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Optional Recap Exercises

Exercise: Formal Proof of a Hoare Triple: while loop
Formally prove the correctness of the following Hoare triple:

$\{\text{int } n \wedge n > 0\}$

```
goal=2;
i=0;
while (i<n){
 goal = 2*goal
 i = i+1
}
```

$\{\text{ goal} = 2^{n+1} \}$

Exercise: Prove Satisfiability using the SAT-algorithm Prove the satisfiability of the propositional logical formula F below by applying the space-efficient SAT algorithm seen in the course.

$$F \equiv (Q \wedge R) \leftrightarrow (R \vee S)$$

Exercise: Applying the Resolution algorithm
Convert the formula F into an equisatisfiable formula F' in CNF using the method seen in class, then apply resolution on F' to prove satisfiability of F.

$$F \equiv P \wedge (\neg Q \rightarrow R)$$

Exercise: Constructing Deterministic Turing Machines
Construct a deterministic TM that accepts if its input is a palindrome. Describe your solution by a finite state diagram and give a short explanation of the behaviour of the TM. The working alphabet is defined as $\Gamma = \{0, 1, B\}$
Hint: *The most effective solution TM will have 7 states and the idea could be to check if the first symbol and the last symbol are the same, if so, replace them by blanks and proceed until one or no symbols remain.*

Exercise: Cooks Theorem Revisited
In Cook's Proof, we constructed a formula $U \wedge N \wedge S \wedge F$. Let M be the TM whose computation is described in the propositional formula above.
Let $p(n)$ be the polynomial restricting the number of computation steps of M on an input of size n.

- What is the purpose of the subformulas U, N, S, F
- What is the structure of U
- What is the site of U and why?

Exercise: NP-Completeness Proofs
You proved that 4TA-SAT $\in \mathcal{NP}$-complete in Exercise Sheet 08.

What argument can you make about the $\mathcal{NP}$-completeness of nTA-SAT, where n is any positive integer, and TA-SAT defined as in Exercise Sheet 8.

Give a proof of nTA-SAT$\in \mathcal{NP}$-complete $\forall n > 0$