Exercise sheet 6

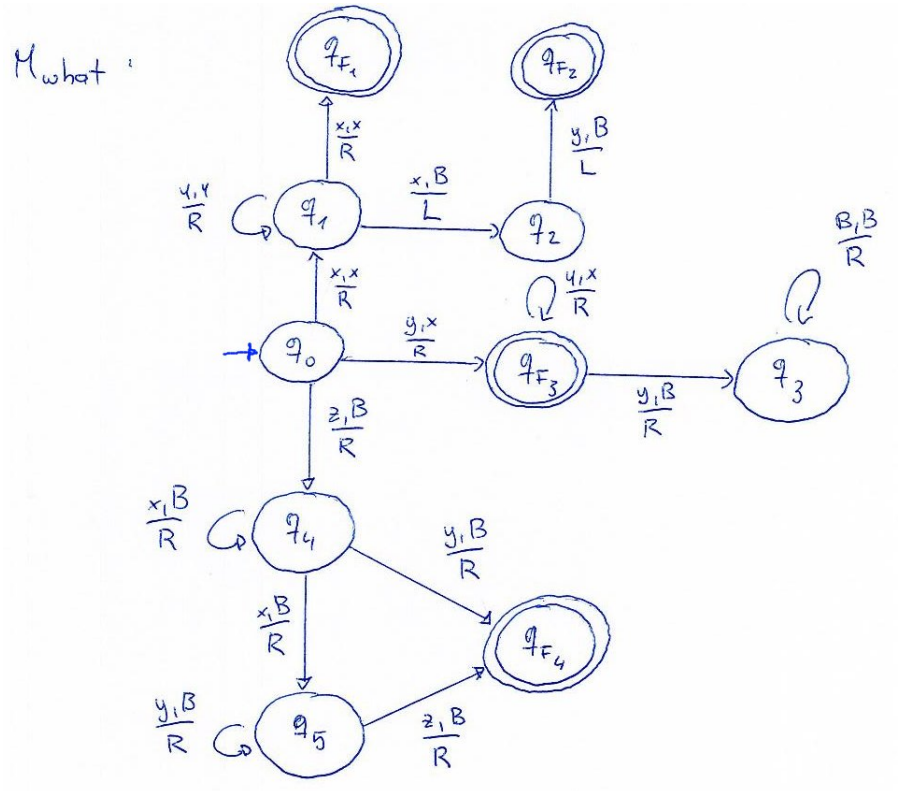
Chapter 4: Computability and Complexity

Exercise 1: Analysing Non-Deterministic Turing Machines

For each of the accepting words $w_1, ..., w_5$ below, write down the steps of the acepting runs and the contents of the tape after such a run on the Turing Machine M_{what}.
Since M_{what} is non-deterministic, multiple possibilities may occur for one given word! All of them need to be written down.

$$w_1 = xyyyxy$$
$$w_2 = xyxx$$
$$w_3 = yyy$$
$$w_4 = yx$$
$$w_5 = zxyz$$

Here is the Turing Machine M_{what}:



8 points

Don't forget/ignore Exercise 2 on next page!

Exercise 2: Complexity classes

For all of the following questions below, give a short answer in your own words:

1. What does it mean if a problem is in $\mathcal{P}$ and what can we say about the TM that solves it?
2. What does it mean if a problem is in $\mathcal{NP}$ and what can we say about the TM that solves it?
3. What problems are in $\mathcal{NP}$-complete and why is this class useful?
4. Are these correct? Are they wrong? Explain shortly and correct the statement if necessary: (where $\mathcal{NPC}$ denotes the set of all $\mathcal{NP}$-complete problems)

$$\begin{aligned} \mathcal{P} &\subseteq \mathcal{NP} \\ \mathcal{NP} &\neq \emptyset \\ \mathcal{NP} &\subset \mathcal{NPC} \\ \mathcal{P} &\cap \mathcal{NPC} \neq \emptyset \end{aligned}$$

5. What would need to be done to prove that $\mathcal{NP} = \mathcal{P}$ and what would this mean?

7 points