FINAL PROJECT

Problem Description

In this project, we aim to design and implement a digital filter within a microcontroller to effectively mitigate noise in signal processing applications. This involves distinguishing essential signal components from noise within the same frequency spectrum to ensure data integrity and accuracy. Utilizing a Butterworth filter design for its flat passband characteristic, the project entails programming a pre-existing difference equation, calculating and applying specific filter coefficients, and embedding the solution in a microcontroller environment. This setup is expected to enhance the signal-to-noise ratio, thus improving the reliability and performance of systems dependent on precise data readings.

Calculations for N and Ω_c

Passband cutoff frequency (ω_p): 0.2π rad/s

Stopband cutoff frequency (ω_s): 0.7π rad/s

Passband ripple: 1 dB (Typical for Butterworth)

Stopband attenuation: -25 dB

Justification for ω_p and ω_s -

The selected passband cutoff frequency (ω_p) 0.2π rad/s and stopband cutoff frequency (ω_s) 0.7π rad/s for the digital filter are strategically chosen to optimize filter performance within design constraints. Setting (ω_p) 0.2π rad/s ensures the essential 10 Hz signal is comfortably included within the passband, preserving its integrity with minimal attenuation, while (ω_s) 0.7π rad/s effectively places unwanted frequencies like the 203 Hz noise well into the stopband, ensuring significant attenuation. This choice balances the need for effective noise reduction with maintaining a manageable filter order ($N \leq 10$), simplifying implementation and minimizing potential side effects such as aliasing and phase distortions in higher frequency components.

Given:

$\delta_p = -1 = 10^{-1/20} = 0.8913$

$\delta_s = -25 = 10^{-25/20} = 0.0562$

Choosing ω_p and ω_s:

$\omega_p = 0.2\pi$ rad/s

$\omega_s = 0.7\pi$ rad/s

Finding k_p and k_s:

$$k_p = \frac{1}{\delta_p^2} - 1 = \frac{1}{(0.8913)^2} - 1 = 0.2588$$

$$k_s = \frac{1}{\delta_s^2} - 1 = \frac{1}{(0.0562)^2} - 1 = 316$$

Finding N and Ω_c:

$$\Omega_p = \frac{\omega_p}{T} = \frac{0.628}{T}$$

$$\Omega_s = \frac{\omega_s}{T} = \frac{2.198}{T}$$

$$|H(j\Omega)^2| = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$\Rightarrow \delta_p^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} \quad , \quad \delta_s^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

$$= \left(\frac{\Omega_p}{\Omega_c}\right)^{2N} \qquad = \left(\frac{\Omega_s}{\Omega_c}\right)^{2N}$$

$$= \frac{1}{\delta_p^2} - 1 = k_p \qquad = \frac{1}{\delta_s^2} - 1 = k_s$$

Hence, $\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = k_p$ and $\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = k_s$

$$\therefore \frac{k_p}{k_s} = \frac{\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}}{\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

$$N = \frac{1}{2} \cdot \frac{\log\left(\frac{k_s}{k_p}\right)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{1}{2} \cdot \frac{\log\left(\frac{316}{0.2588}\right)}{\log\left(\frac{2.198}{0.628}\right)}$$

$$= 2.8367$$

$$\simeq 3 \qquad \Longrightarrow N = 3$$

Now, $\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = k_p \Rightarrow \left(\frac{0.628}{\Omega_c}\right)^{2\times 3} = 0.2588 \Rightarrow \Omega_c = 0.7867$

$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = k_s \Rightarrow \left(\frac{2.198}{\Omega_c}\right)^{6} = 316 \Rightarrow \Omega_c = 0.8422$

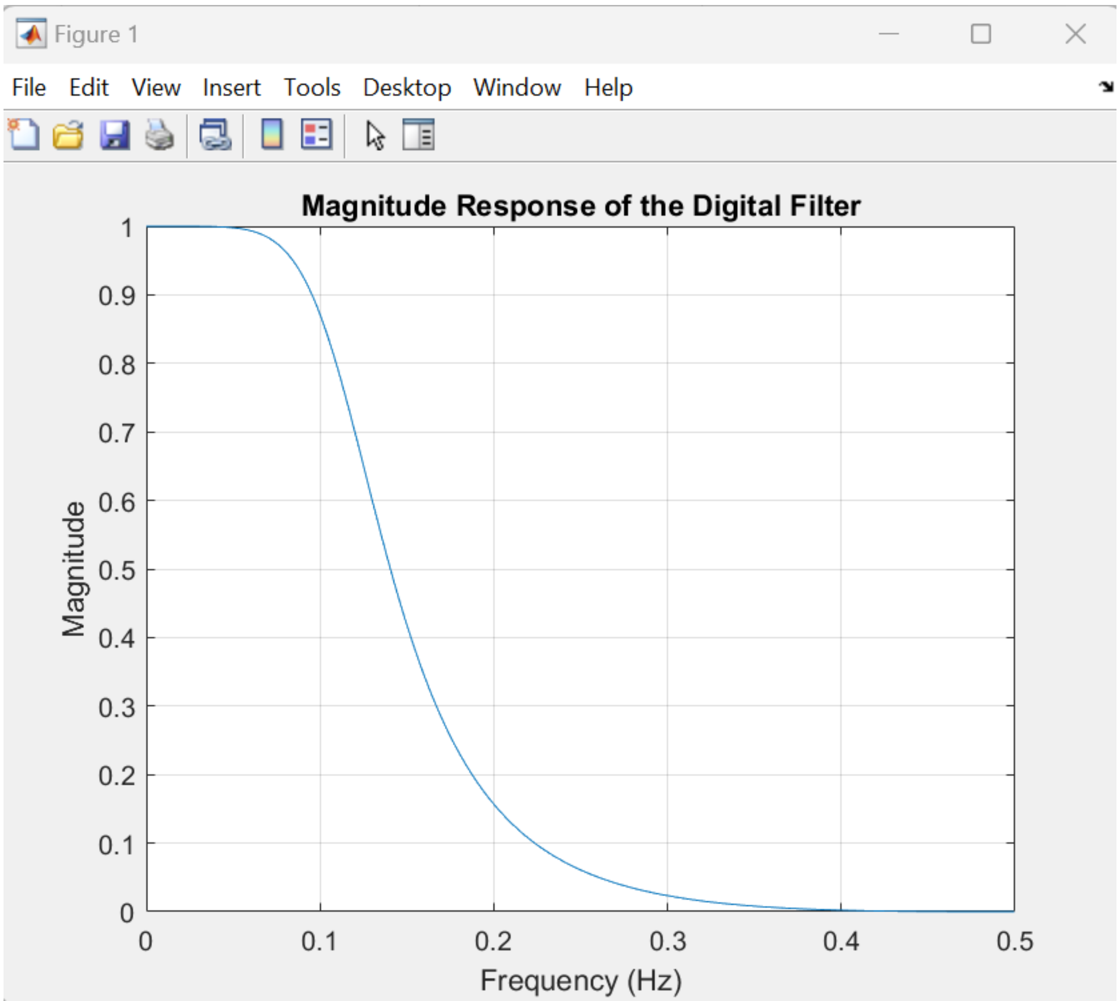
Hence, $N = 3$ and $\Omega_c = 0.7867$

A and B filter coefficients

A = [1.0000 -1.5274 0.9673 -0.2141]

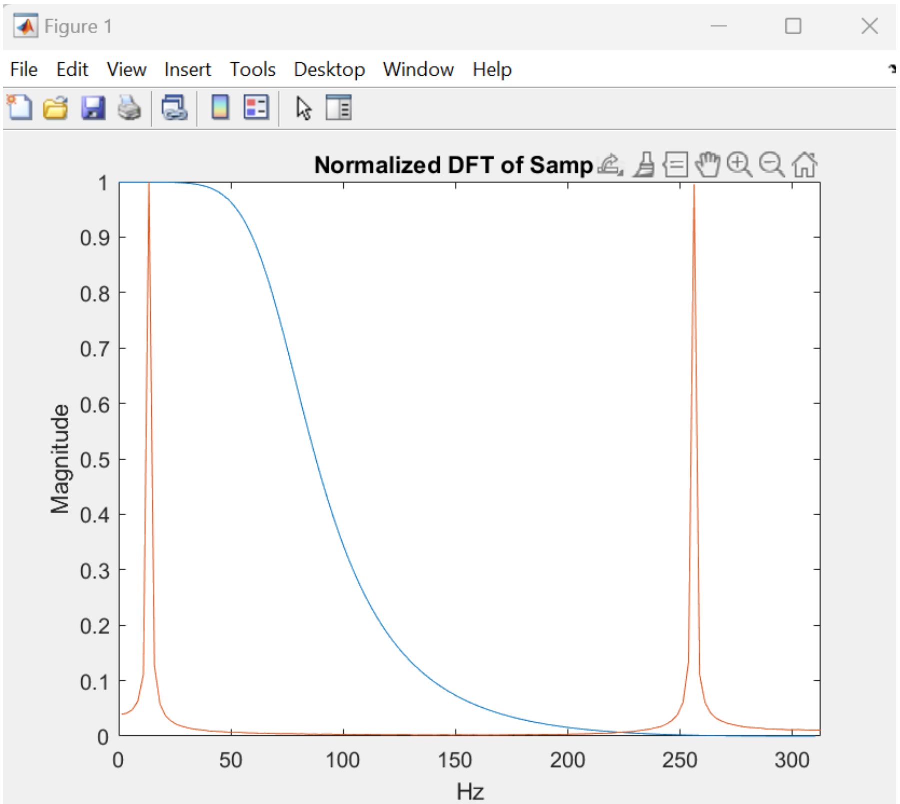
B = [0.0282 0.0846 0.0846 0.0282]

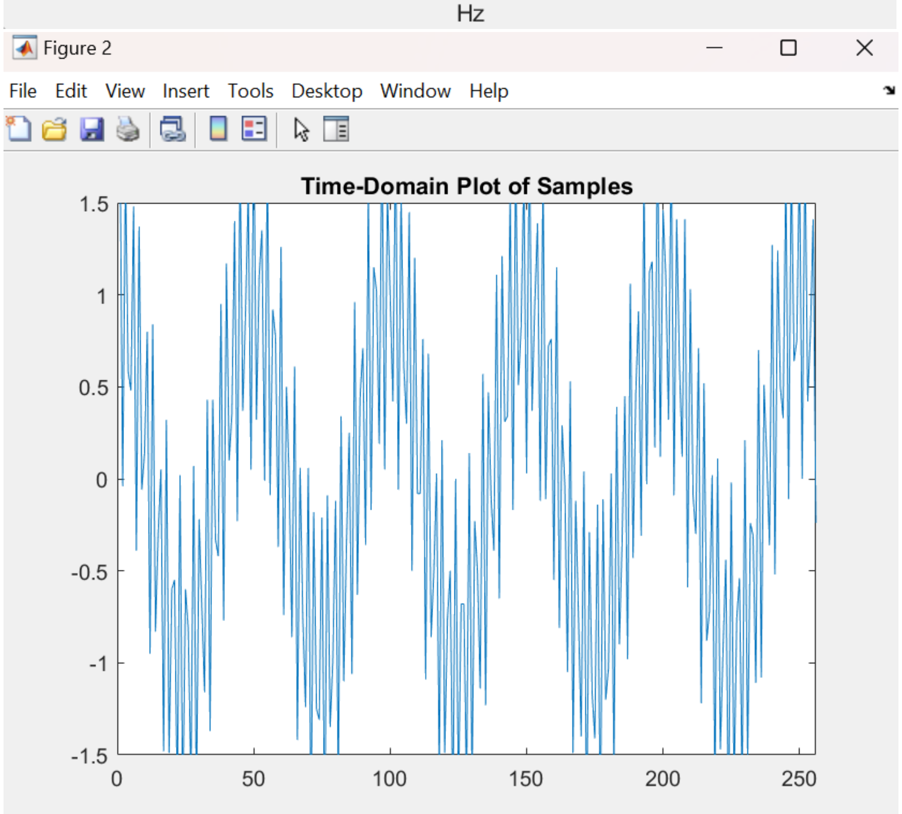
Plot of the magnitude of your digital filter frequency response abs(H) vs digital frequency-



Plots from microcontroller

Plot of the microcontroller output without filtering applied:





Plot of the microcontroller output with filtering applied: