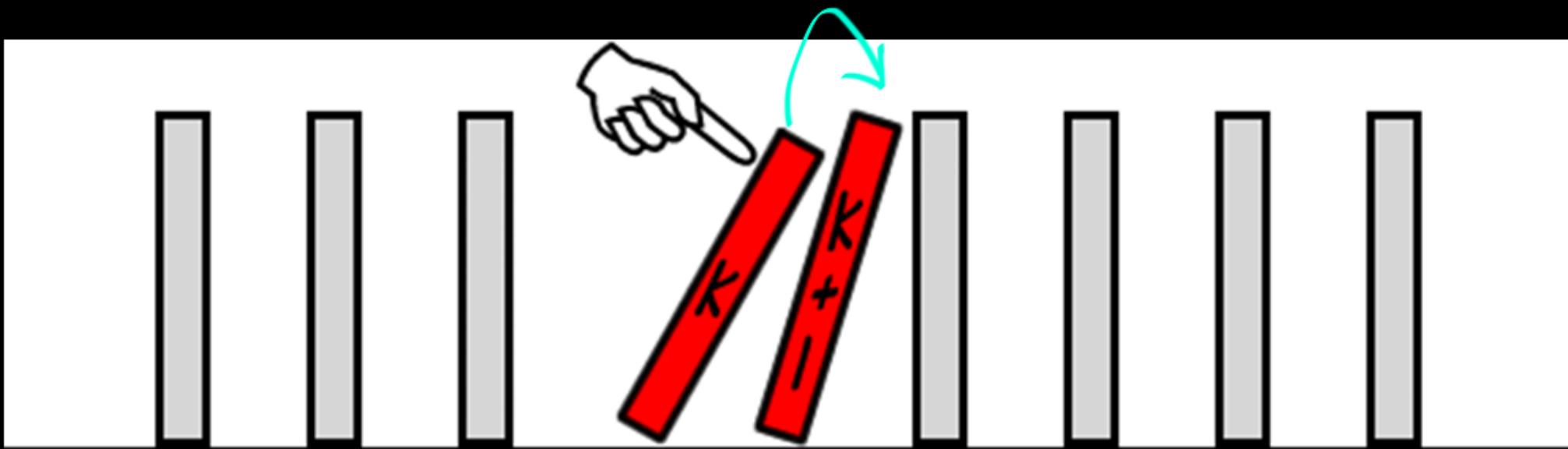




Starting
Point

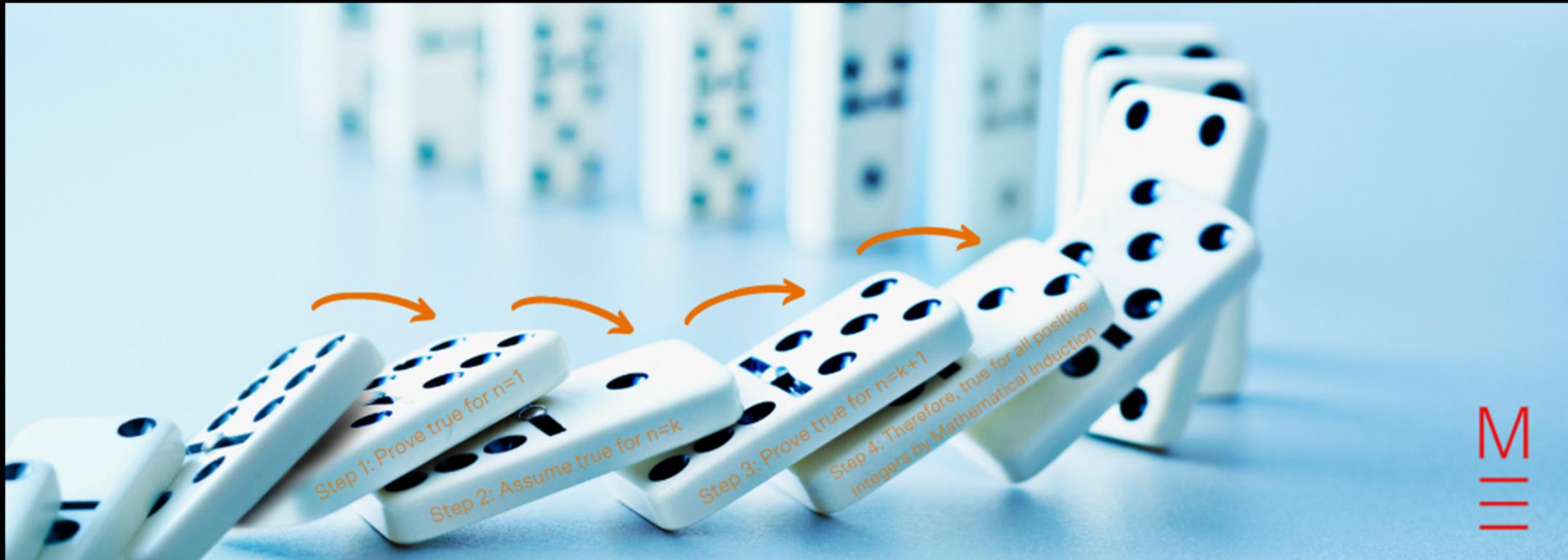


Discrete Mathematics





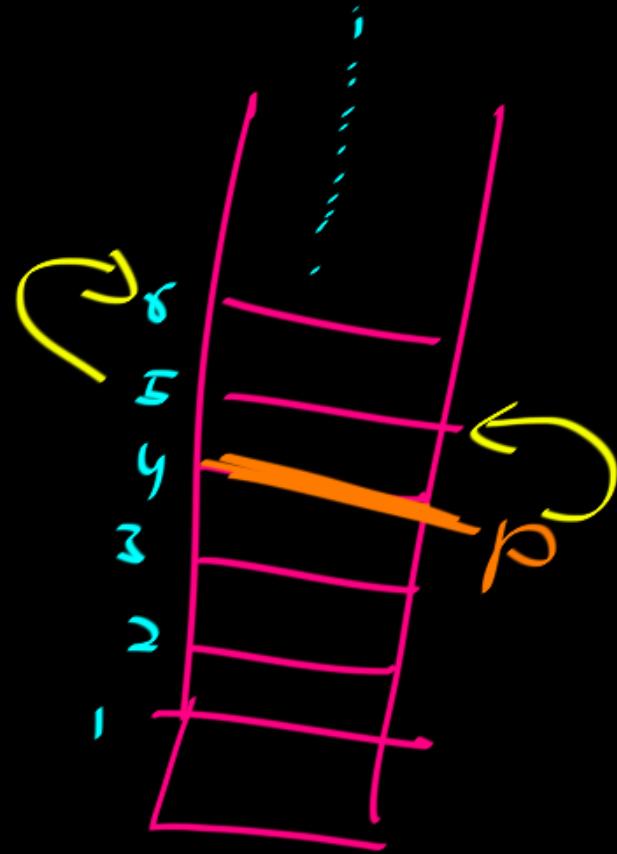
Discrete Mathematics



 (P)

① Can climb on 1st step

② If P is on k^{th} step
P can go to $k+1^{\text{th}}$ step } for every k



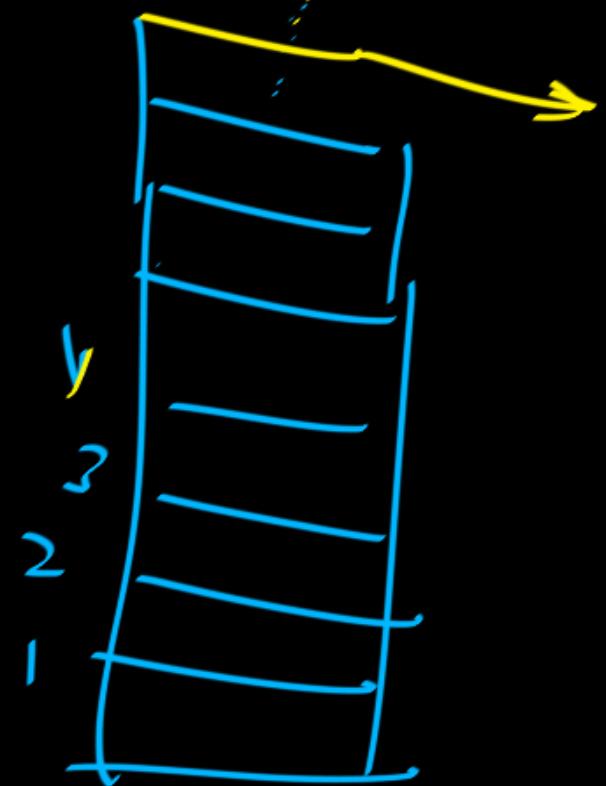
Q: Can ρ go to 100000^{th} step?

Yes.

Q: Can ρ go to $(\underbrace{99999999}_{\text{step 7}})^{99999}$

Yes

Q: Can ρ go to "0th" step?



(999999 99999) 9999

So ρ can go any step n

Where n is natural number.

$$1 \xrightarrow{\curvearrowright} 2 \xrightarrow{\curvearrowright} 3 \xrightarrow{\curvearrowright} 4 \xrightarrow{\curvearrowright} \dots (99) \xrightarrow{\curvearrowright} (99) + 1 \xrightarrow{\curvearrowright} \dots$$

" ∞ " is NOT a number.

∞ is "a Natural number.

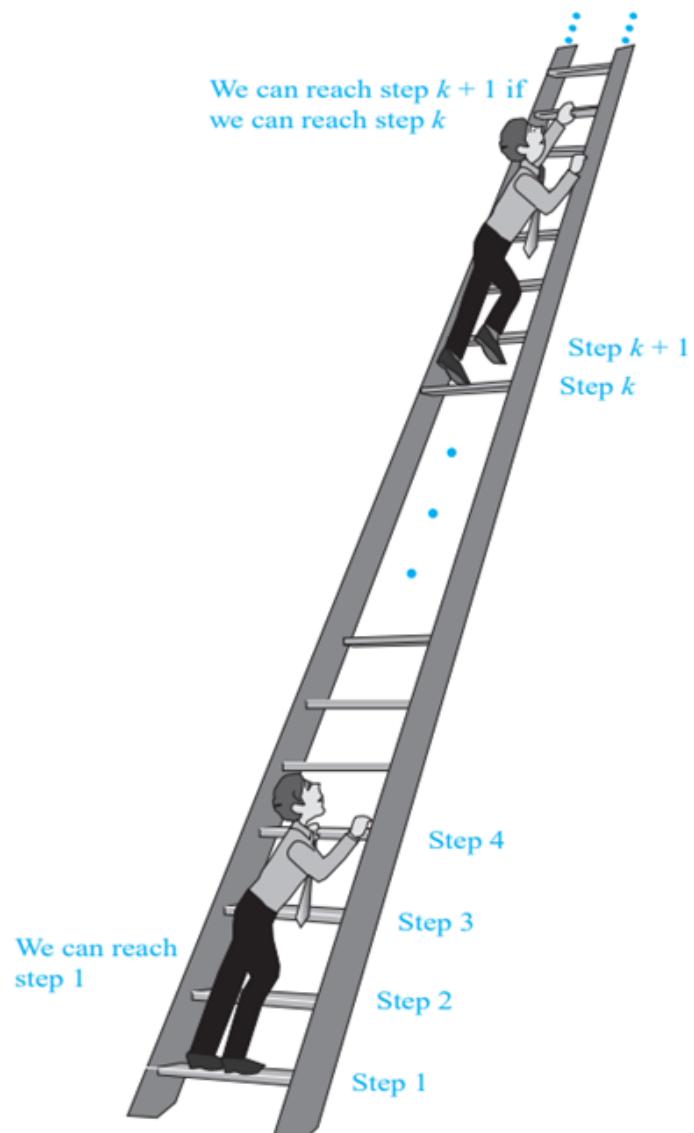


FIGURE 1 Climbing an Infinite Ladder.



Mathematical Induction

Introduction

Suppose that we have an infinite ladder, as shown in Figure 1, and we want to know whether we can reach every step on this ladder. We know two things:

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Can we conclude that we can reach every rung? By (1), we know that we can reach the first rung of the ladder. Moreover, because we can reach the first rung, by (2), we can also reach the second rung; it is the next rung after the first rung. Applying (2) again, because we can reach the second rung, we can also reach the third rung. Continuing in this way, we can show that we



can reach the fourth rung, the fifth rung, and so on. For example, after 100 uses of (2), we know that we can reach the 101st rung. But can we conclude that we are able to reach every rung of this infinite ladder? The answer is yes, something we can verify using an important proof technique called **mathematical induction**. That is, we can show that $P(n)$ is true for every positive integer n , where $P(n)$ is the statement that we can reach the n th rung of the ladder.





To prove:

$P(n)$

for every
 $n \in \mathbb{N}$

Proof by Induction:

① Starting Point : $P(1)$

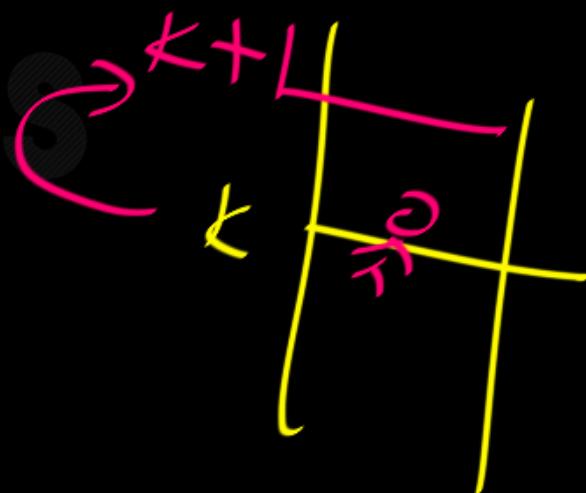
(Base case)

manually check

② Assume $P(k)$ is True

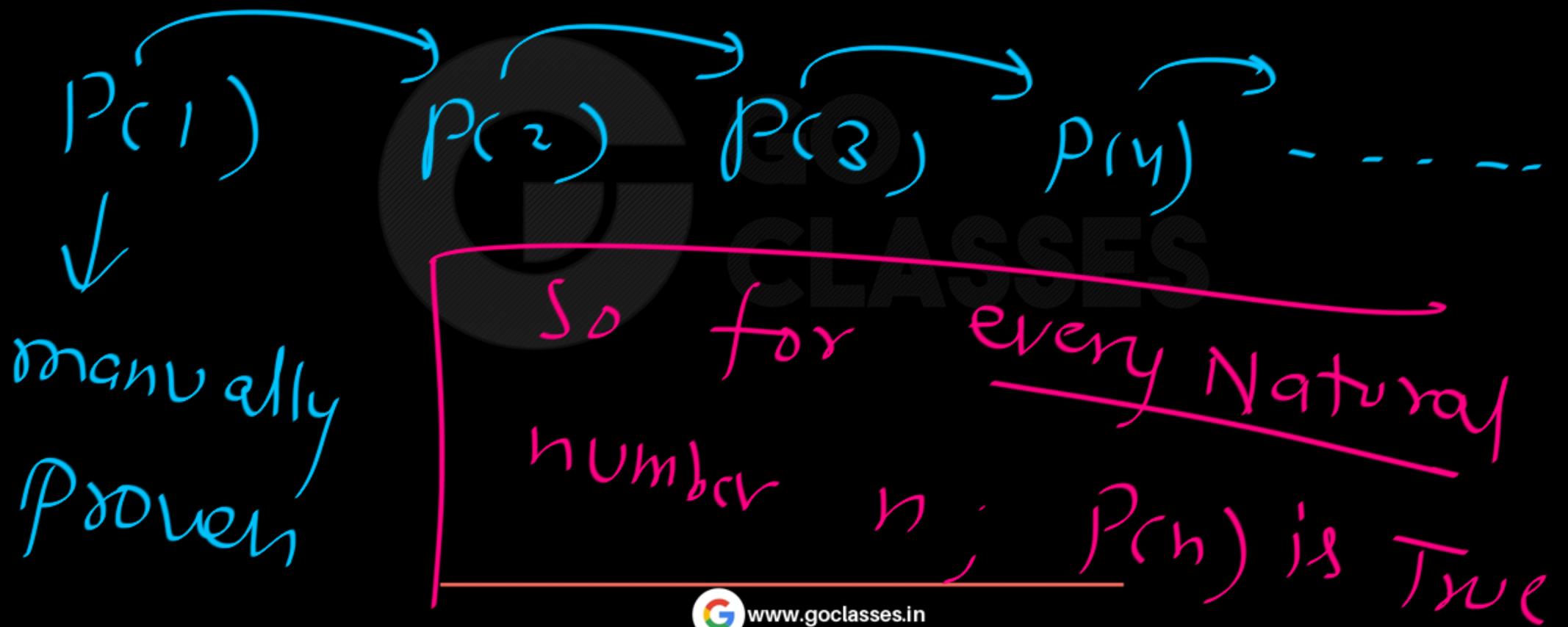
③ Now Prove that

$P(k+1)$





If we can do this then





Mathematical Induction





The principle of mathematical induction states that if for some property $P(n)$, we have that

P(0) is true

and

For any natural number n , $P(n) \rightarrow P(n + 1)$

Then

For any natural number n , $P(n)$ is true.



The principle of mathematical induction states that if for some property $P(n)$, we have that

P(0) is true

If it starts...

and

For any natural number n , $P(n) \rightarrow P(n + 1)$

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The principle of mathematical induction states that if for some property $P(n)$, we have that

If it starts...

P(0) is true

and

and it keeps
going...

For any natural number n , $P(n) \rightarrow P(n + 1)$

Then

For any natural number n , $P(n)$ is true.

The principle of mathematical induction states that if for some property $P(n)$, we have that

P(0) is true

If it starts...

and it keeps
going...

and

For any natural number n , $P(n) \rightarrow P(n + 1)$

...then it's
always true

Then

For any natural number n , $P(n)$ is true.



Induction, Intuitively

- It's true for 0.
- Since it's true for 0, it's true for 1.
- Since it's true for 1, it's true for 2.
- Since it's true for 2, it's true for 3.
- Since it's true for 3, it's true for 4.
- Since it's true for 4, it's true for 5.
- Since it's true for 5, it's true for 6.
- ...



Proof by Induction

- Suppose that you want to prove that some property $P(n)$ holds of all natural numbers. To do so:
- Prove that $P(0)$ is true.
 - This is called the **basis** or the **base case**.
- Prove that for any natural number n , if $P(n)$ is true, then $P(n + 1)$ is true as well.
 - This is called the **inductive step**.
 - $P(n)$ is called the **inductive hypothesis**.
- Conclude by induction that $P(n)$ holds for all n .



Mathematical Induction

To prove $P(x)$ is true for $x \in \mathbb{Z}^+$, where $P(x)$ is a propositional function, we complete two steps :

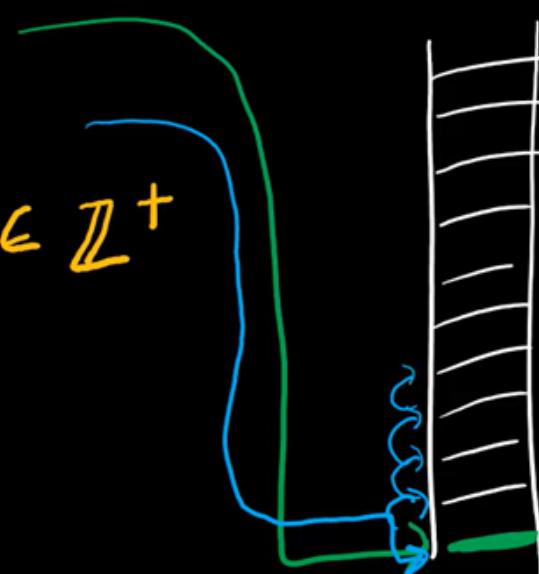
1) Basis step - verify $P(1)$ is true

2) Inductive step - verify if $P(k)$ is true, then $P(k+1)$ is true $\forall k \in \mathbb{Z}^+$

Inductive hypothesis: $P(k)$ is true

Must show: $P(k) \rightarrow P(k+1)$

Conclusion: $P(x)$ is true $\forall k \in \mathbb{Z}^+$





Proving a Summation Formula

Show $1+2+3+\dots+n = \frac{n(n+1)}{2}$





Proving a Summation Formula

Show $1+2+3+\dots+n = \frac{n(n+1)}{2}$

$P(n)$: What you want to prove.

$P(n)$:
$$\boxed{1+2+3+\dots+n = \frac{n(n+1)}{2}}$$



To Prove : $P(n)$ for any $n \in \mathbb{N}$

Base Case : for $n = 1$

$$P(1) : 1 = \frac{1(1+1)}{2} \quad \checkmark$$



$$P(n) : \underbrace{1 + 2 + 3 + 4 + \dots + n}_{n \text{ terms}} = \frac{n(n+1)}{2}$$

$$P(1) : 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

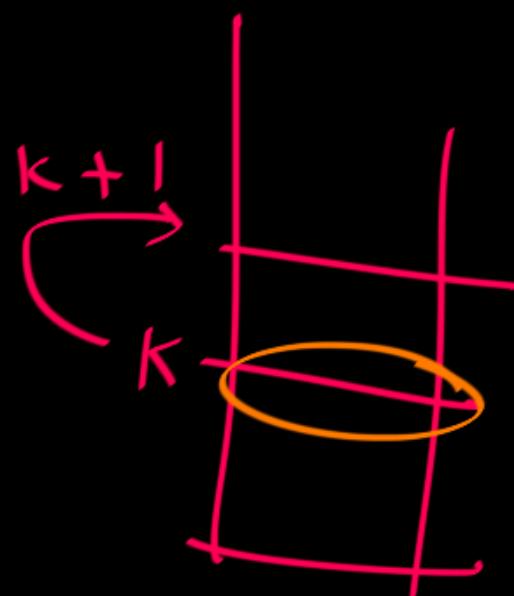
$$P(2) : \underbrace{1 + 2}_{3} = \boxed{\frac{2(2+1)}{2}} = 3$$



Assume $P(k)$ is True.

So

$$1+2+\dots+k = \frac{k(k+1)}{2}$$





Aim:

$P(k)$

$\rightarrow P(k+1)$

Assumed
to be true

Prove this

To prove :

$$\underline{P(k+1)} : 1+2+\dots+k+k+1 = \frac{(k+1)(k+1+1)}{2}$$

LHS :

$$\boxed{1 + 2 + 3 + \dots + k} + k + 1$$

$$\frac{k(k+1)}{2} + k + 1$$

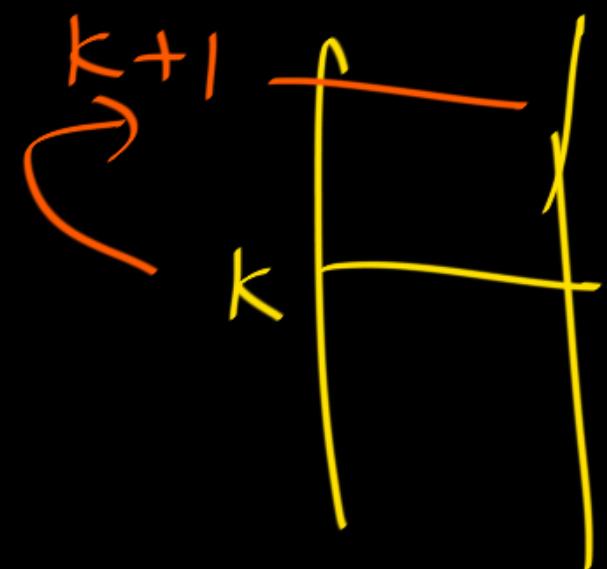
$$= \frac{k^2 + k + 2k + 2}{2} = \boxed{\frac{(k+1)(k+2)}{2}}$$

RHS

So we proved

$$P(k) \rightsquigarrow P(k+1)$$

$$\overbrace{P(1)}^{\curvearrowright} \overbrace{P(2)}^{\curvearrowright} \overbrace{P(3)}^{\curvearrowright} \overbrace{P(4)}^{\downarrow} \\ \cdots \leftarrow \overbrace{P(5)}^{}$$



So for any $n \in N$;

$$P(n) : 1+2+\dots+n = \frac{n(n+1)}{2}$$

is True.



To prove $\underbrace{P(n)}$

Starting Point $P(1)$ ✓

Now prove that If $P(k)$
is True then $P(k+1)$ is True.

- ① $P(k) \rightarrow P(k+1)$
- ② $P(1)$  $P(n)$ for
any $n \in \mathbb{N}$



Use mathematical induction to show that for all non-neg integers n ,

$$1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$$

for all non-negative integers;

$P(n)$

$$1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$$

What you want to prove



Starting Point : (base case)

$n = 0$



$$P(n): 1 + 2^1 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 1$$

n+1 terms

$$P(0) : \underline{\underline{LHS = 1}} ; \underline{\underline{RHS = 2^0 + 1 - 1 = 1}}$$

0+1 term



P(0) ✓

Assume

P(k) is True

$$1 + 2 + 2^2 + \dots + 2^k = 2^{k+1} - 1$$





Prove that $P(k+1)$ is True.

$P(k+1) :$

$$1 + 2 + 2^2 + \dots + 2^k + 2^{k+1} = 2^{k+1+1} - 1$$

To Prove

RHS



LHS :

$$\left[1 + 2 + 2^2 + 2^3 + \dots + 2^k \right] + 2^{k+1}$$

$$2^{k+1} - 1 + 2^{k+1}$$

RHS

$$= 2 \cdot 2^{k+1} - 1 = \boxed{2^{k+2} - 1}$$



So we showed ;

① $P(0)$ ✓

② If $\underline{P(k)}$ is True then

$P(k+1)$ is True .

S_D

$$P(0) \xrightarrow{} P(1) \xrightarrow{} P(2) \xrightarrow{} P(3) \dots$$

\therefore , for every whole number n ,

$P(n)$ is True.



Conjecture and prove a summation formula for the sum of the first n positive odd integers.





Proving an Inequality:

Prove $n < 2^n \quad \forall n \in \mathbb{Z}^+$ using mathematical induction.





Proving an Inequality:

Prove $n < 2^n \quad \forall n \in \mathbb{Z}^+$ using mathematical induction.

Let $P(n) : n < 2^n \quad \forall n \in \mathbb{Z}^+$

① Basis: $P(1)$

$$P(1) : 1 < 2^1 \\ 1 < 2 \checkmark$$

② Inductive $P(k) \rightarrow P(k+1)$

$$\text{IH: } k < 2^k \quad | < 2^k \quad \forall k \in \mathbb{Z}^+$$

$$\text{show: } k+1 < 2^{k+1}$$

$$k+1 < 2^k + 1 < \overbrace{2^k + 2^k} = 2(2^k)$$

$$k+1 < 2^{k+1}$$

□



Prove $2^n < n!$ $\forall n \in \mathbb{Z}^+$ and $n \geq 4$.





Q :

Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

