Indian Statistical Institute

Image Processing - I

M.Tech.(CS): 2022-2023

Full Marks: 40 Time: 2 Hours

Date: 28.02.2023

Answer any **four** questions. All questions carry equal marks.

1. (a) What is a point spread function?

   (b) Derive the following equation

   $$g(\alpha, \beta) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) h(x, \alpha, y, \beta)$$

   where $f(x, y)$ and $g(\alpha, \beta)$ are the $N \times N$ input image and output image, respectively, and h is a point spread function.

   (c) What will be the output if h is (i) shift invariant; (ii) separable; and (iii) both shift invariant and separable?

   [2+4+(1+1+2)=10]

2. Consider a 3-bit image of size 64×64 pixels. It has the intensity distribution as follows:

r_k	0	1	2	3	4	5	6	7
n_k	81	122	245	329	656	850	1023	790

   where r_k denotes the k-th intensity level and n_k is the number of pixels that have intensity r_k.

   (a) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

   (b) Find out the intensity distribution of the histogram-equalized image.

   [7+3=10]

3. (a) Prove that the differentiation of the output of a convolution, of a signal with a filter, can be achieved by convolving the signal with the derivative of that filter.

(b) Write down 3×3 Sobel filters.

(c) Show that the orientation of the edge computed using Sobel filters is equal to its true orientation.

[3+2+5=10]

4. Assume continuous intensity values, and suppose that the histogram of an image can be approximated by the probability density function (PDF)

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{for } 0 \leq r \leq L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(b) Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{for } 0 \leq z \leq L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(c) Express the transformation function from (b) directly in terms of r, the intensities of input image.

[4+4+2=10]

5. (a) Show that the Laplacian is invariant to rotation. You may assume continuous quantities. Consider coordinate rotation by an angle θ is given by

$$x' = x\cos\theta - y\sin\theta; \qquad y' = x\sin\theta + y\cos\theta;$$

where (x, y) and (x', y') are the unrotated and rotated coordinates, respectively.

(b) Suppose an image is filtered with the Laplacian kernel. Prove that the sum of the pixel values in the filtered image is 0.

[6+4=10]