Dynamic System: Study Case

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Dynamic System (GitHub)
Matlab Code (GitHub)

Introduction

This report analyzes the dynamic behavior of a DC motor based on the mathematical model provided in the referenced study by S. Alabaş et al. [1]., specifically the motor component described in Equation (2). The goal is to simulate the transient response of torque, angular velocity, and back electromotive force (EMF) when subjected to pulse voltage inputs. Two test cases are evaluated:

1. A +5 V voltage pulse applied for 5 seconds.
2. A −5 V voltage pulse applied for 5 seconds.

Both simulations use the physical parameters listed in Table 1 of the reference document, which include the armature resistance, inductance, back-EMF constant, torque constant, wheel inertia, and viscous friction coefficient.

The results provide insight into the system's transient dynamics and the influence of electrical and mechanical parameters on the time required to reach steady-state conditions.

DC Motor Mathematical Model

The DC motor model considered in this analysis follows the structure of Equation (2) in the reference document:

$$V = R_a i + L_a \frac{di}{dt} + e_a, \tag{1}$$

$$e_a = K_b \omega, \tag{2}$$

$$\tau = K_t i, \tag{3}$$

where:

- V is the armature input voltage (pulse signal),
- i is the armature current,
- e_a is the back-EMF voltage,
- R_a is the armature resistance,
- L_a is the armature inductance,
- K_b is the back-EMF constant,
- ω is the angular velocity,
- τ is the motor torque,
- K_t is the torque constant.

The mechanical dynamics are modeled as:

$$J_W \frac{d\omega}{dt} + \zeta_W \, \omega = \tau, \tag{4}$$

where J_W is the inertia of the wheel and ζ_W is the viscous damping coefficient.

Simulation Input: Voltage Pulse

Each experiment uses a 5-second voltage pulse defined as:

$$V(t) = \begin{cases} 0, & t < 1\,\text{s}, \\ V_{\text{amp}}, & 1 \leq t < 6\,\text{s}, \\ 0, & t \geq 6\,\text{s}, \end{cases}$$

where $V_{\text{amp}} = +5$ V for the first test case and $V_{\text{amp}} = -5$ V for the second.

Simulation Results

This section presents the dynamic response of the system for both voltage conditions. The results include torque, angular velocity, and back-EMF.

Case 1: +5 V Pulse

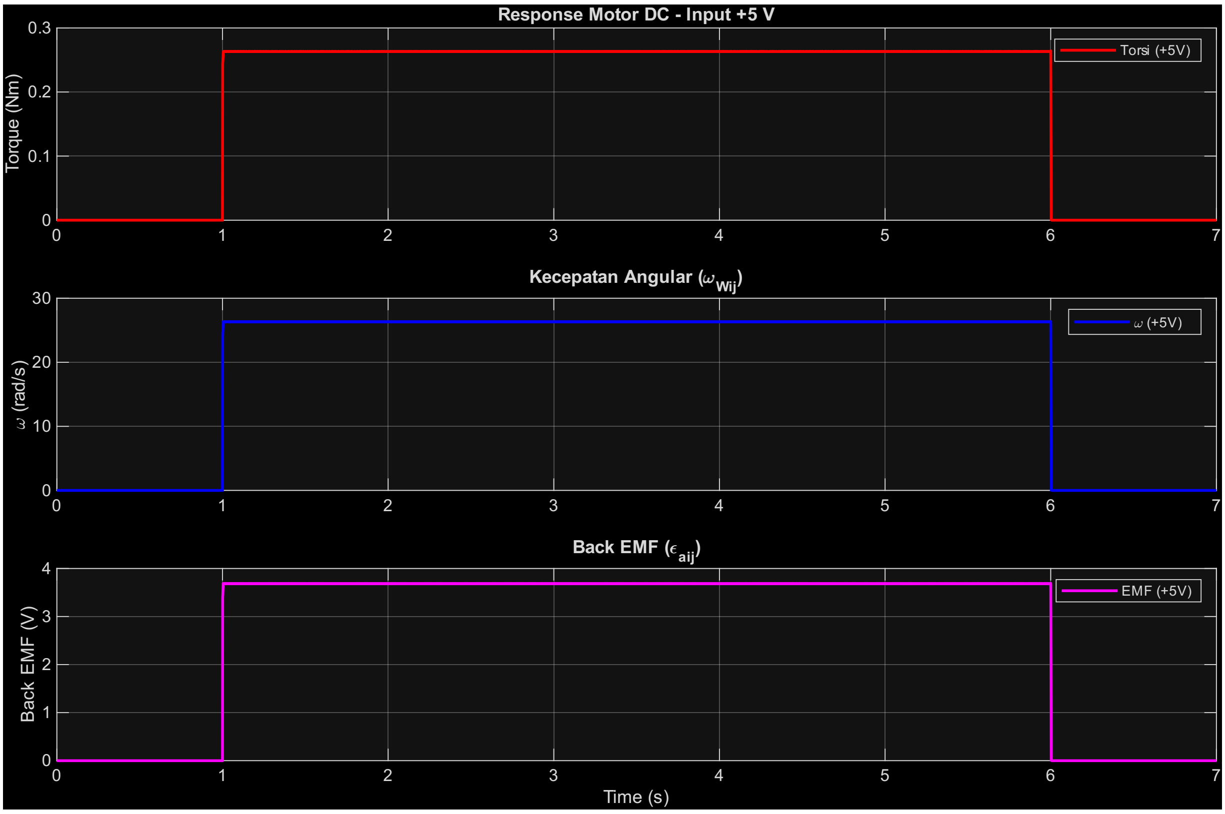

Figure 1: DC motor response for a +5 V input pulse.

Observations:

- Torque τ rises quickly due to sudden increase in current, reaching approximately 0.3Nm.
- Angular velocity increases to around 30rad/s during the pulse interval.
- Back-EMF increases proportionally to the angular velocity, reaching nearly 4V.
- When the voltage returns to 0 at $t = 6$ s, the motor decelerates exponentially due to friction and back-EMF counteraction.

Case 2: -5 V Pulse

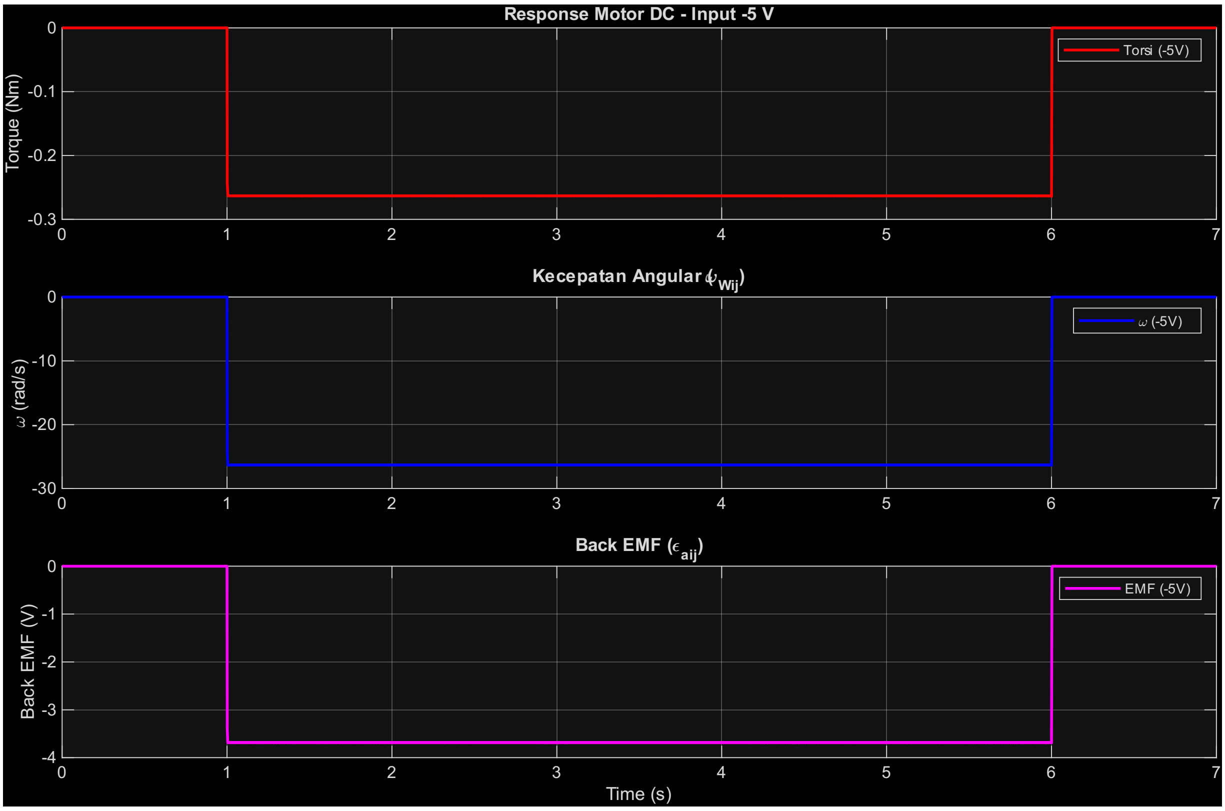

Figure 2: DC motor response for a -5 V input pulse.

Observations:

- Torque becomes negative, with magnitude similar to the +5 V case (around -0.3 Nm).
- Angular velocity reaches negative values, approximately -30 rad/s, indicating reverse rotation.
- Back-EMF becomes negative, mirroring the angular velocity profile.
- After the pulse ends at 6 seconds, the system returns toward zero angular speed following a damped decay.

Comparison Between +5 V and -5 V Cases

Both simulations exhibit symmetrical behavior with respect to voltage polarity:

- Positive voltage produces positive torque and forward rotation.
- Negative voltage produces negative torque and reverse rotation.
- Magnitudes are nearly identical in both directions because the system is linear and symmetric.
- Back-EMF mirrors the sign and magnitude of the angular velocity, confirming the validity of the electrical model.

The transient response shape is dominated by:

- Armature inductance L_a (affects current rise time),
- Mechanical inertia J_W (affects acceleration and deceleration),
- Viscous damping ζ_W (affects deceleration and steady-state speed),
- Back-EMF constant K_b (limits current as speed increases).

Parameters Affecting Time to Reach Steady-State

Based on the governing equations, the parameters most responsible for slowing the motor's approach to steady-state are:

1. **Mechanical Inertia** (J_W): Large inertia slows acceleration, delaying steady-state.
2. **Armature Inductance** (L_a): Higher inductance delays current rise, slowing torque onset.
3. **Viscous Friction** (ζ_W): Higher damping reduces steady-state speed and accelerates decay after voltage removal.
4. **Armature Resistance** (R_a): Higher resistance lowers current, reducing torque.
5. **Back-EMF Constant** (K_b): Limits current as speed increases, reducing acceleration at higher speeds.

References

[1] Hsiu-Ming Wu, Muhammad Qomaruz Zaman. "Enhanced Hierarchical Fuzzy Formation Control with fuzzy collision avoidance behavior for multiple Mecanum wheeled Mobile Robots." *Robotics and Autonomous Systems*, 2025. Available in the provided PDF file: *1-s2.0-S0921889025002210-main.pdf*.