Estimating the change in premium of option contracts using Black-Scholes-Merton Model

Project report submitted to

Visvesvaraya National Institute of Technology, Nagpur

in partial fulfilment of the requirements for the award of the degree

Bachelor of Technology

In

Computer Science and Engineering

by

SNIGDHA PATIL
(BT16CSE066)

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under the guidance of

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VNIT NAGPUR

Department of Computer Science and Engineering

Visvesvaraya National Institute of Technology

Nagpur 440 010 (India)

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Declaration

We, **Snigdha and Varundeep**, hereby declare that this project work titled **"Estimating the change in premium of option contracts using Black-Scholes-Merton Model"** is carried out by us in the Department of Computer Science and Engineering of Visvesvaraya National Institute of Technology, Nagpur. The work is original and has not been submitted earlier whole or in part for the award of any degree/diploma at this or any other Institution / University.

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Certificate

This is to certify that the project titled "**Estimating the change in premium of option contracts using Black-Scholes-Merton Model**", submitted by **Snigdha Patil (BT16CSE066)** and **Varundeep Madari (BT16CSE094)** in partial fulfilment of the requirements for the award of the degree of **Bachelor of Technology in Computer Science and Engineering, VNIT Nagpur**. The work is comprehensive, complete and fit for final evaluation.

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<u>Non-Plagiarism Certificate</u>

Certified that B.Tech. Dissertation titled “**Estimating the change in premium of option contracts using Black-Scholes-Merton Model**” submitted by **Snigdha Patil** (BT16CSE66 / 20188) and **Varundeep Madari** (BT16CSE094 / 19779) is original in nature and not copied from any literature material. Due credit has been given to the original researcher wherever referred.

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ABSTRACT

The four major Greeks provide a substantial measure to gauge risk associated in different dimensions of option contracts. With each Greek, hedging can be done by the trader, to limit the risks. In order to make profits with options, having an insight about the movement of underlying stock, with the help of Greeks the trader/user can estimate the change in premium of an option. The project proposes a solution to use the four Greeks delta, theta, vega and theta of a stock option to measure the sensitivity of an option's price and estimate a new premium. The Black-Scholes-Merton model is used to compute the values of the Greeks

We have developed an algorithm that takes the premium, strike prices and expiry date of a stock as input along with the date at which the user wants to estimate the price. It helps the user by giving him the privilege to examine the new premium based on his bullish/bearish views on the stock and the amount by which he projects the value of stock to change on the expiry day or some day before expiry. By consider all the Greeks and giving them appropriate weightage, the algorithm gives out the estimated new premium and graphs

1. Old premium vs New premium
2. Effect of change in spot price in option chains with different expiry dates.
3. Estimate premium on expiry date or some day before expiry.

The thesis includes the basic background theories and in-depth knowledge of the mathematics involved with the same that have been utilized. In the end, future scope for the project has been listed along with references. Appendix is also included to show mathematics.

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Chapter 1
INTRODUCTION

CHAPTER 1

INTRODUCTION

In the last 40 years, derivatives have gained importance in the area of finance. These which are financial instruments whose value depends on underlying assets. On many exchanges across the world options and futures are actively traded. Those who are working in finance, and those who are not into finance, both should know how derivatives in the stock market work and how their price is decided. This chapter's first section explains what are options and why options trading is important. There is a brief introduction on what are Greeks & the model from which their values are obtained. This is followed by the problem statement explanation, our proposed solution and the challenges we faced throughout our project. In the last section, the flow of the report is mentioned in the form of chapters along with their content introduction.

1.1. Options and other Greeks

Stock options are derivatives whose value depends on the stock price. Trading options is important for traders to protect against an unforeseen decrease in the stock price. Using options, current positions on stocks can be hedged by taking up an opposing position on the stock. Hedging helps an investor to know how options could be leveraged to control the risk exposure of various financial variables.

Consider the example of a trader owning 1,000 shares of some company during the month of January in any year. The cost of one share is $35. The investor thinks that the price of share may decrease in the coming 3 months and hence need to protect or hedge against a loss. The investor can then choose to buy 10 put option contracts of April month of a stock of some company whose strike price is $33. One option contract contains hundred shares and the investor has 1,000 shares. So, such 10 put options make 10*100 = 1,000 shares in total. This gives the investor the right or virtue to sell a set of 1000 shares at $33 even if the stock price goes below $33. If the price of an option is $1, then one options contract would cost 100 * $1 = $100 and the overall amount of the strategy of hedging will cost 10 * $100 = $1,000. This put option has a cost of $1,000 which is the premium but assures that the shares could be sold off for at least at a cost of $33 per share until the option expires. If the price of the stock in the market plunges below $33, the put options can be exercised, and hence 1,000 * $33 = $33,000 can be obtained until options expire. When the premium of options i.e. $1,000 is considered, the amount obtained is $33000 - $1,000 = $32,000. The investor shall not exercise the put options if the market price stays above $33 and they will expire without being exercised. If options expire without exercising, the investor can at maximum lose the options premium. So, options trading safeguards against any potential losses. Put options protect or hedge investors from random and adverse changes in prices in the future, but at the same time it gives an opportunity to benefit from desirable or profitable price movements.

The Greeks is a concept in options trading that is used to measure risks in options as well as financial rewards. With an understanding of these Greeks one can use them for trading strategies. The goal of any trade is to make money and we can estimate this probability of increasing returns or reducing losses by studying various risk dimensions. Since market prices fluctuate continuously, the Greeks help investors to study and analyze the sensitivity of a trade to volatility fluctuations, price changes, and amount of time. With an understanding of the Greeks we can estimate the price of an option (premium) to help the trader decide which call or put options he can buy or sell. The trader can thus choose the most profitable option by making a comparison between the options price(premium) for available expiry dates. Thus, estimating the premium of options using the values of Greeks is crucial for options trading.

Along with Greeks, effects of volatility and time are also important in estimating the premium. The premium always increases with increase in volatility and decreases with decrease in volatility irrespective of the time to expiry. This suggests that one should buy options when volatility is expected to increase and sell options when volatility is expected to decrease. After deciding whether to buy or sell options, the next decision is to choose a specific strike price. Here, the time remaining for contract expiry becomes essential to choose an option with perfect strike price. The number of days remaining till expiry date along with trading days (252) instead of calendar days (365) are used to calculate the life of an option that is further used to calculate the value of Greeks.

Options are priced using mathematical models like the Black-Scholes-Merton Model which takes into consideration the volatility of the underlying security and other values. The values of Greeks are derived from this model. To understand this model, it is crucial to have an understanding of stochastic calculus for finance which includes

Brownian motion in finance, Quadratic variation, symmetric random walk, continuous time stochastic process, Geometric Brownian motion and Ito integrals. A stochastic process is considered a random process which is described as a family of random variables that represents numerical values of a system that changes randomly over a period of time. Random changes in the stock market prices can be modelled and studied using stochastic processes.

1.2. Problem Statement

Estimating the change in premium of an option contract when there is a change in underlying stock value, using four Greeks and Black Scholes Merton model. The basic goal of this project is to give the user / trader an estimation of a new premium which the option will have on the expiry day or some day before expiry after the user enters the change in underlying stocks value.

1.3. Proposed Solution

Each of the four Greeks measures a different dimension of risk associated with an option contract. Along with the Greeks, the current price of underlying security and volatility and the trader's expected difference in the underlying spot price and volatility are also used to calculate the new premium value.

The strike price and its corresponding premium are known along with their current date and expiry date. The values of greeks like delta, gamma, vega and theta calculated from the Black-Scholes-Merton Model are used to reflect their effect on the movement of the premium value. The new premium value calculated for a date before or on the expiry date. The estimated new premium is compared with the old premium value, which is the premium on the current date to see the variation. The premium is calculated for options with the same range of strike prices with fixed current date and varying expiry dates. We have considered trading of European options. These are the options traded only on the expiry date. So this calculated premium can help the trader make better decisions about which call or put options and for which available expiry date he can hold a long position on.

1.4. Challenges

The Black Scholes Merton model cannot be understood using ordinary differential equations as the stock price fluctuates randomly. Such a model which is used for stock price dynamics needs the support of stochastic differential equations. So to understand the model, we had to first learn about mathematics of Stochastic calculus, which includes stochastic processes, stochastic differential equations and Itô integrals. To implement the Black-Scholes-Merton model, basic knowledge about delta hedging is needed. Also, Greeks are valued using Black Scholes Merton formulas.

Historical data of option prices needed, which was taken from www.nseindia.com and saved locally. Using the attributes in saved files, greeks were calculated using Black-Scholes-Merton formulas. After getting values of Greeks, appropriate weighting should be given to each greek's effect on the new premium net value. Day of premium and initial volatility taken on assumption plays a significant role, calculating greeks.

After calculating greeks, it was necessary for the order in which change is premium was modified by each greek. In the end, the new premium is stored as output and the data should be visualisable with different graphs which shows the estimation of premiums with strike prices

1.5. Organisation of Report

We have calculated based the Black-Scholes-Merton formula, with its mathematics explained in the following chapters

Chapter 2 of the thesis contains the background knowledge that is necessary to understand the financial instrument called "Options", a brief section on the Black-Scholes-Merton model and the formulas derived from it, basic knowledge on Greeks.

Chapter 3 of the thesis contains implementation of our proposed solution to the estimation problem.

Chapter 4 of the thesis contains the results of the different scenarios that happen and how our algorithm provides estimation to each of the cases.

Chapter 5 contains the limitations of our implementation, along with future scope of development of this project.

Chapter 6 contains References used by us in the development of this project Appendix in the end contains brief on stochastic calculus and maths behind it.

CHAPTER 2

BACKGROUND PRELIMINARIES

CHAPTER 2
BACKGROUND PRELIMINARIES

Knowing the fundamental of options and the mathematics behind its pricing it important before understanding the formulas. This chapter is divided into three sections. In the first section fundamentals of options are explained with the help of an illustration and various options terminologies are described. Nextly, buying call and put options, that is, what is a long position on options is elaborated with the help of an example. The last sub section defines the moneyness of options.

The second section deals with the Black-Scholes-Merton model. The key idea behind the model is explained, followed by how stock prices can be modelled using stochastic processes. In the remaining sub sections, the assumptions of the model, the BSM formula and properties are stated. The third and final section is about various Options Greeks and their variations with respect to stock price and time till expiry.

2.1. Options

Before understanding options, we should understand what a derivative is. A derivative is considered like a financial tool whose price depends on the values of some underlying variables. An option is a derivative with its value depending on stock price. Nevertheless, derivatives may be dependent on almost any variable.

2.1.1. What are options?

The financial instruments called options are derivatives that are dependent on underlying asset prices like stocks. It is a contract that offers the buyer of options the chance to buy or sell the underlying asset at a decided-upon expiry date and strike price mentioned in the options contract. These option contracts bestow upon owners of option the privilege, but not a compulsion to trade, sell or buy, an underlying asset at a price and date that has been decided upon. Options are of two types:

i. Put options - Put options permit the holder to sell the shares at a fixed price until the options expire.

ii. Call options - Call options permit the holder to buy the shares at a fixed price until the options expire.

These options can be European or American options. The former type of options can only be exercised on the expiration date, as against this the latter type of options can be exercised at any date before the expiration date of the option

2.1.2. Illustration of Call options and their importance:

Consider a scenario where there is a seller called ‘B’ who wants to sell a shop and a buyer called ‘A’ who wishes to buy that shop. The price of the shop is Rs. 5000. ‘A’ receives information that in the next 10 months ‘a residential complex’ is likely to be constructed near this shop. ‘A’ will earn profit from this investment if the residential complex is indeed constructed in the next 10 months and then the price of the shop is also bound to increase. However, if the construction news is not true, then ‘A’ will suffer a loss. The buyer is in a dilemma whether to buy the shop or not. If the option buyer makes a decision to buy, the option seller is ready to sell the shop. ‘A’ does not want to take risk and proposes a structured agreement to ‘B’ as follows:

i. ‘A’ will pay an upfront fee of Rs. 1000 which is non-refundable to ‘B’.

ii. Against these fees, ‘B’ agrees to sell the shop to ‘A’ at the end of 10 months.

iii. The shop costs Rs. 5000 at the end of 10 months.

iv. Only ‘A’ can cancel the agreement by the end of 10 months and ‘B’ can’t because ‘A’ has already paid a fee of Rs. 1000.

v. In case if ‘A’ calls cancels the agreement at the end of 10 months, ‘B’ profits as he keeps the already paid fees by ‘A’ i.e. Rs. 1000.

Now, as options expire in 10 months of this agreement there could be the following three scenarios:

Buy Price = Rs. 5000

Agreement Fees = Rs. 1000

Total fees ‘A’ must pay = Rs. 5000+ Rs. 1000 = Rs. 6000

Scenario 1 – Cost of the shop goes up to Rs. 10000

‘A’ will choose to buy the shop as he shall make profit as follows:

Cost of the shop at the end of 10 months = Rs. 10000

Hence, profit = Rs. 10000 – Rs. 6000 = Rs. 4000

Scenario 2 –Cost of the shop goes down to Rs. 3000

'A' will not choose to buy the shop as he shall make loss as follows:

Cost of the shop at the end of 10 months = Rs. 3000

Hence, loss = Rs. 3000 – Rs. 6000 = - Rs. 3000 if he buys

Loss = Rs. 1000 if he does not buy.

Scenario 3 – Cost of the shop stays at Rs. 5000

'A' will not choose to buy the shop as he shall make loss as follows:

Cost of the shop at the end of 10 months = Rs. 5000 but 'A' would have to spend Rs. 6000

Again, Loss = Rs. 1000 if he does not buy.

2.1.3. Options terminology:

i. Premium - It is the amount required that must be paid by the option buyer to the writer or seller of the option. By paying this premium the buyer buys the right to exercise the option to sell or buy the asset depending on the option type at the strike price when the expiry date is reached.

ii. Strike Price - It is a fixed price for which the seller and buyer agree to follow an options contract or agreement. For call options it is the price at which the stock could be purchased on the expiry day, whereas for put options it is the price for which the stock could be sold on the day of expiration.

For instance, if the option buyer wants to buy XYZ company's call option of Rs.432, where 432 is its strike price then it represents that the buyer would pay a premium today to buy the privilege of 'buying XYZ at Rs.432 on the expiration date'.

iii. Underlying price of stock (Spot Price) - A price of an option is derived from the price of the underlying. This is also called spot price as it is traded in the spot market.

iv. Exercising options - It means to take advantage of the right to sell or buy the shares or stocks before or on the option expiry date.

v. Option expiry - It is the expiration date till which options can be exercised.

With respect to aforementioned illustration, we can summarize the following analogy:

a. The fee paid by ‘A’ to ‘B’ makes sure that ‘A’ has a right to cancel the agreement. ‘B’ is bound to sell the shop at any time before 10 months if ‘A’ chooses to buy. This Rs. 1000 is the option premium.
b. The result of the contract or agreement between ‘A’ and ‘B’ at the end of 10 months depends on the cost of the shop. Here, 10 months’ is the expiration time and the agreement is the options contract.
c. The agreement is an analogy of a derivative and that shop is an analogy of an underlying asset.
d. ‘B’ is called the ‘options seller or writer’ as ‘B’ is going to sell the shop and has received a fee from ‘A’ in advance, and ‘A’ is called the options buyer’ as he wants to buy the shop.
e. Thus, in an option contract the buyer always has a right and the seller is bound to sell or buy depending upon the option type, put or call. Hence, the seller always has an obligation but the buyer is not obligated to exercise the option.

2.1.4. Buying Call and Put options (Long on options):

i**.** *Buying a call option:*

a. The investor must buy a call option when he expects the underlying spot price to increase. This is also called the ‘bullish’ view of the investor.
b. The call option buyer loses money corresponding to the premium he pays to the call option seller. This is the maximum loss (worst case) of the buyer only if the spot price remains consistent or plunges below strike.
c. Intrinsic value - The intrinsic value (IV) at the time of expiry is defined as the non – negative value which the option buyer incurs if he chooses to exercise the call option. It represents how much money a buyer would receive upon expiry, if the call option he holds is profitable. Mathematically it is given as,
IV = Max [0, (underlying asset price – strike price)]

Consider an example of a call option for INFY(Infosys) which is being traded at Rs. 2100 with an exercise price (strike price) of Rs. 2070 and a premium of Rs. 8. The following will be its Profit & Loss(P&L) behaviour for various spot prices:

IV = Max [0, (spot price – strike price)]

P&L = IV + (-) Premium

Sr. no	Possible spot prices	Premium paid	Intrinsic Value (IV)	P&L
1	2040	(-)8	0	-8
2	2055	(-)8	0	-8
3	2060	(-)8	0	-8
4	2070(strike price)	(-)8	0	-8
5	2080	(-)8	10	2
6	2085	(-)8	15	7
7	2090	(-)8	20	12
8	2105	(-)8	35	27
9	2115	(-)8	45	37

Table 2.1.: Profit & Loss(P&L) behaviour for various spot prices for call options

From the table we observe the following:

a. The worst case or maximum loss the buyer of a call option can suffer is equivalent to the option's price or premium paid (i.e. Rs. 8 here). The buyer would suffer a loss as long as the underlying asset price (also called spot price) stays lesser than the strike price.

   Hence, condition to exercise call option is

   strike price < spot price

b. Call option buyers can make unbounded profits only if the underlying asset price becomes surpasses the strike price.

c. At first, call option buyer needs to gain back the options price he has paid and then the call option can make a profit when the underlying spot price further moves higher than the strike price.

ii. *Buying a put option:*

a. The investor must buy a put option when he expects the spot price to decrease. This is also called the ‘bearish’ view of the investor.
b. For a Put option IV is given as follows:
   IV = [Max (0, Strike Price – Spot Price)]
c. The P&L of a Put Option buyer can be calculated as
   P&L = IV + (-) Premium

Consider an example of a put option for INFY(Infosys) which is being traded at Rs. 1900 with a strike price of Rs. 2070 and a premium of Rs. 8. The following will be its Profit & Loss(P&L) behaviour for various spot prices:

Sr. no	Possible spot prices	Premium paid	Intrinsic Value (IV)	P&L
1	2020	(-)8	50	42
2	2030	(-)8	40	32
3	2040	(-)8	30	22
4	2050	(-)8	20	12
5	2060	(-)8	10	2
6	2070(strike price)	(-)8	0	-8
7	2080	(-)8	0	-8
8	2090	(-)8	0	-8
9	2100	(-)8	0	-8

Table 2.2.: Profit & Loss(P&L) behaviour for various spot prices for put options

From the table we observe the following:

a. The buyer of a put option can make a maximum loss which is equivalent to the option price paid (i.e. Rs. 8 here). The buyer of a put option suffers a loss as long as the underlying asset price is above the strike price.
   Hence, condition to exercise put option is
   spot price < strike price
b. The buyer of the put option can gain unlimited profits as the underlying asset price goes below the strike price.
c. At first, the put option buyer needs to gain back the options price(premium) he has paid and then the put option can gain profits when the price of underlying goes below the strike price

2.1.5. Moneyness of options contract:

Moneyness of an option splits the individual options at different strike prices into categories on the grounds of the amount of profit an investor is probable to gain if he decides to exercise the contract on a specific day before the expiry date. It is classified broadly as follows:

a. *In-The-Money (ITM)* - If the strike price of a call option is smaller than the spot price, then the call option is in-the-money. If the strike price of a put option is greater than the spot price, then the put option is in-the-money.
b. *Out-of-The-Money (OTM)* - If the strike price of a call option is greater than the spot price, then the call option is out-of-the-money. If the strike price of a put option is smaller than the spot price, then the put option is out-of-the-money.
c. *At-The-Money (ATM)* - If the strike price of a call or put option is equal to the spot price, then it is an at-the-money option.

2.2. Black-Scholes-Merton Model

Black–Scholes–Merton model is a mathematical model for the dynamics of a financial market containing derivative investment instruments. By deducing the Black-Scholes formula from the partial differential equation Black-Scholes, we get formulas which gives a quantitative approximation of premium of European-style options.

2.2.1. Introducing Black-Scholes-Merton Model:

The options market and options trading have become famous partially due to 'The Black Scholes Merton pricing model'. There wasn't a standard method for options pricing before this model was built. It was very difficult to decide a fair value for the options. As it was very difficult to determine whether there was good value for money available, the options were not observed as acceptable financial instruments by investors and traders.

The Black Scholes Merton model changed this view. It's a mathematical model for financial market fluctuations for deciding the price of options. The formula for Black Scholes can be deduced from the Black-Scholes equation. A theoretical estimate of the price of European options is obtained from this formula and it also indicates that options are uniquely priced. A fair value for an option can be calculated based on certain variables.

2.2.2. Key Idea underlying the Black Scholes Merton (BSM) Model:

The BSM differential equation must be satisfied by the value of any derivative that depends on a non-dividend paying stock. The idea is to develop a riskless portfolio that consists of a position in the derivative to be binomial in position in the stock. The return or gain from the portfolio must be the risk-free interest rate(r), if arbitrage opportunities are not available. This leads to the BSM differential equation.

The rationale that such a portfolio which is riskless can be developed is that the price of stock and that of option depending on it, both are impacted by the same underlying origin of randomness that is the movement of stock price. The loss or profit due to the stock position always counterbalances the loss or profit from the option position so that the total value of the portfolio by the end of a small-time span is obtained with certainty when a correct portfolio of the stock and options is produced.

Consider for instance at a particular time instant, the relationship between the call option price *c* and underlying stock price S is 0.6. A riskless portfolio comprises of:

1. A short position in 100 call options.
2. A long position in 60 shares.

Suppose the stock price increments by 10 paise. The price of the option will go high by 6 paise, and the 60 * 0.1 = ₹6 profit on the stocks is equivalent to 0.06 * 100 = ₹6 amount of loss on the short position. We should also note that, in Black-Scholes-Merton model, the stock position and the option is riskless only for a short duration. It should be rebalanced frequently to remain riskless. It is true that the risk-free interest rate has to be the return from such a portfolio, in a small duration. This is an important factor in the analysis of BSM which gives rise to their formulae for pricing.

2.2.3. The Process of a Stock Price:

The price of a stock follows the generalised Wiener process, and it has a constant variance rate and a constant expected drift rate and. If *S* is the stock price at time *t*, then the expected drift rate should be assumed to be μS for some constant parameter μ, expected drift rate. This means that in s short interval of time, *dt*, the expected increase in *S* is μSdt. The above model is represented by the equation

$$dS = \mu S\,dt \tag{1}$$

On integrating, we get

$$S_T = S_0 e^{\mu T} \tag{2}$$

Where S_0 and S_t are the stock price at time 0 and time T. But equation (2), doesn't show the in practical uncertainty. A considerable presumption is that the change in the return value in a small-time duration *dt*, is just as uncertain of the return when the stock is priced ₹500 as when it is ₹50. This implies that the standard deviation of the change in this small duration *dt* should be proportional to the stock price and it gives rise to the following model which is known as Geometric Brownian Motion.

$$dS = \mu Sdt + \sigma SdW_t \tag{3}$$

The variable μ is the stock's expected or anticipated rate of return (annualized) gained by a trader in a short period of time. In a risk neutral world μ equals the risk-free rate r. The variable σ is the volatility of the stock price. To us the value σ, is critically important for the determination of the value of many derivatives, whereas the value of derivatives that depends on stock is independent of μ. The BSM model is based on the assumption of Geometric Brownian Motion. Refer Appendix for more on Geometric Brownian Motion.

2.2.4. Assumptions:

The Black-Scholes-Merton differential equations are based on following assumptions:

1. The price of stock obeys the process described in (3) with constant σ & μ .
2. The short selling underlying assets with full use of the proceeds is permitted.
3. All securities are perfectly divisible. There are no transaction costs or taxes.
4. During the whole lifespan of the option there are no dividends.
5. None of the opportunities are riskless arbitrage.
6. Trading of securities is continuous.
7. Risk-free rate of interest, *r*, is constant and same for all maturities.

Notations:

1. S_t- underlying asset's price (usually stock) at time t;
2. c - European call option's price;
3. p - European put option's price;
4. K - option's strike price (also known as exercise price);
5. r - risk free interest rate (annualised);
6. μ - annualized drift rate of S;
7. σ, the standard deviation of the stock's returns;
8. T, time remaining till the expiry of the stock option.

The BSM differential equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{4}$$

Equation (4) is the BSM differential equation. There are numerous answers for this equation pertaining to all the different derivatives that can be defined with S as the underlying stock price which is variable. A specific derivative obtained after solving this equation relies on the boundary conditions taken into account. These represent the derivative values at the boundaries of feasible values of S and t.

For a European call option, the key boundary condition is

$$f = max(S - K, 0)\ when\ t = T \tag{5}$$

For a European put option, it is

$$f = max(K - S, 0)\ when\ t = T \tag{6}$$

2.2.5. Black-Scholes-Merton Formula:

The BSM formulas of pricing for the premiums of European call and put options are the most renowned solutions to the differential equation-(4). This price is in accordance with the aforementioned BSM equation which follows since by solving the equation for the corresponding terminal and boundary conditions the formula can be obtained.

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{7}$$

$$p = Ke^{-rt} N(-d_2) - S_0 N(-d_1) \tag{8}$$

where,

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{9}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} \tag{10}$$

In the above equations 7, 8, 9, 10,

1. N(x) is the cumulative probability distribution function for a variable with a standard normal distribution which gives the probability that a variable with a standard normal distribution will be less than x.
2. c is the premium of European call option and p is the European put options
3. S_0 is the stock price at time zero.
4. K is the strike price.
5. r is the continuously compounded risk-free rate,
6. σ is the stock price volatility.
7. T is the time to maturity of the option.

Understanding the Formula: In the formulas (7), (8), (9), (10), the term *$N(d_1)$ and $N(d_2)$*, come from a normal probability distribution curve. If an investor graphed the period daily returns, the resulting graph would be a normal distribution. The BSM model holds an assumption that prices in future are normally distributed. Therefore N(d_1) the area under the bell curve up to some z-score, that gives the probability that the future price will be above the strike price on the expiration date. *$N(d_2)$* is the percentage of probabilities that the option will expire in the money.

$$d_1 = \frac{\overbrace{\ln \frac{S_0}{K}}^{\substack{x \text{ – the value} \\ \text{In this case, the value} \\ \text{above the strike price}}} + \overbrace{(r + \frac{\sigma^2}{2})(T - t)}^{\substack{\mu \text{ – mean of our normal distribution} \\ \text{In this case, a function of volatility, it reflects the} \\ \text{risk-free rate + spread of possible options over time}}}}{\underbrace{\sigma\sqrt{T - t}}_{\substack{\sigma \text{ – standard deviation} \\ \text{In this case, a function of volatility over time}}}}$$

Coming back to equation (7) which gives the call option pricing, the Black-Scholes-Merton formula gives the option buyer's return, after subtracting the option's cost.

$$C(S_0, t) = \overbrace{S_0 N(d_1)}^{\substack{\text{Return} \\ \text{Stock Price X} \\ \text{Probability Function}}} - \overbrace{Ke^{-r(T-t)} N(d_2)}^{\substack{\text{Cost} \\ \text{Strike Price X} \\ \text{Discount to Present Value X} \\ \text{Probability Function}}}$$

1. $Ke^{-rT}N(d_2)$: the exercise price discounted back to present value times the probability that the option is above the strike price at maturity.
2. $S_0N(d_1)$: the stock price today times a probability representing the stock's value if it's above the strike price. The probability is zero, if the Stock Price is below strike price.

2.2.6. Properties of BSM Formula:

Let's assume that we have a very high increase in the stock price of underlying, due to which the price of the call option will increase, whereas value of the put option will decrease. When the stock price becomes very large, both d_1 and d_2, the terms $N(d_1)$ and $N(d_2)$ become close to 1.0. On the other hand, the price of the put option p, becomes close to 0. This complies with equation (8) because $N(-d_1)$ and $N(-d_2)$, are both close to zero.

2.3. The Option Greeks

The Greeks in the options market are the risk variables that measure the risk involved in different dimensions involved in taking an options position. These risk variables signify the relationship of the option with other underlying variables. Some Greeks variables often used to estimate the premium include delta, vega, theta and gamma. The movement of stock options or the risk associated with that option is influenced by the value of Greeks. The first partial derivatives of the BSM options pricing model are Greeks like Delta, Vega, Theta and Gamma. The value of Greeks changes over time. The primary Greeks are as follows:

2.3.1. Delta (Δ):

Delta of an option contact is defined as the ratio of rate of change of the price of an option with respect change in the price of the underlying asset. It measures how sensitive the options price is with respect to the price of the underlying asset. The value of delta for a put option lies between zero and negative one and that of a call option lies between zero and one.

For example, an option with delta 0.7 means that for 1 unit change in stock price, there will be 0.7 change in option premium.

$$0 \leq \Delta_{call} \leq 1 \quad \text{and} \quad -1 \leq \Delta_{put} \leq 0$$

$$\Delta(\text{call}) = N(d_1)$$

$$\Delta(\text{put}) = N(d_1) - 1$$

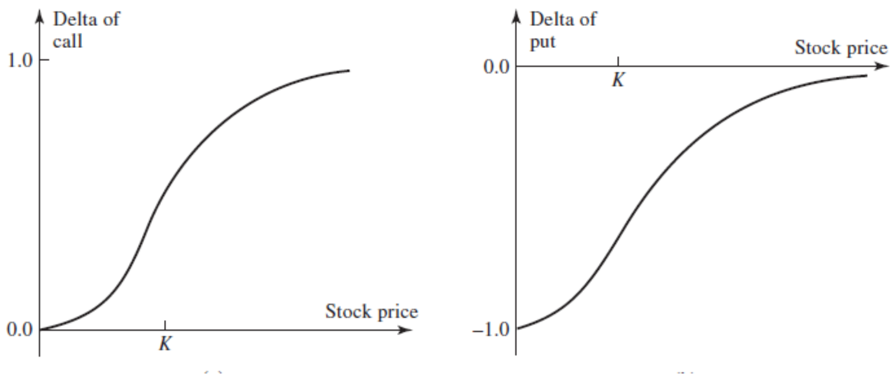


Figure 2.1.: Delta vs Stock price for a call and put option.

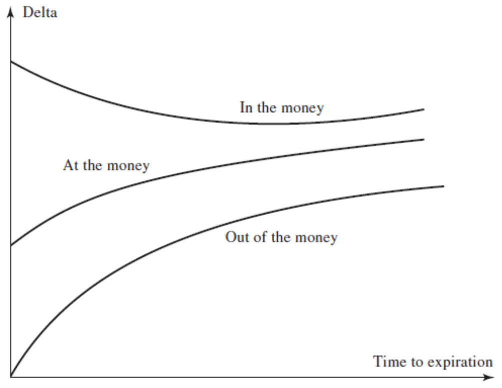


Figure 2.2.: Delta vs time to expiry

2.3.2. Theta (Θ):

The theta of options measures the rate of change of the price of an option with respect to the passage of time with all other variables considered constant. Theta is also referred to as the time decay. As the time to expiration decreases, considering all else equal, theta measures the amount by which an option's price would decrease. Theta decreases when options are in-the money and out-of-the money whereas it increases when options are at-the-money. As options approach closer to expiry date, their time decay increases. Short call and put options have a positive value of theta whereas long call and put options have a negative value of theta.

“per calendar day” and “per trading day” are two ways to measure theta. We have used the latter one. For example, an option with theta -30.2 means that the stock loses -30.2/252 = -0.119 per trading day.

$$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

$$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

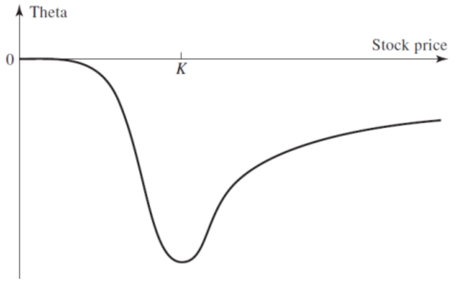


Figure.2.3.: Theta vs Stock Price

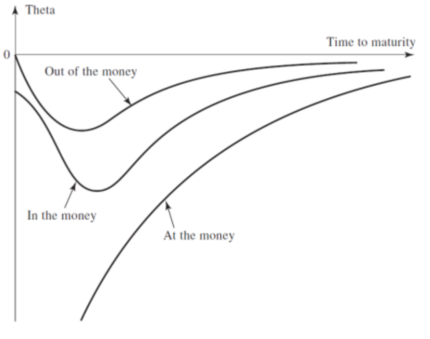


Figure.2.4.: Theta vs Time to expiry

2.3.3. Gamma (Γ):

The Gamma for options is the rate of change of the options delta with respect to the spot price i.e. price of the underlying asset. Gamma indicates the change in delta given a 1-unit change in the underlying asset price. How stable an option's delta is can be determined by the value of Gamma. Delta changes by a significant amount for even small movements in the underlying spot price which is indicated by larger gamma values. Gamma is small for options that are in- and out-of-the-money and is large for options that are at-the-money. As it comes close to the expiry date of the options, the value of gamma increases as changes in the underlying security’s price have a huge influence on the value of gamma. Hence, the options that have further expiry date are less vulnerable to delta fluctuations. The value of gamma tends to increase as it comes closer to the expiry date. For example, if gamma of an option is 0.69, this means that for a change of 1 unit in stock, there will be 0.69 change in option price. Formula of gamma is same for call and put option.

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$

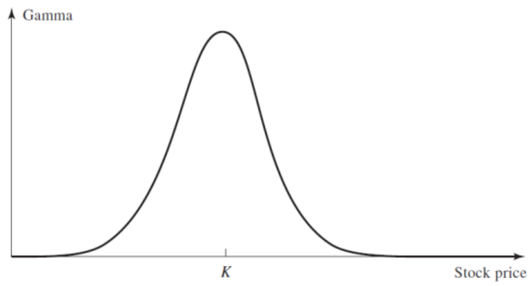


Figure 2.5.: Gamma vs Stock price

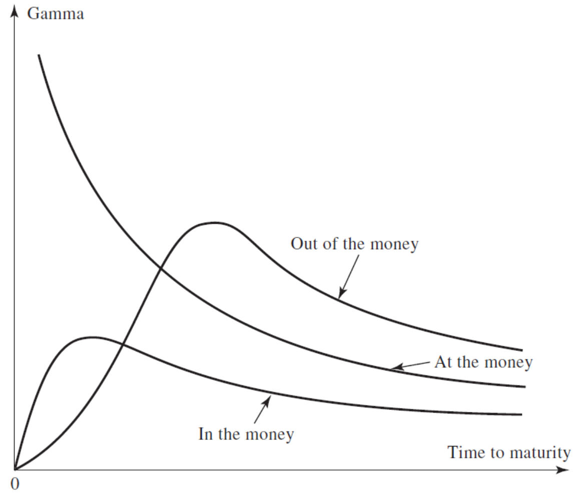


Figure 2.6.: Gamma vs Time to Expiry

2.3.4. Vega (v):

The Vega of an option measures how sensitive the price of an option is to the movements in volatility of the underlying asset. The value of volatility constantly changes with time and hence the price of an option is likely to change because of fluctuations in volatility, underlying spot price and the progress of time. Volatility changes have a small effect on the value of the options if the Vega value is close to zero. As against this, the value of an option is very sensitive to small changes in volatility if the value of Vega is extremely positive or extremely negative. For example, if option's Vega is 13.2, then a 1% increase (0.01) in volatility increases value of option by $0.01 \times 13.2 = 0.132$

$$\mathcal{V} = S_0\sqrt{T}\,N'(d_1)$$

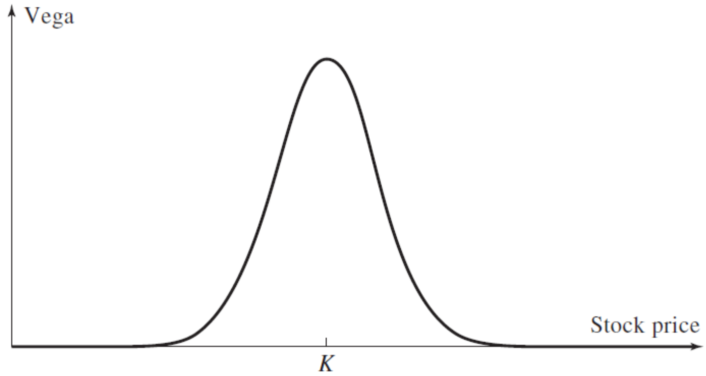


Figure 2.7.: Vega vs Stock price

CHAPTER 3

IMPLEMENTATION

CHAPTER 3

IMPLEMENTATION

In this chapter, the implementation of our algorithm has been explained. The steps followed are explained starting with from where we collected data, inputs for the algorithm, the main algorithm which calculates the new premium price, the outputs of the algorithm and stating the RMSE evaluation values which tells how good our algorithm works.

3.1. Data collection:

The data for call and put options is taken from the NSE Option Chain for NIFTY. From the current date till the expiry date, a range of strike prices from 8000 to 11000 is chosen. Current date at the time of testing is 20th May, 2020. Different expiry dates taken for the options are 28th May, 4th June, 11th June and 18th June in the year 2020. For calculation of RMSE the actual premiums are also collected dated 22nd May 2020.

3.2. Inputs:

From the user:

1. Volatility - It is the rate at which the value of a stock moves or fluctuates due to market conditions. Before starting a trade, it's good to have a clue of how the stock price traded is likely to change. Hence, volatility is so important to traders, as it is a prime factor that helps to predict what shall happen to the future price of a security.

   Hence, the user is asked to enter volatility and expected change in volatility.
2. Spot Price - It is the underlying security price on the current date. The user is asked to enter this spot price and expected change in the spot price.
3. The date on which the user wants to know the premium if he wants to get the value of premium before the expiry date.

From file:

1. File contains strike price, premium (old premium), current date, expiry date, moneyness and option type.
2. File containing observed real price of the option, which is used to calculate RMSE.

3.3. Algorithm:

From the BSM model, the values of options Greeks are obtained. The Greeks are calculated from BSM formula the formulas obtained as discussed in Chapter 2.3 and are used in calculating the new premium value. The following steps are followed for both call and put options:

1. The value of spot price and volatility are updated with respect to their expected changes respectively.

    volatility = volatility + change_in_volatilty

    spot_price = spot_price + change_in_spot_price

2. The value of the greek letter 'gamma' is calculated.
3. The BSM model formula for 'delta' returns the value of the greek letter 'delta' that needs to be updated according to the calculated value of gamma to reflect the effect of change in the delta value. This is because gamma is the 2nd order derivative and measures the delta change with regards to the price of the underlying asset.
4. 'Theta' for options is calculated.
5. 'Vega' for options is calculated and then effect of vega is calculated as follows:

    vega_effect = vega * change_in_volatility

6. The new premium value is calculated as a cascading effect of new_delta, theta and vega effect as follows:

    new_premium = old_premium + ((old_delta + new_delta) / 2.0)* change_in_spot_price

    new_premium = new_premium + num_day * theta + vega_effect

The following is the algorithm:

```
            # Step 1:
d.spot_price = d.spot_price + change_in_spot_price
spot_price = d.spot_price
v = v + change_in_volatility

old_premium_list.append(d.old_premium)

# Step 2:
gamma = c_gamma(spot_price, int(float(d.strike_price)), t, v, 0.07)
d.gamma = gamma# print("Gamma is ", gamma)
    # Step 3:
old_delta = c_delta(spot_price, int(float(d.strike_price)), t, v, 0.07)
d.old_delta = old_delta# print("Old Delta is ", old_delta)
new_delta = old_delta + gamma * change_in_spot_price# print("New Delta is ", new_delta)
d.new_delta = new_delta
    # Step 4:
theta = c_theta(spot_price, int(float(d.strike_price)), t, v, 0.07)# print("Theta is ", theta)
d.theta = theta
    # Step 5:
vega = c_vega(spot_price, int(float(d.strike_price)), t, v, 0.07)
vega_effect = vega * change_in_volatility
    # Step 6:
#print("In Theta, Numday = ",num_day)
new_premium = int(float(d.old_premium)) + ((old_delta + new_delta) / 2.0) * change_in_spot_price# print("Old premium is ", d.old
new_premium = new_premium + num_day * theta + vega_effect# addition cuz theta already has a negative value
if new_premium < 0:
    new_premium = 0

d.new_premium = new_premium
```

Figure 3.1.: A snapshot of algorithm

3.4. Outputs:

1. New premiums for the corresponding strike prices are stored as output for both call and put options along with the change in the old and new premium.
2. Two types of graphs are plotted:
    a. Plots of call options and put options for all expiry dates in a single graph to compare the variation in prices of premium for different expiry dates
    b. Individual plots of initial and final curve in one graph for call and put options for each expiry date to compare the old premium price i.e. premium price on the current date and the estimated new premium price on the expiry date or the date for which the user wishes to know the premium price.

3.5. Performance evaluation using RMSE:

1. The Root-Mean-Square-Error is calculated between the actual premium and the estimated premium for a specific date before the expiry date to evaluate the performance of our algorithm.
2. The actual values of premium for the strike prices for NIFTY are taken from NSE for a specific date and the same date is fed to our algorithm to calculate the estimated premium for that date.

The RMSE Results for the date: 22-05-2020

Option type	Current date	Expiry date	Actual date (User date)	RMSE
CALL	20-05-2020	28-05-2020	22-05-2020	18.6808
PUT	20-05-2020	28-05-2020	22-05-2020	57.03236
CALL	20-05-2020	04-06-2020	22-05-2020	108.5871
PUT	20-05-2020	04-06-2020	22-05-2020	125.799
CALL	20-05-2020	11-06-2020	22-05-2020	153.757
PUT	20-05-2020	11-06-2020	22-05-2020	206.5959
CALL	20-05-2020	18-06-2020	22-05-2020	220.5614
PUT	20-05-2020	18-06-2020	22-05-2020	127.2938

Table 3.1.: Greeks Formulae for call and put options

CHAPTER 4

RESULTS

CHAPTER 4

RESULTS

Results of the implementation, which was discussed in the previous chapter have been examined in this chapter. Various cases are considered to estimate the new premium based on the combination of inputs given by the user.

The chosen underlying asset here, is NIFTY, with snapshot of data as on 20-05-2020 Stock price of NIFTY as of the snapshot was 9066.55.
Stock price of NIFTY as on 22-05-2020 was 9039.

For the following scenarios, option premiums were estimated using the implementation:

1. On expiry date, with an increase of 200 points in the stock value.
2. On expiry date, with a decrease of 200 points in the stock value.
3. On 22-05-2020, with an increase of 250 points in the stock value.
4. On 22-05-2020, with a decrease of 250 points in the stock value.
5. On 22-05-2020, when stock price fell to 9039 from 9067 in 2 days.
6. On 24-05-2020, with an increase of 250 points with different expiry dates
7. On 24-05-2020, with a decrease of 250 points with different expiry dates

Sr. No.	Style	Meaning
1	Dotted Line	Old Premium
2	Smooth Line	New Premium
3	Vertical Blue Line	Spot Price
4	Green Line	New ITM options
5	Blue Line	Old ITM options
6	Red Line	New OTM options
7	Cyan Line	Old OTM options

Table 4.1.: Notation for Graphs

Sr. No.	File Name	Option Type	Expiry Date
1	file1c.csv	Call	28-05-2020
2	file1p.csv	Put	28-05-2020
3	file2c.csv	Call	04-06-2020
4	file2p.csv	Put	04-06-2020
5	file3c.csv	Call	11-06-2020
6	file3p.csv	Put	11-06-2020
7	file4c.csv	Call	18-06-2020
8	file4p.csv	Put	18-06-2020

Table 4.2.: Input Files

4.1. Case 1: On expiry date when there is an increase

Estimating the premium of option on expiry day when there is an increase of 200 points in the underlying stock value.

Initial Stock Price: 9066.55

Change in Stock Price: +200

Initial volatility: 25%

Change in volatility: +2%

Expiry Date of Option: 28-05-2020

Date of estimated new premium: 28-05-2020

In figure 4.1, we observe that:

The premiums of ATM and OTM call and put options is zero, this is because at the time of expiry the intrinsic value of each option is near zero and trader would rather not exercise the options than take a loss. This is shown by $\boldsymbol{\theta}$ theta, as the value an option loses per day.

Call option premium of ITM to Deep ITM options increases, and the increase in Deep ITM options is more than ITM options. As we move from away from spot price (left of blue line), we enter from ITM to Deep ITM call options. Whereas in case of put options this is opposite, the decrease in premium price is more in Deep ITM options then ITM options.

This is shown by Δ as,

$$0 \leq \Delta_{call} \leq 1 \quad \text{and} \quad -1 \leq \Delta_{put} \leq 0$$

Which has opposite effects on premium of call and put options. Moreover, the price of ATM and OTM options is near zero, as the intrinsic value of OTM and ATM options is near zero. Gamma is used to get Delta value after change in price and volatility of underlying. Change in volatility is affected into new premium by vega which is minor.The estimation holds true as per Black-Scholes-Merton pricing model.

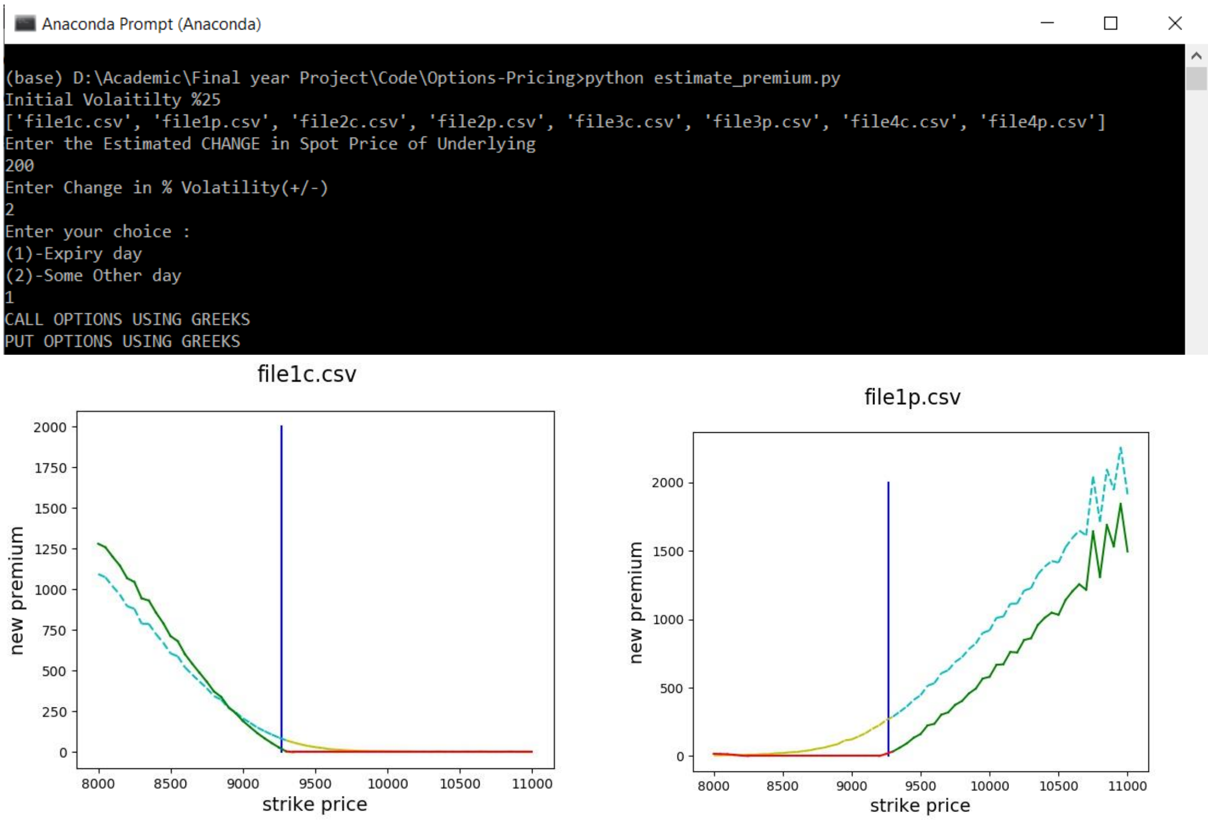


Figure 4.1.: Premium trends on +200 increase, on Expiry day

4.2. Case 2: On expiry date when there is a decrease

Estimating the premium of option on expiry date with a decrease of 200 points in the underlying stock value

Initial Stock Price: 9066.55

Change in Stock Price: -200

Initial volatility: 25%

Change in volatility: +2%

Expiry Date of Option: 28-05-2020

Date of estimated new premium: 28-05-2020

In figure 4.2, we observe the opposite result of Case 1. With a 200-point decrease in the premium of call options ITM call option decreases, whereas OTM and ATM call options reaches zero.

In case of put options the premiums of ITM option have decreased slightly as option is near expiry. OTM and ATM put options also reaches zero. However, the price of the put option has slightly decreased because of theta.

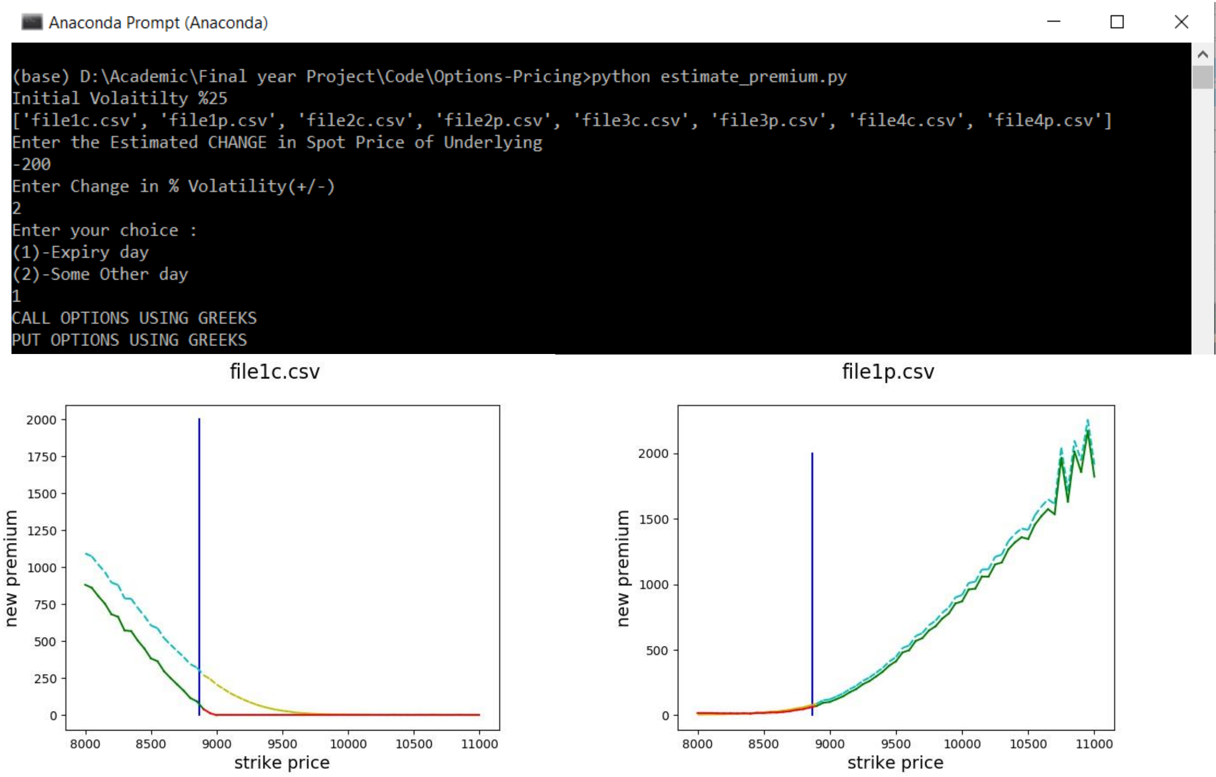


Fig 4.2.: Premiums trends on -200 change, on Expiry day

4.3. Case 3: Near the current day before expiry when there is an increase

Estimating the premium of option on some date before expiry day and near the current date when there is an increase of 250 points in the underlying stock value.

Initial Stock Price: 9066.55

Change in Stock Price: +250

Initial volatility: 25%

Change in volatility: +2%

Expiry Date of Option: 28-05-2020

Date of estimated new premium: 22-05-2020

With a 250-point increase in the stock price, on the date 22-05-2020. It is observed in the output that the premium of call options increases whether it is OTM, ATM, ITM options increases. The increase in premium is more in ITM options as compared to ATM and OTM options.

Correspondingly, the premiums of put options decrease more in OTM than ATM than ITM. The main difference from Case 1 & 2 is the date of premium estimation. As premium loses theta value per day, the effect of theta is more near expiry. So, options prices still have intrinsic value. As date of expiry comes near the intrinsic value drops, which is characterised by theta in our implementation.

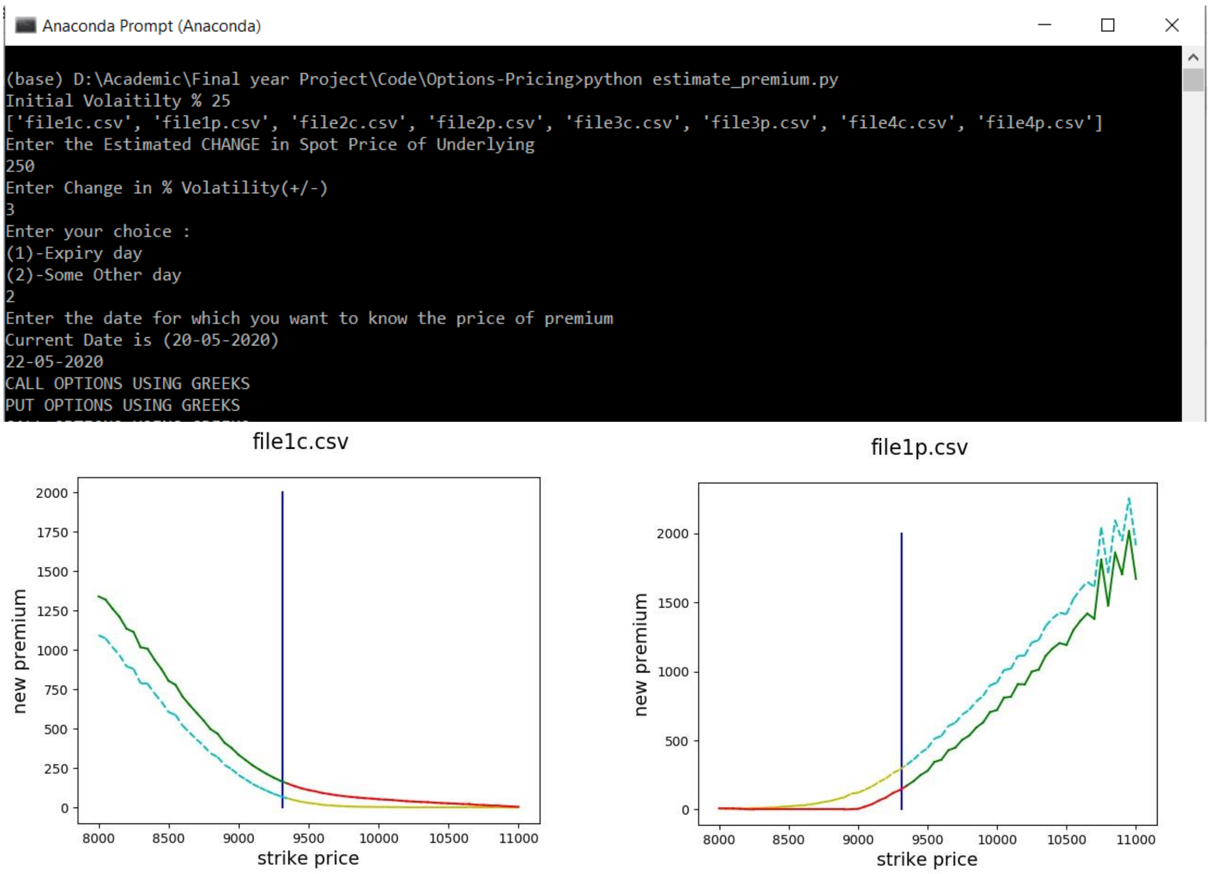


Fig 4.3.: Premium trends on +250 change, on non-expiry date

4.4. Case 4: Near the current day before expiry when there is a decrease

Estimating the premium of option on some date before expiry day and near the current date when there is a decrease of 250 points in the underlying stock value.

Initial Stock Price: 9066.55 Initial volatility: 25%

Change in Stock Price: -250 Change in volatility: +2%

Expiry Date of Option: 28-05-2020 Date of estimated new premium: 22-05-2020

This case is a mirror opposite of Case 3, with a 250-point decrease in the stock price, on the date 22-05-2020, Premiums of put options contracts increase regardless of its moneyness, because of their high intrinsic value and time left for expiry. Call option prices have decreased as they lose their intrinsic value and better not be exercised than take a loss. As compared to case 2, because of less theta deductions the put option premiums have increased.

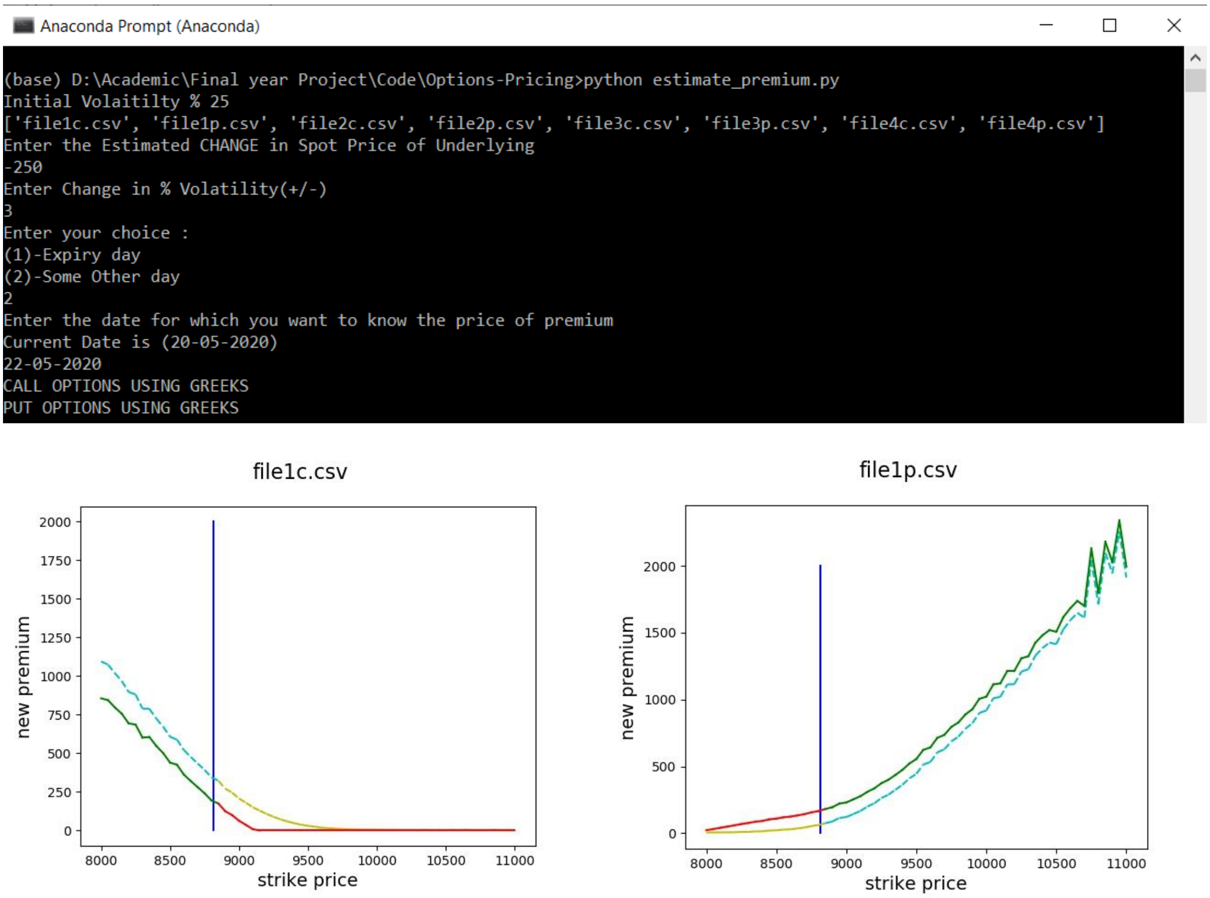


Fig 4.4: Premium trends on -250 change, on non-expiry date

4.5. Case 5: Estimating on small change and calculating error

Estimating the premium of options on 22th May 2020 when stock prices fell from 9066.55 to 9039 in 2 days, and comparing it with actual values to calculate root mean square error. For this case, the actual prices which were observed are required as input.

Initial Stock Price: 9066.55

Change in Stock Price: -27.55

New Stock Price: 9037

Initial volatility: 25%

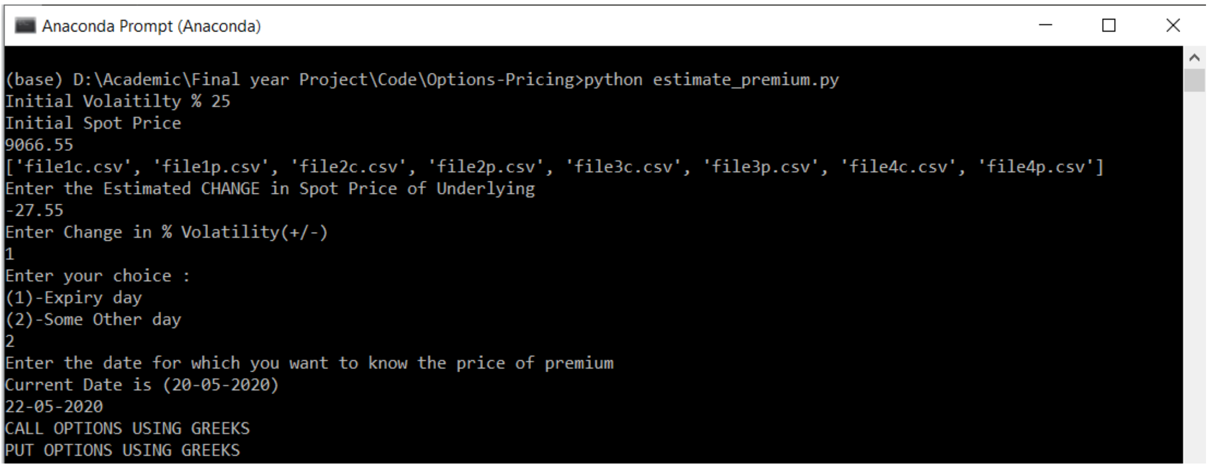
Change in volatility: +2%

Expiry Date of Option: 28-05-2020, 04-06-2020, 11-06-2020, 18-06-2020

Date of estimated new premium: 22-05-2020

In this case, we are estimating the premium value as on 22-05-2020 with the actual value the premium on that day. When the stock prices of NIFTY changed from 9066.55 to 9039 in span of 2 days, from our algorithm we observe slight decrease in prices of call options and a small increase in the premiums of put options.

However, the RMSE value we get for our estimation is not negligible, which is because the prices heavily depend on user input of volatility and the change in it.




```
(base) D:\Academic\Final year Project\Code\Options-Pricing>python estimate_premium.py
Initial Volaitilty % 25
Initial Spot Price
9066.55
['file1c.csv', 'file1p.csv', 'file2c.csv', 'file2p.csv', 'file3c.csv', 'file3p.csv', 'file4c.csv', 'file4p.csv']
Enter the Estimated CHANGE in Spot Price of Underlying
-27.55
Enter Change in % Volatility(+/-)
1
Enter your choice :
(1)-Expiry day
(2)-Some Other day
2
Enter the date for which you want to know the price of premium
Current Date is (20-05-2020)
22-05-2020
CALL OPTIONS USING GREEKS
PUT OPTIONS USING GREEKS
```

Figure 4.5.: Estimation on small change, on 22-05-2020 (Part 1)

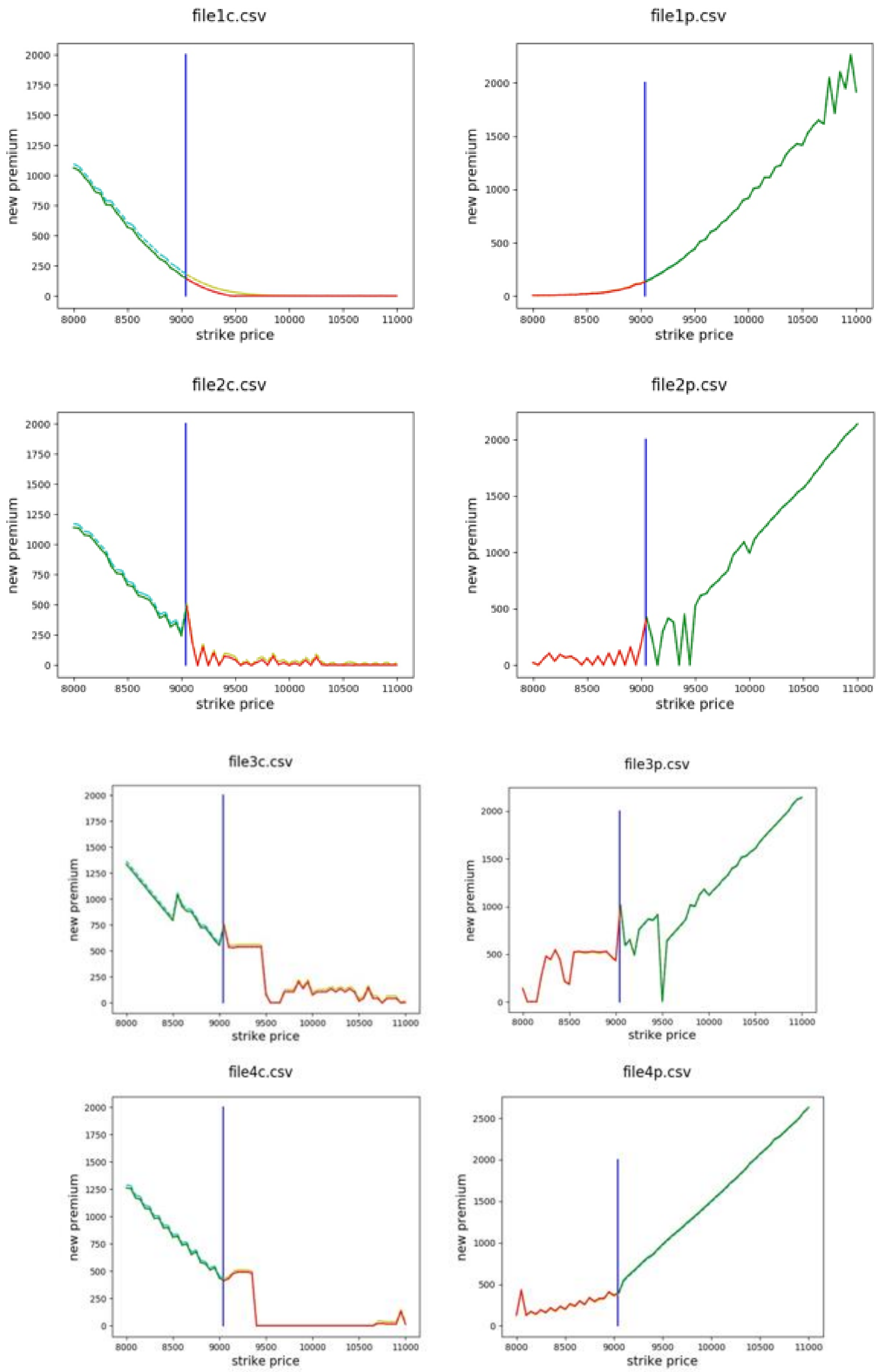
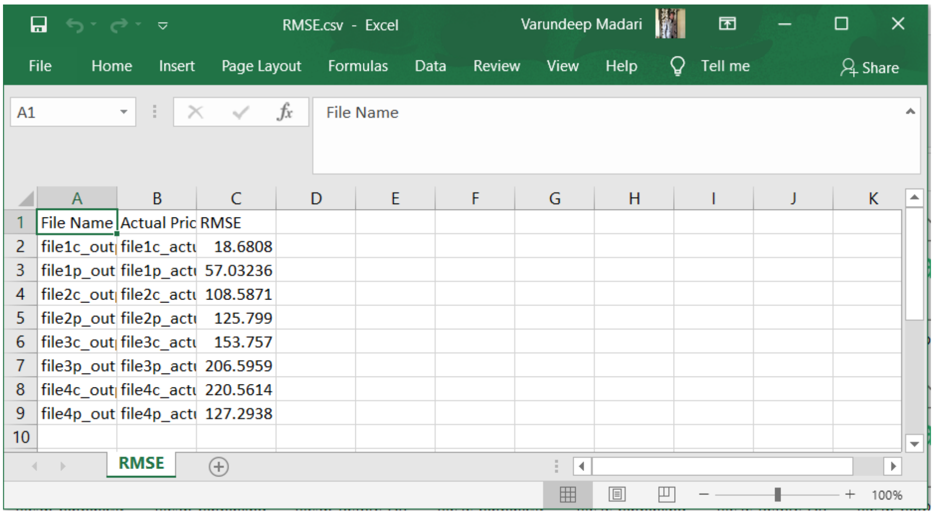


Figure 4.5.: Estimation on small change, on 22-05-2020 (Part 2)



File Name	Actual Pric	RMSE
file1c_out	file1c_actu	18.6808
file1p_out	file1p_actu	57.03236
file2c_out	file2c_actu	108.5871
file2p_out	file2p_actu	125.799
file3c_out	file3c_actu	153.757
file3p_out	file3p_actu	206.5959
file4c_out	file4c_actu	220.5614
file4p_out	file4p_actu	127.2938

Figure 4.6.: RMSE calculation for each expiry day

4.6. Case 6: Examining the change on different expiry dates on increase

Estimating the premium of options on some date before expiry date when there is an increase of 250 points. Also, studying the changes observed in each option chain with different expiry comparatively.

Initial Stock Price: 9066.55

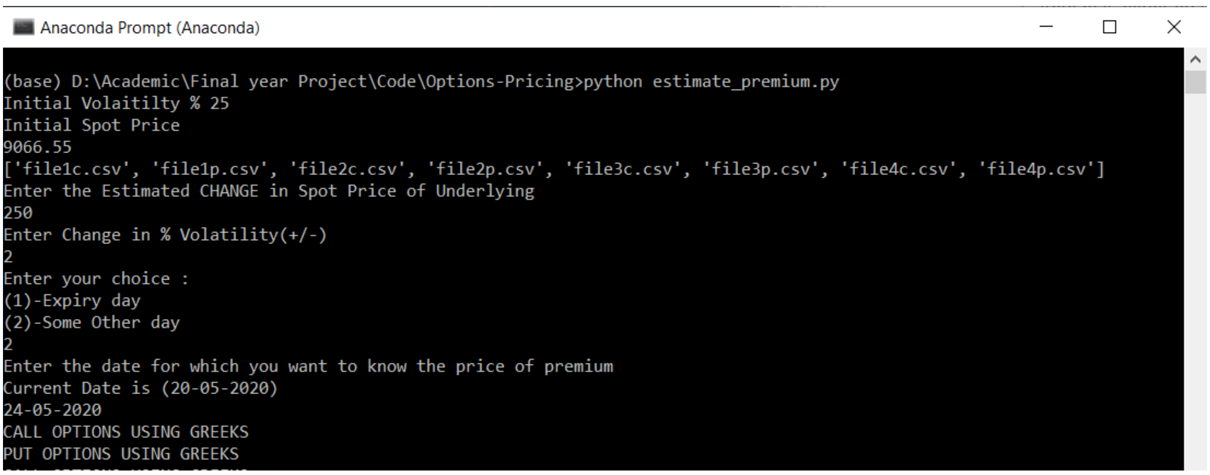
Change in Stock Price: +250

Initial volatility: 25%

Change in volatility: +2%

Expiry Date of Option: 28-05-2020, 04-06-2020, 11-06-2020, 18-06-2020

Date of estimated new premium: 24-05-2020



```
(base) D:\Academic\Final year Project\Code\Options-Pricing>python estimate_premium.py
Initial Volaitilty % 25
Initial Spot Price
9066.55
['file1c.csv', 'file1p.csv', 'file2c.csv', 'file2p.csv', 'file3c.csv', 'file3p.csv', 'file4c.csv', 'file4p.csv']
Enter the Estimated CHANGE in Spot Price of Underlying
250
Enter Change in % Volatility(+/-)
2
Enter your choice :
(1)-Expiry day
(2)-Some Other day
2
Enter the date for which you want to know the price of premium
Current Date is (20-05-2020)
24-05-2020
CALL OPTIONS USING GREEKS
PUT OPTIONS USING GREEKS
```

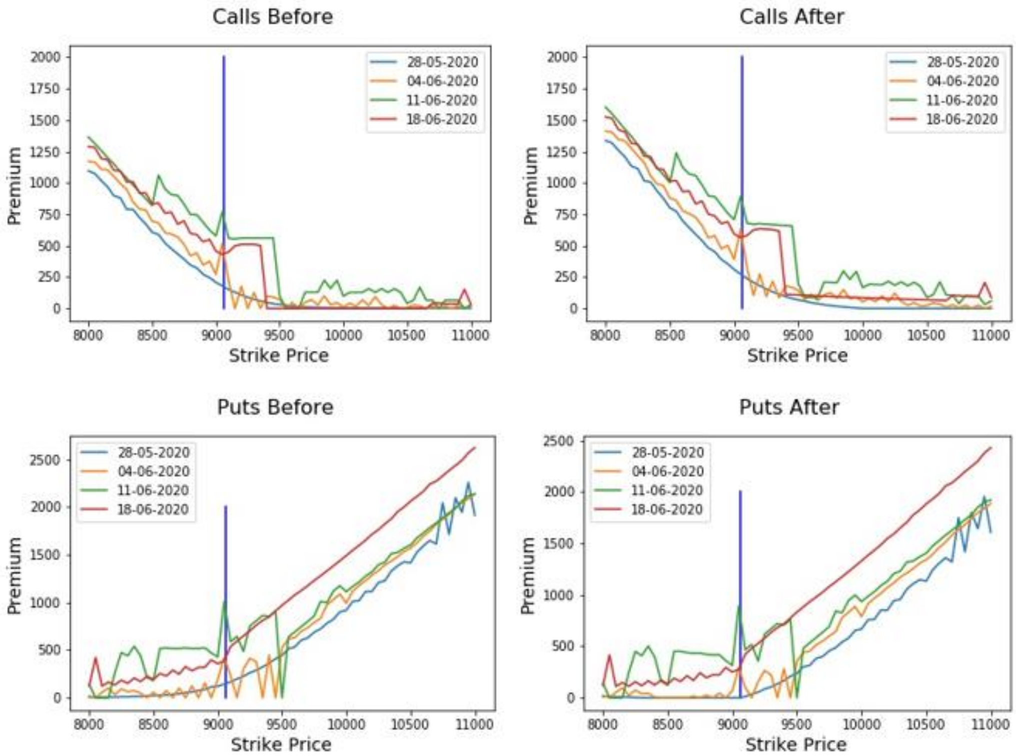


Fig 4.7.: Comparative results for a 250-point increase

In Fig 4.7, we observe the changes in premiums of options when there is a +250-point change in the underlying value of the stock. As it an increase, as observed in previous cases the premium of call option increases and premium of put option decreases.

When we compare the changes in premiums of different dates, it is always observed consistently that the premium change greatly depends on the date of expiry and affect each Greek add to the new premium. With an increase of 250 in stock prices, the option premiums of various expiry dates are shown above as estimated on 24-05-2020. Options with more time remaining to maturity have more option premium, and are correctly estimated by our implementation.

4.7. Case 7: Examining the change on different expiry dates on decrease

Estimating the premium of option on some date before expiry date when there is a decrease of 250 points. Also, studying the changes observed in each option chain with different expiry comparatively.

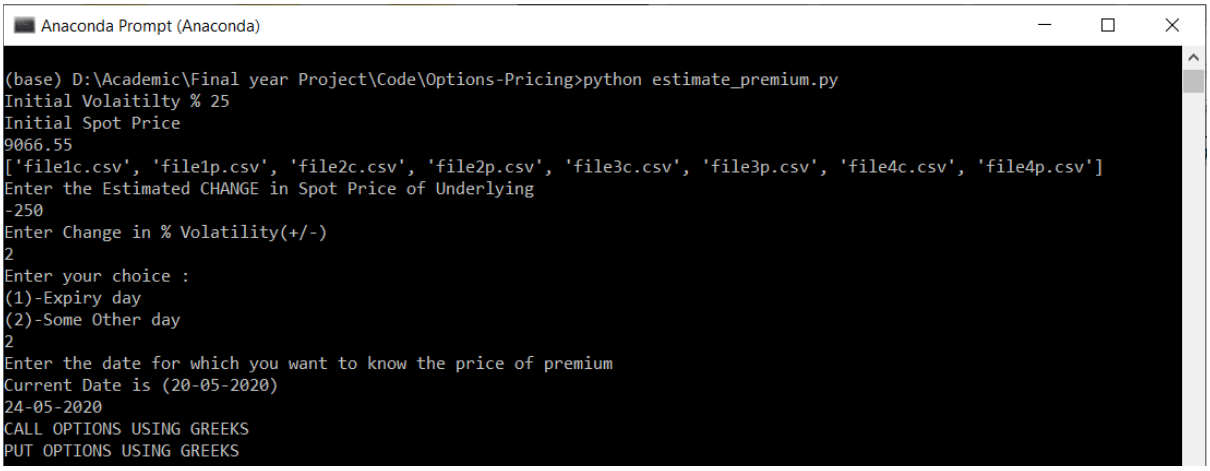
Initial Stock Price: 9066.55 Change in Stock Price: -250

Initial volatility: 25% Change in volatility: +2%

Expiry Date of Option: 28-05-2020, 04-06-2020, 11-06-2020, 18-06-2020

Date of estimated new premium: 24-05-2020

With a decrease in spot price by 250, this case is total opposite of Case 6. We observe that prices of put options increases and, premiums those who have more time left to maturity is gets more increase than those who are near maturity.



Anaconda Prompt (Anaconda)

```
(base) D:\Academic\Final year Project\Code\Options-Pricing>python estimate_premium.py
Initial Volaitilty % 25
Initial Spot Price
9066.55
['file1c.csv', 'file1p.csv', 'file2c.csv', 'file2p.csv', 'file3c.csv', 'file3p.csv', 'file4c.csv', 'file4p.csv']
Enter the Estimated CHANGE in Spot Price of Underlying
-250
Enter Change in % Volatility(+/-)
2
Enter your choice :
(1)-Expiry day
(2)-Some Other day
2
Enter the date for which you want to know the price of premium
Current Date is (20-05-2020)
24-05-2020
CALL OPTIONS USING GREEKS
PUT OPTIONS USING GREEKS
```

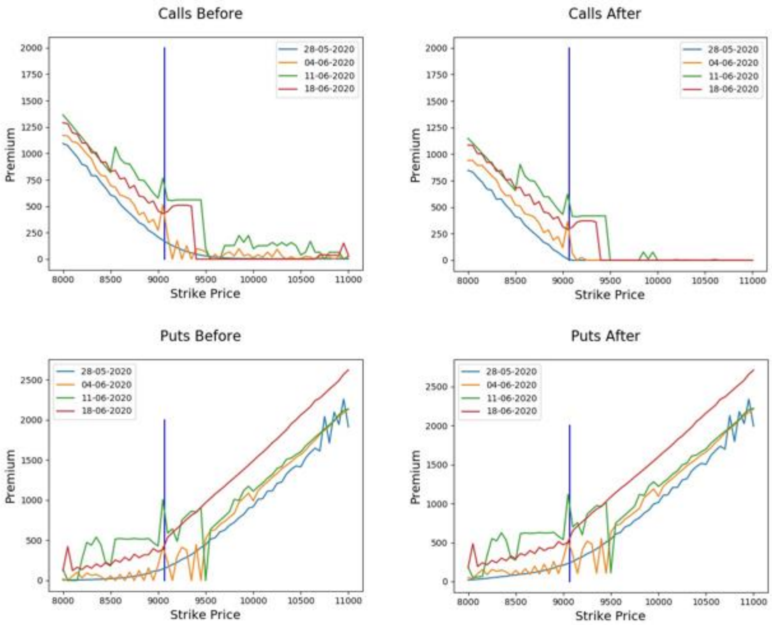


Fig 4.8.: Comparative results for a 250-point decrease

4.8.: Observations:

For every positive change in underlying stock:

a. For call options

   i. On Expiry / Near Expiry

      1. Premium of ITM call options which have strike price less than stock price increases.
      2. Premium of ATM call options approaches zero.
      3. Premium of OTM call option is zero.

   ii. Far from Expiry

      1. Premium of ITM call options increases significantly
      2. Premium of ATM call options increase.
      3. Premium of OTM is near zero.

b. For put Options

   i. On Expiry / Near Expiry

      1. Premium of ITM put options which have strike price less than stock price decreases.
      2. Premium of ATM put options approaches zero.
      3. Premium of OTM put options are zero.

   ii. Far from Expiry

      1. Premium of ITM put option decreases significantly
      2. Premium of ATM put option decreases.
      3. Premium of OTM put option approaches zero.

For every negative change in underlying stock:

a. For call options:

   i. On Expiry / Near Expiry

      1. Premium of ITM call options decreases significantly.
      2. Premium of ATM call options is almost near zero.
      3. Premium of OTM call option zero.

   ii. Far from Expiry

      1. Premium of ITM call options decreases
      2. Premium of ATM call options decreases significantly.
      3. Premium of OTM call option approaches zero.

b. For put Options

i. On Expiry / Near Expiry

1. Premium of ITM put options increases.
2. Premium of ATM put options increases.
3. Premium of OTM put options are near zero.

ii. Far from Expiry

1. Premium of ITM put option increases significantly
2. Premium of ATM put option increases.
3. Premium of OTM put option approaches zero.

CHAPTER 5

CONCLUSIONS, LIMITATIONS AND FUTURE SCOPE

CHAPTER 5

CONCLUSIONS, LIMITATIONS AND FUTURE SCOPE

The goal of the project was to help the trader estimate the change in premium of call and put options for various strike prices and expiry dates using Greeks. This will ease the option trader's strategic decision-making to safeguard against losses due to random or unforeseen changes in the spot prices by choosing which options to buy or sell. The solution was designed by taking into account the effects of various Greeks, volatility, expiry dates, and the trader's view of change in the spot price and volatility. This solution produced the new estimated premium and comparison of premiums for different expiry dates for a selected range of strike prices. The results abide by the BSM model with assumptions. The RMSE evaluation compares the estimated values of premium with the actual values of premium for a past date. As the premium not only depends on the Greeks but also on the expected change in volatility and spot price by the trader, the RMSE values are subject to changes and correct input of changes in volatility and stock price.

5.1. Limitations:

1. Estimation by using BSM model is only valid for European style options and not American style options.
2. Change in spot price and volatility is not known to the trader/user, he enters the values according to his view on the market (bearish/bullish) and experience of market trends.
3. Values of Greeks in reality are constant for a short instant of time and change continuously and stochastically.
4. In real life, values of σ and μ in equation (3) are not constant and vary stochastically.
5. Dividends yield rate is not considered when pricing.
6. Risk-free rate of interest is considered constant in the period of pricing.
7. Historical data used for input, and not real time data.

5.2. Future Scope:

1. Considering the rate of dividend yield when pricing.
2. Employing Implied Volatility, for better estimation.
3. Using Data Stream of stock prices in place of historical data.
4. Improving the accuracy of estimation.

CHAPTER 6

REFERENCES

CHAPTER 6

REFERENCES

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2. https://zerodha.com/varsity/chapter/greek-interactions/

   To understand about Options, Greeks and their risk assessment.
3. https://brilliant.org/wiki/black-scholes-merton/

   To understand the Black-Scholes-Merton formula and its assumptions.
4. Stochastic Differential Equations: An Introduction with Applications –by Bernt Øksendal

   To develop an understanding on what Stochastic Differential equation is.
5. Stochastic Calculus for Finance II – by Steven Shreve

   To understand about Ito integrals.
6. https://www.youtube.com/playlist?list=PLS3zAvd8Oxexb09MgPf7STaJWvBUkaH-Y

   To visualize the Stochastic calculus for clearer understanding.

APPENDIX

APPENDIX

A.1. Stochastic Processes

A set containing random variables ordered and indexed by a mathematical set such that each random variable is uniquely linked to an element in the set, is called a stochastic process. Index set is defined as a mathematical set, which is used to index the random variable. If the index set of a stochastic process contains a finite number of elements that can explicate the index set as time, then the stochastic process is discrete time. A stochastic process takes it value from a set called state space. It is continuous, if the index set is an interval of the real number line. Stochastic processes are represented as $\{ P_t \}$ or $\{P(t)\}_{t \in T}$, where P(t) is referred to as a random variable with index *t*.

Formally, A stochastic process is defined as a collection of random variables defined on a common probability space (**Ω , F , P)** , where **Ω** is the sample space, **F** is the **σ**-algebra, and **P** is a probability measure; and the random variables, indexed by some set **T**, all take values in the same mathematical space **S**, which must be measurable with respect to some **σ**-algebra

Some examples of Stochastic processes are Random Walk, Wiener process, and Poisson process. In our implementation we have taken the Random Walk and Wiener process.

A.2. Random Walk

These are a type of stochastic processes which are given as summation of independent and identically distributed random, so these are processes that show changes in discrete intervals of time. When the state space of a random walk consists of integers, this stochastic process in discrete time is called a simple random walk. This is carried out on the integers. Its value increments by 1 unit with a probability '***q'***, or decrements by 1 unit with probability '***1-q'***. State space of a simple random walk is integers and its index set is of natural numbers. It is known as a symmetric random walk when the value of ***q=0.5***, that is probability of increments and decrements are equal.

A.3. Wiener process

The Wiener process characterises the Brownian motion into a mathematical equation. It is a stochastic process which has independent and stationary increments which are normally distributed based on the size of the increments. This process is also called Brownian motion. Its state space and index set both are non-negative real numbers.

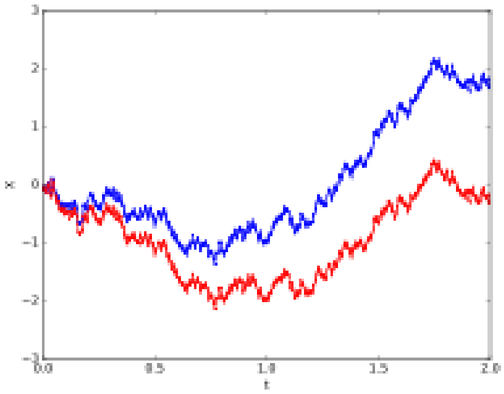

Figure A.1: Wiener Process graph

The Wieners process W_t has the following properties:

1. $W_0 = 0$
2. W has independent increments. The future increments W_{t+i} - W_t , $i \geq 0$, are independent of the past values W_q , for every $t > 0$, such that $q \leq t$.
3. W_{t+i} - W_t is normally distributed with mean 0 and variance *u*. $W_{t+i} - W_t \sim \mathcal{N}(0, i)$
4. W_t is continuous in *t*.

A sample path of a Wiener process is nowhere differentiable but almost surely continuous everywhere. Simple random walk can be considered as a continuous version of Wiener process's path, which has been shown below.

Stochastic calculus is one of the many fields that has many applications of Wiener process. In various fields, such as natural sciences and divisions of social sciences, it is used as a mathematical model to model such random phenomena. Similarly, it performs a vital part in the field of quantitative finance. One of its major applications in quantitative finance is in the Black-Scholes-Merton model.

A.4. Defining Wiener Process as a limit of Symmetric Random walk

To show Wiener as a limit of Symmetric random walk. Consider a random process W, where each w_i has value of Head or Tail

$$W = w_1 w_2 w_3 . . . . w_n$$

Let $W = HHTTTHHHHTT.....$

Now we construct a function to map a random variable to an integer value,

$$X(j) = \begin{cases} 1, & w(j) = H \\ -1, & w(j) = T \end{cases}$$

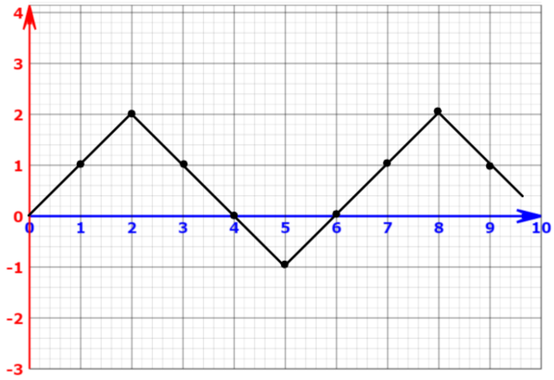

Figure A.2.: Plotting the Symmetric random walk.

For the above random process *W*, we get graph as below, from which we make observations as follows:

1. The above process has independent increments.
2. The random variable's present value doesn't depend on previous random variable's values.

If we increase the number of intervals to infinitely large values, the graph in Fig A.2 will look similar to the Fig A.3, which is how stock market value looks like.

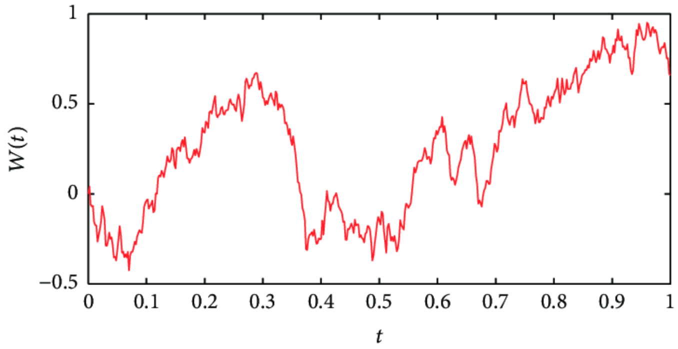

Figure A.3.: Wiener Process or Brownian Motion

A specific type of stochastic process is a Markov process wherein only the variable's present value is important to predict the future's value. The variable's history and the manner the present has turned up from the past are unimportant. Prices of stocks are believed to obey a Markov process. As the future predictions of stock price are not certain they ought to be demonstrated in the form of probability distributions. The Markov property suggests that the stock price's probability distribution at any instant in future is independent of the path taken by the stock price in the past.

A.5. Introducing Stochastic Differential Equations

Before moving to SDE, let us have a look at Differential equations.

An Ordinary differential equation shortly called ODE consists of a single or more than one function of one independent variable & the derivative of those functions.

Equation (11) is example of an ODE is the population growth model for a village.

$$\frac{dP}{dt} = \alpha(t)P(t) \tag{11}$$

$$P(0) = P_0 \;\; (a\ constant\ value) \tag{12}$$

Here *P(t)* is the population of village at time instant *t* and$\alpha(t)$ is the growth rate of population at time *t*. However, we may see that $\alpha(t)$is not a fixed value, and may vary depending on some random environmental factors that may alter the population rate , for example immigration, birth, epidemics, etc, so we have

$$\alpha(t) \;=\; \gamma(t) \;+\; \omega(t) \tag{13}$$

Where we do not know the exact behaviour of the $\omega(t)$ term it behaves as noise, but only its probability distribution. Ths function $\gamma(t)$ is assumed to be a non-random function. After including the noise term, our equation (5) can be written as

$$\frac{dP}{dt} = (\,\gamma(t)\; +\; \omega(t)\,)\,P(t) \tag{14}$$

or in a more generally as

$$\frac{dX}{dt} = b(\,t\,, X(t)\,)\; +\; \sigma(\,t\,, X(t)\,). \text{"}noise\text{"} \tag{15}$$

Where b and σ are some given functions. When the noise is 1-dimensional. We use a stochastic process W_t which shows the properties of Wiener process to add the effect of random noise to the equations.

A.6. Stochastic Differential Equations

A stochastic differential equation also shortly called SDE is a differential equation wherein one or more than one terms are a form of stochastic process, this results in a solution that is a stochastic process. They are used for modelling of numerous different phenomena which characterise some randomness like unstable prices of stocks. Generally, stochastic differential equations have a variable that renders the random white noise. This random noise is computed as a sample of Wiener process or the derivative of Brownian motion. They are usually comprehended as continuous time limits of corresponding stochastic differential equations. However, such a way of understanding SDEs is vague & ambiguous, that’s why it must be supplemented with a proper interpretation. The most distinguished interpretations are given by Stratonovich and Itô. Itô calculus is used in mathematics and quantitative finance and Stratonovich calculus is used in Physics

A.7. Stochastic Calculus

It is a branch of math which works on stochastic processes. It paves way to define a consistent theory of integration for integrals of stochastic nature with regards to stochastic processes. Wiener process which is used to model Brownian motion is the most used stochastic process, to which stochastic calculus is applied. The stochastic Wiener process, or the Brownian motion was found to be exceptionally complex in terms of mathematics. It is almost surely continuous everywhere but not at all differentiable, thus it requires its own calculus rules. As the value of noise has considerable variation in smaller periods of time, the point of integration chosen in an interval gives different answers depending on the point. The most dominant versions of stochastic calculus are the Itô stochastic calculus and the Stratonovich stochastic calculus. Each has its own set of pros and cons. Our project domain of Mathematical finance uses Itô calculus

A.8. Itô integral

When we integrate a stochastic process, on dividing the in intervals of the function domain, we see that the quadratic variation of a stochastic process keeps on increasing, while in case of non-stochastic process, the quadratic variation tends to zero. The fact that the randomness of a stochastic process keeps increasing on dividing the intervals, makes it different from Non-stochastic or the integration we follow. The point in the interval [t_k , t_{k+1}] of integration gives different results. We see this in the illustration below.

We are integrating a Wiener process with itself.

$$\int_0^t W(s)\, dW_s$$

We divide the intervals into *n* equal parts.

$\Delta t = \frac{t-0}{n} = \frac{t}{n}$ such that $0 = t_0 < t_1 < t_2 < \ldots < t_n = t$

We denote $t_k = k\frac{t}{n} = k\Delta t$

$$\int_0^t W(s)\, dW_s = \lim_{n\to\infty} \sum_1^n W_{t_{k'}}(W_{t_k} - W_{t_{k-1}}) \qquad (16)$$

As we have discussed above, we have 3 options to consider the point of integration in a interval $t_{k-1} \leq t_{k'} \leq t_k$

1. if $t_{k'} = t_{k-1}$ then $\sum_{k=1}^n W_{t_{k-1}}(W_{t_k} - W_{t_{k-1}})$ (17)
2. if $t_{k'} = (t_{k-1} + t_k)/2$ then $\sum_1^n \left(\frac{W_{t_{k-1}} + W_k}{2}\right)(W_{t_k} - W_{t_{k-1}})$ (18)
3. if $t_{k'} = t_k$ then $\sum_1^n W_{t_k}(W_{t_k} - W_{t_{k-1}})$ (19)

Case (1) (Equation 17) is known as the Itô integral, when we consider the station of the interval as the point of integration. This is used in Financial Mathematics as the trading of financial instruments is done at the starting of intervals. Itô integral does not follow the Newton Calculus rules

Case (2) (Equation 18) is known as Stratonovich integral, which is used in Physics. Stratonovich calculus follows Newtonian calculus rules.

Coming back to solving from equation (17),

$$\int_0^t W(s)\, dW_s = \lim_{n\to\infty} \sum_1^n W_{t_{k-1}}(W_{t_k} - W_{t_{k-1}})$$

To solve it, we use a simple function $\Delta_n(t)$, defines value for every interval. Simple functions can be understood as step functions.

$$\Delta_n(t) = \begin{cases} W(0) = 0 & \text{if } 0 \leq t < \frac{T}{n}, \\ W\left(\frac{T}{n}\right) & \text{if } \frac{T}{n} \leq t < \frac{2T}{n}, \\ \vdots & \\ W\left(\frac{(n-1)T}{n}\right) & \text{if } \frac{(n-1)T}{n} \leq t < T, \end{cases}$$

$$\begin{aligned} \int_0^T W(t)\, dW(t) &= \lim_{n\to\infty} \int_0^T \Delta_n(t)\, dW(t) \\ &= \lim_{n\to\infty} \sum_{j=0}^{n-1} W\left(\frac{jT}{n}\right)\left[W\left(\frac{(j+1)T}{n}\right) - W\left(\frac{jT}{n}\right)\right]. \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\sum_{j=0}^{n-1}(W_{j+1} - W_j)^2 &= \frac{1}{2}\sum_{j=0}^{n-1} W_{j+1}^2 - \sum_{j=0}^{n-1} W_j W_{j+1} + \frac{1}{2}\sum_{j=0}^{n-1} W_j^2 \\ &= \frac{1}{2}\sum_{k=1}^{n} W_k^2 - \sum_{j=0}^{n-1} W_j W_{j+1} + \frac{1}{2}\sum_{j=0}^{n-1} W_j^2 \\ &= \frac{1}{2}W_n^2 + \frac{1}{2}\sum_{k=0}^{n-1} W_k^2 - \sum_{j=0}^{n-1} W_j W_{j+1} + \frac{1}{2}\sum_{j=0}^{n-1} W_j^2 \\ &= \frac{1}{2}W_n^2 + \sum_{j=0}^{n-1} W_j^2 - \sum_{j=0}^{n-1} W_j W_{j+1} \\ &= \frac{1}{2}W_n^2 + \sum_{j=0}^{n-1} W_j(W_j - W_{j+1}). \qquad \textbf{(4.3.5)} \end{aligned}$$

$$\sum_{j=0}^{n-1} W_j(W_{j+1} - W_j) = \frac{1}{2}W_n^2 - \frac{1}{2}\sum_{j=0}^{n-1}(W_{j+1} - W_j)^2. \qquad (19)$$

Equation (19) is solution of equation (17), where the second term of the RHS is the quadratic variation, which makes Stochastic calculus different than Newtonian calculus.

A.9. Itô's lemma

Itô's lemma is an identity used in Itô Calculus to find the differential of a time-dependent function of a stochastic process. Due to complex calculation involving Itô Calculus, Itô's lemma eases the calculation in integration. It is like a chain rule's counterpart in stochastic calculus. The lemma has many applications in finance of math, and its renowned application is the derivation of the Black-Scholes equation for values of options. This lemma is also referred to as the Itô-Doeblin theorem.

Let $f(t,x)$ be a function for which the partial derivatives $f_t(t,x), f_x(t,x)$ and $f_{xx}(t,x)$ are defined and continuous, and let $W_t$be a Brownian motion.

Then, for every $T \geq 0$,

$$f(T, W(T)) = f(0, W(0)) + \int_0^T f_t(t, W(t))\, dt + \int_0^T f_x(t, W(t))\, dW(t) + \frac{1}{2}\int_0^T f_{xx}(t, W(t))\, dt \tag{20}$$

Equation (20) is the Itô's lemma.

A.10. Geometric Brownian Motion

Brownian motion is capable of taking negative values as well, which makes it non-usable to model stock price, so to avoid the negative value, we use exponential, which gives us Geometric Brownian motion. It is a continuous-time stochastic process where the log of quantity that changes randomly obeys a Brownian motion with diffusion and drift. In particular, it is used in mathematical finance to model stock prices in Black-Scholes-Merton model

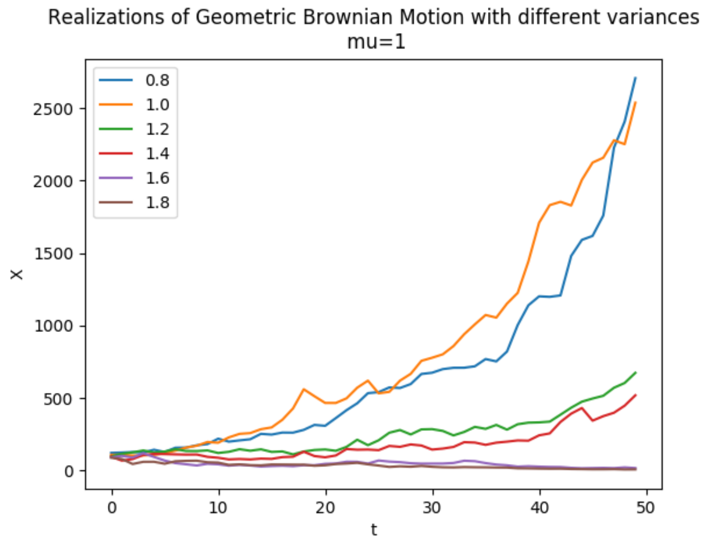

Figure A.4.: Geometric Brownian Motion

A stochastic process S_t obeys a GBM, if it satisfies the following SDE:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{21}$$

where W_t is a Wiener process, μ is the percentage drift and σ is the percentage volatility. μ and σ are constants. μ is used to model the deterministic trends and σ is used to model the randomness of unpredictable events which take place during this process.

The equation can be solved using Itô's formula and getting the solution

$$S_t = S_0 exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right) \tag{22}$$

GBM is used to model stock prices in BSM model and is implemented as a model that exhibits movements in stock price. Some reason to use it are as follows:

1. The expected returns of GBM are independent of the value of the process (stock price), which agrees with what we would expect in reality.
2. Just like real stock prices, a GBM process only assumes positive values.
3. A GBM process shows the same kind of 'zigzagness' and 'roughness' in its paths as we see in real stock prices.
4. Calculations with GBM processes are relatively easy.

However, in real life, volatility of a stock price changes stochastically in real time and not constant as assumed in our equation. GBM also failed to showcase sudden jumps caused by unpredictable events and news.

A.11. Black-Scholes-Merton equation derivation:

Consider this GBM process for stock price process (from equation 3)

$$dS = \mu S dt + \sigma S dz \tag{23}$$

Where z is Brownian motion. Suppose that f is the price of a derivative dependent on S. We consider a call option. The variable f must be some function of S and t. Hence from Itô lemma we get

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S\, dz \tag{24}$$

The discrete versions of equations (23) and (24) are

$$\Delta S = \mu S \Delta t + \sigma S \Delta z \tag{25}$$

$$\Delta f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \tfrac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial f}{\partial S}\sigma S\,\Delta z \tag{26}$$

where Δf and ΔS are the changes in f and Sin a small-time interval Δt. From this, a portfolio can be constructed of the stock and the derivative such that the Wiener process can be eliminated. The portfolio is:

- $\partial f/\partial S$: shares
- -1 : derivative

The portfolio owner is long $\partial f/\partial S$ of shares and short one derivative.

Let Πbe the value of the portfolio. By definition

$$\Pi = -f + S \times (\partial f/\partial S) \tag{27}$$

In the time interval Δt, the change in the value of the portfolio $\Delta\Pi$ is given by

$$\Delta\Pi = -\Delta f + \Delta S \times (\partial f/\partial S) \tag{28}$$

Substituting equations 15.10 and 15.11 into equation 15.13 gives

$$\Delta\Pi = \left(-\frac{\partial f}{\partial t} - \tfrac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t \tag{29}$$

As the above equation doesn't contain any stochastic term, the portfolio of the investor must be riskless during the time Δt. The assumptions listed in the above section imply that the portfolio must instantaneously earn the same rate of return as other short-term risk-free securities. If it earned less than this return, arbitrageurs could make a riskless profit by shorting the portfolio and buying risk-free securities; if it earned more , they could make a riskless profit by borrowing money to buy the portfolio. It follows that change in value of portfolio must be equal to risk-free rate of interest r

$$\Delta\Pi = r\Pi\Delta t \tag{30}$$

Substituting from equations (27) and (29) into (30) we get

$$\left(\frac{\partial f}{\partial t} + \tfrac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t \tag{31}$$

Which can be re-written as

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \tfrac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf \tag{32}$$

The above equation (32) is the Black-Scholes-Merton equation

A.12. Data Sources:

The data can be collected from different sources namely,

- NSE India
  https://www1.nseindia.com/live_market/dynaContent/live_watch/option_chain
- Money Control
  https://www.moneycontrol.com/markets/fno-market-snapshot
- Sharekhan
  https://www.sharekhan.com/market/market-derivatives

The data which was used in this implementation was collected from the NSE - National Stock Exchange of India Ltd. One can find the options data under Option Chain (Equity Derivatives). Here one can view Options Contracts for any symbol such as NIFTY or any other company and filter them by expiry dates OR by strike price. For example, in our data collection, we chose NIFTY and filtered by expiry dates. On the upper right side, 'Underlying index' shows the underlying value of the stock price for that trading day. In the middle is the 'STRIKE PRICE' column and options corresponding to several listed strike prices can be chosen. On the Left Side of STRIKE PRICE is call option data and on the Right Side, we find put option data.