INDIAN STATISTICAL INSTITUTE

Second-Semestral Examination: 2022-2023

Image Processing - I

M.Tech.(Computer Science)

Date: 09.05.2023 **Full Marks: 100** **Time: 3 Hours**

Answer any **ten** questions. All questions carry equal marks.

1. The histogram of an image can be approximated by the probability density function

$$p_r(r) = Ae^{-r},$$

where r is the grey level variable taking values between 0 and b, and A is a normalizing factor. Calculate the transformation $s = T(r)$, such that the transformed image has the probability density function

$$p_s(s) = Bse^{-s^2},$$

where s is the grey level variable in the transformed image taking values between 0 and b, and B is some normalizing factor. [10]

2. The grey values of the object and the background pixels of an image are distributed according to the probability density function

$$p(x) = \begin{cases} \frac{3}{4a^3}[a^2 - (x-b)^2] & \text{for } b-a \le x \le b+a \\ 0 & \text{otherwise} \end{cases}$$

with $a = 1$, $b = 5$ for background, $a = 2$, $b = 7$ for object, and x is the grey value of the pixel.

(a) Sketch the two probability density functions.

(b) If the number of object pixels is eight-ninths of the total number of pixels, determine
   (i) the minimum error threshold; and
   (ii) the fraction of misclassified object pixels by optimal thresholding. [3+(4+3)=10]

3. Consider the following equation

$$\tan^2\theta + \frac{\overline{m}_{20} - \overline{m}_{02}}{\overline{m}_{11}}\tan\theta - 1 = 0$$

where $\overline{m}_{ij}$ denotes the (i,j)-th central moment of an image f and θ represents the slope of the principal axis.

(a) Show that the above equation is equivalent to

$$(\overline{m}_{11}\tan\theta + \overline{m}_{20})^2 - (\overline{m}_{20} + \overline{m}_{02})(\overline{m}_{11}\tan\theta + \overline{m}_{20}) + (\overline{m}_{20}\overline{m}_{02} - \overline{m}_{11}^2) = 0.$$

(b) Hence, show that $(\overline{m}_{11}\tan\theta + \overline{m}_{20})$ is an eigenvalue of the matrix

$$\begin{pmatrix} \overline{m}_{20} & \overline{m}_{11} \\ \overline{m}_{11} & \overline{m}_{02} \end{pmatrix}$$

(c) Show that the principal axis is in the direction of the eigenvector corresponding to the larger eigenvalue of this matrix. [4+3+3=10]

4. (a) Write down the three criteria introduced by J. Canny for optimal edge detection.

   (b) Show that to achieve good signal-to-noise ratio, an odd filter must be chosen to enhance the edges of an image.

   (c) Prove that the partial derivative of a 2-D Gaussian kernel is separable. [3+4+3=10]

5. (a) Define Laplacian of a Gaussian (LoG).

   (b) Write an algorithm to find zero-crossings of an image.

   (c) Define Difference-of-Gaussians (DoG).

   (d) Prove that the DoG function provides a close approximation to the scale-normalized LoG.

   [2+3+2+3=10]

6. Consider the following 4×4 block of grey levels:

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 2 & 2 & 3 & 3 \end{pmatrix}$$

   Construct the grey level co-occurrence matrices for angle $\theta = 0^\circ$ and 90°, considering unit pixel distance, and compute the angular second moment for each case. [(4+4)+2=10]

7. (a) Consider the block of grey levels of Question 6. Encode the above grey levels with strings of 0's and 1's based on Huffman coding.

   (b) Calculate the average code-word length. [8+2=10]

8. (a) Define local binary pattern (LBP) and rotation invariant LBP.

   (b) Consider the following 3×3 block of grey levels:

$$\begin{pmatrix} 7 & 6 & 1 \\ 6 & 5 & 2 \\ 9 & 3 & 7 \end{pmatrix}$$

   Compute LBP and rotation invariant LBP for the central pixel. [(2+2)+(2+4)=10]

9. Consider the following 6×6 block of grey levels:

$$\begin{pmatrix} 1 & 3 & 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 2 & 1 & 0 \\ 0 & 2 & 2 & 2 & 1 & 1 \\ 0 & 2 & 3 & 2 & 0 & 1 \\ 1 & 3 & 3 & 2 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 \end{pmatrix}$$

Construct the run-length matrices for angle $\theta = 0°$ and $90°$, and compute the run percentage for each case. [(4+4)+2=10]

10. Consider the following 4×4 block of grey levels:

$$\begin{pmatrix} 9 & 8 & 2 & 1 \\ 7 & 6 & 2 & 3 \\ 8 & 4 & 3 & 6 \\ 4 & 2 & 7 & 8 \end{pmatrix}$$

(a) Calculate the compressed and reconstructed representation of this block using Block Truncation Coding.

(b) Calculate PSNR and bpp. [(4+3)+(2+1)=10]

11. Assume that an image, with minimum grey value $I_{\min}$, has a bright object on a dark background. Show that Otsu thresholding method finds the optimal threshold for segmenting the image by maximizing the following objective function:

$$J(t) = \frac{[\mu(t) - \mu\theta(t)]^2}{\theta(t)[1 - \theta(t)]}$$

where $t > I_{\min}$ denotes the threshold, μ is the mean grey value of the image, $\theta(t)$ denotes the fraction of pixels having grey values between $I_{\min}$ and t, and $\mu(t) = \tilde{\mu}\theta(t)$, $\tilde{\mu}$ being the mean grey value of the pixels having grey values between $I_{\min}$ and t.

(b) How can Otsu method be extended to obtain multiple thresholds? [8+2=10]

12. (a) Show that the histogram equalised version of an image conveys the maximum possible information the image may convey.

(b) Show that when the range of grey values of an image increases, its information content also increases.

(c) Is it possible for the following matrix to represent the autocovariance matrix of a color image? Justify your answer.

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -2 & 2 & 0 \end{pmatrix}$$

(d) Discuss the advantages and disadvantages of using the principal components of a color image, instead of the original color bands. [3+2+2+3=10]