

Mathematical Methods for Computer Science 2
Spring 2018

Series 2

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1

a)

We have

$$(1 - x + 2x^3) = (a_0 + a_1x + a_2x^2 + a_3x^3 + \dots)$$

we use the following equation to find a_0 , a_1 , a_2 and a_3

$$(1 - x + 2x^3)(a_0 + a_1x + a_2x^2 + a_3x^3 + \dots) = 1$$

$$\begin{aligned} (1 - x + 2x^3)(a_0 + a_1x + a_2x^2 + a_3x^3 + \dots) = \\ a_0 - a_0x + 2a_0x^3 + \\ a_1x - a_1x^2 + 2a_1x^4 + \\ a_2x^2 - a_2x^3 + 2a_2x^5 + \\ a_3x^3 - a_3x^4 + 2a_3x^6 + \\ a_4x^4 - a_4x^5 + 2a_4x^7 + \dots = 1 \end{aligned}$$

So we can extract the following system :

$$\begin{cases} a_0 = 1 \\ -a_0x + a_1x = 0 \\ -a_1x^2 + a_2x^2 = 0 \\ 2a_0x^3 - a_2x^3 + a_3x^3 = 0 \end{cases}$$

which leads to the following :

$$\begin{aligned} a_0 &= 1 \\ -1x + a_1x &= 0 \longrightarrow a_1 = 1 \\ -x^2 + a_2x^2 &= 0 \longrightarrow a_2 = 1 \\ 2x^3 - x^3 + a_3x^3 &= 0 \longrightarrow a_3 = -1 \end{aligned}$$

b)

TODO

2

a)

$$\begin{aligned}
 a_0 &= 1 & a_{n+1} &= 2a_n + 3 \\
 a_1 &= 5 \\
 a_2 &= 13 \\
 a_3 &= 29 \\
 &\dots \\
 a_{n+1} - 2a_n &= 3
 \end{aligned}$$

$$\begin{aligned}
 F(x) &= a_0 + a_1x + a_2x^2 + \dots \\
 xF(x) &= a_0x + a_1x^2 + \dots \\
 F(x) - 2xF(x) &= (a_0 + a_1x + a_2x^2 + \dots) - (2a_0x + 2a_1x^2 + \dots) \\
 &= a_0 + (a_1 - 2a_0)x + (a_2 - 2a_1)x^2 + \dots \\
 &= 1 + \underbrace{(5 - 2)}_3x + \underbrace{(13 - 10)}_3x^2 + \dots \\
 &= 1 + 3x + 3x^2 + \dots \\
 &= 1 + 3x(1 + x + x^2 + x^3 + \dots) \\
 &= 1 + \frac{3x}{1 - x} = \frac{(1 - x) + 3x}{1 - x} = \frac{2x + 1}{1 - x}
 \end{aligned}$$

$$\begin{aligned}
 F(x) &= \frac{2x + 1}{(1 - x)(1 - 2x)} = \frac{A}{1 - x} + \frac{B}{1 - 2x} \\
 2x + 1 &= A(1 - 2x) + B(1 - x) = A - 2Ax + B - Bx = 2x + 1
 \end{aligned}$$

We extract the following system :

$$\begin{cases} A + B = 1 \\ -2A - B = 2 \end{cases}$$

$$\begin{aligned}
 A &= 1 - B \\
 (-2)(1 - B) - B &= 2 \\
 -2 + 2B - B &= 2 \\
 B &= 4 \\
 A &= 1 - 4 = -3
 \end{aligned}$$

$$F(x) = \frac{-3}{1 - x} + \frac{4}{2x + 1} =$$

b)

$$\begin{aligned}
F(x) &= \frac{-3}{1-x} + \frac{4}{2x+1} = -3 \frac{1}{1-x} + 4 \frac{1}{2x+1} \\
&= -3 \frac{1}{1-x} + 4 \frac{1}{1-(-2x)} \\
&= -3 \sum_{k=0}^{\infty} x^k + 4 \sum_{k=0}^{\infty} (-1)^k 2^k x^k \\
&= 4 \sum_{k=0}^{\infty} (-1)^k 2^k x^k - 3 \sum_{k=0}^{\infty} x^k \\
&= \sum_{k=0}^{\infty} (-1)^k \cdot 4 \cdot 2^k x^k - \sum_{k=0}^{\infty} 3 \cdot 1^k \cdot x^k \\
&= \sum_{k=0}^{\infty} 4(2^k - 1) - 3x^k
\end{aligned}$$

$$a_n = 4(2^n - 1) + 1$$

3

$$\frac{x^2 + x}{(x-1)(x+2)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x-1)}$$

$$\begin{aligned}
x^2 + x &= A(x-1)(x+2) + B(x-1) + C(x+2)^2 \\
&= A(x^2 - 2 + x) + Bx + B + C(x^2 + 4 + 4x) \\
&= Ax^2 + Ax - 2A + Bx + B + Cx^2 + 4C + 4Cx
\end{aligned}$$

We extract the following system :

$$\begin{cases} -2A - B + 4C = 0 \\ A + B + 4C = 1 \\ A + C = 1 \end{cases}$$

$$A + C = 1 \longrightarrow A = 1 - C$$

\longrightarrow

$$\begin{cases} -2 + 2C - B + 4C = 0 \\ 1 + 3C + B = 1 \end{cases}$$

\longrightarrow

$$B = -3C$$

$$\begin{aligned}
-2 + 2C + 3C + 4C &= 0 \\
9C &= 2 \\
C &= \frac{2}{9}
\end{aligned}$$

$$\begin{aligned}
B &= -3\frac{2}{9} = -\frac{6}{9} = -\frac{2}{3} \\
A &= 1 - C \\
A &= 1 - \frac{2}{9} = \frac{7}{9}
\end{aligned}$$

$$\frac{x^2 + x}{(x-1)(x+2)} = \frac{7}{9(x+2)} - \frac{2}{3(x+2)^2} + \frac{2}{9(x-1)}$$

4

a)

$$\begin{aligned}
(2n-1)!! &= \frac{(2n)!}{2^n n!} \\
&\longrightarrow \\
(2n-1)!! 2^n n! &= (2n)!
\end{aligned}$$

$$\begin{aligned}
(2n-1)!! 2^n n! &= ((2n-1)(2n-3)\dots(1)) \cdot 2^n \cdot n! \\
&\longrightarrow \\
2^n &= \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot \dots \cdot 2}_n \\
n! &= \underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n}_n \\
2^n \cdot n! &= \underbrace{2 \cdot 4 \cdot 6 \cdot 8 \cdot \dots \cdot 2n}_n = (2n)(2n-2)(2n-4)\dots(2) \\
(2n-1)!! 2^n n! &= (2n-1)(2n-2)(2n-3)(2n-4)\dots(2)(1) = (2n)!
\end{aligned}$$

Therefore

$$(2n-1)!! = \frac{(2n)!}{2^n n!}$$

b)

c)

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a)

b)

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