Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -6 \\ -6 \end{pmatrix} \quad (1)$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 1 \\ -8 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -9 \\ -2 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 8 \\ 10 \end{pmatrix}$	CA
$\|A-B\|$	8.06	length of AB
$\|B-C\|$	9.21	length of BC
$\|C-A\|$	12.81	length of CA
rank $\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} -8 \\ -1 \end{pmatrix}$	AB
c_1	-12	
$\mathbf{n_2}$	$\begin{pmatrix} -2 \\ 9 \end{pmatrix}$	BC
c_2	30	
$\mathbf{n_3}$	$\begin{pmatrix} 10 \\ -8 \end{pmatrix}$	CA
c_3	-108	
Area	37	Area of Triangle
$\angle A$	45.78°	Angles
$\angle B$	95.40°	
$\angle C$	38.81°	

TABLE 1: Vectors.

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -1.5 \\ -5 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$(-2, -1)$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 2.5 \\ 0 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} -3.5 \\ -9 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -9 \\ 3.5 \end{pmatrix}$	
c_4	-4	
$\mathbf{m_5}$	$\begin{pmatrix} -5 \\ 3 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$	
c_5	-11	
$\mathbf{m_6}$	$\begin{pmatrix} 8.5 \\ 6 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 6 \\ -8.5 \end{pmatrix}$	
c_6	15	
$\mathbf{G}$	$\begin{pmatrix} -0.33 \\ -2 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
rank $\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
rank $\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
rank $\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

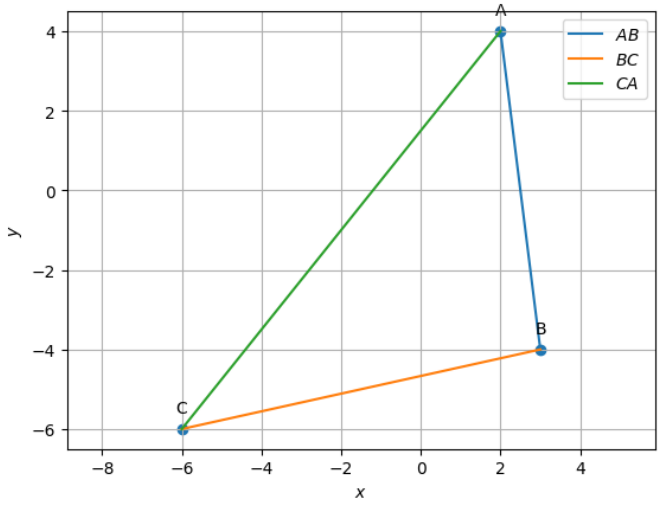

Fig. 1: triangle plotted using python

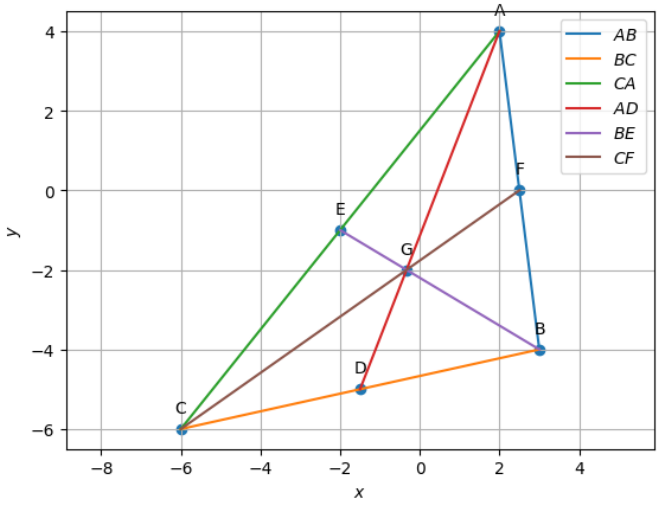

Fig. 2: medians plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}3.74\\-3.83\end{pmatrix}$	Foot of altitude from $\mathbf{A}$
$\mathbf{E_1}$	$(-1.51, -0.39)$	Foot of altitude from $\mathbf{B}$
$\mathbf{F_1}$	$\begin{pmatrix}3.11\\-4.86\end{pmatrix}$	Foot of altitude from $\mathbf{C}$
$\mathbf{m_7}$	$\begin{pmatrix}1.74\\-7.83\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-7.83\\-1.74\end{pmatrix}$	
c_7	-8.7	
$\mathbf{m_8}$	$\begin{pmatrix}-4.51\\3.61\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}3.61\\4.51\end{pmatrix}$	
c_8	-7.21	
$\mathbf{m_9}$	$\begin{pmatrix}9.1\\1.14\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}1.14\\-9.1\end{pmatrix}$	
c_9	47.82	
$\mathbf{H}$	$\begin{pmatrix}3.95\\-4.76\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-2\\9\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}9\\2\end{pmatrix}$	
c_{10}	-23.50	
$\mathbf{m_{11}}$	$\begin{pmatrix}10\\-8\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}-8\\-10\end{pmatrix}$	
c_{11}	26	
$\mathbf{m_{12}}$	$\begin{pmatrix}-8\\-1\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-1\\8\end{pmatrix}$	
c_{12}	-2.5	
$\mathbf{O}$	$\begin{pmatrix}-2.47\\-0.62\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	6,43	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	91.57°	$\angle BOC = 2\angle BAC$
$\angle BAC$	45.78°	
$\angle AOC$	169.19°	$\angle AOC = 2\angle ABC$
$\angle ABC$	95.40°	
$\angle AOB$	282.37°	$\angle AOB = 2\angle BCA$
$\angle BCA$	$^\circ$	

TABLE 4: Perpendicular Bisector.

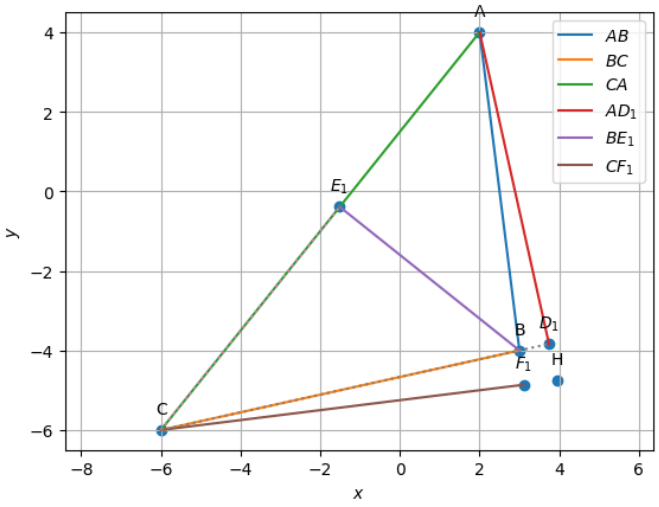

Fig. 3: altitudes plotted using python

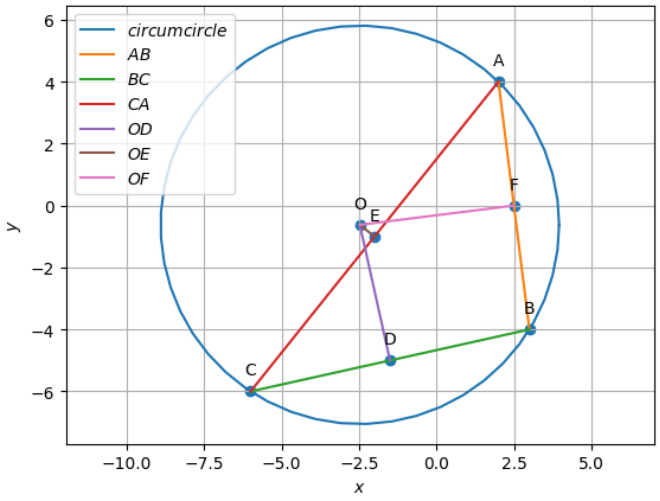

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

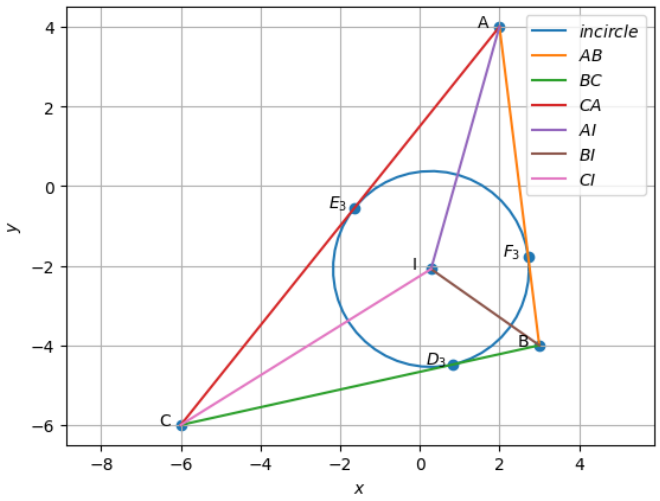

Fig. 5: Angle bisectors plotted using python

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-0.5\\1.77\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}1.77\\-0.5\end{pmatrix}$	
c_{13}	1.54	
$\mathbf{m_{14}}$	$\begin{pmatrix}-1.10\\0.77\end{pmatrix}$	BI
$\mathbf{m_{14}}$	$\begin{pmatrix}-0.77\\-1.10\end{pmatrix}$	
c_{14}	2.07	
$\mathbf{m_{15}}$	$\begin{pmatrix}-1.60\\-0.99\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}0.99\\-1.60\end{pmatrix}$	
c_{15}	3.62	
$\mathbf{I}$	$\begin{pmatrix}0.28\\-2.08\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}0.82\\-4.49\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-1.64\\-0.55\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}2.72\\-1.78\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	2.46	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	22.89°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	47.70°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	19.40°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.