Q-9.3.30

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Question: The probability of guessing correctly at least 8 out of 10 answers on a true-false type examination is

Solution: Defining variables:

Parameter	Value	Description
n	10	Number of questions
p	0.5	probability of guessing correctly
$\mu = np$	5	mean of the distribution
$\sigma^2 = np(1-p)$	2.5	variance of the distribution
Y	0-10	denotes number of questions guessed correctly

1) **Binomial distribution:** the probability of getting exactly 8 correct answers is

$$= \binom{10}{8} \times 0.5^8 \times 0.5^2 \tag{1}$$

$$= 0.043946 \tag{2}$$

2) **Gaussian Distribution:**
The gaussian distribution for Y is

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

For getting exactly 8 correct answers

$$Y = 8 \tag{4}$$

Substituting in equation (3), probability for getting exactly 8 correct answers is

$$p_Y(8) = \frac{1}{\sqrt{2\pi \times 2.5}} e^{\frac{-(8-5)^2}{2\times 2.5}} \tag{5}$$

$$= 0.05204 \tag{6}$$

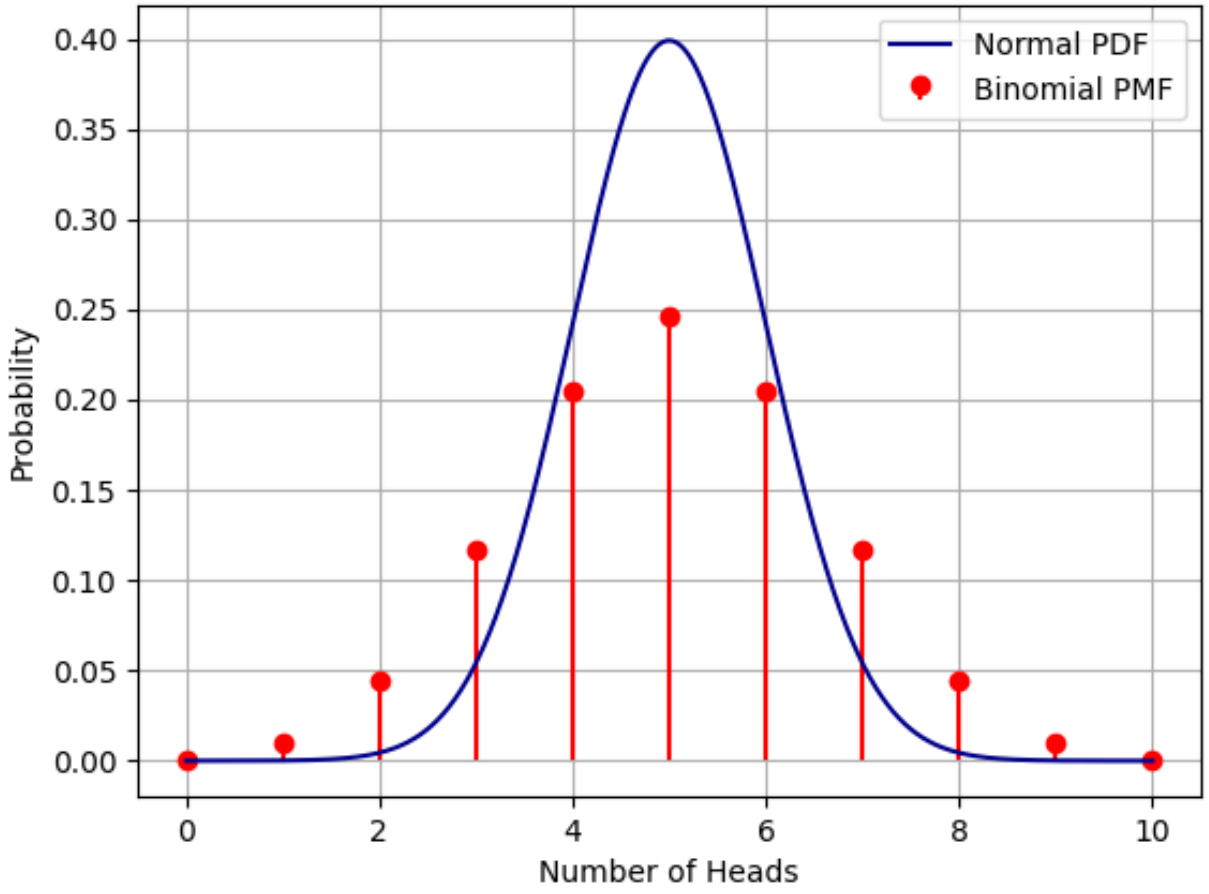

Fig. 1. Binomial distribution vs Gaussian distribution

3) **Using Q function:** Defining a gaussian random variable Z such that

$$Z \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{7}$$

Due to continuity correction, $\Pr(Z = x)$ can be approximated as

$$p_Z(x) \approx \Pr(x - 0.5 \leq Z < x + 0.5) \tag{8}$$
$$\approx \Pr(Z < x + 0.5) - \Pr(Z < x - 0.5) \tag{9}$$
$$\approx F_Z(x + 0.5) - F_Z(x - 0.5) \tag{10}$$

CDF of Z is defined as

$$F_Z(x) = \Pr(Z < x) \tag{11}$$
$$= \Pr\left(\frac{Z - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \tag{12}$$

As

$$\frac{Z-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{13}$$

$$\implies F_Z(x) = 1 - \Pr\left(\frac{Z-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) \tag{14}$$

$$= \begin{cases} 1 - Q(\frac{x-\mu}{\sigma}) & x \geq \mu \\ Q(\frac{\mu-x}{\sigma}) & x < \mu \end{cases} \tag{15}$$

$\therefore$ Gaussian approximation for $\Pr(Z = 8)$ is

$$p_Z(8) = 1 - Q(1.63273) \tag{16}$$

$$= 0.051263 \tag{17}$$