Mathematical Methods for Computer Science I

Fall 2017

Series 9 – Hand in before Monday, 27.11.2017 - 12.00

We are using a proof system with the set of axioms given at the bottom of this page and modus ponens as the only inference rule.

1. a) Show that $A \to B, B \to C, A \vdash C$ by explicitly writing a deduction of the conclusion from the premises. Explain every step. Which axioms did you use?
   b) With the help of the deduction lemma show that the formula
   $$(A \to B) \to ((B \to C) \to (A \to C))$$
   is provable.

2. Show that if $\Gamma_1 \vdash A$ and $\Gamma_2, A \vdash B$, then $\Gamma_1 \cup \Gamma_2 \vdash B$. (This is called the *cut rule*.)

3. With the help of the deduction lemma show that the formulas
   a) $((A \wedge B) \to C) \to (A \to (B \to C))$; b) $(A \wedge B) \to (B \wedge A)$
   are provable in our proof system.

4. With the help of the deduction lemma and the case distinction lemma show that the formula
   $$((A \wedge C) \vee (B \wedge C)) \to ((A \vee B) \wedge C)$$
   is provable in our proof system.

5. Change the axiom set by replacing the axiom (9) with $\neg\neg A \to A$. Show that this does not change the set of provable formulas.

(1) $A \to (B \to A)$;

(2) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$;

(3) a) $(A \wedge B) \to A$; b) $(A \wedge B) \to B$;

(4) $A \to (B \to (A \wedge B))$;

(5) a) $A \to (A \vee B)$; b) $B \to (A \vee B)$;

(6) $(A \to C) \to ((B \to C) \to (A \vee B \to C))$;

(7) $\neg A \to (A \to B)$;

(8) $(A \to B) \to ((A \to \neg B) \to \neg A)$;

(9) $A \vee \neg A$.