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Exercise sheet 9

Chapter 7: First Order Logic

Exercise 1: Analysing FOL Formulas

For the following 3 formulas, please indicate

a) the scope of the quantifiers
b) the free variables (if there are any)
c) whether the formula is closed or not

1. $\forall e.(S(e)) \rightarrow \exists d.(P(d))$
2. $\forall e.P(a,e) \rightarrow \forall e.P(b,e)$
3. $\exists x.(S(x,b) \wedge \forall y.(S(y,b) \rightarrow (x = y)))$

6 points

Exercise 2: Translating FOL Formulas

For the 3 formulas in Exercise 1, use the following vocabulary for the variables, constants and functions to translate them into English:

1. constants: none
   variables: e: 'exam', d: 'day'
   functions: S: 'successful', P: 'party'
2. constants: a: Anna, b: Bob
   variables: e: 'exam'
   functions: P: 'passes'
3. constants: b: 'Bill'
   variables: x, y: 'persons'
   functions: S: 'isSister'

3 points

Exercise 3: Interpretation Check

Which one(s) of these three formulas are true under the interpretation $I = (\{\text{anna}, \text{bob}\}, \alpha_1)$ described below. Please explain your answer.

- $\forall x.\forall y.(cat(x,y) = cat(x,x) \vee (cat(x,y) = cat(y,y))$
- $\forall x.\exists y.(cat(x,y) = cat(x,x) \vee (cat(x,y) = cat(y,y))$
- $\exists x.\exists y.(cat(x,y) = cat(x,x) \vee (cat(x,y) = cat(y,y)).$

$$\begin{aligned} I : \alpha_1[cat](\text{anna}, \text{anna}) &= \text{annaanna} \\ \alpha_1[cat](\text{anna}, \text{bob}) &= \text{annabob} \\ \alpha_1[cat](\text{bob}, \text{anna}) &= \text{bobanna} \\ \alpha_1[cat](\text{bob}, \text{bob}) &= \text{bobbob} \end{aligned}$$

(interpreting '=' as usual).

3 points