Université de Fribourg — Département de Mathématiques
Universität Freiburg — Mathematatik Departement

Mathematical Methods for Computer Science I

Fall 2017

Series 11 – Hand in before Monday, 11.12.2017 - 12.00

1. Use the predicate specifications

$H(x)$:	x is an ice hockey team
$B(x, y)$:	x beats y
$L(x, y)$:	x loses to y
$G(x, y)$:	x is a goalkeeper of y

and the constant symbols

f:	Fribourg-Gottéron
b:	CP Berne

to translate the following into predicate logic:

a) Every ice hockey team has a goalkeeper.

b) Nobody can be goalkeeper in two different teams. (Here you will need the binary predicate "=".)

c) If Gottéron beats Berne, then Gottéron does not lose to every team.

d) Gottéron beats some team, which beats Berne.

2. a) Find appropriate predicates and their specifications to translate the following into predicate logic.

   i) All red things are in the box.

   ii) Only red things are in the box.

   b) Translate the following sentences into predicate logic using three constants a, g, s and a unary function f.

   i) Ben is a grandfather.

   ii) Arthur and Gaspard have the same father.

   c) Translate the sentences from b) using the same constants and a binary predicate.

3. a) Let m be a constant, f a unary function symbol, and P a unary predicate symbol, and Q a binary predicate symbol. Which of the following expressions are formulas of the predicate logic? Specify a reason for failure for expressions which aren't.

   i) $P(f(x, y))$ ii) $Q(m, f(m))$ iii) $Q(Q(m, x), y)$ iv) $Q(x, y) \to \exists z(Q(z, y))$

   b) Identify all free and bound variable occurrences in the formula

$$\exists x(P(y, z) \wedge (\forall y(\neg Q(y, x) \wedge P(y, z))))$$

   c) Change variable symbols so that no symbol is used in different contexts and form the universal closure of the formula from b).

4. a) Consider the formula

$$\forall x \exists y(P(x, y) \wedge Q(y))$$

   Which of the following interpretations satisfy this formula?

   i) $U = \mathbb{Z}, \quad P = \{(x, y) \mid x < y\}, \quad Q = \{x \mid x > 0\}$

   ii) $U = \mathbb{Z}, \quad P = \{(x, y) \mid x > y\}, \quad Q = \{x \mid x > 0\}$

   b) Show that the formula

$$\exists x \forall y(P(x, y) \wedge Q(y))$$

   is satisfiable but not valid.

please see overleaf

5. Let P and Q be binary predicate symbols. Consider the formulas

$$\phi = \forall x \exists y (P(x, y) \rightarrow Q(x, y))$$
$$\psi = \forall x \exists y (Q(x, y) \rightarrow P(x, y))$$

and the interpretation

$$U = \{a, b, c\}$$
$$P = \{(a, b), (b, c)\}$$
$$Q = \{(a, b), (b, a), (c, a), (c, b), (c, c)\}$$

For each of the formulas ϕ and ψ determine if they are satisfied by the given interpretation. Justify your answer.