FORMATIVE 2 : Eigen Values & Vectors.

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} = A$$

Group Project
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$\det(A - \lambda I) = 0$, where A is the matrix, λ is the Eigen Value & I is the identity matrix.

$$\det\left[\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] = 0$$

$$\det\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix} = 0$$

Matrices of Minors.

$$4-\lambda\left(\det\begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix}\right) - 8\left(\det\begin{pmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{pmatrix}\right)$$

$$+ -1\left(\det\begin{pmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{pmatrix}\right) - -2\left(\det\begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}\right)$$

Simplification

$$4-\lambda \det\begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix} = -9-\lambda\begin{vmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{vmatrix} - -2\begin{vmatrix} 10 & -10 \\ -13 & -13-\lambda \end{vmatrix} + -4\begin{vmatrix} 10 & 5-\lambda \\ -13 & -14 \end{vmatrix}$$

$$-9-\lambda((5-\lambda)(-13-\lambda) - 140) + 2(10(-13-\lambda) - 130) - 4(-140 + (13)(5-\lambda))$$

$$-9-\lambda(\lambda^2 + 8\lambda - 65) - 140) + 2(-130 - 10\lambda - 130) - 4(-140 + 65 - 13\lambda)$$

$$(-9-\lambda)(\lambda^2 + 8\lambda - 205) + (-520 - 20\lambda) - (-300 - 52\lambda)$$

$$(-\lambda^3 - 17\lambda^2 + 133\lambda + 1845) + (-520 - 20\lambda) - (-300 - 52\lambda)$$

$$(4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) \quad \text{Equation 1}$$

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$$-8\left(\det\begin{pmatrix}-2 & -2 & -4\\ 0 & 5-\lambda & -10\\ -1 & -14 & -13-\lambda\end{pmatrix}\right) = -2\begin{vmatrix}5-\lambda & -10\\ -14 & -13-\lambda\end{vmatrix} - -2\begin{vmatrix}0 & -10\\ -1 & -13-\lambda\end{vmatrix} -4\begin{vmatrix}0 & 5-\lambda\\ -1 & -14\end{vmatrix}$$

$= -2((5-\lambda)(13-\lambda) - 140) + 2(-10) \quad -4(5-\lambda)$

$-2((\lambda^2 + 8\lambda - 65) - 140) + (-20) + (-20 + 4\lambda)$

$(-2\lambda^2 - 16\lambda + 410) + (-20) + (-20 + 4\lambda)$

$-8 \times (-2\lambda^2 - 12\lambda + 370)$

$= +16\lambda^2 + 96\lambda - 2960$ Equation 2

$$-1\left(\det\begin{pmatrix}-2 & -9-\lambda & -4\\ 0 & 10 & -10\\ -1 & -13 & -13-\lambda\end{pmatrix}\right) = -2\begin{vmatrix}10 & -10\\ -13 & -13-\lambda\end{vmatrix} -(-9-\lambda)\begin{vmatrix}0 & -10\\ -1 & -13-\lambda\end{vmatrix} -4\begin{vmatrix}0 & 10\\ -1 & -13\end{vmatrix}$$

$= -2((-130 - 10\lambda) - 130) - (-9-\lambda)(-10) \quad -4(+10)$

$= -2(-260 - 10\lambda) - (90 + 10\lambda) + (-40)$

$= (520 + 20\lambda) + (-90 - 10\lambda) + (-40)$

$= 520 - 90 - 40 + 20\lambda - 10\lambda$

$= 390 + 10\lambda$

$= (-1)(390 + 10\lambda) = -390 - 10\lambda$ Equation 3

$$2\left(\det\begin{pmatrix}-2 & -9-\lambda & -2\\ 0 & 10 & 5-\lambda\\ -1 & -13 & -14\end{pmatrix}\right) = -2\begin{vmatrix}10 & 5-\lambda\\ -13 & -14\end{vmatrix} -(-9-\lambda)\begin{vmatrix}0 & 5-\lambda\\ -1 & -14\end{vmatrix} -2\begin{vmatrix}0 & 10\\ -1 & -13\end{vmatrix}$$

$= -2(-140 - (-13)(5-\lambda)) - (-9-\lambda)(5-\lambda) \quad -2(10)$

$= -2(-140 + 65 + -13\lambda) + (\lambda + 9)(5 - \lambda) + (-20)$

$= (280 - 130 + 26\lambda) + (-\lambda^2 + -4\lambda + 45) + (-20)$

$= -\lambda^2 + 26\lambda - 4\lambda + 280 - 130 + 45 - 20$

$= -\lambda^2 + 22\lambda + 175$

$= 2(-\lambda^2 + 22\lambda + 175) = -2\lambda^2 + 44\lambda + 350$ Equation 4

~~Expanding~~ Expanding Equation 1

$(4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625)$

$4(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) \quad -\lambda(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625)$

$= -4\lambda^3 - 68\lambda^2 + 660\lambda + 6500 + \lambda^4 + 17\lambda^3 - 165\lambda^2 - 1625\lambda$

Emmanuel Dania

$= -4\lambda^3 - 68\lambda^2 + 600\lambda + 6500 + \lambda^4 + 17\lambda^3 - 165\lambda^2 - 1625\lambda$

$= \lambda^4 - 4\lambda^3 + 17\lambda^3 - 68\lambda^2 - 165\lambda^2 + 660\lambda - 1625\lambda + 6500$

$= \lambda^4 + 13\lambda^3 - 233\lambda^2 - 965\lambda + 6500$

$= \lambda^4 + 13\lambda^3 - 233\lambda^2 - 965\lambda + 6500$ Equation 1 Expanded

Sum Up all the Equations.

$(\lambda^4 + 13\lambda^3 - 233\lambda^2 - 965\lambda + 6500) + (16\lambda^2 + 96\lambda - 2960) + (-340 - 10\lambda)$
$+ (-2\lambda^2 + 44\lambda + 350)$

$= \lambda^4 + 13\lambda^3 (-233\lambda^2 + 16\lambda^2 - 2\lambda^2) + (-965\lambda + 96\lambda - 10\lambda + 44\lambda) + (6500 - 2960 - 340 + 350)$

$= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

$\therefore$ The determinant of

$$\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

is $\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

To find the roots of the equation we have a number of steps which we can use.

- ☆ Ferrari's Method. (incredibly complex)
- ☆ Graphing (which is more visually appealing)

I put the equation into a graphing software & found the roots kindly find the attached graph.

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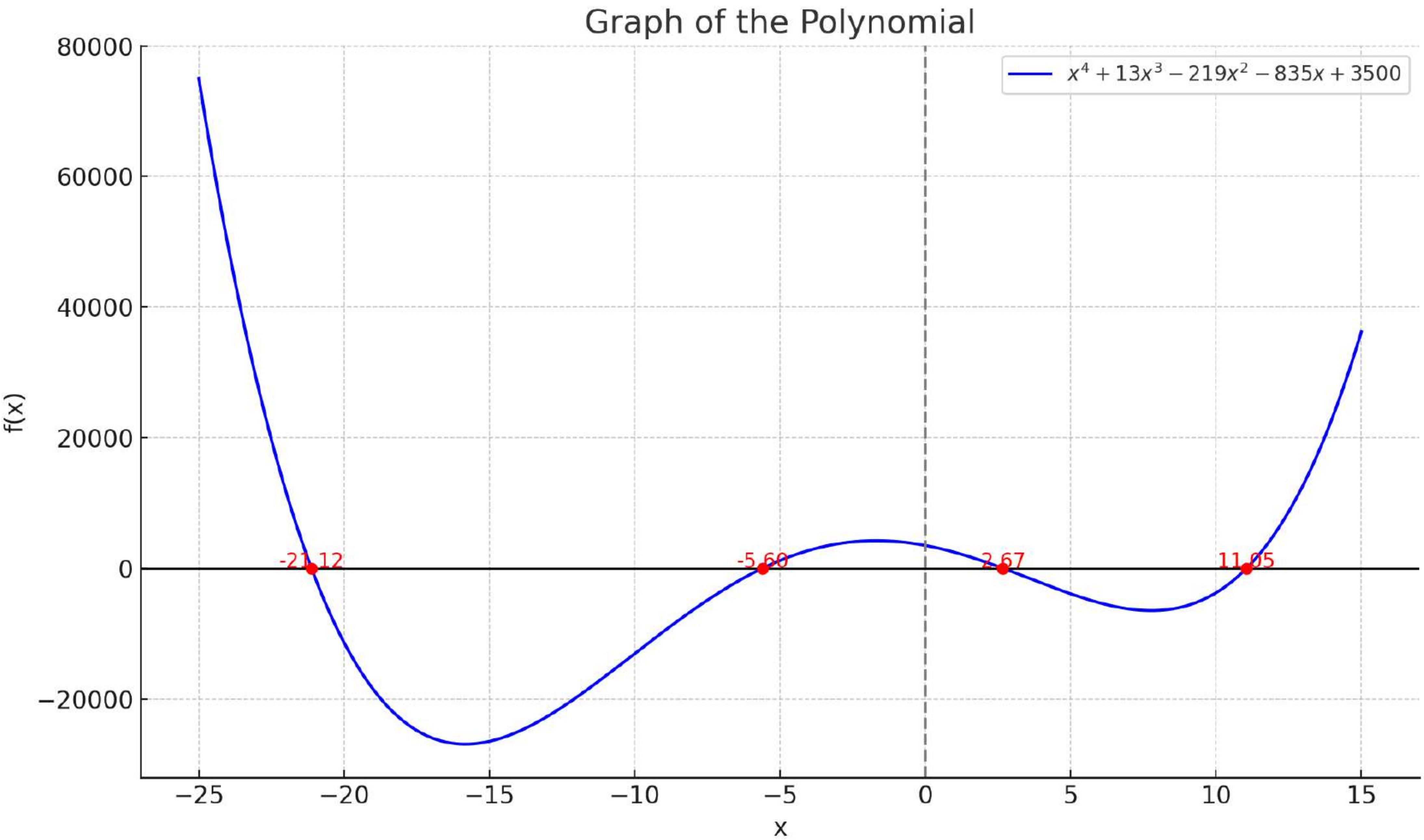

Graph of the Polynomial
$x^4 + 13x^3 - 219x^2 - 835x + 3500$
80000
60000
40000
20000
0
−20000
f(x)
-21.12
-5.60
2.67
11.05
−25
−20
−15
−10
−5
0
5
10
15
x

So Our EigenValues for the equation $\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$ are

$\lambda_1 = -21.12$

$\lambda_2 = -5.60$

$\lambda_3 = 2.67$

$\lambda_4 = 11.05$

Emmanuel's Eigen Vector

So using the original equation

$$(A - \lambda I)\vec{V} = 0$$

I will be using 11.05 as my λ

$\lambda = 11.05$

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - (11.05) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-11.05 & 8 & -1 & -2 \\ -2 & -9-11.05 & -2 & -4 \\ 0 & 10 & 5-11.05 & -10 \\ -1 & -13 & -14 & -13-11.05 \end{bmatrix}$$

$$= \begin{bmatrix} -7.05 & 8 & -1 & -2 \\ -2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ -1 & -13 & -14 & -24.05 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = 0$$

Gaussian Elimination

$$\left(\begin{array}{cccc|c} -7.05 & 8 & -1 & -2 & 0 \\ -2 & -20.05 & -2 & -4 & 0 \\ 0 & 10 & -6.05 & -10 & 0 \\ -1 & -13 & -14 & -24.05 & 0 \end{array}\right)$$

Divide Row 1 by -7.05

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right)$$ Subtract Row 2 by $-2 \times$ Row 1

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right)$$ Subtract Row 4 by $-1 \times$ Row 1

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right)$$ Divide Row 2 by -22.322

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right)$$ Subtract Row 3 by $-10 \times$ Row 2

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.54 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right)$$ Subtract Row 4 by $-14.134 \times$ Row 2

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right)$$ Divide Row 3 by -6.823

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right)$$ Subtract Row 4 by $-12.771 \times$ Row 3

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Row2 − 0.077 × Row 3

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Subtract Row1 by −0.142 × Row3

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0 & 0.044 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Row1 −(−1.134) × Row 2

$$\left(\begin{array}{cccc|c} 1 & -0 & 0 & 0.071 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} 1 & -0 & 0 & 0.071 \\ 0 & 1 & 0 & 0.024 \\ 0 & 0 & 1 & 1.691 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Expand.

$$V_1 + 0.071 V_4 = 0$$
$$V_2 + 0.024 V_4 = 0$$
$$V_3 + 1.691 V_4 = 0$$

$$V_1 = -0.071 V_4$$
$$V_2 = -0.024 V_4$$
$$V_3 = -1.691 V_4$$
$$V_4 = V_4$$

let $V_4 = t$

$$\vec{V} = \begin{pmatrix} -0.071t \\ -0.024t \\ -1.691t \\ t \end{pmatrix}$$

$$\vec{V} = \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix} t$$

Eigen Vector 4

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$\lambda_3 = 2.675$

$A - \lambda I = 0$

$$A - \lambda_3 = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix} = 0$$

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.66 & 0 \end{array}\right] \times 0.754$$

$$\left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \begin{array}{l} \\ \times 2 \\ \\ \times 1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.75 & -17.184 & 0 \end{array}\right] \begin{array}{l} \\ \times 2.518 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times -10$$

$$\left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & 166.688 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times 6.964$$

$$\left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & 166.688 & 0 \\ 0 & 0 & -76.278 & -140.231 & 0 \end{array}\right] \times 0.0\ldots$$

$$\begin{bmatrix} 1 & 6.036 & -0.754 & -1.509 \\ 0 & 1 & -8.834 & -17.669 \\ 0 & 0 & 1 & 1.838 \\ 0 & 0 & -76.278 & -140.231 \end{bmatrix} \times 76.2\ldots$$

$$\begin{bmatrix} 1 & 6.036 & -0.754 & -1.509 \\ 0 & 1 & -8.834 & -17.669 \\ 0 & 0 & 1 & 1.838 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times 8.834$$

$$\begin{bmatrix} 1 & 6.036 & -0.754 & -1.509 \\ 0 & 1 & 0 & -1.428 \\ 0 & 0 & 1 & 1.838 \\ 0 & 0 & 0 & 0 \end{bmatrix} 0.754$$

$$\begin{bmatrix} 1 & 6.036 & 0 & -0.122 \\ 0 & 1 & 0 & -1.428 \\ 0 & 0 & 1 & 1.838 \\ 0 & 0 & 0 & 0 \end{bmatrix} -6.036$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 8.494 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 8.494x_4 = 0$

$x_2 + -1.428x_4 = 0$

$x_3 + 1.838x_4 = 0$

$x_3 = -1.838x_4$

$x_2 = 1.428x_4$

$x_1 = -8.494x_4$

$x_4 = x_4$

if $x_4 = 1$

$x_3 = -1.838$

$x_2 = 1.428$

$x_1 = -8.494$

$x_4 = 1$

thus vector for eigen value $\lambda = 2.675$

$$V_3 = \begin{bmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{bmatrix}$$

SENGA

Emmanuel

$\lambda_3 = -5.60$

$A - \lambda I = 0$

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} = \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 4+5.6 & 8 & -1 & -2 \\ -2 & -9+5.6 & -2 & -4 \\ 0 & 10 & 5+5.6 & -10 \\ -1 & -13 & -14 & -13+5.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 9.6 & 8 & -1 & -2 \\ -2 & -3.4 & -2 & -4 \\ 0 & 10 & 10.6 & -10 \\ -1 & -13 & -14 & -7.4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 9.6 & 8 & -1 & -2 \\ -2 & -3.4 & -2 & -4 \\ 0 & 10 & 106 & -10 \\ -1 & -13 & -14 & -7.4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = 0 \qquad \times \frac{1}{9.6}$$

$$\begin{bmatrix} 9.6 & 8 & -1 & -2 \\ -2 & -3.4 & -2 & -4 \\ 0 & 10 & 10.6 & -10 \\ 0 & 12.16 & -13.8 & 7.6 \end{bmatrix} \qquad \times \frac{12.16}{8}$$

$$\begin{bmatrix} 9.6 & 8 & -1 & -2 \\ -2 & -3.4 & -2 & -4 \\ 0 & 10 & 10.6 & -10 \\ 0 & 0 & 12.38 & -10.64 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = 0$$

$$9.6y_1 + 8y_2 - y_3 - 2y_3 = 0$$

$$-2y_1 - 3.4y_2 - 2y_3 - 4y_4 = 0$$

$$12.38y_3 - 10.64y_4 = 0$$

$$Y_3 = \frac{10.64}{12.38} = \underline{\underline{0.86}}$$

$$12.38(0.86) - 10.64 Y_4 = 0$$

$$Y_4 = \frac{10.64}{10.64}$$

$$Y_4 = \underline{\underline{1}}$$

$$10Y_2 + 10.6(0.86) - 10(1) = 0$$

$$10Y_2 + 9.116 - 10 = 0$$

$$10Y_2 - 0.884 = 0$$

$$Y_2 = \frac{0.884}{10}$$

$$Y_2 = 0.0884$$

$$-2Y_1 - 3.4(0.0884) - 2(0.86) - 4(1) = 0$$

$$-2Y_1 - 0.30056 - 1.72 - 4 = 0$$

$$-2Y_1 - 6.02056 = 0$$

$$Y_1 = \frac{-6.02056}{2}$$

$$Y_1 = -3.01028$$

$y_1 = -3.01028$

$y_2 = 0.0884$

$y_3 = 0.86$

$y_4 = \underline{1}$

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IMPORTANCE OF THE EIGENVALUES.

$\lambda_1 = -21.12$
$\lambda_2 = -5.60.$
$\lambda_3 = 2.67$
$\lambda_4 = 11.05$

To calculate the importance we take the sum of the values and it has to be the absolute values.

$$21.12 + 5.60 + 2.67 + 11.05 = 40.44$$

$$\text{Importance } (\%) = \frac{\text{Absolute Eigenvalue}}{\text{Total}} \times 100.$$

$$\lambda_1 = \frac{21.12}{40.44} \times 100 = 52.24\%$$

$$\lambda_2 = \frac{5.60}{40.44} \times 100 = 13.85\%$$

$$\lambda_3 = \frac{2.67}{40.44} \times 100 = 6.60\%$$

$$\lambda_4 = \frac{11.05}{40.44} \times 100 = 27.31\%$$

$\therefore$ Therefore the importance of λ_1 is 52.24%, λ_2 is 13.85%, $\lambda_3 = 6.60\%$, $\lambda_4 = 27.31\%$

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