Mathematical Methods for Computer Science 2 Spring 2018

Series 4

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1

a)

$$p(n; k) = p(n; \leq k) - p(n; \leq k-1)$$
$$p(n; \leq k) = p(n; k) + p(n; \leq k-1)$$

By definition, the sum of the number of partition of n in exactly k parts and the number of partition of n in at most $k-1$ parts is the number of partition of n in at most $k-1$ parts. The set P_1 is the partitions of n in exactly k parts and the set P_2 is the partitions of n in at most $k-1$ parts are different : $P_1 \cap P_2 = \emptyset$.

b)

We know that $p(n-k; \leq k) = p(n; k)$. So, by substition of this equation in the previous one, we obtain

$$p(n; \leq k) = p(n; \leq k-1) + p(n-k; \leq k)$$

2

a)

$$q(n; k) = p(n - \binom{k}{2}; k)$$

At first, $\binom{k}{2}$ is used to compute the area of the top-left triangle (in blue on the figure provided) :

$$\binom{k}{2} = \frac{k!}{(k-2)!2!} = \frac{k(k-1)}{2} = 1 + 2 + \cdots + k$$

Then, because we remove the triangle, we have a partitions of $n - \binom{k}{2}$, the number of row in the triangle is not touch so there is k parts left.
So there is a bijection from each partition of $q(n; k)$ to $p(n - \binom{k}{2}; k)$.

3

The proof is by bijection. We take any partition of $n - l$ and take its Ferrers diagrams. The diagram has $k-1$ parts and none of them is exceeding l.

By adding a new top row of k nodes and deleting the first column (in this order), we obtain a new diagram with k parts and none of them is exceeding $l-1$.
Then, by taking the conjugate of the new diagram, we obtain a partition of $n-k$ into exactly $l-1$ parts of size $\leq k$.
It is a one-to-one transformation so the bijection is established.
Note : I have work with Mr. Lauper and Mr. Papinutto for this exercice.

4

In order to weight any integer, we can find a way to encode any integer using the ternary system. We use a representation similar to the encoding in binary, except that we don't encode power of 2 with 0 and 1, but power of 3 with 1, 0 and -1.
We encode the first 10 numbers as follow :

$$\begin{aligned}
0 &= 0 \\
1 &= (1)3^0 \\
2 &= (1)3^1 + (-1)3^0 \\
3 &= (1)3^1 \\
4 &= (1)3^1 + (1)3^0 \\
5 &= (1)3^2 + (-1)3^1 + (-1)3^0 \\
6 &= (1)3^2 + (-1)3^1 + (0)3^0 \\
7 &= (1)3^2 + (-1)3^1 + (1)3^0 \\
8 &= (1)3^2 + (0)3^1 + (-1)3^0 \\
9 &= (1)3^2 + (0)3^1 + (0)3^0
\end{aligned}$$

Then, to encode any integer (in base 10) in this format, we can use the following generating function.

$$(a_n a_{n-1} \dots a_1 a_0)_{10} = \sum_{k=0}^{\infty} a_k 10^k$$

Where a_k are the digit of the original representation.

5

The proof is to show how to encore any integer n into a sum of fibonnaci numbers :

1. Find the greatest fibonnaci number f_m which is smaller than n.
2. If $n - f_m$ is a fibonnaci number, then we are done.
3. Else, we repeat the process until the substraction is 0.

The encoding does not hold any two consecutive fibonnaci number, because if it is the case, we can replace both of them by another one due to the recurence relation of the fibonnaci numbers : $a_{n-2} + a_{n-1} = a_n$