NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
A.A. 2023/2024

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HOMEWORK 2

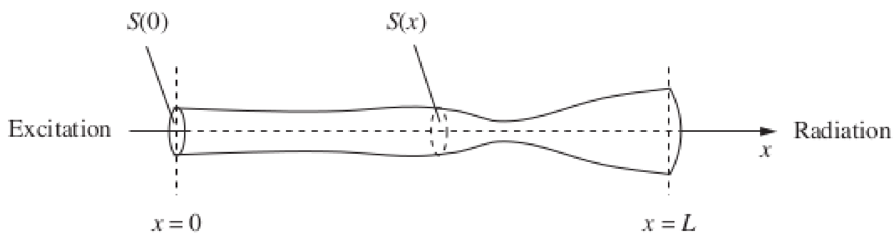

Figure 1: Example of a 1D acoustic tube, with excitation at the left end, and radiated sound produced at the right end.

Consider the following wave propagation problem (Webster's equation) in a longitudinal domain having variable cross section $S(x)$, cf. Figure 1, and lenght $L > 0$:

$$\begin{cases} S\varphi_{tt} = \gamma^2(S\varphi_x)_x + f & (x,t) \in (0,1) \times (0,T], \\ \varphi_x(0,t) = 0 & t \in (0,T], \\ \varphi_t(1,t) = 0 & t \in (0,T], \\ \varphi(x,0) = \varphi_0(x) & x \in (0,1), \\ \varphi_t(x,0) = \varphi_1(x) & x \in (0,1), \end{cases} \tag{1}$$

where $\gamma = c/L$ and f, φ_0 and φ_1 are given functions.

1. Introduce the variables p and u s.t.

$$p = \frac{1}{\gamma}\varphi_t, \quad u = -S\varphi_x.$$

Verify that the system (1) is equivalent to the following

$$\begin{cases} Sp_t = -\gamma u_x + \frac{1}{\gamma} f & (x,t) \in (0,1) \times (0,T], \\ \dfrac{1}{S} u_t = -\gamma p_x & (x,t) \in (0,1) \times (0,T], \\ u(0,t) = 0 & t \in (0,T], \\ p(1,t) = 0 & t \in (0,T], \\ u(x,0) = -S\varphi_0'(x) & x \in (0,1), \\ p(x,0) = \frac{1}{\gamma}\varphi_1(x) & x \in (0,1), \end{cases} \tag{2}$$

2. Propose a finite difference scheme for the solution to (1) and a finite difference staggered scheme for the solution to (2). Write precisely how to apply the boundary conditions.

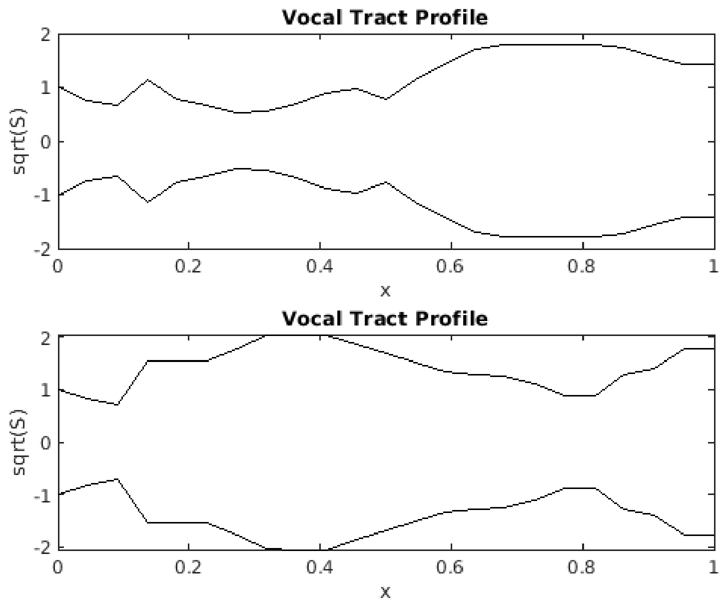

Figure 2: Vocal tract profiles exercise 4. You can obtain the plots above as well as the definition of sections S by using the conde `Sections.m`.

3. Write two Matlab codes that implement the schemes at the previous step. Verify your implementation by considering the following data:

   - $L = 1$, $c = 1$, $S(x) = 1$,
   - $L = 1$, $c = 1$, $S(x) = (1 + 2x)^2$.

   Consider as exact solution to problem (1) the function $\varphi_{ex}(x, t) = \cos(2\pi x)\cos(3\pi t)$ and compute the remaining data accordingly. The exact solution to problem (2) as well as its the missing data can be derived from φ_{ex}. Report the plot of the computed solution obatined as well ase the computed discretization errors in a Table. Consider both space and time grid refinements. Comment on the results.

4. Solve problem (2) with the following data:

   - $L = 1$, $c = 2$, $f = \varphi_0 = \varphi_1 = 0$,

   and

   $$p(1, t) = 0, \quad u(0, t) = u_{in} = \frac{1}{2}\left(\sin(\pi(t - 0.1)/0.05) + |\sin(\pi(t - 0.1)/0.05)|\right), \qquad (3)$$

   by considering the staggered scheme at point 2. Consider $S(x) = 1$ and the sections S reported in the file `Sections.m`, cf. also Figure 2. Compute the acoustic output velocity/input velocity transfer function $|\frac{\hat{u}_0}{\hat{u}_{in}}|$, and the spectrum of the output pressure waveform $|\hat{p}_0|$, by choosing a suitable probe inside the tube. Report the results and comment on them.