Indian Statistical Institute

Image Processing - I

M.Tech.(CS): 2021-2022

Full marks: 100 Time: 3 Hours

Date: 07.06.2022

Answer any **ten** questions. All questions carry equal marks.

1. Assume continuous intensity values, and suppose that the intensity values of an image have the probability density function

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image, where $L-1$ represents the maximum intensity value.

(b) Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity probability density function is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{for } 0 \le z \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

(c) Express the transformation function from (b) directly in terms of r, the intensities of input image. [3+4+3=10]

2. The gray values of the object and the background pixels are distributed according to the probability density function

$$p(x) = \begin{cases} \frac{3}{4a^3}[a^2 - (x-b)^2] & \text{for } b-a \le x \le b+a \\ 0 & \text{otherwise} \end{cases}$$

with $a = 1$, $b = 5$ for background, $a = 2$, $b = 7$ for object, and x is the gray value of the pixel.

(a) Sketch the two probability density functions.

(b) If the number of object pixels is eight-nineths (8/9) of the total number of pixels, determine the fraction of misclassified object pixels by optimal thresholding. [3+7=10]

3. (a) Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

(b) An image is filtered with a kernel whose coefficients sum to 1. Show that the sum of the pixel values in the original and filtered images is the same.

(c) Show that the Laplacian is invariant to rotation. [3+2+5=10]

4. (a) Write down three criteria proposed by Canny for optimal edge detection.

(b) Show that to achieve good signal-to-noise ratio (SNR), we must choose an odd filter if the feature we want to enhance is an odd function.

(c) Prove that the partial derivative of a 2-D Gaussian kernel is separable. [3+4+3=10]

5. Consider the following equation

$$\tan^2\theta + \frac{\overline{m}_{20} - \overline{m}_{02}}{\overline{m}_{11}} \tan\theta - 1 = 0$$

where $\overline{m}_{ij}$ denotes the (i,j)-th central moment of an image f and θ represents the slope of the principal axis.

(a) Show that the above equation is equivalent to

$$(\overline{m}_{11}\tan\theta + \overline{m}_{20})^2 - (\overline{m}_{20} + \overline{m}_{02})(\overline{m}_{11}\tan\theta + \overline{m}_{20}) + (\overline{m}_{20}\overline{m}_{02} - \overline{m}_{11}^2) = 0.$$

(b) Hence, show that $(\overline{m}_{11}\tan\theta + \overline{m}_{20})$ is an eigenvalue of the matrix

$$\begin{pmatrix} \overline{m}_{20} & \overline{m}_{11} \\ \overline{m}_{11} & \overline{m}_{02} \end{pmatrix}$$

(c) Show that the principal axis is in the direction of the eigenvector corresponding to the larger eigenvalue of this matrix. [4+3+3=10]

6. Consider the following block of gray levels:

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 2 & 2 & 3 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{pmatrix}$$

Construct the gray level co-occurrence matrices for angle $\theta = 0^\circ$ and 90°, considering unit pixel distance, and compute the angular second moment for each case. [(4+4)+2=10]

7. (a) Consider the block of gray levels of Question 6. Encode the above gray levels with strings of 0's and 1's based on Huffman coding.

(b) Calculate the average code-word length. [8+2=10]

8. (a) Define local binary pattern (LBP) and rotation invariant LBP.

   (b) Consider the following 3×3 block of gray levels:

$$\begin{pmatrix} 6 & 5 & 2 \\ 7 & 6 & 1 \\ 9 & 3 & 7 \end{pmatrix}$$

   Compute LBP and rotation invariant LBP for the central pixel. [(2+2)+(2+4)=10]

9. Consider the following block of gray levels:

$$\begin{pmatrix} 9 & 8 & 2 & 1 \\ 7 & 6 & 2 & 3 \\ 8 & 4 & 3 & 6 \\ 4 & 2 & 7 & 8 \end{pmatrix}$$

   (a) Calculate the compressed and reconstructed representation of the block using Block Truncation Coding.

   (b) Calculate PSNR and bpp. [(4+3)+(2+1)=10]

10. (a) Write down the derivation of Otsu's thresholding method.

    (b) How do you extend Otsu method to obtain multiple thresholds? [7+3=10]

11. (a) Describe the HSI model for color image processing.

    (b) Write down the expressions for converting colors from RGB to HSI models and HSI to RGB models.

    (c) What is pseudocolor image processing? [2+(3+3)+2=10]

12. Write short notes on any **two**:

    (a) Hough transform;

    (b) α-trimmed mean filtering;

    (c) Region growing (splitting and merging). [5+5=10]