Notes about TFNP

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1 Complexity class in TFNP

1.1 TFNP

1.1.1 Function problem

Definition 1.1. *A functional problem* $\boldsymbol{P}$ *is defined by a relation R over strings of an arbitrary alphabet* Σ : $\boldsymbol{R} \subseteq \Sigma^* \times \Sigma^*$.

An algorithm solves $\boldsymbol{P}$ if for every input x such that there exists a y satisfying $(x, y) \in \boldsymbol{R}$, the algorithm produces one such y, and if there are no such y, it rejects. The output for function problems is more complex than that of a decision problem, which means it's not simply 'yes' or 'no'.

1. FP: the set of binary relations for which there is a polynomial time algorithm that, given x, finds some y for which $R(x, y)$ holds.
2. FNP: the set of binary relations for which there is a polynomial time algorithm that can determine whether $R(x, y)$ holds given both x and y.
3. TFNP: a subset of FNP for which are total relation.

1.1.2 Black-Box model