Numerical Integration

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Introduction

Key Concept

Integration is a fundamental operation in calculus used to compute areas under curves, accumulated values of a function, or physical quantities such as displacement, probability, and energy. In many practical cases, evaluating an integral analytically is difficult or impossible due to the complexity of the function.
Numerical integration provides a set of techniques to approximate the value of definite integrals when exact solutions are unavailable. This report introduces the basic concepts of numerical integration and compares several common methods, including Riemann Sum (Left, Midpoint, and Right), Trapezoidal Rule, Simpson's Rule, and Monte Carlo Integration.

Github and Code can Access in,
Numerical Integration (GitHub)
Matlab Code (GitHub)

Numerical Methods

Riemann Sum Methods

Riemann Sum approximates an integral by dividing the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$ and summing up the areas of rectangles formed by the function values.

Left Riemann Sum

$$I \approx \sum_{i=0}^{n-1} f(x_i)\Delta x$$

Right Riemann Sum

$$I \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

Midpoint Riemann Sum

$$I \approx \sum_{i=0}^{n-1} f\left(\frac{x_i + x_{i+1}}{2}\right)\Delta x$$

Trapezoidal Rule

The Trapezoidal Rule approximates the area under the curve using trapezoids, averaging the function values at the endpoints of each subinterval. It generally provides higher accuracy than simple Riemann sums.

Simpson's Rule

Simpson's Rule uses parabolic interpolation across pairs of subintervals, yielding high accuracy for smooth functions. The method requires an even number of subintervals and is particularly effective when the function exhibits smooth curvature.

Monte Carlo Integration

Monte Carlo Integration relies on random sampling to estimate the integral:

$$I \approx (b-a) \cdot \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

This probabilistic method is powerful in high-dimensional integrals but less efficient for simple one-dimensional problems.

Experiment and Results

Experimental Setup

The experiment evaluates the accuracy of different numerical integration methods on the function:

$$f(t) = \sin(2\pi t) - \cos(3\pi t)$$

- Integration interval: $[0, 5]$
- Number of subintervals/points: $N = 100$
- Exact integral calculated analytically:

$$F(t) = -\frac{\cos(2\pi t)}{2\pi} - \frac{\sin(3\pi t)}{3\pi}$$

Result

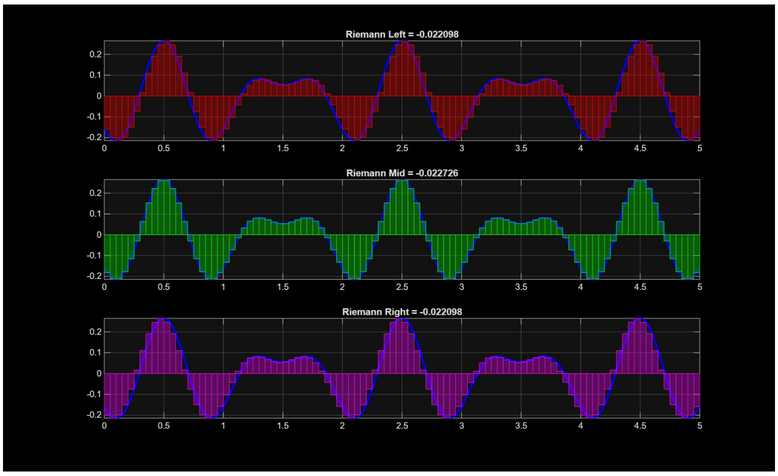

Figure 1: Reimann Sum Method Result

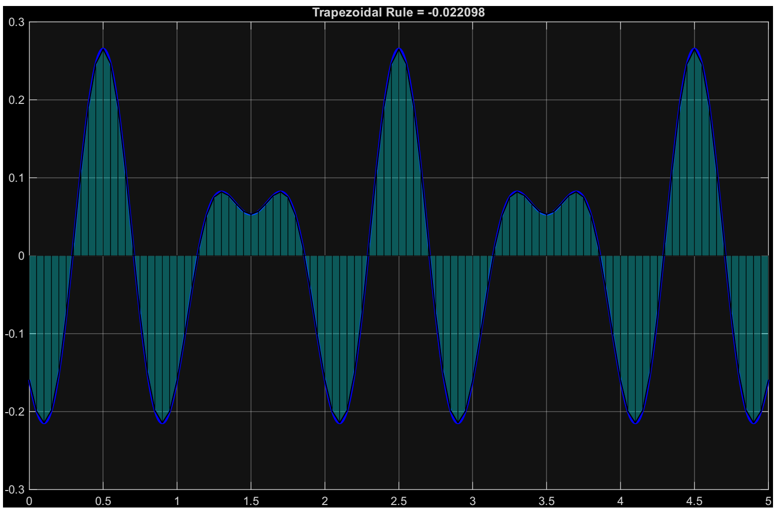

Figure 2: Trapezoidal Rule Method Result

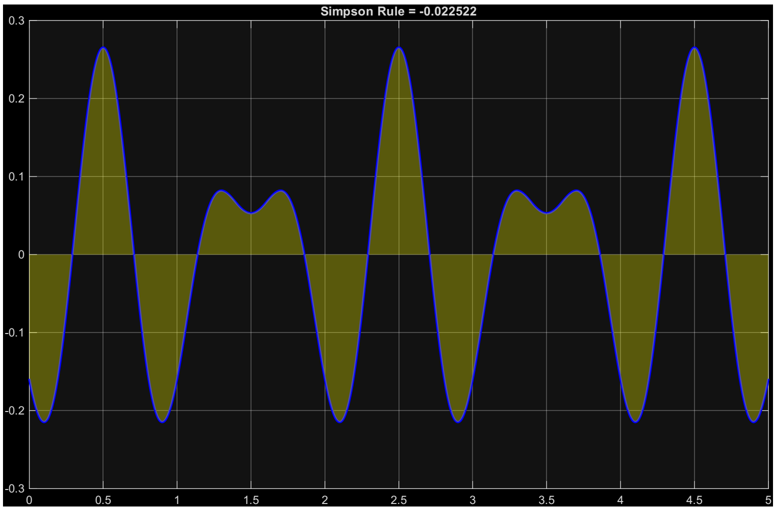

Figure 3: Simpson Rule Method Result

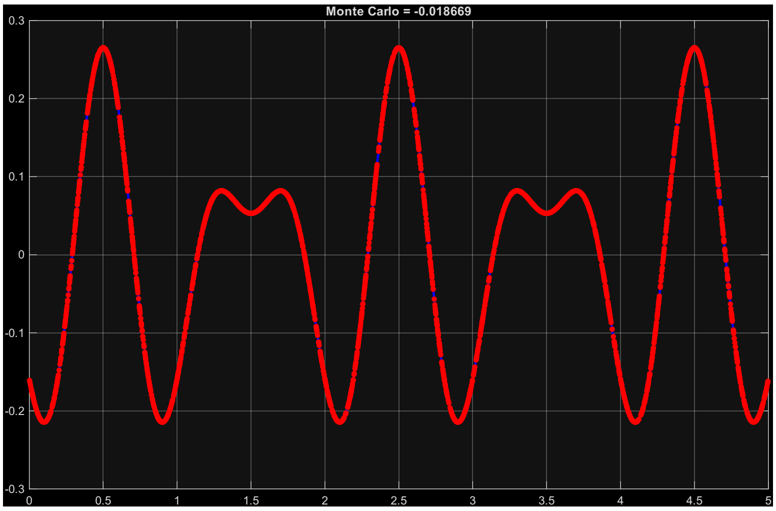

Figure 4: Monte Carlo Method Result

Comparison of Methods

The table below summarizes approximate integral values and relative errors. Simpson's Rule achieves the most accurate result, while Monte Carlo shows the largest error due to its probabilistic nature in 1D.

Method	**Approximate Value**	**Relative Error (%)**
Riemann Left	-0.022098	1.8574
Riemann Midpoint	-0.022726	0.9313
Riemann Right	-0.022098	1.8574
Trapezoidal	-0.022098	1.8574
Simpson's Rule	-0.022522	0.0281
Monte Carlo	-0.018669	17.083
Exact Value	**-0.022516**	—

Table 1: Comparison of numerical integration results.

Conclusion

Conclusion

Simpson's Rule provides the highest accuracy for smooth, one-dimensional functions because of its parabolic approximation. Riemann and Trapezoidal methods are simpler but less precise due to their linear or rectangular approximations. Monte Carlo Integration is powerful for multi-dimensional problems but less effective in simple 1D scenarios with limited sample points. These results emphasize the importance of selecting an appropriate numerical method based on the function's characteristics and desired accuracy.