

GATE BM 2023 Question 9

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Question :

Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:

- 1) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.
 - 2) Mix the samples within each set and test the mixed sample for covid.
 - 3) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.
 - 4) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.
- Given this strategy, no more than testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped

- 1) 700
- 2) 600
- 3) 800
- 4) 1000

Solution :

Given :

Parameter	Value
Number of individuals	1000
Strenght of each group	5
Number of groups	200
Number of Covid positive individuals	100

$$\text{Number of tests initially to examine each group} = 200 \quad (1)$$

Probability	Value
Probability of individual testing positive	0.1
Probability of individual testing negative	0.9

Parameter	Value	Expression
n	5	Number of individuals in group
p	0.1	Probability of testing positive
q	0.9	Probability of testing negative

Using binomial expansion to find the probability of a group testing positive

$$\begin{aligned} Pr(X = i) &= {}^n C_i (q)^i (p)^{n-i} \\ &= {}^5 C_i (0.9)^i (0.1)^{5-i} \end{aligned} \quad (2)$$

(3)

And the CDF is

$$F_X(k) = Pr(X \leq k) \quad (4)$$

$$= \sum_{i=0}^k {}^nC_i(q)^i(p)^{n-i} \quad (5)$$

$$= \sum_{i=0}^k {}^5C_i(0.9)^i(0.1)^{5-i} \quad (6)$$

$$(7)$$

Where i is number of individuals tested positive Evaluating probability of every case in a group

Parameter	Calculation	Probability
All positive	${}^5C_5(0.1)^5$	0.00001
4 positive 1 negative	${}^5C_4(0.1)^4(0.9)$	0.00045
3 positive 2 negative	${}^5C_3(0.1)^3(0.9)^2$	0.00810
2 positive 3 negative	${}^5C_2(0.1)^2(0.9)^3$	0.07290
1 positive 4 negative	${}^5C_1(0.1)^1(0.9)^4$	0.32805
All negative	${}^5C_0(0.9)^5$	0.5904

$$P_{(ofagrouptestingpositive)} = F_X(4) \quad (8)$$

$$= \sum_{i=0}^4 {}^5C_i(0.9)^i(0.1)^{5-i} \quad (9)$$

$$= {}^5C_5(0.1)^5 + {}^5C_4(0.1)^4(0.9) + {}^5C_3(0.1)^3(0.9)^2 + {}^5C_2(0.1)^2(0.9)^3 + {}^5C_1(0.1)^1(0.9)^4 \quad (10)$$

$$= 0.00001 + 0.00045 + 0.0081 + 0.0729 + 0.32805 \quad (11)$$

$$= 0.4096 \quad (12)$$

Now calculating the expected number of sets:

$$Expectednumberoftestsforsoneset = 1 + 5 * P_{(ofagrouptestingpositive)} \quad (13)$$

$$= 1 + 5 * 0.4096 \quad (14)$$

$$= 3.048 \quad (15)$$

$$Totalexpectednumberofsets = 200 * (Expectednumberoftestsforsoneset) \quad (16)$$

$$= 200 * 3.048 \quad (17)$$

$$= 609.6 \quad (18)$$

∴ So 609.6 is the number of tests required to detect all the covid positive individuals.

According to the options, no more than 700 tests will be required.

So, the correct answer is option A 700.