Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

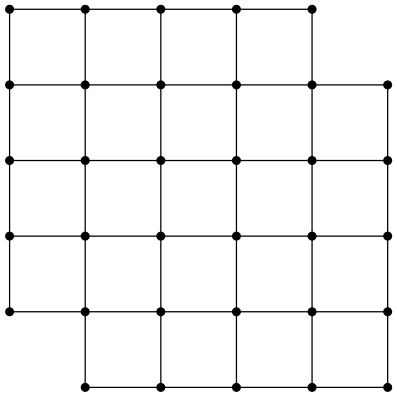
Mathematical Methods for Computer Science I

Fall 2017

Series 7 – Hand in before Monday, 13.11.2017 - 12.00

1. a) How many different perfect matchings does the complete bipartite graph $K_{n,n}$ have?
   b) How many different perfect matchings does the complete graph K_{2n} have?

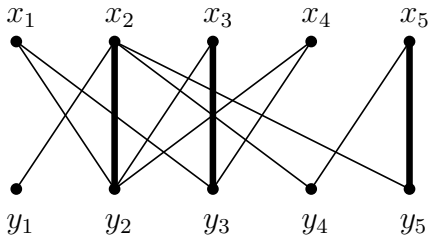
2. a) Show that every Hamiltonian graph with an even number of vertices has a perfect matching.
   b) Does the graph on the figure below have a perfect matching?



3. Let $G = (X \sqcup Y, E)$ be a k-regular bipartite graph.
   a) Show that $|X| = |Y|$.
   b) Show that G has a perfect matching.

4. One deals a standard deck of cards into 13 piles of four cards each. Show that it is always possible to select exactly one card from each pile in such a way that among the 13 selected cards there is exactly one card of each rank (ace, 2, 3, . . . , queen, king).

5. Starting with the matching shown on the figure below, apply the Hungarian method in order to either find a perfect matching or to establish its non-existence. During the search for an augmenting path, keep track of the sets S and T with the help of an alternating tree.



please see overleaf

6.* Two people play a game on a graph G by alternately selecting distinct vertices $v_1, v_2, \ldots$ so that, for $i \geq 1$, v_{i+1} is adjacent to v_i. The player who cannot select a vertex loses the game.

a) Show that if G has a perfect matching, then the second player has a winning strategy.

b) Show that if G has no perfect matching, then the first player has a winning strategy.