Formal Methods
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S09

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Exercise 1

(1)

$$\phi = \forall e.(S(e) \rightarrow \exists d.(P(d)))$$

- The scope of $\forall e$ is $F[e] = S(e) \rightarrow \exists d.(P(d))$
- The scope of $\exists d$ is $F[d] = P(d)$
- There is no free variable
- The formula ϕ is closed, because e is bound in $F[e]$ by the $\forall$ quantifier and d is bound in $F[d]$ by the $\exists$ quantifier

(2)

$$\phi = \forall e.P(a, e) \rightarrow \forall e.P(b, e)$$

- The scope of $\forall e.P(a, e)$ is $F[e] = P(a, e)$
- The scope of $\forall e.P(b, e)$ is $F[e]' = P(b, e)$
- a and b are free variables
- The formula ϕ is not closed because a and b are free variables

(3)

$$\phi = \exists x.(S(x, b) \wedge \forall y.(S(y, b) \rightarrow (x = y)))$$

- The scope of $\exists x$ is $F[x] = S(x, b) \wedge \forall y.(S(y, b) \rightarrow (x = y))$
- The scope of $\forall y$ is $F[y] = S(y, b) \rightarrow (x = y)$
- x is a free variables in ϕ
- The formula ϕ is not closed because x is a free variable

Exercise 2

(1)

"For all successfull exam, we are going to make a party one day".

(2)

"If anna passes any exam, bill would pass that exam too".

(3)

"Bill has a sister and if anyone is the sister of Bill, this is the same person".

Maybe one can argue that "Bill has only one sister", because Bill can't have less than 1 sister, and all the person that are a sister of Bill is the same person.

Exercise 3

The only part that differ from the three given formula is the type of quantifier used in front of their scope. So each formula is solved the same ways and only after we indicates which formula is true or not.

$$I \lhd \{x = anna, y = anna\} \overset{?}{\models} \phi$$
$$\longrightarrow$$
$$cat(anna, anna) = cat(anna, anna) \lor cat(anna, anna) = cat(anna, anna) \Leftrightarrow$$
$$annaanna = annaanna \lor annaanna = annaanna \Leftrightarrow$$
$$true \lor true \Leftrightarrow$$
$$true$$

$$I \lhd \{x = anna, y = bob\} \overset{?}{\models} \phi$$
$$\longrightarrow$$
$$cat(anna, bob) = cat(anna, anna) \lor cat(anna, bob) = cat(bob, bob) \Leftrightarrow$$
$$annabob = annaanna \lor annabob = bobbob \Leftrightarrow$$
$$false \lor false \Leftrightarrow$$
$$false$$

$$I \lhd \{x = bob, y = anna\} \overset{?}{\models} \phi$$
$$\longrightarrow$$
$$cat(bob, anna) = cat(bob, bob) \lor cat(bob, anna) = cat(anna, anna) \Leftrightarrow$$
$$bobanna = bobbob \lor bobanna = annaanna \Leftrightarrow$$
$$false \lor false \Leftrightarrow$$
$$false$$

$$I \lhd \{x = bob, y = bob\} \overset{?}{\models} \phi$$
$$\longrightarrow$$
$$cat(bob, bob) = cat(bob, bob) \lor cat(bob, bob) = cat(bob, bob) \Leftrightarrow$$
$$bobbob = bobbob \lor bobbob = bobbob \Leftrightarrow$$
$$true \lor true \Leftrightarrow$$
$$true$$

In resume, we have the following :

- $I \lhd \{x = anna, y = anna\} \models \phi$
- $I \lhd \{x = anna, y = bob\} \not\models \phi$
- $I \lhd \{x = bob, y = anna\} \not\models \phi$
- $I \lhd \{x = bob, y = bob\} \models \phi$

`(1)`

$$\phi = \forall x.\forall y.((cat(x, y) = cat(x, x)) \lor (cat(x, y) = cat(y, y)))$$

ϕ is not true under the interpretation I.

`(2)`

$$\phi = \forall x.\exists y.((cat(x, y) = cat(x, x)) \lor (cat(x, y) = cat(y, y)))$$

ϕ is not true under the interpretation I because we can't find a x such that for all x, ϕ is true.

`(3)`

$$\phi = \exists x.\exists y.((cat(x, y) = cat(x, x)) \lor (cat(x, y) = cat(y, y)))$$

ϕ is true under the interpretation I for the following assignement :

- $I \lhd \{x = anna, y = anna\} \models \phi$
- $I \lhd \{x = bob, y = bob\} \models \phi$