Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science II

Spring 2018

Series 10 – Hand in before Monday, 07.05.2018 - 12.00

1. Let L be a language in a one-letter alphabet $\{0\}$. Show that either all words in $\{0\}^*$ are pairwise L-inequivalent or the number of their L-equivalence classes is finite.

2. (8 points) Let $L \subset \Sigma^*$ be a language, and let $a \in \Sigma$ be a letter of the alphabet. The *quotient of L by a* is the language defined by
$$L/a = \{w \in \Sigma^* \mid wa \in L\}.$$
   a) Show that $L/a \cdot \{a\} \subset L$, but not necessarily $L/a \cdot \{a\} = L$. Here $\cdot$ stands for the concatenation of languages.
   b) Show that $u \sim_L v \Rightarrow u \sim_{L/a} v$.
   c) Given a DFA that accepts L, explain how to modify it to a DFA that accepts L/a.
   d) Show that the number of states of the minimal automaton for L/a is less or equal to the number of states of the minimal automaton for L. Do these two automata always have the same number of states?

3. For any language L, call the number of L-equivalence classes the *index* of L. Denote it by $\operatorname{ind}(L) \in \mathbb{N} \cup \{\infty\}$.
   a) Let $h\colon \Sigma^* \to \Delta^*$ be a homomorphism, let $L \subset \Delta^*$ be a regular language, and let $L' = h^{-1}(L) \subset \Sigma^*$ be its inverse homomorphic image. Show that
$$\operatorname{ind}(L') \le \operatorname{ind}(L).$$
   Give an example of h and L when this inequality is strict.
   b) Let $L_1, L_2 \subset \Sigma^*$ be two regular languages. Show that
$$\operatorname{ind}(L_1 \cap L_2) \le \operatorname{ind}(L_1)\operatorname{ind}(L_2).$$
   Give an example of L_1 and L_2 when this inequality is strict.

4. Minimize the DFA given by the following table.

	0	1
q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_7	q_1
q_3	q_6	q_7
q_4	q_7	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_7	q_0

   Here q_0 is the initial state, and q_7 is the only final state.