Mathematical Methods for Computer Science I

Fall 2017

Series 2 – Hand in before Monday, 02.10.2017 - 12.00

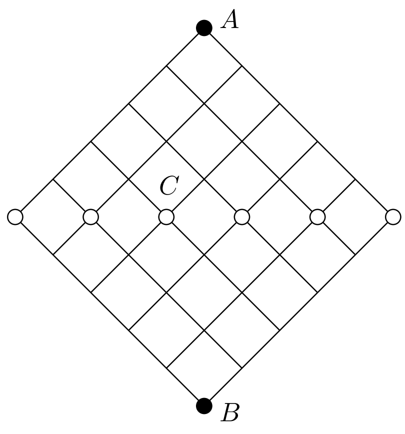
1. A kiosk sells postcards with 10 different views.
   a) In how many ways can one buy 8 (not necessarily different) postcards?
   b) In how many ways can one buy 15 postcards so that to have at least one postcard of every kind?
   c) In how many ways can one buy 8 different postcards?
   d*) You have bought 8 different postcards. In how many ways can you send them to 5 friends of yours so that everybody gets at least one postcard? ⋆

2. a) You are playing a lottery where 6 balls are drawn out of 49. How many different ways are there to fill the lottery card? How many of them will guess exactly 3 of 6 numbers? How many of them will guess at least 3 of 6 numbers?
   b) Let m, n, k be positive integers such that $k \leq m$ and $k \leq n$. Prove:

$$\binom{m+n}{k} = \binom{m}{0}\binom{n}{k} + \binom{m}{1}\binom{n}{k-1} + \cdots + \binom{m}{k}\binom{n}{0}.$$

   (*Hint:* think of taking k balls from a bag with m black and n white balls. In how many different ways can you get i black and $k-i$ white balls?)

3. a) How many lattice paths are there that go from A to B moving only downwards and passing through the point C?



   b) Prove the identity

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

   using the fact that any path from A to B passes through one of the white points.
   c) Prove the same identity using the result of Problem 2b).

⋆Problems marked with $*$ are bonus problems.

4. Recall that a composition of a positive integer n is a representation of n as the sum of positive integers, taking into account the order of summands. The number of summands can range between 1 and n, in particular the compositions

$$n = n \quad \text{and} \quad n = \underbrace{1 + \cdots + 1}_{n}$$

are possible. How many different compositions of n are there?

5. a) Represent the identity

$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+k-1}{k-1} = \binom{n+k}{k-1}$$

graphically on the Pascal triangle and check it for some values of n and k.

b) Prove this identity for all n and k.

$$\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\
1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1 \\
1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1
\end{array}$$

The exercise sheets can also be found at
`http://homeweb.unifr.ch/Izmestie/pub/Teaching/MMCS/index.html`.