Mathematical Methods for Computer Science 1
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Series 1

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a)

The number of permutation f of the set $s = \{1, 2, 3, 4, 5\}$ where $f(1) \neq 1$ is computed by using the formula on the number of permutation of a set : $NbPerm(s) = |s|!$ where $f(1) \neq 1$, so we modify the formula : $NbPerm'(s) = (|s| - 1) * ((|s| - 1)!)$.
Finally we can compute $NbPerm'(s) = 4 * (4!) = 96$.

b)

The number of 10-digit numbers that have at least two equal digit can be computated by substracting the number of digit which have no duplicated digit, noted $N_{noDuplicate}$, to the total number of 10-digit numbers, noted N_{tot}.
Normally, the total number of a n-digit number could be compute by using the function $N_{tot}(n) = 10^n$, but a number can't start with 0 sowe have to slightly modify to formula : $N_{tot}(n) = 9 * 10^{n-1}$ where $n \geq 0$.
Now we can compute $N_{tot}(10) = 9 * 10^9$.
Then, the number of 10-digit number that have no duplicate's digit is the same has the number of permutation f of the set $s = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ where $f(0) \neq 0$. So we take the formula $NbPerm'(s) = (|s| - 1) * ((|s| - 1)!)$ and we compute $N_{noDuplicate} = NbPerm'(10) = 9 * (9!)$.
Finally, we can compute the total number of 10-digit number that have at least two equal digits : $N_{withDuplicate} = N_{tot} - N_{noDuplicate} = 9 * 10^9 - 9 * (9!) = 8'996'734'080$.

2

a

From any square on the chessboard, a rook is threatening $7 * 2 = 14$ another square. The total number of square on a chessboard is $8 * 8 = 64$ so there is $64 - 14 = 50$ free square for the other rook. So we have two choice to made : put the first rook on any square (64 possibility) and put the second rook on a non-threatened square ($64 - 14 = 50$ possibility). Finally the number of position that satisfy the constraint are $64 * 50 = 3200$.

b

Putting two rooks of the same color on a chessboards is the same as computing $\binom{64}{2} = \frac{64!}{2! * 62!} = \frac{64 * 63}{2} = 2016$.

The other way is to make a first choice from 64 squares and a second choice of 63 squares. But because the two rooks are undistinguishable, we need to divide the total by 2 : $\frac{64*63}{2} = 2016$

c

First, we place a rook on one of the 64 squares of the chessboards and then we choose a square to put the second, that gives two differents situation :

1. The second rook is places on a square already threatened by the other rook
2. The second rook is places on a square that it is not threatened by the other rook

In the first situation, the total number of square controlled $N_{ControlledSquare}$ by the two rooks is $14 + 14 - 7 - 1 = 20$. In the second situation, the total number of square controlled by the two rooks is $14 + 14 - 2 = 26$, because two square are threatened twice.
If we consider the first situation only, we have $64 * 63 * (62 - 20) * (61 - 20) = 6943104$ situations that satisfies the constraints and in the second situation we have $64 * 63 * (62 - 26) * (61 - 26) = 5080320$ situations that satisfies the constraints.
The sum of the two gives the total number of position that satisfies the constraints : $5'080'320 + 6'943'104 = 12'023'424$

3

a

The number of natural divisors of 60 is 12 (10 is 1 and 60 are not counted). This number is equal to the number of possible sub-set of the (multi-)set $\{2, 2, 3, 5\}$, but we have to remove some sub-set due to the fact that there is two 2 in the (multi-)set : $2^4 - 4 = 16 - 4 = 12$. We remove 4 because 2 and $2'$ are the same so any time 2 could be replace by $2'$ we remove one from 2^4.

b

For any natural number n such that $\nexists k$ where $k^2 = n$, the number of natural divisor of n is even because each operand o_1 of the multiplication that gives n needs a second operand o_2 where $o_1 \neq o_2$. For example, we take $n = 12$:

$$1 * 12 = 12 \qquad\qquad 2 * 6 = 123 * 4 = 12$$

For any natural number n such that $\exists k$ where $k^2 = n$, the number of natural divisor is increased by exactly one because $k * k = n$.

4

a

The total number of different words in the language is $4^3 = 4 * 4 * 4 = 64$.

b

The total number of different words that start with a vowels is $4 * 5 * 3 * 4 * 2 = 480$.
The total number of different words that start with a consonants is $5 * 4 * 4 * 3 * 3 = 720$.
The total number of different words is $480 + 720 = 1200$.

5

a

The number of different ways to separate $2n$ persons into two teams is the same as taking n from $2n$ and dividing the whole by 2 because a groups of n have a symetric groups : $\binom{2n}{n} * 2^{-1}$.

b

First, we count the number of permutation of n stone and then we divive the whole by n because the bracelets is circular, so circular permutation don't count : $\frac{n!}{n}$.