

Indian Statistical Institute

Image Processing - I

M.Tech.(CS): 2021-2022

Full marks: 50

Time: 2 Hours

Date: 06.04.2022

Answer any **five** questions. All questions carry equal marks.

1. (a) What is a point spread function?

(b) Derive the following equation

$$g(\alpha, \beta) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) h(x, \alpha, y, \beta)$$

where $f(x, y)$ and $g(\alpha, \beta)$ are the $N \times N$ input image and output image, respectively, and h is a point spread function.

(c) What will be the output if h is (i) shift invariant; (ii) separable; and (iii) both shift invariant and separable? [2+4+(1+1+2)=10]

2. (a) Show that the stacking operator is linear.

(b) How can a separable transform be written in matrix form?

(c) Write down the derivation of (b) in terms of stacking operator.

[3+3+4=10]

3. Consider a 3-bit image of size 64×64 pixels. It has the intensity distribution as follows:

r_k	0	1	2	3	4	5	6	7
n_k	790	1023	850	656	329	245	122	81

where r_k denotes k -th intensity level and n_k is the number of pixels that have intensity r_k .

(a) Find the transformation function that will map the input intensity values, r , into values, s , of a histogram-equalized image.

(b) Find out the intensity distribution of the histogram-equalized image.

[7+3=10]

4. The histogram of an image can be approximated by the probability density function

$$p_r(r) = Ae^{-r},$$

where r is the grey level variable taking values between 0 and b , and A is a normalizing factor. Calculate the transformation $s = T(r)$, where s is grey level value in the transformed image, such that the transformed image has the probability density function

$$p_s(s) = Bse^{-s^2},$$

where s takes values between 0 and b , and B is some normalizing factor.

[10]

5. (a) Prove that the differentiation of the output of a convolution, of a signal with a filter, can be achieved by convolving the signal with the derivative of that filter.

(b) Suppose an image is filtered with the Laplacian kernel. Prove that the sum of the pixel values in the filtered image is 0.

(c) Define difference of Gaussian (DoG). [3+4+3=10]

6. (a) How can we choose the weights of a 3×3 mask for edge detection?

(b) What is Sobel operator?

(c) Write down the derivation of Sobel operator. [3+2+5=10]

7. (a) Define Laplacian of a Gaussian (LoG).

(b) Describe Marr-Hildreth edge detection algorithm.

(c) Write an algorithm to find zero-crossings of an image. [2+4+4=10]