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Physics Assignment

1) $\psi(x, y, z) = 2xz^2 - x^2y$

Find the value of $\vec{\nabla}\psi$ and $|\vec{\nabla}\psi|$ at the point $(2, -2, -1)$

→ we are to compute

i) the gradient vector $\nabla\psi$

ii) the magnitude $|\nabla\psi|$ at the point $(2, -2, -1)$

compute gradient $\nabla\psi$

$$\nabla\psi = \left(\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z} \right)$$

$$\frac{\partial\psi}{\partial x} = 2z^2 - 2xy$$

$$\frac{\partial\psi}{\partial y} = -x^2$$

$$\frac{\partial\psi}{\partial z} = 8xz$$

step 2: Evaluate at $(x, y, z) = (2, -2, -1)$

$$\frac{\partial\psi}{\partial x} = 2(-1)^2 - 2(2)(-2) = 2 + 8 = 10$$

$$\frac{\partial\psi}{\partial y} = -2^2 = -4$$

$$\frac{\partial\psi}{\partial z} = 8(2)(-1) = -16$$

$$\text{so, } \nabla\psi(2, -2, -1) = (10, -4, -16)$$

step - 3 magnitude of gradient

$$|\nabla\psi| = \sqrt{10^2 + (-4)^2 + (-16)^2} = \sqrt{100 + 16 + 256}$$

$$= \sqrt{372}$$

$$|\nabla\psi| = 2\sqrt{93}$$

2) what is Gauss's divergence theorem?

→ Gauss divergence theorem is a fundamental result in vector calculus that relates the flux of a vector field through a ~~calculus~~ close surface to the divergence of the field inside the volume bounded by the surface.

Let \vec{F} be a continuously differentiable vector field on region V enclosed by a closed surface S , then:

$$\boxed{\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V (\nabla \cdot \vec{F}) dv}$$

3) Starting from the consideration that the rate of loss of energy is equal to power dissipated to the damping establish the constitutive equation of damping.

→ Let E be mechanical energy of the system.

P_d be the power dissipated due to damping

$$\frac{\partial E}{\partial t} = -P_d \quad [\text{this represents that energy is lost due to damping}]$$

power dissipated by damping force.

$$F_d = -c\dot{x}$$

then power dissipated:

$$P_d = F_d \cdot \dot{x} = (-c\dot{x}) \cdot \dot{x} = -c\dot{x}^2$$

Equality rate of energy loss & power dissipated

$$\frac{\partial E}{\partial t} = -P_d = c\dot{x}^2$$

This shows how energy is lost at rate proportional to the square velocity with proportional constant c

constitutive eqn of damping

$$\boxed{F_d = -c\dot{x}}$$

It is derived by equating the rate of energy loss to the power dissipated due to the damping force which is proportional to the square velocity.