TOH

$T_n = T_{n-1} + 1 + T_{n-1}$

$= 2T_{n-1} + 1 \longrightarrow$ Open form

$T_4 = 2T_3 + 1$

$= 2(2T_2 + 1) + 1$

$= 4T_2 + 3$

$= 4(2T_1 + 1) + 3$

$= 8T_1 + 7$

$= 8(2T_0 + 1) + 7$

$= 15$

$T_0 = 0$, $T_1 = 1$, $T_2 = 3$, $T_3 = 7$, $T_4 = 15$

$T_n = 2^n - 1 \longrightarrow$ Close form

Prove by mathematical induction

Basis $\longrightarrow$ Hypothesis $\longrightarrow$ Induction [Steps]

Basis: for the lowest value of n.

$T_0 = 2^0 - 1 = 0$

Hypothesis: Assuming for all values of n ranging from 1 to n-1, $2^n - 1$ is true.

Induction:

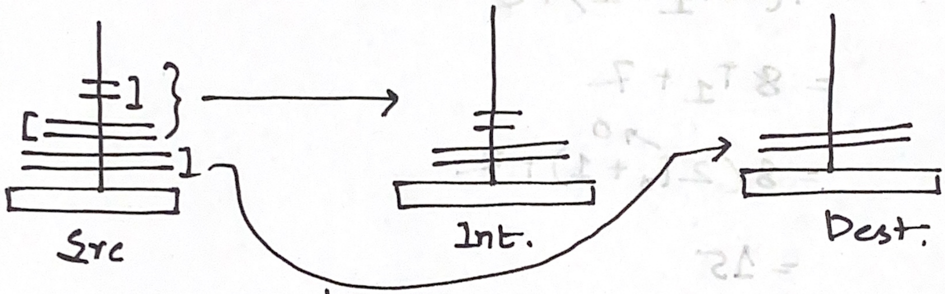
from the reccurence relation,

$$T_n = 2T_{n-1} + 1$$

$$= 2(2^{n-1} - 1) + 1$$

$$T_n = 2^n - 1$$

DTOH



$$T_n = T_{n-1} + 2 + T_{n-1}$$

$$= 2T_{n-1} + 2$$

$$= 2(2T_{n-2} + 2) + 2 = 2^2 T_{n-2} + 2^2 + 2$$

$$= 2^3 T_{n-3} + 2^3 + 2^2 + 2$$

.....

.....

$$= 2^n T_{n-n} + 2^n + 2^{n-1} + \dots + 2^1$$

$$= 2^n.0 + 2^n + 2^{n-1} + \dots + 2^2 + 2$$

$$= 2(2^{n-1} + \dots + 2^1 + 2^0)$$

$$= 2\left(\frac{2^{n-1+1} - 1}{2-1}\right)$$

$$= 2^{n+1} - 2 \longrightarrow \text{Closed form/Eq}^n$$

Prove by mathematical induction for DTOH that $T_n = 2(2^n - 1)$ Holds.

Basis: $n=0 \ldots$

Hypothesis: Assuming for $(n-1)$ true, $T_{n-1} = 2\cdot(2^{n-1}-1)$

Induction:

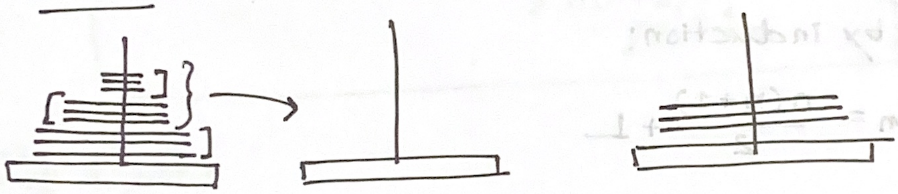
from Recurrence $T_n = 2T_{n-1} + 2$

$$= 2(2(2^{n-1}-1)) + 2$$

$$= 2(2^n - 2) + 2$$

$$= 2(2^n - 1)$$

TTOH



$$T_n = T_{n-1} + 3 + T_{n-1}$$

$$= 2T_{n-1} + 3$$

$$= 2(2T_{n-2} + 3) + 3$$

$$= 2^2 T_{n-2} + 2\cdot 3 + 3$$

$$= 2^3 T_{n-3} + 2^2 \cdot 3 + 2^1 \cdot 3 + 2^0 \cdot 3$$

$\ldots$

$$= 2^n T_{n-n} + 2^{n-1}\cdot 3 + 2^{n-2}\cdot 3 + \ldots + 2^1 \cdot 3 + 2^0 \cdot 3$$

$$= 3(2^0 + 2^1 + \ldots + 2^{n-1})$$

$$= 3\left(\frac{2^{n-1+1}-1}{2-1}\right) = 3(2^n - 1) \longrightarrow \text{Closed form}$$

$\rightarrow$ Prove by math. induction

Lines in the plane

$L_n = L_{n-1} + n$ — Recurrence

$= L_{n-2} + (n-1) + n$

$= L_{n-3} + (n-2) + (n-1) + n$

$\cdots = L_{n-n} \cdots$

$= L_0 + 1 + 2 + \cdots + (n-2) + (n-1) + n$

$= 1 + \dfrac{n(n+1)}{2}$

Prove by induction:

$$l_n = \frac{n(n+1)}{2} + 1$$

Basis: $n = 0$

$l_0 = 1$

Hypothesis: Assuming until $(n-1)$, the eq^n holds

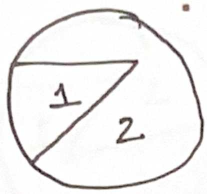
Induction:

$$L_n = L_{n-1} + n$$

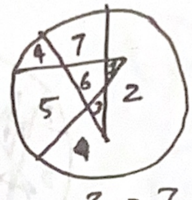
$$= \frac{(n-1)(n-1+1)}{2} + 1 + n$$

$$= \frac{n^2 - n + 2 + 2n}{2} = \frac{n(n+1)}{2} + 1$$

Zig :



$Z_1 = 2$



$Z_2 = 7$

no. of Zig	no. of lines	Line Region	Zig Region	lost Region
1	2	4	2	2
2	4	11	7	4
3	6	22	16	6

$\hookrightarrow \frac{n(n+1)}{2} + 1$

Zig Regions

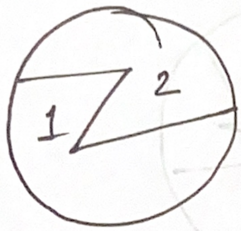
$$Z_n = L_{2n} - 2n$$

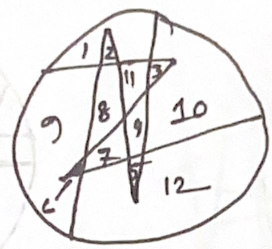
$$= \frac{2n(2n+1)}{2} + 1 - 2n$$

$$= 2n^2 + n + 1 - 2n$$

$$= 2n^2 - n + 1$$

Zig Zag:





$ZZ_2 = 12$

$ZZ_1 = 2$

$ZZ_3 = 31$

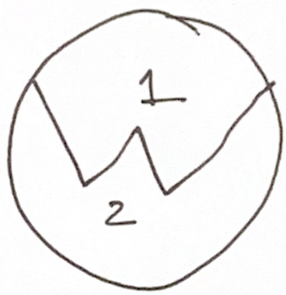
ZZ	Lines	no. of Line Region	no. of ZZ Region	lost Region
1	3	7	2	5
2	6	22	12	10
3	9	46	31	15

$$ZZ_n = l_{3n} - 5n = \frac{3n(3n+1)}{2} + 1 - 5n$$

$$= \frac{9n^2 + 3n + 2 - 10n}{2}$$

$$= \frac{9n^2 - 7n + 2}{2}$$

W-Object:



$w_1 = 2$

$w_2 = 19$

W	no. of Line	no of Line Region	w Region	lost Region
1	4	11	2	9
2	8	37	19	18

$$w_n = l_{4n} - 9n$$

$$= 1 + \frac{4n(4n+1)}{2} - 9n$$

$$= 1 + 2n(4n+1) - 9n$$

$$= 8n^2 + 2n + 1 - 9n$$

$$= 8n^2 - 7n + 1$$