Exercise sheet 8

Chapter 6: Proofs of NP-Complete Problems

Exercise: NP-Completeness Proofs

4TA-SAT:
The problem 4TA-SAT is stated as follows: Given a propositional logic formula F, does F have at least 4 different satisfying truth assignments?
To show: **4TA-SAT** $\in \mathcal{NP}$-complete!
Hint: *To show the $\mathcal{NP}$-hardness of **4TA-SAT**, reduce **SAT** to **4TA-SAT***

3 points

NAE-4-SAT:
Given a propositional formula F in 4-CNF (all conjunctions contain 4 literals), does F have a satisfying truth assignment such that in every clause not all literals are true (i.e. at least one literal has to be true and at least one has to be false).
To show: **NAE-4-SAT** $\in \mathcal{NP}$-complete!
Hint: *To show the $\mathcal{NP}$-hardness of **NAE-4-SAT**, reduce **3-SAT** to **4TA-SAT**. This observation might help: Satisfying truth assignments for any NAE-SAT formulas come on complementary pairs, i.e. if I is a satisfying truth assignment, then I', where each variable gets assigned the opposite value compared to I, is also a satisfying truth assignment*

7 points

General Hint about proving $\mathcal{NP}$-completeness:
For each of the exercises above, you must prove a certain problem to be $\mathcal{NP}$-complete. To do so, first show that the problem is itself in the complexity class $\mathcal{NP}$ (an informal argument is enough, but you still have to justify it!). Then you need to prove that the problem is $\mathcal{NP}$-hard. The way to achieve that is to reduce another known $\mathcal{NP}$-hard problem to the problem you are dealing with. You need to show that your reduction can be done in polynomial time and that your reduction is indeed correct (i.e. solving an instance of one problem directly solves the instance of the other problem and vice versa).