ASSIGNMENT 1

EE22BTECH11050 - Snehil Singh

Question 1.5.5

Suppose the equations AB, BC and CA are respectively given by
$\mathbf{n}_i^T\mathbf{x} = \mathbf{c_i}$ $i = 1, 2, 3$

The equation of the respective angle bisectors are then given by

$$\frac{\mathbf{n}_i^T - \mathbf{c_i}}{||\mathbf{n_i}||} = \pm\frac{\mathbf{n}_j^T - \mathbf{c_j}}{||\mathbf{n_j}||} \tag{1}$$

Substitute numerical values and find the equation of the angle bisectors of A, B and C. If I be the point of intersection of angle bisectors of B and C, find the distance of I from Ab and AC.

Solution:

Given - vertices of the triangle
$\mathbf{A}\begin{pmatrix}1\\-1\end{pmatrix}\mathbf{B}\begin{pmatrix}-4\\6\end{pmatrix}\mathbf{C}\begin{pmatrix}-3\\-5\end{pmatrix}$

1) The direction vector of AB

$$= \mathbf{B} - \mathbf{A} \tag{2}$$
$$= \begin{pmatrix}-4\\6\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \tag{3}$$
$$= \begin{pmatrix}-4-1\\6-(-1)\end{pmatrix} \tag{4}$$
$$= \begin{pmatrix}-5\\7\end{pmatrix} \tag{5}$$

2) The direction vector of AC

$$= \mathbf{C} - \mathbf{A} \tag{6}$$
$$= \begin{pmatrix}-3\\-5\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \tag{7}$$
$$= \begin{pmatrix}-3-1\\-5-(-1)\end{pmatrix} \tag{8}$$
$$= \begin{pmatrix}-4\\-4\end{pmatrix} \tag{9}$$

Finding the normals of vectors AB and AC As we know -

$$\mathbf{n} = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\mathbf{m} \tag{10}$$

1) vector normal to AB

$$\mathbf{n}_1 = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}-5\\7\end{pmatrix} \tag{11}$$
$$= \begin{pmatrix}7\\5\end{pmatrix} \tag{12}$$

2) vector normal to AC

$$\mathbf{n}_2 = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}-4\\-4\end{pmatrix} \tag{13}$$
$$= \begin{pmatrix}-4\\4\end{pmatrix} \tag{14}$$

$\mathbf{I_1}$ = unit vector of $\mathbf{n_1}$

$$= \frac{\begin{pmatrix}7\\5\end{pmatrix}}{\sqrt{7^2+5^2}} \tag{15}$$
$$= \frac{1}{\sqrt{74}}\begin{pmatrix}7\\5\end{pmatrix} \tag{16}$$

$\mathbf{I_2}$ = unit vector of $\mathbf{n_2}$

$$= \frac{\begin{pmatrix}-4\\4\end{pmatrix}}{\sqrt{(-4)^2+4^2}} \tag{17}$$
$$= \frac{1}{\sqrt{32}}\begin{pmatrix}-4\\4\end{pmatrix} \tag{18}$$

From previous question the incentre is-

$$\mathbf{i} = \frac{1}{\sqrt{37}+4+\sqrt{67}}\begin{pmatrix}\sqrt{61}-16-\sqrt{37}\\-\sqrt{61}+24-5\sqrt{37}\end{pmatrix} \tag{19}$$

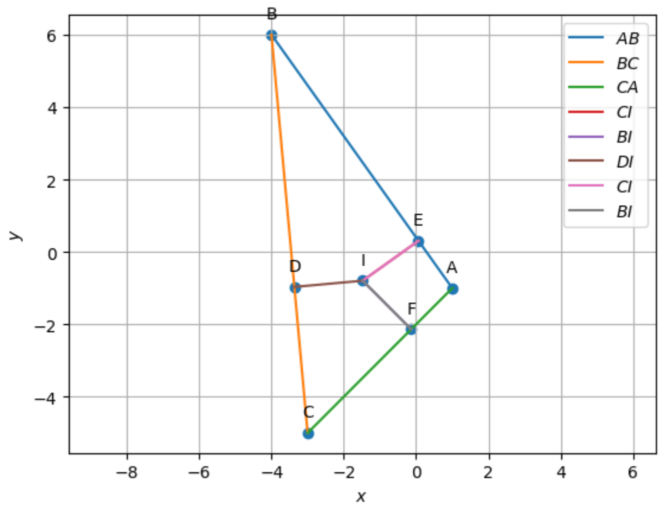

Fig. 2. Points $\mathbf{E}_3$ and $\mathbf{F}_3$ plotted using python

1) Distance of AB from **i**

$$r_1 = \mathbf{I_1}.(\mathbf{B}-\mathbf{i}) \tag{20}$$
$$= \frac{1}{\sqrt{74}}\begin{pmatrix}7\\5\end{pmatrix}.(\begin{pmatrix}-4\\6\end{pmatrix} - \frac{1}{\sqrt{37}+4+\sqrt{67}}\begin{pmatrix}\sqrt{61}-16-\sqrt{37}\\-\sqrt{61}+24-5\sqrt{37}\end{pmatrix}) \tag{21}$$
$$= 1.897 \tag{22}$$

2) Distance of AC from **i**

$$r_2 = \mathbf{I_2}.(\mathbf{C}-\mathbf{i}) \tag{23}$$
$$= \frac{1}{\sqrt{32}}\begin{pmatrix}-4\\4\end{pmatrix}.(\begin{pmatrix}-3\\-5\end{pmatrix} - \frac{1}{\sqrt{37}+4+\sqrt{67}}\begin{pmatrix}\sqrt{61}-16-\sqrt{37}\\-\sqrt{61}+24-5\sqrt{37}\end{pmatrix}) \tag{24}$$
$$= 1.897 \tag{25}$$