

TOPIC NAME: Machine Learning DAY: _____

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ANN (Artificial Neural Network)

■ Single Neuron/Perception:

Basic Structure:

Inputs: $x = [x_1, x_2, \dots, x_n]$

Weights: $w = [w_1, w_2, \dots, w_n]$

Bias: b

Activation Function: ϕ

■ Single Neuron/Perception:

* Pre activation step:

$$z = \sum_{i=0}^n w_i x_i + b$$

$$z = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n + b$$

* Activation Function:

$$y = \phi(z)$$

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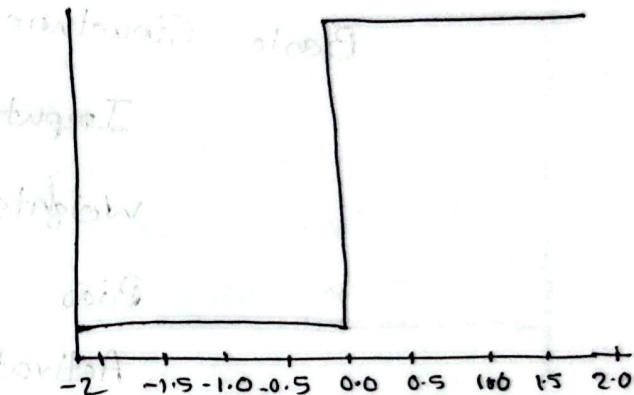
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Binary Step:

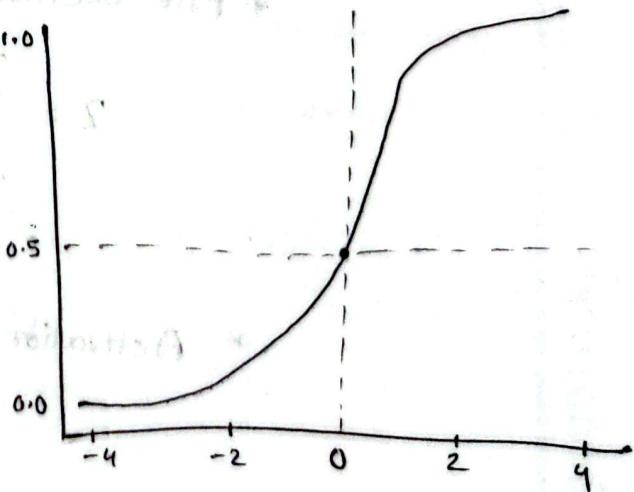
$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$$



Activation Functions (Cont)

Sigmoid/Logistic

$$f(x) = \frac{1}{1 + e^{-x}}$$



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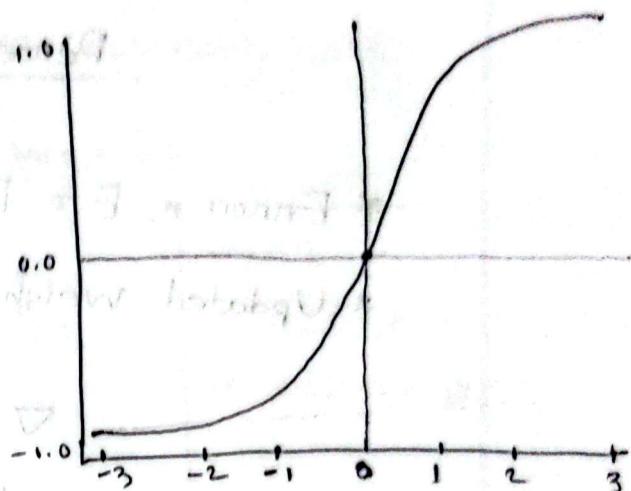
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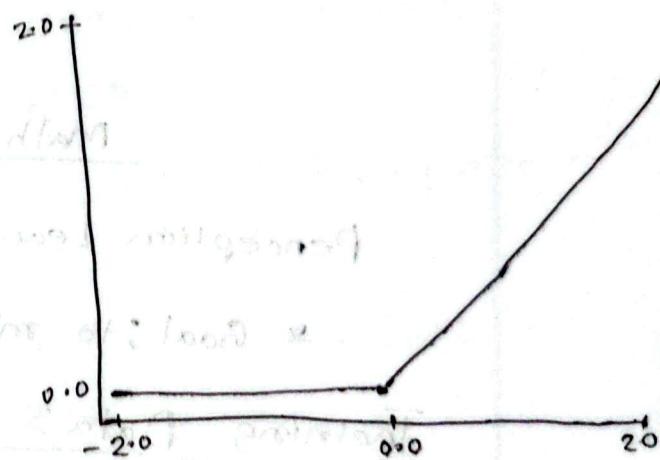
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Tanh

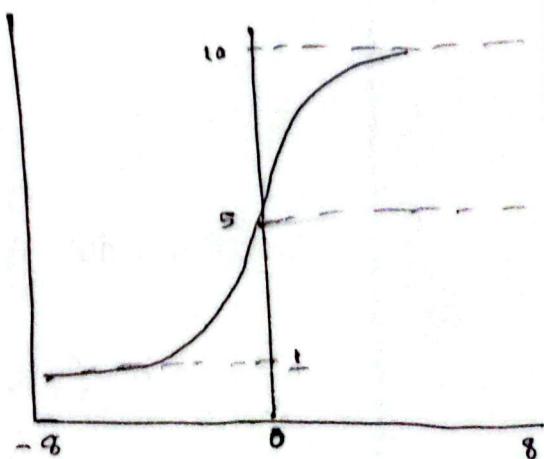
$$f(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

ReLU

$$f(x) = \max(0, x)$$



$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum \exp(z_i)}$$



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Perception of Learning Rule

- * Error ϵ , $E = \text{Target} - \text{Output}$
- * Updated weights & Bias:

$$\nabla w_i = \alpha \cdot E \cdot x_i$$

$$b_{\text{new}} = b_{\text{old}} + \alpha \cdot E$$

α is the new learning rate.

Math

Perception Learning example: AND Gate

- * Goal: to mimic AND logic, $y = x_1 \cdot x_2$

Training Data:

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

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$$w_1x_1 + w_2x_2 + w_3x_3 + \dots + b$$

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Hence, initialize parameters (randomly)

Weights, $w_1 = 0.5, w_2 = 0.5$

Bias, $b = -0.7$

Learning rate, $\alpha = 0.1$

Decision,
if $z \geq 0 \Rightarrow y = 1$
if $z < 0 \Rightarrow y = 0$

Epoch 1:

Sample (0, 0):

$$z = (0.5 \times 0) + (0.5 \times 0) - 0.7 = -0.7$$

$$y = \text{Step}(-0.7) = 0$$

$\therefore \text{Error } (t - y) = 0 - 0 = 0$ $\boxed{\text{No update}}$

Sample (0, 1):

$$z = (0.5 \times 0) + (0.5 \times 1) - 0.7 = -0.2$$

$$y = \text{Step}(-0.2) = 0$$

$\therefore \text{Error } (t - y) = 0 - 0 = 0$ $\boxed{\text{No Update}}$

Sample (1, 0):

$$z = (0.5 \times 1) + (0.5 \times 0) - 0.7 = -0.2$$

$$y = \text{Step}(-0.2) = 0$$

$\therefore \text{Error } (t - y) = 0 - 0 = 0$ $\boxed{\text{No Update}}$

Sample (1, 1):

$$z = (0.5 \times 1) + (0.5 \times 1) - 0.7 = 0.3$$

$$y = \text{Step}(0.3) = 1$$

$\therefore \text{Error } (t - y) = 1 - 1 = 0$ $\boxed{\text{No Update}}$

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: Epoch 2:

* All prediction correct \rightarrow Convergence achieved.

* Decision Boundary: $0.5x_1 + 0.5x_2 - 0.7 = 0$

Example 2

Weights, $w_1 = 0.25$

$w_2 = 0.25$

Bias, $b = -0.8$

Learning rate, $\alpha = 0.5$

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

: Epoch 1:

Sample (0,0):

$$z = (0.25 \times 0) + (0.25 \times 0) - 0.8 = -0.8$$

$$y = \text{step}(-0.8) = 0$$

$\therefore \text{Error}(t-y) = (0-0) = 0$ $\boxed{\text{No Update}}$

Sample (0,1):

$$z = (0.25 \times 0) + (0.25 \times 1) - 0.8 = -0.55$$

$$y = \text{step}(-0.55) = 0$$

$\therefore \text{Error}(t-y) = (0-0) = 0$ $\boxed{\text{No Update}}$



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Sample (1, 0):

$$\therefore z = (0.25 \times 1) + (0.25 \times 0) - 0.8 = 0.5$$

$$y = \text{step}(-0.5) = 0$$

$$\therefore \text{Error}(t-y) = (0-0) = 0 \rightarrow \text{No Update}$$

Sample (1, 1):

$$z = (0.25 \times 1) + (0.25 \times 1) - 0.8 = 0.3$$

$$y = \text{step}(-0.3) = 0$$

$$\therefore \text{Error}(t-y) = (1-0) = 1 \rightarrow \text{No Update}$$

Updated Weights and Bias:

$$\Delta w_1 = \alpha \cdot (t-y) \cdot x_1 \rightarrow w_1 = 0.25 + 0.5 = 0.75$$
$$= 0.5 \times 1 \times 1 = 0.5$$

$$\Delta w_2 = \alpha \cdot (t-y) \cdot x_2 \rightarrow w_2 = 0.25 + 0.5 = 0.75$$
$$= 0.5 \times 1 \times 1 = 0.5$$

$$\therefore \Delta b = \alpha \cdot b_{\text{old}} + \alpha \cdot E$$

$$= -0.8 + 0.5 \times 1 = -0.3$$

After epoch 1:

$$w_1 = 0.75$$

$$w_2 = 0.75$$

$$b = -0.3$$

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Epoch 2Given Sample (0, 0), t = 0

$$z = (0.75 \times 0) + (0.75 \times 0) - 0.3 = -0.3$$

$$y = \text{step}(-0.3) = 0$$

$$\text{Enron} = (t - y) = 0 \quad \boxed{\text{No Update}}$$

Given Sample (0, 1), t = 0

$$z = (0.75 \times 0) + (0.75 \times 1) - 0.3 = 0.45$$

$$y = \text{step}(0.45) = 1$$

$$\therefore \text{Enron} = (t - y) = -1$$

$$\therefore \nabla w_1 = \alpha \cdot E \cdot x_i \\ = 0.5 \cdot (-1) \cdot 0 = 0 \quad \Rightarrow w_1 = 0.75 + 0 = 0.75$$

$$\therefore \nabla w_2 = \alpha \cdot E \cdot x_i \\ = 0.5 \cdot (-1) \cdot 1 = -0.5 \quad \Rightarrow w_2 = 0.75 + 0.5 = 0.25$$

$$\therefore b = b_{old} + \alpha \cdot E$$

$$= -0.3 + (0.5 \cdot (-1)) = -0.8$$

$$\therefore w_1 = 0.75$$

$$w_2 = 0.25$$

$$b = -0.8$$



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Q) Sample (1,0), t=0

$$z = (0.75 \times 1) + (0.25 \times 0) - 0.8$$

$$z = 0.05 \geq 0 \text{ No Update}$$

$$\therefore y = \text{step}(z=0.05) = 1 \quad ; \quad \text{Ennout}(t-y) = (1-1)=0$$

Q) Sample (1,1), t=1

$$z = (0.75 \times 1) + (0.25 \times 1) - 0.8$$

$$z = 1 - 0.8 = 0.2 \geq 1$$

$$\therefore y = \text{step}(z) = 1$$

$$\therefore \text{Ennout}(t-y) = (1-1)=0 \therefore \text{No Update}$$

$$\therefore w_1 = 0.75$$

$$w_2 = 0.25$$

$$w_b = -0.8$$

\therefore Decision Function:

$$z = w_1x_1 + w_2x_2 + b$$

$$z = 0.75x_1 + 0.25x_2 - 0.8$$

Fundamental Question

Q1: What is Artificial Neural Network (ANN)? How is it inspired by the human brain?

Ans: ANN is a computer model that works like human brain to solve problems by learning from data. It uses artificial neurons that mimic how brain neurons send signals.

Method Learning

Q2: Explain the perceptron learning rule. How are weights & Bias Updated?

Ans: It is a method used to adjust the 'weights' and bias of a neuron based on the error (difference between expected and actual output) using a formula.

Updated w and b :

$$\nabla w_i = \alpha \cdot E \cdot x_i$$

$$b_{\text{new}} = b_{\text{old}} + \alpha \cdot E$$

↳ learning rate.

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Q3: Write the mathematical expression for the output for a single neuron?

Ans: pre-activation:

$$z = \sum_{i=0}^n w_i x_i + b$$

$$= w_0 x_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n + b$$

Activation Function:

$$y = \phi(z)$$

Q4: Why is the bias term important & What role does the learning rate (α) play in training a perceptron?

Ans: It helps shift the output so the neuron can learn better * The learning rate controls how big the updates to weights and bias are.

Q5: What is activation function?

Ans: It decides whether a neuron should be "activated" or not by applying a non-linear transformation to the input.

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Q6: Difference between Step, sigmoid, and ReLU?

Ans: Step: Outputs 0 or 1, used in basic models.

Sigmoid: Smooth S-shaped curve, output between 0 and 1.

ReLU: Outputs 0 if input is negative, otherwise outputs the input itself

Q7: Why is ReLU better than Sigmoid?

Ans: ReLU is faster and helps avoid the vanishing gradient problem, making it better for deep networks.



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ML Math



Given,

weights, $w_1 = 0.25$

$w_2 = 0.25$

Bias, $b = -0.8$

Learning rate, $\alpha = 0.5$

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

: AND gate training steps:

Epoch 1:

Sample $(0, 0), t=0$:

$$z = (0.25 \times 0) + (0.25 \times 0) - 0.8 = -0.8$$

$$y = \text{step}(-0.8) = 0$$

$$\text{Enron} = (t - y) = (0 - 0) = 0 \quad \boxed{\text{No Update.}}$$

Sample $(0, 1), t=0$:

$$z = (0.25 \times 0) + (0.25 \times 1) - 0.8 = -0.55$$

$$y = \text{step}(-0.55) = 0$$

$$\text{Enron} = (t - y) = (0 - 0) = 0 \quad \boxed{\text{No Update}}$$

Sample $(1, 0), t=0$:

$$z = (0.25 \times 1) + (0.25 \times 0) - 0.8 = -0.55$$

$$y = \text{step}(-0.55) = 0$$

$$\text{Enron} = (t - y) = (0 - 0) = 0 \quad \boxed{\text{No Update.}}$$

Sample $(1, 1), t=1$:

$$z = (0.25 \times 1) + (0.25 \times 1) - 0.8 = -0.3$$

$$y = \text{step}(-0.3) = 0$$

$$\therefore \text{Enron} = (t - y) = (1 - 0) = 1 \quad \boxed{\text{Enron}}$$

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Updated Weight and Bias:

Given,

$$\alpha = 0.5$$

$$\Delta w_1 = \alpha \cdot E \cdot x_1$$

$$= 0.5 \times 1 \times 1$$

$$= 0.5$$

$$\therefore w_1 = 0.5 + 0.25 = 0.75$$

Updated $\Delta w_2 = \alpha \cdot E \cdot x_2$:

$$= 0.5 \times 1 \times 1$$

$$= 0.5$$

$$\therefore w_2 = 0.5 + 0.25 = 0.75$$

$$\text{Epoch 2: } \therefore b = b_{\text{old}} + \alpha \cdot E^{(2)} - 0.8 + 0.5 \times 1 = -0.3$$

Sample: (0, 0), t = 0:

$$z = (0.75 \times 0) + (0.75 \times 0) - 0.3 = -0.3$$

$$y = \text{step}(-0.3) = 0 \text{ No Update.}$$

$$\text{Error} = t - y = 0 - 0 = 0 \rightarrow$$

Sample: (0, 1), t = 0:

$$z = (0.75 \times 0) + (0.75 \times 1) - 0.3 = 0.45$$

Sample: (1, 0), t = 0:

$$y = \text{step}(0.45) = 1$$

$$\text{Error} = t - y = 0 - 1 = -1$$

Sample: (1, 1), t = 1:

$$z = (0.75 \times 1) + (0.75 \times 1) - 0.3 = 1$$

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∴ Updated w & b:

$$\Delta w_1 = \alpha \cdot E \cdot x_1$$

$$= 0.5 \times (-1) \times 0.2 = 0$$

$$\therefore w_1 = 0.75 + 0 = 0.75$$

~~$$\Delta w_2 = \alpha \cdot E \cdot x_2$$~~

$$= 0.5 \times (-1) \times 1$$

$$= -0.5$$

$$\therefore w_2 = 0.75 - 0.5 = 0.25$$

$$\therefore b = b_{old} + \alpha \cdot E \cdot 1$$

$$= -0.3 + 0.5 \times (-1) = -0.8$$

∴ Sample (1, 0), t = 0:

$$z = (0.75 \times 1) + (0.25 \times 0) = 0.75$$

$$= 0.05$$

$$\therefore y = \text{step}(0.05) = 0$$

∴ Error = t - y = 0 $\boxed{\text{No Update}}$

∴ Sample (1, 1), t = 1:

$$z = (0.75 \times 1) + (0.25 \times 1) = 0.75 + 0.25 = 0.2$$

$$\therefore y = \text{step}(0.2) = 1$$

∴ Error = t - y = 1 - 1 = 0 $\boxed{\text{No Update}}$

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Epoch 3:

Sample (0,0), t=0

$$z = (0.75x_0) + (0.25x_1) - 0.8 = -0.8$$

$$\gamma = \text{step}(-0.8) = 0$$

$$\therefore \text{Enron} = t - \gamma = 0$$

No Update

Sample (0,1), t=0

$$z = (0.75x_0) + (0.25x_1) - 0.8 = -0.55$$

$$\gamma = \text{step}(-0.55) = 0$$

$$\text{Enron} = t - \gamma = 0$$

No Update

Sample (1,0), t=0

$$z = (0.75x_0) + (0.25x_1) - 0.8 = -0.05$$

$$\gamma = \text{step}(-0.05) = 0$$

$$\therefore \text{Enron} = 0$$

No Update

Sample (1,1), t=1

$$z = (0.75x_0) + (0.25x_1) - 0.8 = 0.2$$

$$\gamma = \text{step}(0.2) = 1$$

$$\therefore \text{Enron} = 1 - 1 = 0$$

No Update

∴ All prediction correct \rightarrow Convergence Achieved

∴ Decision Boundary:

$$0.75x_0 + 0.25x_1 - 0.8 = 0$$

Ques 1: What is an Artificial Neural Network (ANN)?

Ans: An ANN is a computer model inspired by the human brain. It has neurons (perceptions) that take inputs, multiply them by weights, add a bias, and pass them through an activation function to produce an output.

Ques 2: Explain the role of activation functions in ANN.

Ans: Activation functions decide if a neuron should "fire" (activate) or not.

* Step function: Simple (0 or 1). $f(x) = \frac{1}{1+e^{-x}}$

~~sigmoid~~

* Sigmoid/Tanh: Smooth curves, good for probabilities

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

* ReLU: Fast learning, avoids negative outputs $f(x) = \max(0, x)$

* Softmax: Used for classification

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Q3. How does a perceptron learn? Explain the weight update rule.

Ans: It calculates error and it's related to weight.

1. It calculates Error = Target - Output

2. It updates weights: $w_i = w_i + \alpha \cdot E \cdot x_i$

3. It updates bias: $b = b + \alpha \cdot E$

Q4: difference between sigmoid and ReLU.

Sigmoid

Output: 0 to 1

Slow for deep networks

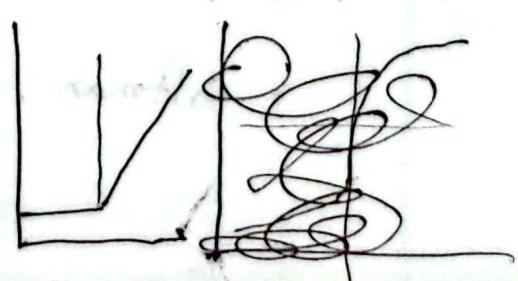
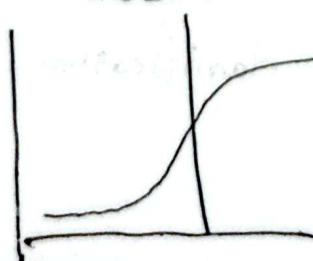
Used in old models

ReLU

Output: 0 or linear

Faster, avoids vanishing gradient

popular in deep learning



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