Automata on Infinite Structure
Fall 2018

Exercice Sheet 10

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Exercise 1

$$\phi = \forall x.(x \in Q_a)$$

$$x \in X_0 \implies x \in Q_a$$

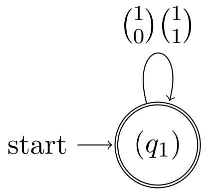
$$x \notin X_0 \implies x \in Q_b$$

$$\forall x.(x \in Q_a) \implies \forall x.(x \in X_0) \implies$$

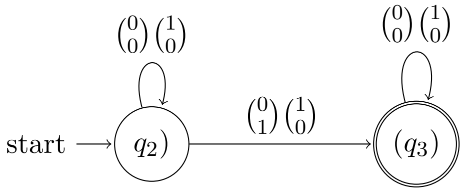
$$\forall X_1.(sing(X_1) \to X_1 \subseteq X_0) = \forall X_1.(\neg(sing(X_1) \lor X_1 \subseteq X_0)) =$$

$$\neg\exists\neg X_1.(\neg(sing(X_1) \lor X_1 \subseteq X_0)) =$$

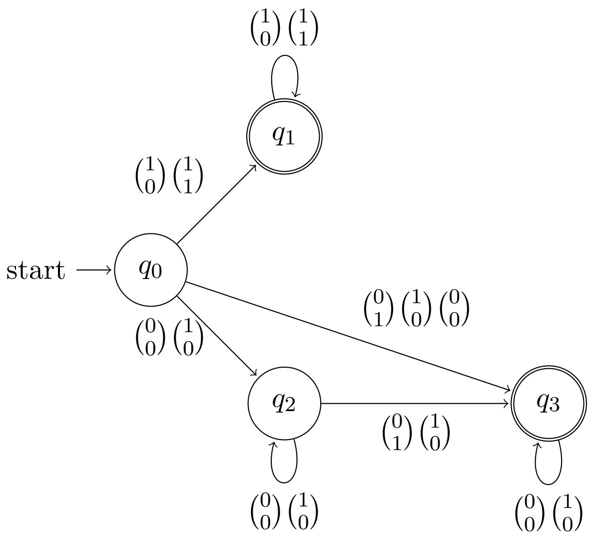
Automata for $L_1 = X_1 \subseteq X_0$



Automata for $L_2 = sing(X_1)$

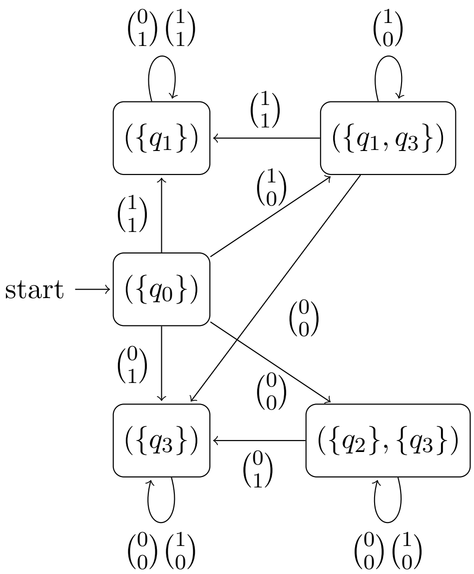


Automata for $L_1 \cup L_2$

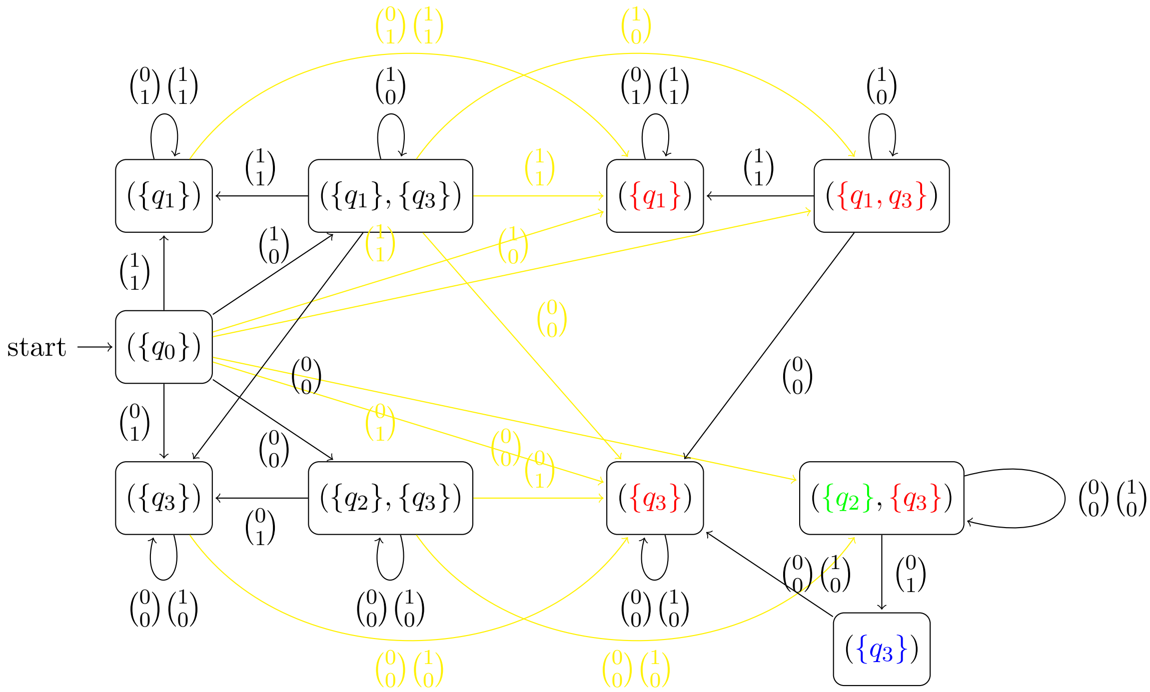


Automata for $\overline{L_1 \cup L_2}$

Upper part :



Full automata :



There are no accepting state...which seems weird and indicates that I have made a mistake...

Exercise 2

$\forall x \,(x \in Q_a \to x+1 \in Q_b)$

$x \in X_0 \Rightarrow x \in Q_a$

$x \notin X_0 \Rightarrow x \in Q_b$

$\forall x \,(x \in Q_a \to x+1 \in Q_b) =$

$\forall x \,(x \in Q_a \to succ(x, y)) \quad =$

$\forall X_1 \,((sing(X_1) \wedge X_1 \subseteq X_0) \to (succ(X_1, X_2)$
$\quad \wedge X_2 \not\subseteq X_0) =$

$\forall X_1 \,(\neg(sing(X_1) \wedge X_1 \subseteq X_0) \vee (succ(X_1, X_2)$
$\quad \wedge (X_2 \subseteq X_0) =$

$\forall X_1 \,(\neg sing(X_1) \vee X_1 \subseteq X_0 \vee \neg(\neg succ(X_1, X$
$\quad \vee \neg(X_2 \subseteq X_0))$

$=$

$\neg \exists \neg X_1 (\neg sing(X_1) \vee X_1 \subseteq X_0 \vee \neg(\neg succ($
$\quad \vee \neg(X_2 \subseteq X_0))$

$L_1 = sing(X_1)$

$L_2 = X_1 \subseteq X_0$

$L_3 = X_2 \subseteq X_0$

$L_4 = succ(X_1, X_2)$

$$L = \overline{\left(\overline{L_1} \cup L_2 \cup \overline{\overline{L_3} \cup \overline{L_4}} \right)}$$

Something like that...