Formal Methods
Fall 2017

S02 : Hoare Logic and Propositional Logic

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Exercise 2.1 :

In order to check the satisfiability of a formula, we just have to demonstrate his validity.

(a)

We have to find an interpretation I such that

$$I \models (A \to B) \leftrightarrow (\neg B \to \neg A)$$

Using the inference rules, we get

$$I \models (A \to B) \leftrightarrow (\neg B \to \neg A)$$

$$I \models (A \to B) \land (\neg B \to \neg A) \qquad\qquad I \not\models (A \to B) \lor (\neg B \to \neg A)$$

$$\begin{array}{c} I \models (A \to B) \\ I \models (\neg B \to \neg A) \end{array} \qquad\qquad \begin{array}{c} I \not\models (A \to B) \\ I \not\models (\neg B \to \neg A) \end{array}$$

$$\begin{array}{c} I \not\models A \\ I \models (\neg B \to \neg A) \end{array} \qquad \begin{array}{c} I \models B \\ I \models (\neg B \to \neg A) \end{array} \qquad \begin{array}{c} I \models A \\ I \not\models B \\ I \not\models (\neg B \to \neg A) \end{array}$$

$$\begin{array}{c} I \not\models A \\ I \not\models \neg B \end{array} \quad \begin{array}{c} I \not\models A \\ I \models \neg A \end{array} \quad \begin{array}{c} I \models B \\ I \not\models \neg B \end{array} \quad \begin{array}{c} I \models B \\ I \models \neg A \end{array} \qquad \begin{array}{c} I \models A \\ I \not\models B \\ I \models \neg B \\ I \not\models \neg A \end{array}$$

$$\begin{array}{c} I \not\models A \\ I \models B \end{array} \quad \begin{array}{c} I \not\models A \\ I \not\models A \end{array} \quad \begin{array}{c} I \models B \\ I \models B \end{array} \quad \begin{array}{c} I \models B \\ I \not\models A \end{array} \qquad \begin{array}{c} I \models A \\ I \not\models B \\ I \not\models B \\ I \models A \end{array}$$

$$I \not\models A \qquad I \models B \qquad \begin{array}{c} I \models A \\ I \not\models B \end{array}$$

And by unifiying the lefting formula, we have

$$\begin{array}{c} I \not\models A \\ I \models B \end{array} \Bigg| \; I \not\models A \; \Bigg| \; I \models B \; \Bigg| \begin{array}{c} I \models B \\ I \not\models A \end{array} \Bigg| \begin{array}{c} I \models A \\ I \not\models B \end{array}$$

we have 5 ways to find an interpretation I that satisfies the base formula. Because we have $I \models A$ and $I \not\models A$, A could take any value *true* or *false* and satisfies the formula, and it is the same for B, so the formula is satisfiable for any value for A and B so the formula is valid.
To verify this, we could use the truth table of the formula and find out that the result is always 1 :

a	b	(a	→	b)	↔	(¬	b	→	¬	a)
T	T	T	T	T	T	F	T	T	F	T
T	F	T	F	F	T	T	F	F	F	T
F	T	F	T	T	T	F	T	T	T	F
F	F	F	T	F	T	T	F	T	T	F

(b)

We have to find an interpretation I such that

$$I \models (A \lor B) \to (A \land B)$$

Using the inference rules, we get

$$\begin{array}{cc} \multicolumn{2}{c}{I \models (A \lor B) \to (A \land B)} \\ \diagup & \diagdown \\ I \not\models (A \lor B) & I \models (A \land B) \\ | & | \\ I \not\models A & I \models A \\ I \not\models B & I \models B \end{array}$$

So the formula is satisfies for $I : \{A \mapsto true, B \mapsto true\}$ or $I : \{A \mapsto false, B \mapsto false\}$, but it is not valid, for example, the interpretation $I : \{A \mapsto true, B \mapsto false\}$ do not satisfies $(A \lor B) \to (A \land B)$.

Exercise 2.2 :

(a)

NNF:

$$\begin{aligned} \neg((\neg P \lor Q) \to \neg R) &= \neg(\neg(\neg P \lor Q) \lor \neg R) \\ &= \neg((\neg\neg P \land \neg Q) \lor \neg R) \\ &= \neg((P \land \neg Q) \lor \neg R) \\ &= \neg(P \land \neg Q) \land \neg\neg R \\ &= \neg(P \land \neg Q) \land R \\ &= (\neg P \lor \neg\neg Q) \land R \\ &= (\neg P \lor Q) \land R \end{aligned}$$

CNF:

$$(\neg P \lor Q) \land R$$

DNF:

$$(\neg P \lor Q) \land R = (\neg P \land R) \lor (Q \land R)$$

(b)

$$\begin{aligned} ((P \land Q) \to (Q \to (P \land Q))) \land P &= ((\neg(P \land Q)) \lor (Q \to (P \land Q))) \land P \\ &= ((\neg P \lor \neg Q) \lor (\neg Q \lor (P \land Q))) \land P \\ &= ((\neg P \lor \neg Q) \land P) \lor ((\neg Q \lor (P \land Q)) \land P) \\ &= \underbrace{(\neg P \land P)}_{false} \lor (\neg Q \land P) \lor ((\neg Q \lor (P \land Q)) \land P) \\ &= (\neg Q \land P) \lor (\neg Q \land P) \lor (P \land Q) \\ &= (\neg Q \land P) \lor (P \land Q) \\ &= \underbrace{(\neg Q \lor Q)}_{true} \land P \\ &= P \end{aligned}$$

P is in NNF, CNF and DNF form.