Mathematical Methods for Computer Science 1 Fall 2017

Series 7

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(a)

In order to show that $\{\to, \neg\}$ is complete, we have to found the equivalent formula of $a \lor b$ and $a \land b$ ($\neg a$ and $a \to b$ are trivial...).

$a \lor b$

$$(\neg a) \to b = (\neg\neg a) \lor b = a \lor b$$

and the truth table to validate the result :

a	b	¬	a	→	b
T	T	F	T	T	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	F	F	F

$a \land b$

$$\begin{aligned} \neg(a \to \neg b) &= \neg(\neg a \lor \neg b) \\ &= \neg\neg(a \land b) \\ &= (a \land b) \end{aligned}$$

and the truth table to validate the result :

a	b	¬	(	a	→	¬	b	)
T	T	T		T	F	F	T	
T	F	F		T	T	T	F	
F	T	F		F	T	F	T	
F	F	F		F	T	T	F	

(b)

In order to show that $\{\uparrow\}$ is complete, we have to found the equivalent formula of $a \to b$ and $\neg a$, because we just showed that $\{\neg, \to\}$ is complete.
We know that $a \uparrow b \leftrightarrow \neg(a \land b)$.

$\neg a$

$$a \uparrow a = \neg(a \wedge a) = \neg a$$

and the truth table to validate the result :

a	¬	(	a	&	a	)
T	F		T	T	T	
F	T		F	F	F	

$a \to b$

$$\begin{aligned} a \to b &= \neg a \vee b \\ &= \neg\neg(\neg a \vee b) \\ &= \neg(a \wedge \neg b) \\ &= \neg(a \wedge \underbrace{\neg(b \wedge b)}_{b \uparrow b}) \\ &= \underbrace{\neg(a \wedge (b \uparrow b))}_{a \uparrow (b \uparrow b)} \\ &= a \uparrow (b \uparrow b) \end{aligned}$$

and the truth table to validate the result :

a	b	a	↑	(	b	↑	b	)
T	T	T	T		T	F	T	
T	F	T	F		F	T	F	
F	T	F	T		T	F	T	
F	F	F	T		F	T	F	

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(a)

(b)

3

(a)

(b)

(c)

4

(a)

(b)

(c)

5

(a)

(b)

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