

① $g(x) = \frac{1}{1+e^{-x}}$

$$g'(x) = \frac{0 \cdot (1 + e^{-x}) - 1(0 - e^{-x})}{(1 + e^{-x})^2} = \frac{e^{-x}}{1 + 2e^{-x} + e^{-2x}} =$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$(g(x))(1-g(x)) = \frac{1}{1+e^x} \cdot \left(1 - \frac{1}{1+e^x}\right) =$$

$$= \frac{1}{1+e^{-x}} \left(\frac{1+e^{-x}-1}{1+e^{-x}} \right) = \boxed{\frac{e^{-x}}{(1+e^{-x})^2}}$$

$$(2) \quad l(w) = \frac{1}{N} \sum_{t=1}^N \log(p(y_t|x_t)) =$$

$$= \frac{1}{N} \sum_{t=1}^N \log(g(w^T x_t)^{y_t} (1 - g(w^T x_t))^{1-y_t})$$

$$\frac{\partial l(w)}{\partial w} = \frac{1}{N} \sum_{t=1}^N \frac{1}{g(w^T x_t)^{y_t} (1-g(w^T x_t))^{1-y_t}} \cdot (y_t g(w^T x_t)^{y_t-1} \cdot g(w^T x_t) - (1-g(w^T x_t))^{1-y_t} \cdot (1-g(w^T x_t))^{1-y_t})$$

$$(-g(w^T x_t) \cdot (1 - g(w^T x_t)) \cdot x_t)$$

$$= \frac{1}{N} \sum_{t=1}^N y_t \cdot (1 - g(\omega^T x_t)) \cdot x_t + (1 - y_t) \cdot (-g(\omega^T x_t)) \cdot x_t \quad \checkmark$$

$$= \frac{1}{N} \sum_{t=1}^N (y_t - y_t g(w^T x_t) - g(w^T x_t) + y_t g(w^T x_t)) x_t$$

$$= \frac{1}{N} \sum_{t=1}^N (y_t - g(w^T x_t)) x_t$$