Mathematical Methods for Computer Science 1
Fall 2017

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a)

The number of permutation f of the set $s = \{1, 2, 3, 4, 5\}$ where $f(1) \neq 1$ is computed by using the formula on the number of permutation of a set : $NbPerm(s) = |s|!$ where $f(1) \neq 1$, so we modify the formula : $NbPerm'(s) = (|s| - 1) * ((|s| - 1)!)$.
Finally we can compute $NbPerm'(s) = 4 * (4!) = 96$.

b)

The number of 10-digit numbers that have at least two equal digit can be computated by substracting the number of digit which have no duplicated digit, noted $N_{noDuplicate}$, to the total number of 10-digit numbers, noted N_{tot}.
Normally, the total number of a n-digit number could be compute by using the function $N_{tot}(n) = 10^n$, but a number can't start with 0 sowe have to slightly modify to formula : $N_{tot}(n) = 9 * 10^{n-1}$ where $n \geq 0$.
Now we can compute $N_{tot}(10) = 9 * 10^9$.
Then, the number of 10-digit number that have no duplicate's digit is the same has the number of permutation f of the set $s = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ where $f(0) \neq 0$. So we take the formula $NbPerm'(s) = (|s| - 1) * ((|s| - 1)!)$ and we compute $N_{noDuplicate} = NbPerm'(10) = 9 * (9!)$.
Finally, we can compute the total number of 10-digit number that have at least two equal digits : $N_{withDuplicate} = N_{tot} - N_{noDuplicate} = 9 * 10^9 - 9 * (9!) = 8'996'734'080$.

2

a

From any square on the chessboard, a rook is threatening $7 * 2 = 14$ another square. The total number of square on a chessboard is $8 * 8 = 64$ so there is $64 - 14 = 50$ free square for the other rook. So we have two choice to made : put the first rook on any square (64 possibility) and put the second rook on a non-threatened square ($64 - 14 = 50$ possibility). Finally the number of position that satisfy the constraint are $64 * 50 = 3200$.

b

Putting two rooks of the same color on a chessboards is the same as computing $\binom{64}{2} = \frac{64!}{2!*62!} = \frac{64*63}{2} = 2016$.
The other way is to make a first choice from 64 squares and a second choice of 63 squares. But because the two rooks are undistinguishable, we need to divide the total by 2 : $\frac{64*63}{2} = 2016$

c

3

a

b

4

a

b

5

a

b