ASSIGNMENT 1

EE22BTECH11050 - Snehil Singh

Question 1.5.5

Suppose the equations AB, BC and CA are respectively given by

$$\mathbf{n}_i^T x = c_i \quad i = 1, 2, 3 \tag{1}$$

The equation of the respective angle bisectors are then given by

$$\frac{\mathbf{n}_i^T - c_i}{||n_i||} = \pm \frac{\mathbf{n}_j^T - c_j}{||n_j||} \tag{2}$$

Substitute numerical values and find the equation of the angle bisectors of A, B and C. If I be the point of intersection of angle bisectors of B and C, find the distance of I from Ab and AC.

Solution :

Given - vertices of the triangle

$$A\begin{pmatrix}1\\-1\end{pmatrix} \quad B\begin{pmatrix}-4\\6\end{pmatrix} \quad C\begin{pmatrix}-3\\-5\end{pmatrix} \tag{3}$$

(a) The direction vector of AB

$$= B - A \tag{4}$$
$$= \begin{pmatrix}-4\\6\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \tag{5}$$
$$= \begin{pmatrix}-4-1\\6-(-1)\end{pmatrix} \tag{6}$$
$$= \begin{pmatrix}-5\\7\end{pmatrix} \tag{7}$$

(b) The direction vector of AC

$$= C - A \tag{8}$$
$$= \begin{pmatrix}-3\\-5\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix} \tag{9}$$
$$= \begin{pmatrix}-3-1\\-5-(-1)\end{pmatrix} \tag{10}$$
$$= \begin{pmatrix}-4\\-4\end{pmatrix} \tag{11}$$

Finding the normals of vectors AB and AC As we know -

$$n = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} m \tag{12}$$

(a) vector normal to AB

$$\mathbf{n}_1 = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}-5\\7\end{pmatrix} \tag{13}$$
$$= \begin{pmatrix}7\\5\end{pmatrix} \tag{14}$$

(b) vector normal to AC

$$\mathbf{n}_2 = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}-4\\-4\end{pmatrix} \tag{15}$$
$$= \begin{pmatrix}-4\\4\end{pmatrix} \tag{16}$$

I_1 = unit vector of n_1

$$= \frac{\begin{pmatrix}7\\5\end{pmatrix}}{\sqrt{7^2 + 5^2}} \tag{17}$$
$$= \frac{1}{\sqrt{74}}\begin{pmatrix}7\\5\end{pmatrix} \tag{18}$$

I_2 = unit vector of n_2

$$= \frac{\begin{pmatrix}-4\\4\end{pmatrix}}{\sqrt{(-4)^2 + 4^2}} \tag{19}$$
$$= \frac{1}{\sqrt{32}}\begin{pmatrix}-4\\4\end{pmatrix} \tag{20}$$

From previous question the incentre is-

$$O = \frac{1}{\sqrt{37} + 4 + \sqrt{67}}\begin{pmatrix}\sqrt{61} - 16 - \sqrt{37}\\-\sqrt{61} + 24 - 5\sqrt{37}\end{pmatrix} \tag{21}$$

Distance of AB from O

$$r_1 = I_1.(B - O) \tag{22}$$
$$= \frac{1}{\sqrt{74}}\begin{pmatrix}7\\5\end{pmatrix}.(\begin{pmatrix}-4\\6\end{pmatrix} - \frac{1}{\sqrt{37} + 4 + \sqrt{67}}\begin{pmatrix}\sqrt{61} - 16 - \sqrt{37}\\-\sqrt{61} + 24 - 5\sqrt{37}\end{pmatrix}) \tag{23}$$
$$= 1.897 \tag{24}$$

Distance of AC from O

$$r_2 = I_2.(C - O) \tag{25}$$

$$= \frac{1}{\sqrt{32}}\begin{pmatrix}-4\\4\end{pmatrix}.(\begin{pmatrix}-3\\-5\end{pmatrix} - \frac{1}{\sqrt{37}+4+\sqrt{67}}\begin{pmatrix}\sqrt{61}-16-\sqrt{37}\\-\sqrt{61}+24-5\sqrt{37}\end{pmatrix}) \tag{26}$$

$$= 1.897 \tag{27}$$