ASSIGNMENT 1

EE22BTECH11050 - Snehil Singh

Question 1.5.5 - Suppose the equations AB, BC and CA are respectively given by

$$\mathbf{n}_i^T x = c_i \quad i = 1, 2, 3 \tag{1}$$

The equation of the respective angle bisectors are then given by

$$\frac{\mathbf{n}_i^T - c_i}{\|n_i\|} = \pm \frac{\mathbf{n}_j^T - c_j}{\|n_j\|} \tag{2}$$

Substitute numerical values and find the equation of the angle bisectors of A, B and C.

If I be the point of intersection of angle bisectors of B and C, find the distance of I from Ab and AC.

Solution :

Given - vertices of the triangle

$$A\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad B\begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad C\begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

(a) The direction vector of AB

$$= B - A \tag{4}$$

$$= \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} -4 - 1 \\ 6 - (-1) \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{7}$$

(b) The direction vector of AC

$$= C - A \tag{8}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} -3 - 1 \\ -5 - (-1) \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{11}$$

Finding the normals of vectors AB and AC As we know -

$$n = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} m \tag{12}$$

(a) vector normal to AB

$$\mathbf{n}_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{14}$$

(b) vector normal to AC

$$\mathbf{n}_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{15}$$

$$= \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{16}$$

$$I_1 = unit vector of n_1$$

$$= \frac{\begin{pmatrix} 7 \\ 5 \end{pmatrix}}{\sqrt{7^2 + 5^2}} \tag{17}$$

$$= \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{18}$$

$$I_2 = unit vector of n_2$$

$$= \frac{\begin{pmatrix} -4 \\ 4 \end{pmatrix}}{\sqrt{(-4)^2 + 4^2}} \tag{19}$$

$$= \frac{1}{\sqrt{32}} \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{20}$$

From previous question the incentre is-

$$O = \frac{1}{\sqrt{37} + 4 + \sqrt{67}} \begin{pmatrix} \sqrt{61} - 16 - \sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \tag{21}$$

Distance of AB from O

$$r_1 = I_1.(B - O) \tag{22}$$

$$= \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5 \end{pmatrix}.(\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \frac{1}{\sqrt{37} + 4 + \sqrt{67}} \begin{pmatrix} \sqrt{61} - 16 - \sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix}) \tag{23}$$

$$= 1.897 \tag{24}$$

Distance of AC from O

$$r_2 = I_2.(C - O) \tag{25}$$

$$= \frac{1}{\sqrt{32}} \begin{pmatrix} -4 \\ 4 \end{pmatrix} \cdot \left(\begin{pmatrix} -3 \\ -5 \end{pmatrix} - \frac{1}{\sqrt{37} + 4 + \sqrt{67}} \begin{pmatrix} \sqrt{61} - 16 - \sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \right) \tag{26}$$

$$= 1.897 \tag{27}$$