

Exemplar - 10.13.3.21

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Question 10.13.1.19

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is less than 9

Solution:

Product: Let the random variables be defined as:

Variable	Values	Description
X	$1 \leq X \leq 6$	First Dice Roll
Y	$1 \leq Y \leq 6$	Second Dice Roll

Assuming both dice rolls are equally likely,:

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The probability mass function is:

$$p_{XY}(k) = \Pr(XY = k) \quad (3)$$

$$= \Pr\left(X = \frac{k}{Y}\right) \quad (4)$$

$$= E\left(p_X\left(\frac{k}{Y}\right)\right) \quad (5)$$

$$= \sum_{i=1}^6 p_X\left(\frac{k}{i}\right) p_Y(i) \quad (6)$$

$$= \frac{1}{6} \sum_{i=1}^6 p_X\left(\frac{k}{i}\right) \quad (7)$$

$$= \frac{1}{6} \sum_{i=1}^6 \frac{[k \bmod i = 0]}{6} \left[\frac{k}{i} \leq 6 \right] \quad (8)$$

$$= \frac{1}{36} \sum_{i=1}^6 [k \bmod i = 0] \left[\frac{k}{i} \leq 6 \right] \quad (9)$$

Thus, the probability of Apoorv rolling a 36 is:

$$\Pr(XY < 9) = \sum_{i=1}^8 \left(\frac{1}{36} \sum_{j=1}^6 [j \bmod i = 0] \left[\frac{j}{i} \leq 6 \right] \right) \quad (10)$$

$$= \frac{1}{36} (1 + 2 + 2 + 3 + 2 + 4 + 2) \quad (11)$$

$$= \frac{16}{36} \quad (12)$$

$$= \frac{4}{9} \quad (13)$$