

Exercise sheet 7

Chapter 5: Cook's Theorem

Exercise: Understanding Cook's Theorem

In the course, we have seen the proof to Cook's Theorem, showing that SAT is \mathcal{NP} -complete. Review the proof, make sure you understand the different steps taken in the proof and answer the following questions in your own words:

1. Why is Cook's Theorem so important for complexity theory?
2. Why are we constructing the propositional formula Φ , what are the propositional variables of Φ and what does it mean if an interpretation of Φ sets to true?
3. Φ consists of 4 sub-formulas, what do these four sub-formulas encode and why are these all necessary?
4. Discuss the structure of sub-formula N and their sub-sub-formulas which are used to correctly encode N . How is this construction still in polynomial time?
5. If we had a TM M with alphabet $\Gamma = \{0, 1, B\}$ and starting state q_0 , how, using the propositional variable notation given in the course, would we encode the following sentence: M starts on input "1001"

8 points

Exercise 2: Explaining Cook's Proof

Write a short presentation (short text or some bullet points) to explain the basic outline of the proof of Cook's Theorem by including every important information (why is SAT \mathcal{NP} -complete, what do we construct, why does this construction work, why does this prove the theorem), but not getting too specific for such a short presentation (e.g. no need to talk in detail about the structure of the sub-formulas).

Don't worry, you will not need to present it, I just believe it helps the understanding of a complicated proof a lot to extract the most valuable information on your own without getting lost in the details. So take a good look at the proof and write down the general goal of the proof and how this is achieved.

7 points