Resume : Description Logic

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C is the set of concept names and R the set of role names. Every concept names is a concept description (CD).

Formal notation :

$$
\begin{aligned}
&C \sqcap D \rightarrow \text{conjonction}\\
&C \sqcup D \rightarrow \text{disjonction}\\
&\neg C \rightarrow \text{negation}\\
&\exists r.C \rightarrow \text{existential restriction}\\
&\forall r.C \rightarrow \text{value restriction}
\end{aligned}
$$

An interpretation $I = (\Delta, \cdot)$ such that $\Delta \neq \emptyset$, (Δ is called the domain of the interpretation) and with the following :

$$
\begin{aligned}
&A \in C \rightarrow A^I \subseteq \Delta\\
&r \in R \rightarrow r^I \subseteq \Delta \times \Delta\\
&\top^I = \Delta\\
&\bot^I = \Delta\\
&(C \sqcap D)^I = C^I \cap D^I\\
&(C \sqcup D)^I = C^I \cup D^I\\
&(\neg C)^I = \Delta \setminus C^I\\
&(\exists r.C)^I = \{d \in \Delta \mid \exists e \in \Delta \text{ with } (d, e) \in r^I \text{ and } e \in C^I\}\\
&(\forall r.C)^I = \{d \in \Delta \mid \forall e \in \Delta,\ \text{if } (d, e) \in r^I,\ \text{then } e \in C^I\}
\end{aligned}
$$

We call C^I the extension of C in I and $b \in \Delta^I$ an $r - filler$ of a in I if $(a, b) \in r^I$.

Lemma 1. *Let I be an interpretation, C, D concepts and r a role. Then*

$$
\begin{aligned}
&\top^I = (C \sqcup \neg C)^I\\
&\bot^I = (C \sqcap \neg C)^I\\
&(\neg\neg C)^I = C^I\\
&(\neg(C \sqcap D))^I = (\neg C \sqcup \neg D)^I\\
&(\neg(C \sqcup D))^I = (\neg C \sqcap \neg D)^I\\
&(\neg(\exists r.C))^I = (\forall r.\neg C)^I\\
&(\neg(\forall r.C))^I = (\exists r.\neg C)^I
\end{aligned}
$$

Because the $\sqcup$ operator can be tricky sometimes, we use the following relation to replace the $\sqcup$ by $\sqcap$: $C \sqcup D \rightarrow \neg(\neg C \sqcap \neg D)$.

ALC $TBoxes$

For C and D possibly compound ALC concepts, an expression of form $C \sqsubseteq D$ is called an ALC general concept inclusion (GCI). We use $C \equiv D$ has an abbreviation for $C\tau \sqsubseteq D, D \sqsubseteq C$. A finite set of GCI is called an ALC $TBox$ and noted τ.

Lemma 2. *If $\tau \in \tau'$ for two $TBox$ τ and τ', then each model of τ' is also a model of τ.*

Example of a $TBox$ T_{ex} :

$$\begin{aligned} T_{ex} = \{Course &\sqsubseteq \neg Person, \\ UGC &\sqsubseteq Course, \\ PGC &\sqsubseteq Course, \\ Teacher &\equiv Person \sqcap \exists teaches.Course, \\ \exists teaches.T &\sqsubseteq Person, \\ Student &\equiv Person \sqcap \exists attends.Course, \\ \exists attends.T &\sqsubseteq Person\} \end{aligned}$$

ALC $ABoxes$

Let I be a set of individual names disjoint from R and C. For $a, b \in I$ individual names, C a possibly compound ALC concept, and $r \in R$ a role name, an expression of the form $a : C$ is called an ALC concept assertion and $(a, b) : r$ is called and ALC role assertion.

A finite set of concept and role assertion is called an ALC $ABox$. An interpretation function $\cdot^I$ is additionally required to map every individual name $a \in I$ to an element a^I $in\Delta^I$. An interpretation I satisfies a concept assertion $a : C$ if $a^I \in C^I$ and a role assertion $(a, b) : r$ if $(a^I, b^I) \in r^I$.

An interpretation that satisfies each concept assertion and each role assertion in a $ABox$ A is called a model of A.

Example of a $ABox$ A_{ex} :

$$\begin{aligned} A_{ex} = \{Mary &: Person, \\ CS600 &: Course, \\ Ph456 &: Course \sqcap PGC, \\ Hugo &: Person, \\ Betty &: Person \sqcap Teacher, \\ (Mary, CS600) &: teaches, \\ (Hugo, Ph456) &: teaches, \\ (Betty, Ph456) &: attends, \\ (Mary, Ph456) &: attends\} \end{aligned}$$

Then we can create an interpretation I of this $ABox$ which is a model:

$$\begin{aligned}
\Delta^I &= \{h, m, c6, p4\}, \\
Mary^I &= m \\
Betty^I &= Hugo^I = h, \\
CS600^I &= c6, \\
Ph456^I &= p4, \\
Person^I &= \{h, m, c6, p4\}, \\
Teacher^I &= \{h, m\}, \\
Course^I &= \{c6, p4\}, \\
PGC^I &= \{p4\}, \\
UGC^I &= \{cs\}, \\
Student^I &= \emptyset, \\
teaches^I &= \{(m, c6), (h, p4)\}, \\
attends^I &= \{(h, p4), (m, p4)\}
\end{aligned}$$

Note that isn't a model of the previous example of a $TBox$.