Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

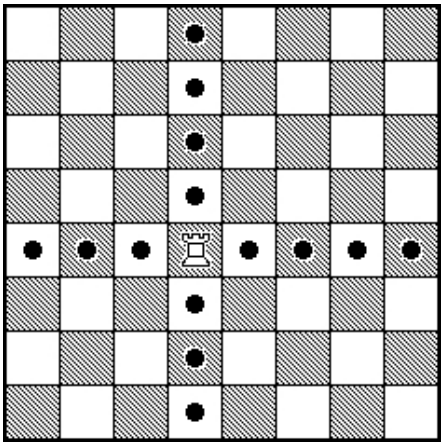
Mathematical Methods for Computer Science I

Fall 2017

Series 1 – Hand in before Monday, 25.09.2017 - 12.00

1. a) How many permutations f of the set $\{1, 2, 3, 4, 5\}$ satisfy $f(1) \neq 1$?
   b) How many 10-digit numbers have at least two equal digits? A number cannot start with 0.

2. The chessboard has the size 8×8. Two rooks on a chessboard are called threatening each other if they are situated in the same vertical or horizontal row.
   a) In how many different ways can one put a white and a black rook on the chessboard so that they don't threaten each other?
   b) In how many different ways can one put two white rooks on the chessboard? (They may threaten each other. Besides, the rooks are undistinguishable.)
   c) In how many different ways can one put two white and two black rooks on the chessboard so that no two rooks of different colors are threatening each other?



3. a) How many different natural divisors does the number 60 have? (*Hint:* $60 = 2^2 \cdot 3 \cdot 5$. What can the prime factorization of a divisor of 60 look like?)
   b) Let n be a natural number. Show that the number of different divisors of n is odd if and only if n is a complete square, that is $n = k^2$ for some integer k.

4. a) In the language of a stone age tribe there were four sounds A, E, O, U. Every word was a sequence of three sounds. How many different words did this language have?
   b) In the language of an iron age tribe there were 4 vowels and 5 consonants. All words had length 5 and consisted of alternating vowels and consonants (a word could start with a vowel or with a consonant). How many different words were there?

5. a) In how many different ways can one split $2n$ persons into two equal teams?
   b) How many different bracelets can one make out of n different stones? The bracelet should contain all n stones.