Mathematical Methods for Computer Science I

Fall 2017

Series 3 – Hand in before Monday, 09.10.2017 - 12.00

1. a) Show that for $n = k + l + m$ we have
$$\binom{n}{k, l, m} = \binom{n-1}{k-1, l, m} + \binom{n-1}{k, l-1, m} + \binom{n-1}{k, l, m-1}.$$
   b) Show that for every n we have
$$\sum_{\substack{k+l+m=n \\ k,l,m \geq 0}} \binom{n}{k, l, m} = 3^n.$$

2. Prove
$$\binom{n}{k_1, \ldots, k_r} = \binom{n}{k_r}\binom{n - k_r}{k_1, \ldots, k_{r-1}}$$
   a) using the formula for multinomial coefficients;
   b) by a combinatorial argument.

3. a) A group of seven friends is renting a holiday apartment with three rooms. One room has four beds, the other two beds, and the third one bed. In how many different ways can the friends distribute the rooms?
   b) In how many different ways can one divide 10 people into pairs? (There is no order on the set of pairs and no order inside each pair.)

4. Of 33 students, 20 ski, 15 climb, 8 play ice hockey. Besides, 6 ski and climb, 2 ski and play ice hockey, and 3 climb and play ice hockey. How many students do all three sports?

5. How many monotone maps $f\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$ are there? (A map f is called monotone if $f(i) \leq f(j)$ for all $i \leq j$.)

6.* There are $m \geq 2k$ seats at a round table. In how many different ways can k married couples sit at this table so that every couple sits next to each other?

7.* In how many different ways can one place n married couples at a round table with $2n$ seats so that neither of the couples sit next to each other?