CSE 427 - Note

Noor Mohammed Priom

14, August 2025

Contents

1 Linear Regression **2**

1 Linear Regression

Example - 1

Fit a linear regression model ($\hat{y} = Wx + b$) using gradient descent and derive the best fit line to predict house rent based on size. Where $W = 0$, $b = 0$ & $\alpha = 0.2$ [in exam Error Tolerance = $1 \times 10^{-}2$ will be given or iteration = 5 will be given.

House size (sq.fit)	Rent ($1000)
0.0	1.5
0.4	2.0
0.7	2.5
1.0	3.0

Table 1: Table

Answer:
we know,
$W = W - \alpha[-\frac{2}{n}\sum_{i=1}^{n} x_i(y_i - \hat{y}_i)]$
$b = b - \alpha[-\frac{2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)]$
Cost Function (MSE) = $[\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2]$
and Line for linear regression,

$$\hat{y} = Wx + b$$

1^{st} **Iteration**

here, $W = 0$ & $b = 0$ and $\alpha = 0.2$

i	$\hat{y}_i = Wx + b$	$y_i - \hat{y}_i$	$(y_i - \hat{y}_i)^2$	$x_i(y_i - \hat{y}_i)$
1	0	1.5	2.25	0.00
2	0	2.0	4.00	0.80
3	0	2.5	6.25	1.75
4	0	3.0	9.00	3.00
1 to 4	-	$\sum_{i=1}^{n}(y_i - \hat{y}_i) = 9$	$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = 19.25$	$\sum_{i=1}^{n} x_i(y_i - \hat{y}_i) = 5.55$

Table 2: 1st iteration

so,

new $W = W - \alpha[-\frac{2}{n}\sum_{i=1}^{n} x_i(y_i - \hat{y}_i)]$
$= 0 - 0.2(-\frac{2}{4} \times 5.55)$
$= 0.555$

new $b = b - \alpha[-\frac{2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)]$
$= 0 - 0.2(-\frac{2}{4} \times 9)$
$= 0.90$

MSE $\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
$= \frac{1}{4} \times 21.5$
$= 5.375$

2^{nd} **Iteration**

here, $W = 0.555$ & $b = 0.90$ and $\alpha = 0.2$

i	$\hat{y_i} = Wx + b$	$y_i - \hat{y_i}$	$(y_i - \hat{y_i})^2$	$x_i(y_i - \hat{y_i})$
1	0.900	0.600	0.360	0.000
2	1.122	0.878	0.771	0.351
3	1.289	1.212	1.468	0.848
4	1.455	1.545	2.387	1.545
1 to 4	-	$\sum_{i=1}^{n}(y_i - \hat{y_i}) = 4.235$	$\sum_{i=1}^{n}(y_i - \hat{y_i})^2 = 4.986$	$\sum_{i=1}^{n} x_i(y_i - \hat{y_i}) = 2.744$

Table 3: 2nd iteration

so,

$$\begin{aligned} \text{new } W &= W - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n} x_i(y_i - \hat{y_i})] \\ &= 0.555 - 0.2(-\tfrac{2}{4} \times 2.744) \\ &= 0.8294 \end{aligned}$$

$$\begin{aligned} \text{new } b &= b - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})] \\ &= 0.9 - 0.2(-\tfrac{2}{4} \times 4.235) \\ &= 1.3235 \end{aligned}$$

$$\begin{aligned} \text{MSE } &\tfrac{1}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})^2 \\ &= \tfrac{1}{4} \times 1.2465 \\ &= 5.375 \end{aligned}$$

3^{rd} **Iteration**

here, $W = 0.8294$ & $b = 1.3235$ and $\alpha = 0.2$

i	$\hat{y_i} = Wx + b$	$y_i - \hat{y_i}$	$(y_i - \hat{y_i})^2$	$x_i(y_i - \hat{y_i})$
1	1.324	0.177	0.031	0.000
2	1.655	0.345	0.119	0.138
3	1.904	0.596	0.355	0.417
4	2.153	0.847	0.718	0.847
1 to 4	-	$\sum_{i=1}^{n}(y_i - \hat{y_i}) = 1.964$	$\sum_{i=1}^{n}(y_i - \hat{y_i})^2 = 1.223$	$\sum_{i=1}^{n} x_i(y_i - \hat{y_i}) = 1.402$

Table 4: 3rd iteration

so,

$$\begin{aligned} \text{new } W &= W - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n} x_i(y_i - \hat{y_i})] \\ &= 0.8294 - 0.2(-\tfrac{2}{4} \times 1.402) \\ &= 0.9696 \end{aligned}$$

$$\begin{aligned} \text{new } b &= b - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})] \\ &= 1.3235 - 0.2(-\tfrac{2}{4} \times 1.964) \\ &= 1.5199 \end{aligned}$$

$$\begin{aligned} \text{MSE } &\tfrac{1}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})^2 \\ &= \tfrac{1}{4} \times 1.223 \\ &= 0.3056 \end{aligned}$$

4^{th} **Iteration**

here, $W = 0.9696$ & $b = 1.5199$ and $\alpha = 0.2$

i	$\hat{y_i} = Wx + b$	$y_i - \hat{y_i}$	$(y_i - \hat{y_i})^2$	$x_i(y_i - \hat{y_i})$
1	1.520	-0.020	0.000	0.000
2	1.908	0.092	0.009	0.037
3	2.199	0.301	0.091	0.211
4	2.490	0.511	0.261	0.511
1 to 4	-	$\sum_{i=1}^{n}(y_i - \hat{y_i}) = 0.884$	$\sum_{i=1}^{n}(y_i - \hat{y_i})^2 = 0.360$	$\sum_{i=1}^{n} x_i(y_i - \hat{y_i}) = 0.758$

Table 5: 4th iteration

so,

$$\begin{aligned} \text{new } W &= W - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n} x_i(y_i - \hat{y_i})] \\ &= 0.9696 - 0.2(-\tfrac{2}{4} \times 0.758) \\ &= 1.0454 \end{aligned}$$

$$\begin{aligned} \text{new } b &= b - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})] \\ &= 1.5199 - 0.2(-\tfrac{2}{4} \times 0.884) \\ &= 1.6083 \end{aligned}$$

$$\begin{aligned} \text{MSE } &\tfrac{1}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})^2 \\ &= \tfrac{1}{4} \times 0.360 \\ &= 0.0900 \end{aligned}$$

5^{th} **Iteration**

here, $W = 0.0454$ & $b = 1.6083$ and $\alpha = 0.2$

i	$\hat{y_i} = Wx + b$	$y_i - \hat{y_i}$	$(y_i - \hat{y_i})^2$	$x_i(y_i - \hat{y_i})$
1	1.608	-0.108	0.012	0.000
2	2.026	-0.026	0.001	-0.011
3	2.340	0.160	0.026	0.112
4	2.654	0.346	0.120	0.346
1 to 4	-	$\sum_{i=1}^{n}(y_i - \hat{y_i}) = 0.371$	$\sum_{i=1}^{n}(y_i - \hat{y_i})^2 = 0.158$	$\sum_{i=1}^{n} x_i(y_i - \hat{y_i}) = 0.448$

Table 6: 4th iteration

so,

$$\begin{aligned} \text{new } W &= W - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n} x_i(y_i - \hat{y_i})] \\ &= 1.0454 - 0.2(-\tfrac{2}{4} \times 2.098) \\ &= 1.0902 \end{aligned}$$

$$\begin{aligned} \text{new } b &= b - \alpha[-\tfrac{2}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})] \\ &= 1.6083 - 0.2(-\tfrac{2}{4} \times 0.884) \\ &= 1.6454 \end{aligned}$$

$$\begin{aligned} \text{MSE } &\tfrac{1}{n}\textstyle\sum_{i=1}^{n}(y_i - \hat{y_i})^2 \\ &= \tfrac{1}{4} \times 0.157 \\ &= 0.0395 \end{aligned}$$

So the equation for the best fit line to predict house rent based on size final or the Model is $= \hat{y} = 1.0902x + 1.6454$