Automata on Infinite Structure
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Exercice Sheet 7

Author : Sylvain Julmy

Professor : Ultes-Nitsche Ulrich
Assistant : Stammet Christophe

Exercise 1

Theorem. *$\mathcal{PSPACE}$ is closed under union, intersection and complement.*

Proof. Assume $L_1, L_2 \in \mathcal{PSPACE}$, hence there exist TM M_1 and M_2 that are using polynomial space, such that M_1 decides L_1 in nondeterministic time $O(n^k)$ and in polynomial space $O(n^{k'})$, and M_2 decides L_2 in nondeterministic time $O(n^l)$ and in polynomial space $O(n^{l'})$. We show that

1. there exist a decider M in nondeterministic polynomial time and polynomial space such that $L(M) = L_1 \cup L_2$.
2. there exist a decider M in nondeterministic polynomial time and polynomial space such that $L(M) = L_1 \cap L_2$.

The constructions are standard ones.

1. Intersection : we run the word ω on M_1, if M_1 reject it, then M reject it too, else we run ω on M_2, if M_2 reject it, then M reject it too, else M accept ω. Clearly, M is a poly-time nondeterministic decider and a poly-space decider for $L_1 \cap L_2$, for the input ω of length n, the time complexity is $O(n^{max(k,l)})$ and the space complexity is $O(n^{max(k',l')})$.
2. Union : we run the word ω on M_1, if M_1 accept it, then M accept it too, else we run ω on M_2, if M_2 accept it, then M accept it too, else M reject ω. Clearly again, M is a poly-time nondeterministic decider and a poly-space decider for $L_1 \cap L_2$, for the input ω of length n, the time complexity is $O(n^{max(k,l)})$ and the space complexity is $O(n^{max(k',l')})$. We could also choose non-deterministicaly either M_1 or M_2 and use only the selected machine.

To prove that $\mathcal{PSPACE}$ is closed under complement, we use the fact that every language $L \in \mathcal{PSPACE}$ has a deterministic poly-space TM M, we can swap the accepting and non-accepting states in M in polynomial time and get a poly-space decider for $\bar{L}$, hence $\bar{L} \in \mathcal{PSPACE}$. $\square$

Exercise 2

w_1

$$q_0 1010 \longrightarrow 1q_0 010 \longrightarrow 10q_0 10 \longrightarrow 101q_0 0 \longrightarrow 1010q_0 \longrightarrow 101q_1 0 \longrightarrow 10q_1 11 \longrightarrow 101q_2 1$$

w_2

$$q_0 0100 \longrightarrow 0q_0 100 \longrightarrow 01q_0 00 \longrightarrow 010q_0 0 \longrightarrow 0100q_0 \longrightarrow 010q_1 0 \longrightarrow 01q_1 01 \longrightarrow 0q_1 111 \longrightarrow$$
$$01q_2 11$$