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Exercise sheet 6

Chapter 4: Computability and Complexity

Exercise 1: Constructing Deterministic Turing Machines

Construct deterministic TM that do the following tasks, where for both, the working alphabet is $\Gamma = \{x, y, B\}$:

1. TM M_1 that accepts the language $L_1 = (xxx \mid yyy)^n$, for $n > 0$

   e.g. $xxxyyyxxx \in L_1$, $yyyyyy \in L_1$, $yyyyxxxx \notin L_1$, $\emptyset \notin L_1$, ...

2. TM M_2 accepting words with at most 2 x's, and outputting on the tape only the x's (NOT separated by blanks)

   e.g.
   on input $yyxyyyyx$, the start tape would be ...$BBByyxyyyyxBBB$..., the TM would accept and at the end the tape would contain ...$BBBxxBBB$...

   on input $yyyyyy$, the start tape would be ...$BBByyyyyyBBB$..., the TM would accept and at the end the tape would contain ...BBB...

   on input $yyxxyyyxy$, the start tape would be ...$BBByyxxyyyxyBBB$..., the TM would not accept.

6 points

Exercise 2: Non-computability

In the lecture, we showed that the Busy Beaver function BB is not computable. On the slides, we use the statement "$BB(2m) > BB(m + c)$, for $m > c$", to prove non-computability.
Explain the meaning of this statement and formally prove its correctness!
Hint: *Proving $BB(a) > BB(b)$, for $a > b$ is enough to prove the statement, since $2m > m + c$, if $m > c$. To show this, construct a Busy Beaver TM with $n+1$ states that produces more 1's than any Busy Beaver TM with n states, for an arbitrary n.*

6 points

Exercise 3: Recursive and recursively enumerable languages

1. Prove that for a recursive language L, $\bar{L}$ is recursive as well ($\bar{L}$ denotes the complement of L), i.e. that L is closed under the complement.

2. Prove that recursive languages are closed under union and intersection.
   Hint: *To do this, let L_1 and L_2 be arbitrary recursive languages, i.e. $\exists$ TM M_1, M_2 such that M_1 and M_2 always halt and $L(M_1) = L_1$ and $L(M_2) = L_2$. Now show on a high level that there exists a TM M that will always halt and $L(M) = L_1 \cup L_2$ for the union and $L(M) = L_1 \cap L_2$ for the intersection.*

8 points