Exercice Sheet 5

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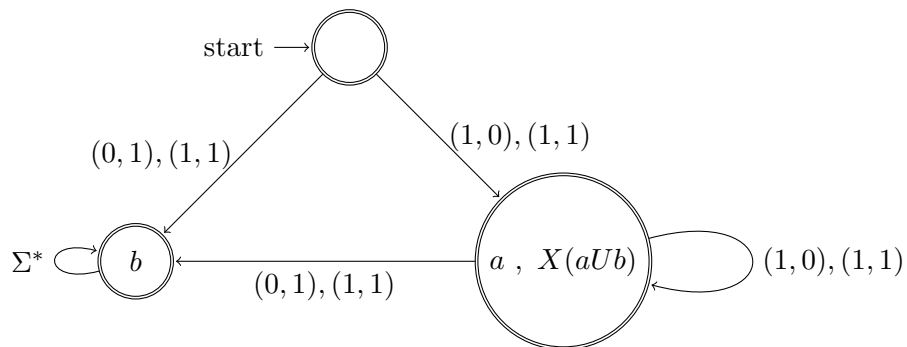
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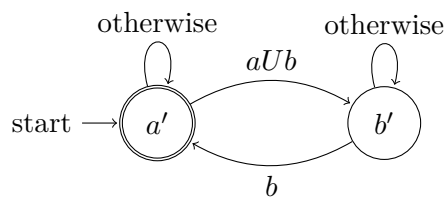
Exercice 1
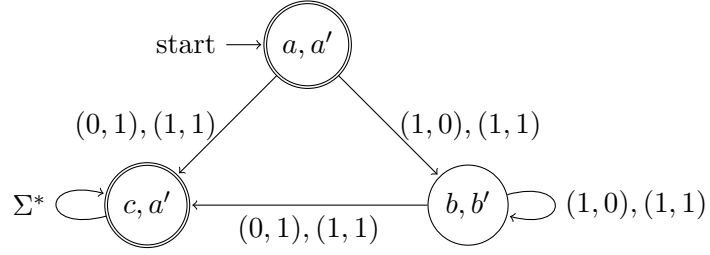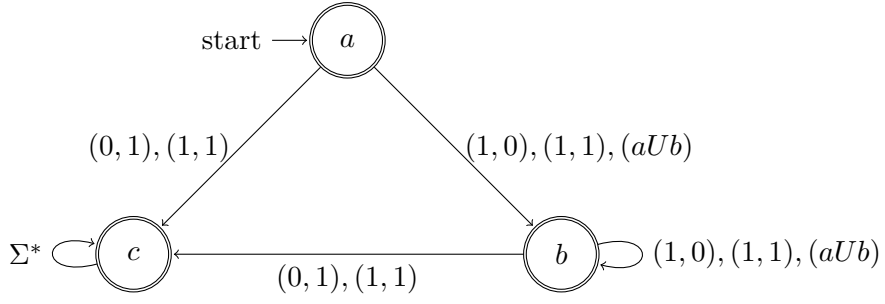
(1)

Local automata :

Eventuality automata :

Complete automaton :

start $\longrightarrow$ a

$(0,1),(1,1)$

$(1,0),(1,1),(aUb)$

Σ^* c

b $(1,0),(1,1),(aUb)$

$(0,1),(1,1)$

start $\longrightarrow$ a,a'

$(0,1),(1,1)$

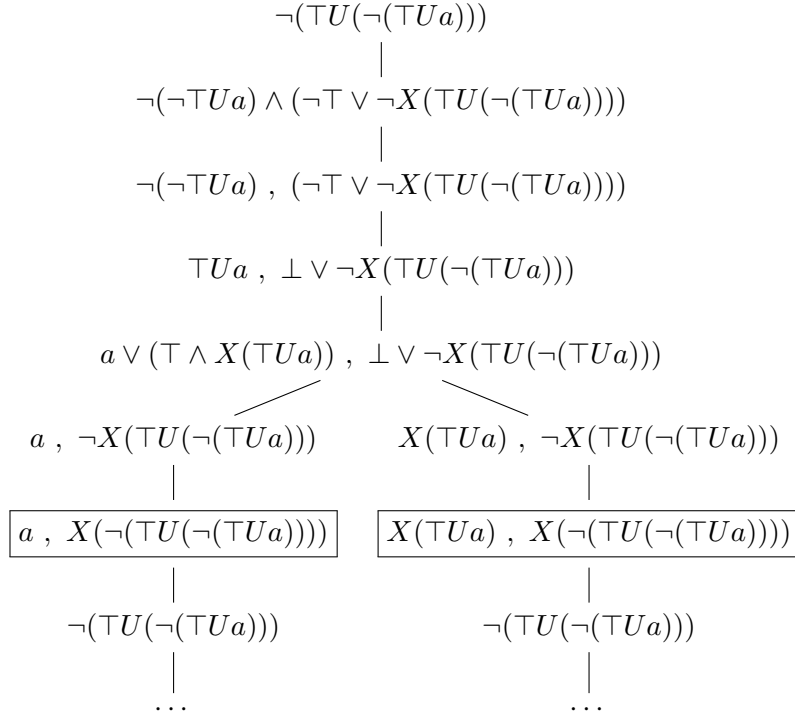$(1,0),(1,1)$

Σ^* c,a'

b,b' $(1,0),(1,1)$

$(0,1),(1,1)$

(2)

First, we transform

$$\Box\Diamond a$$

into

$$\Box\Diamond a \equiv \neg\Diamond\neg(\Diamond a)$$
$$\equiv \neg(\top U(\neg(\Diamond a)))$$
$$\equiv \neg(\top U(\neg(\top U a)))$$

Algorithmic sugar (note : we simplify formulae like $\top \wedge a \equiv a$ and $\bot \vee a \equiv a$) :

$$\neg(\top U(\neg(\top U a)))$$

$$|$$

$$\neg(\neg\top U a) \wedge (\neg\top \vee \neg X(\top U(\neg(\top U a))))$$

$$|$$

$$\neg(\neg\top U a) \ , \ (\neg\top \vee \neg X(\top U(\neg(\top U a))))$$

$$|$$

$$\top U a \ , \ \bot \vee \neg X(\top U(\neg(\top U a)))$$

$$|$$

$$a \vee (\top \wedge X(\top U a)) \ , \ \bot \vee \neg X(\top U(\neg(\top U a)))$$

$$a \ , \ \neg X(\top U(\neg(\top U a))) \qquad\qquad X(\top U a) \ , \ \neg X(\top U(\neg(\top U a)))$$

$$| \qquad\qquad\qquad\qquad\qquad\qquad |$$

$$\boxed{a \ , \ X(\neg(\top U(\neg(\top U a))))} \qquad \boxed{X(\top U a) \ , \ X(\neg(\top U(\neg(\top U a))))}$$

$$| \qquad\qquad\qquad\qquad\qquad\qquad |$$

$$\neg(\top U(\neg(\top U a))) \qquad\qquad\qquad \neg(\top U(\neg(\top U a)))$$

$$| \qquad\qquad\qquad\qquad\qquad\qquad |$$

$$\dots \qquad\qquad\qquad\qquad\qquad\qquad \dots$$

Automata :
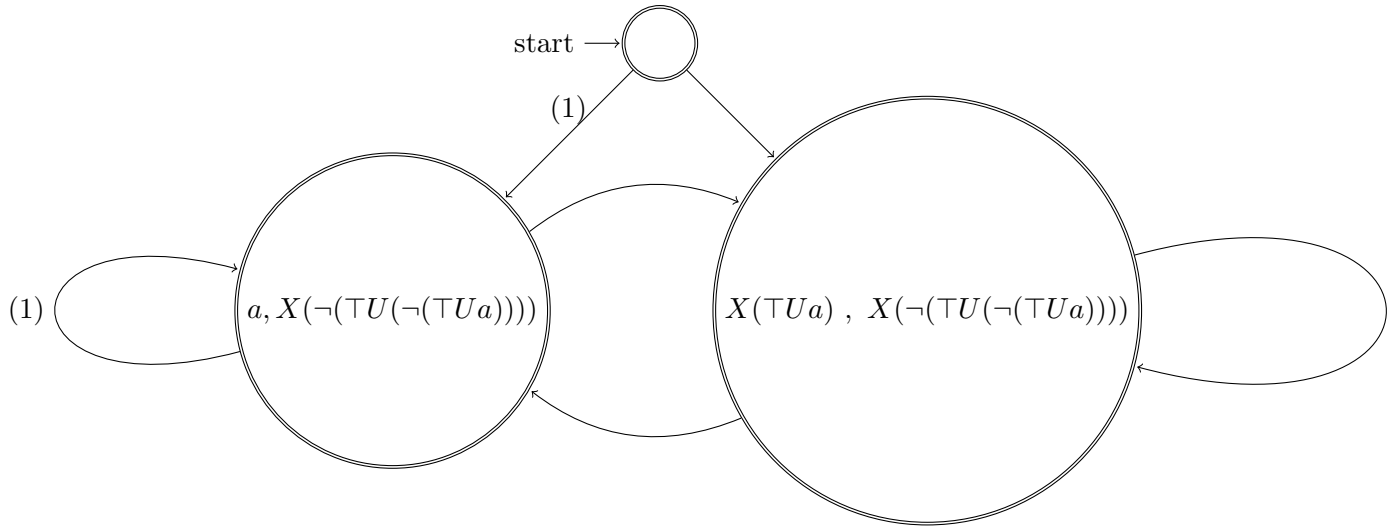
Exercice 2

(1)

$$\Box\Diamond(x == 1)$$

It is not a safety property, because it does not exist a finite execution of the system that would not satisfies this formula, because we always need to "know" if $x == 1$ or not in order to unvalidate the property.

It is a liveness property, because the property is checking, at any moment, if $x == 1$ would occurs now or in the future.

(2)

$$\neg(\Box\Diamond(x < 36))$$

It is not a safety property, because it does no exist a finite execution of the system that would not satisfies the formula. For example, if $x < 36$ occurs 2 times, we don't know if the property would hold or not in the future.

It is a liveness property, we could transform the formula to $\neg(\Box\Diamond(x < 36)) = \neg(\neg(\Diamond\neg(\Diamond(x < 36)))) = \Diamond\neg(\Diamond(x < 36))$, and here the property would check if $\Diamond(x < 36)$ would not happen.

(3)

$$\Diamond(\neg(x > 10))$$

It is a liveness property, we check if $\neg(x > 10)$ would eventually happen or not. We could transform $\neg(x > 10)$ to $x \leq 10$, and $\Diamond(x \leq 10)$ is a liveness property.

It is not a safety property, because it is a liveness property.

(4)

$$\neg\Diamond(x == 12)$$

It is a safety property, because it exist a finite execution of the system that does not satisfy this property, for example if $x == 12$ now, the property does not hold and any further execution of the system would satisfies the property.

It is not a liveness property, because it is a safety property.