Higher order differential equation

General form,

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{(n-1)}}{dx^{(n-1)}} + \cdots + a_{(n-1)}(x)\frac{dy}{dx} + a_n(x)y = R(x)$$

or, 0

Solution, $y = y_c + y_p$

* If R.H.S $= 0$ then $y = y_c$
* If R.H.S $= R(x)$ then $y = y_c + y_p$

Here,

y_c = Complementary function which contains arbitrary constant

y_p = Particular integral which has no con

Distinct Root → Root are different

Repeated ,, → Root are same repeated.

Complex Root → Imaginary number as

2 bar differentiation

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$y = e^{mx}$

Ans: let $y = e^{mx}$

$\Rightarrow \frac{dy}{dx} = me^{mx}$

$\Rightarrow \frac{d^2y}{dx^2} = m^2e^{mx}$

∴ Auxilary Solution of (i)

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$\Rightarrow m^2e^{mx} - 3me^{mx} + 2e^{mx} = 0$

$\Rightarrow e^{mx}(m^2 - 3m + 2) = 0$

$\Rightarrow m_1 = 1, \; m_2 = 2$

$\therefore y = y_c$

$= C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$= C_1 e^{x} + C_2 e^{2x}$

2 $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$ —(i)

let $y = e^{mx}$

$\Rightarrow \frac{dy}{dx} = me^{mx}$

$\Rightarrow \frac{d^2y}{dx} = m^2e^{mx}$

$\Rightarrow \frac{d^3y}{dx} = m^3e^{mx}$

∴ Auxilary Solution of (i).

$m^3e^{mx} - 4m^2e^{mx} + me^{mx} + 6e^{mx} = 0$

$\Rightarrow e^{mx}(m^3 - 4m^2 + m + 6) = 0$

$e^{mx} \neq 0$

$\therefore m^3 - 4m^2 + m + 6 = 0$

$m_1 = -1, \; m_2 = 2$

$m_3 = 3$

$m^3 - 4m^2 + m +$

$= m^3 - m^2 - 5m^2 - 5$

$=$

$\therefore y = y_c$

$= C_1 e^{-x} + C_2 e^{2x} + C_3 e^{3x}$

Q3) $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^3} - 3\frac{dy}{dx} + 18y = 0$

Auxilary Solution,

$m^3e^{mx} - 4m^2e^{mx} - 3me^{mx} + 18e^{mx} = 0$

$\Rightarrow e^{mx}(m^3 - 4m^2 - 3m + 18) = 0$

$\therefore e^{mx} \neq 0$

$\therefore m^3 - 4m^2 - 3m + 18 = 0$

$\Rightarrow m^3 + 2m^2 - 6m^2 - 12m + 9m + 18 = 0$

$\Rightarrow m^2(m+2) - 6m(m+2) + 9(m+2) = 0$

$\Rightarrow (m+2)(m^2 - 6m + 9) = 0$

$\Rightarrow (m+2)(m-3)^2 = 0$

$m_1 = -2,\ m_2 = 3,\ m_3 = 3$

$y_c = c_1e^{-2x} + c_2e^{3x} + c_3xe^{3x}$

For repeated root:

4) Given, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

$y = e^{mx}$

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx^2} = m^2e^{mx}$

$m^2e^{mx} - 6e^{mx} + 9e^{mx}$

$\Rightarrow e^{mx}(m^2 - 6 + 9) = 0$

$e^{mx} \neq 0$

$\therefore m^2 - 6 + 9 = 0$

$m_1 = 3,\ m_2 = 3$

As $m_1 = m_2$

$y_c = c_1e^{m_1x} + c_2xe^{mx}$

$= c_1e^{3x} + c_2xe^{3x}$ Ans

Dc → 134, 190-mr

C.F

If $(m_1 \neq m_2)$

$y = y_c$
$= c_1 e^{-x} + c_2 e^{2x} + c_3 e^{3x}$

If $(m_1 = m_2)$

$y = y_c$
$= c_1 e^{mx} + c_2 x e^{mx} + c_3 x^2 e^{mx} \ldots$
$= (C_n x^{n-1}) e^{mx}$

Denis; Ex 4.3 (1-18)

if $m = \alpha \pm i\beta$

The solution $y_c = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

given, $y'' + 4y' + 7y = 0$

$\Rightarrow \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y = 0$ ——(1)

Auxilary Solution:

$m^2 e^{mx} + 4m e^{mx} + 7e^{mx} = 0$

$\Rightarrow e^{mx}(m^2 + 4 + 7) = 0$

$\therefore e^{mx} \neq 0$

$\therefore m^2 + 4 + 7 = 0$

$\therefore m_1 = -2 + \sqrt{2}i$, $m_2 = -2 - \sqrt{3}i$

Here,
$\alpha = -2$, $\beta = \sqrt{3}$

$\therefore y_c = e^{-2x}(c_1 \cos \sqrt{3}\, x + c_2 \sin \sqrt{3}\, x)$

19.03-২২

Class - 2

$$\boxed{\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}}$$

If $f(a)=0$ then the above rule ~~fits~~ fails

$$\boxed{\frac{1}{f(D)} e^{ax} = x \frac{1}{f'(a)} e^{ax}}$$

If $f'(a)=0$ then $\boxed{\frac{1}{f(D)} e^{ax} = x^2 \frac{1}{f''(a)} e^{ax}}$

Note: যেখানে D সেখানে a এর মান বসাবো

if $a=0$ $\quad x \frac{1}{f'(a)} e^{x}$

$a=0$ $\quad x^2 \frac{1}{f''(a)} e^{x}$

$CF = (c_1 + c_2 x)\boxed{e^{x}} = P.I$

Ex:

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$$

$$\Rightarrow (D^2 + 6D + 9)y = 5e^{3x}$$

Direct calculator এ করা যাবে।

Auxiliary equation:

$$m^2 e^{mx} + 6e^{mx} + 9e^{mx}$$

$$e^{mx}(m^2 + 6 + 9) = 0$$

$$\therefore m_1 = -3, \quad m_2 = -3$$

$$\therefore C.F = (c_1 + c_2 x) e^{-3x}$$

$$P.I = \frac{1}{f(D)}$$

$$= \frac{1}{D^2+6D+9} \, 5e^{3x}$$

$$= 5 \frac{e^{3x}}{(3)^2 + 6(3) + 9}$$

এই স্থানে [illegible] value 3

$$= \frac{5e^{3x}}{36}$$

19.03-২৭

Class - 2

$$\boxed{\frac{1}{F(D)} e^{ax} = \frac{1}{f(a)} e^{ax}}$$

If $f(a)=0$ then the above rule ~~fits~~ fails

$$\boxed{\frac{1}{f(D)} e^{ax} = x \frac{1}{f'(a)} e^{ax}}$$

If $f'(a)=0$ then $\boxed{\frac{1}{F(D)} e^{ax} = x^2 \frac{1}{f''(a)} e^{ax}}$

Note: যেখানে D সেখানে a এর মান বসাবে

if $a=0$ $x \frac{1}{f'(a)} e^{x}$

আবার differentiate করে $a=0$ $x^2 \frac{1}{f''(a)} e^{x}$

$CF = (c_1 + c_2 x) e^{x} = P.I$

Ex:

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$$

$$\Rightarrow (D^2 + 6D + 9)y = 5e^{3x}$$

Direct calculate এ করা যাবে

Auxiliary equation:

$$m^2 e^{mx} + 6e^{mx} + 9e^{mx}$$

$$e^{mx}(m^2 + 6 + 9) = 0$$

$$\therefore m_1 = -3, \quad m_2 = -3$$

$$\therefore C.F = (c_1 + c_2 x) e^{-3x}$$

$$P.I = \frac{1}{F(D)}$$

$$= \frac{1}{D^2 + 6D + 9} 5e^{3x}$$

$$= 5 \frac{e^{3x}}{(3)^2 + 6(3) + 9}$$

এই স্থানে m এর value 3 বসিয়েছি

$$= \frac{5e^{3x}}{36}$$

a এর মান D তে বসানোর সব যদি ... আর সামনে x গুণ হয়
এর eqⁿ কে derivative করতে হবে।

The complete solution is $y = (c_1 + c_2 x)e^{-3x} + \frac{5e^{3x}}{36}$

Example:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + \cancel{\frac{1}{D^2-6D+9}}\, 7e^{-2x} - \log_2$$

Soln:

$$(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} + \log 2$$

এই কথা বলেছে। m এর value D তে বসালে 0 হলে Derivative

Auxilary equation is =

$$m^2 - 6m + 9 = 0$$

$$\therefore m = 3, 3$$

$$\therefore CF = e^{mx}(c_1 + c_2 x)$$

$$= e^{3x}(c_1 + c_2 x)$$

$$PI = \frac{1}{F(D)}$$

$$= \frac{1}{D^2 - 6D + 9} 6e^{3x} + \frac{1}{D^2-6D+9} 7e^{-2x} + \frac{1}{D^2-6D+9}(-\log 2)$$

Here D এর যায়গায়
3 বসাইলে 0 হয়ে যায় তাই এজন্য
Derivative করেছে।

$$= x\frac{1}{2D-6} 6e^{3x} + \frac{1}{4+12+9} 7e^{-2x} - \log 2 \frac{1}{D^2-6D+9} e^{0x}$$

আগে D এর Derivative করে x গুণ

$$= x^2 \frac{1}{2} 6e^{3x} + \frac{1}{25} 7e^{-2x} - \log 2 \left(\frac{1}{9}\right)$$

$$= 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \frac{1}{9}\log 2$$

The complete equation is $y = (c_1 + c_2 x)e^{3x} + 3x^2 e^{3x} + \frac{7}{25}e^{-2x} - \frac{1}{9}\log 2$

The complete equation = CF + PI

we out CF from auxilary and

Ex.1) $[D^2+5D+6][y] = e^x$

Auxiliary equation $= m^2+5m+6=0$

$\therefore m = -3, -2$

As $m_1 \neq m_2$ So,

$C.F = e^{-3x} c_1 + e^{-2x} c_2$

$PI = \dfrac{1}{D^2+5D+6} e^{ax}$

Here $a = 1$
So Derivative
না করে মান বসিয়ে
যোগ করছি

$= \dfrac{1}{1+5+6} e^x$

$= \dfrac{1}{12} e^x$

The complete equation is $y = \left(e^{-2x} c_1 + e^{-3x} c_2 + \frac{1}{12} e^x\right)$

Ex. 2) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = e^{3x}$

$\therefore (D^2-3D+2)y = e^{3x}$

$\therefore$ Auxiliary equation $= m^2-3m+2$

$\therefore m = 2, 1$

As $m_1 \neq m_2$

So,

$CF = c_1 e^x + c_2 e^{2x}$

$\therefore PI = \dfrac{1}{f(D)}$

$= \dfrac{1}{D^2-3D+2} e^{3x}$

$= \dfrac{1}{2} e^{3x}$

$3^2 - 3\times 3 + 2$
$\Rightarrow 9-9+2$
$= 2$
$\therefore a \neq 0$

$\therefore$ The complete equation, $= c_1 e^x + c_2 e^{2x} + \dfrac{e^{3x}}{2}$

Ex 3: $(D^3 + 2D^2 - D - 2)y = e^x$

$A.E = m^3 + 2m^2 - m - 2$

$\Rightarrow m_1 = 1,\ m_2 = -1,\ m_3 = -2$

$C.F = e^x c_1 + e^{-x} c_2 + e^{-2x} c_3$

$\therefore PI = \frac{1}{F(D)} e^{mx}$

$= \frac{1}{D^3 + 2D^2 - D - 2} e^x$ | $1 + 2 - 1 - 2 = 0$, $a = 0$

$= x \frac{1}{3D^2 + 4D - 1} e^x$

$\Rightarrow x \frac{1}{6} e^x$

$\therefore$ The complete auxilary equation

$y = e^x c_1 + e^{-x} c_2 + e^{-2x} c_3 + \frac{x}{6} e^x$

4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin hx$

$\Rightarrow (D^2 + 2D + 2)y = \sin hx$

$\Rightarrow (D^2 + 2D + 2)y = \frac{e^x - e^{-x}}{2}$

$\left[\frac{e^x - e^{-x}}{2},\ \cos h = \frac{e^x}{} \right.$

$\Rightarrow (D^2 + 2D + 2)y = \frac{e^x}{2} - \frac{e^{-x}}{2}$

$\therefore AE = m^2 + 2D + 2$

$\Rightarrow m_1 = -1 + i,\ m_2 =$

$\therefore CF = e^{-x}(c_1 \cos x + c_2 \sin x)$

$PI = \frac{1}{D^2 + 2D + 2} \frac{e^x}{2} + \frac{1}{D^2 + 2D + 2}\left(-\frac{e^{-x}}{2}\right)$

$= \frac{1}{5} \frac{e^x}{2} - \frac{1}{2} e^{-x}$

$= \frac{1}{10} e^x - \frac{1}{2} e^{-x}$

The complete Solution is $y = e^{-x}[c_1 \cos x + c_2 \sin x]$ $e^x - \frac{1}{2} e^x$

24/03/2...

Higher ORDER D·E

(1) $(D^2+D-2)y = 2(1+x-x^2)$ → algebraic form.

Sol^n: Aux $= e^{mx}(m^2+m-2)=0$.

$\therefore m_1 = 1,\ m_2 = -2$

$\therefore y_c = C_1e^{x} + C_2e^{-2x}$

$$\boxed{\frac{1}{F(D)}x^n = [F(D)]^{-1}x^n}$$

$$y_{PI} = \frac{1}{D^2+D-2}\,2(1+x-x^2)$$

$$= \frac{1}{-2\left[1-\left(\frac{D^2+D}{2}\right)\right]}\,2(1+x-x^2)$$

$$= -\left[1-\left(\frac{D^2+D}{2}\right)\right]^{-1}(1+x-x^2)$$

$$= -\left[1+\frac{D+D^2}{2}+\left(\frac{D+D^2}{2}\right)^2+\cdots\right](1+x-x^2)$$

$$-\left[1+\frac{D}{2}+\frac{3}{4}D^2+\cdots\right](1+x-x^2)$$

$$= -\left[(1+x-x^2)+\frac{1}{2}(1-2x)+\frac{3}{4}(-2)\right]$$

$$= -\left[1-x-x^2+\frac{1}{2}+x-\frac{3}{2}\right]$$

$$= -[-x^2]$$

$$= x^2$$

$\therefore y = C_1e^{x}+C_2e^{-2x}+x^2$

(2) $(D^3-D^2-6D)y = x^2+1$

Sol^n: Aux Sol^n $\Rightarrow e^{mx}(m^3-m^2-6m)=0$

$m_1 = 0,\ m_2 = -2,\ m_3 = 3$

$\therefore y_c = C_1e + C_2e^{-2x} + C_3e^{3x}$

$= C_1 + C_2e^{-2x} + C_3e^{3x}$

$$\therefore y_{PI} = \frac{1}{D^3-D^2-6D}(x^2+1)$$

$$= \frac{1}{-6D\left[1+\left(\frac{D-D^2}{6}\right)\right]}(x^2+1)$$

24/03/2

Higher ORDER D.E

① $(D^2+D-2)y = 2\underbrace{(1+x-x^2)}$ → algebraic form.

Sol^n: Aux $= e^{mx}(m^2+m-2)=0$

$\therefore m_1 = 1,\ m_2 = -2$

$\therefore y_c = c_1 e^{x} + c_2 e^{-2x}$

$$\boxed{\frac{1}{F(D)} x^n = [F(D)]^{-1} x^n}$$

$$y_{PI} = \frac{1}{D^2+D-2} 2(1+x-x^2)$$

common

$$= \frac{1}{-2\left[1-\left(\frac{D^2+D}{2}\right)\right]} 2(1+x-x^2)$$

nt Part

non form.

$$= -\left[1-\left(\frac{D^2+D}{2}\right)\right]^{-1}(1+x-x^2)$$

$$= \left[1+\frac{D+D^2}{2}+\left(\frac{D+D^2}{2}\right)^2+\cdots\right](1+x-x^2)$$

$$-\left[1+\frac{D}{2}+\frac{3}{4}D^2+\cdots\right](1+x-x^2)$$

$$= -\left[(1+x-x^2)+\frac{1}{2}(1-2x)+\frac{3}{4}(-2)\right]$$

$$= -\left[1-x-x^2+\frac{1}{2}+x-\frac{3}{2}\right]$$

$$= -[-x^2]$$

$$= x^2$$

$\therefore y = c_1 e^{x} + c_2 e^{-2x} + x^2$

Q2) $(D^3-D^2-6D)y = x^2+1$

Sol^n: Aux Sol^n ⇒ $e^{mx}(m^3-m^2-6m)=0$

$m_1 = 0,\ m_2 = -2,\ m_3 = 3$

$\therefore y_c = c_1 e^{0x} + c_2 e^{-2x} + c_3 e^{3x}$

$= c_1 + c_2 e^{-2x} + c_3 e^{3x}$

$$\therefore y_{PI} = \frac{1}{D^3-D^2-6D}(x^2+1)$$

$$= \frac{1}{-6D\left[1+\left(\frac{D-D^2}{6}\right)\right]}(x^2+1)$$

$D \Rightarrow$ Differentiation

$\frac{1}{D} \Rightarrow$ Integration.

$$= -\frac{1}{6D}\left(1+\frac{D-D^2}{6}\right)^{-1}(x^2+1)$$

$$= -\frac{1}{6D}\left(1-\frac{D-D^2}{6}+\left(\frac{D-D^2}{6}\right)^2 - -\right)(x^2+1)$$

$$= -\frac{1}{6D}\left(1-\frac{D}{6}+\frac{D^2}{6}+\frac{D^2}{36}-\frac{D^3}{18}+\frac{D^4}{36}---\right)(x^2+1)$$

$$\left(1-\frac{D}{6}+\frac{D^2}{6}+\frac{D^2-2D^3+D^4}{36}\right)$$

$$= \frac{1}{6D}\left(1-\frac{D}{6}+\frac{7D^2}{36}\right)(x^2+1)$$

$$= -\frac{1}{6D}\left(1-\frac{D}{6}+\frac{7D^2}{36}\right)(x^2+1)$$

$$= -\frac{1}{6D}\left((x^2+1)-\frac{1}{6}(2x)+\frac{7}{36}(2)\right)$$

$$= -\frac{1}{6D}\left(x^2+1-\frac{1}{3}x+\frac{7}{18}\right)$$

$$= -\frac{1}{6D}\left(x^2-\frac{1}{3}x+\frac{25}{18}\right)$$

$$= -\frac{1}{6}\left(\frac{x^3}{3}-\frac{1}{3}\cdot\frac{x^2}{2}+\frac{25}{18}x\right)$$

$$= -\frac{x^3}{18}+\frac{x^2}{36}-\frac{25x}{108}$$

$$y = C_1 + C_2e^{-2x} + C_3e^{3x} - \frac{x^3}{18} + \frac{x^2}{36} - \frac{25x}{108}$$

HW Day

① $(D^2+5D+4)y = 3-2x$

$$A.e. = e^{mx}(m^2+5m+4) = 0$$

$$m_1 = -1, \; m_2 = -4$$

$$y_c = c_1e^{-x} + c_2e^{-4x}$$

$$P.I. = \frac{1}{F(D)} = \frac{1}{D^2+5D+4}(3-2x)$$

$$= \frac{1}{4\left(\frac{D^2}{4}+\frac{5D}{4}+1\right)}(3-2x)$$

$$= \frac{1}{4\left(\frac{5D+D^2}{4}\right)+1}(3-2x)$$

$$= \frac{1}{4}\left[1+\left(\frac{5D+D^2}{4}\right)\right]^{-1}(3-2x)$$

$$= \frac{1}{4}\left[1 - \frac{5D+D^2}{4} + \left(\frac{5D+D^2}{4}\right)^2 - \cdots\right](3-2x)$$

$$= \frac{1}{4}\left(1 - \frac{5D}{4} - \frac{D^2}{4} + \frac{25D^2+10D^3+D^4}{16} \cdots\right)$$

$$= \frac{1}{4}\left[(3-2x) - \frac{5}{4}(-2) + D\right]$$

$$= \frac{1}{4}\left(3 - 2x + \frac{5}{2}\right)$$

$$= \frac{1}{4}\left(-2x + \frac{11}{2}\right)$$

$$= \frac{1}{8}(-4x + 11)$$

$$= \frac{1}{8}(11 - 4x)$$

$$y = c_1 e^{-x} + c_2 e^{-4x} + \frac{1}{8}(11 - 4x) \quad \text{Ans}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x$$

$$\text{Aux} = e^{mx}(m^2 + 2m + 1) = 0$$

$$m_1 = -1, \quad m_2 = -1$$

$$y_c = c_1 e^{-x} + c_2 e^{-x}$$

$$= e^{-x}[c_1 + x c_2)$$

$$PI = \frac{1}{F(D)} = \frac{1}{\left(\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y\right)} x$$

$$= \frac{1}{1\left[1 + \frac{2D+D^2}{1}\right]}(x)$$

$$= [1 + (2D + D^2)]^{-1}(x)$$

$$= [1 - (2D+D^2) + (2D+D^2)^2 - \cdots]x$$

$$= (1 - 2D - D^2 + 4D^2 + 4D^3 + D^4)x$$

$$= x - 2 + 0$$

$$= x - 2$$

$$y = (c_1 + x c_2)e^{-x} + x - 2 \quad \text{Ans}$$

Math Assignment

$\boxed{22101095}$

Exercise 3-20 :

1) $(D^2+5D+6)[y] = e^x$

$m^2+5m+6 = e^x$

Aux Soln $= (m+2)(m+3) = 0$

$= m = -2, -3$

$CF = c_1 e^{-2x} + c_2 e^{-3x}$

$PI = \frac{1}{D^2+5m+6} e^x \quad [e^{ax} = a = 1]$

$= \frac{1}{1+5+6} = \frac{1}{12} e^x$

Complete Solution

$y = c_1 e^{-2x} + c_2^{-3x} + \frac{1}{12} ex$

2) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = -e^{3x}$

$D^2 - 3D + 2y = e^{3x}$

$m^2 - 3m + 2 = e^{3x}$

Aux Soln : $(m-1)(m-2) =$

$\therefore m_1 = 1$

$m_2 = 2$

$CF = c_1 e^{x} + c_2 e^{2x}$

$PI = \frac{1}{D^2 - 3D + 2y} e^{3x}$

$= \frac{1}{1 - 9 + 2} e^{3x}$

$= \frac{1}{2} e^{3x}$

Complete solution: $c_1 e^{x} + c_2 e^{2x} + \frac{1}{2} e^{3x}$

$(D^3 + 2D^2 - D - 2)y = e^{x}$

$m^3 + 2m^2 - m - 2 = e^{x}$

Aux Soln: $(m-1)(m+1)(m+2) = 0$

$m = 1, -1, 2$

$CF = c_1 e^{x} + c_2^{-x}, c_3^{-2x}$

$PI = \frac{1}{D^3 + 2D^2 - D - 2} e^{x}$

$= x \frac{1}{3D^2 + 4D - 1} e^{x}$ [differentiation

$= x \frac{1}{6} e^{x}$

$= \frac{xe^{x}}{6}$

Complete solution $= y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x} + \frac{1}{6} xe^{x}$

4) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin hx$

$= (D^2 - 2D + 2y)y = \sin hx$

Aux Soln $= m^2 + 2m + 2 = 0$

$= m_1 = -1 + i$

$m_2 = 1 - i$

$CF = e^{-x}(c_1 \cos x + c_2 \sin x]$

$PI = \frac{1}{D^2 - 2D + 2} \sin hx$

$= \frac{1}{D^2 + 2D + 2}\left(\frac{e^{x}}{2} - \frac{e^{-x}}{2}\right)$

$= \frac{e^{x}}{2D^2 + 4D + 4} - \frac{e^{-x}}{-2 + 4 + 2}$

$= \frac{e^{x}}{10} - \frac{e^{-x}}{2}$

Complete Soln $= e^{-x}\left[c_1 \cos x + c_2 \sin x\right) + \frac{e^{x}}{10} - \frac{e^{-x}}{e}$

5) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$

$(D^2 + 4D + 5)y = 2\cosh x$

$m^2 + 4m + 5 = 2\cosh x$

Aux Soln : $m = -2 + i, \; -2 - i$

$C_F = e^{-2x}\left[c_1 \cos x + c_2 \sin x\right)$

$PI = \frac{1}{D^2 - 4D + 5} \times -2\cosh x$

$PI = \frac{1}{D^2 + 4D + 5} \times -2\left(\frac{e^{x}}{2} + \frac{e^{-x}}{2}\right)$

$= \frac{1}{D^2 - 4D + 5} \times e^{x} - \frac{1}{D^2 + 4D + 5} e^{-x}$

$= \left(\frac{1 \cdot e^{x}}{10} + \frac{1}{2} e^{x}\right)$

$\therefore$ Sol: $e^{-2x}(c_1 \cos x + c_2 \sin x) - \left(\frac{1}{10} e^{x} + \frac{1}{2} e^{-x}\right)$

6) $(D^3 - 2D^2 - 5D + 6)y = e^{3x}$

$= m^3 - 2m^2 - 5m + 6 = e^{3x}$

Aux Soln $= m = -2, 3, 1$

$C_F = c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x}$

$PI = \frac{1}{D^3 - 2D^2 - 5D + 6} e^{3x}$

$= \frac{x}{3D^2 - 4D - 5} e^{x}$

$= \frac{x e^{x}}{10}$

Complete sol ; $c_1 e^{x} + c_2 e^{-2x} + c_3 e^{3x} + \frac{1}{10}$

7) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = e^{x}$

$(D^3 - D^2 + 4D - 4)y = e^{x}$

Aux Soln : $m^3 - m^2 + 4m - 4 = 0$

$m_1 = 1, \; m_2 = 2i, \; m_3 = -2$

$\alpha = 0$

$\beta = 2$

$C_F = c_1 e^{x} + c_2 \cos 2x + c_3 \sin 2x$

$$PI = \frac{1}{D^3 - D^2 + 4D - 4} e^x$$

$$= \frac{x}{2D^2 - 2D + 4} e^x$$

$$= \frac{x}{2 - 1 + 4} e^x$$

$$= \frac{xe^x}{5}$$

Complete Soln : $C_1 e^x + C_2 \cos 2x + C_3 \sin 2x + \frac{xe^x}{5}$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$

$D^2 - 6D + 9 = e^{3x}$

$m^2 - 6m + 9 = e^{3x}$

Aux Soln $= m^2 - 6m + 9$

$m = 3, 3$

$CF = (C_1 + C_2 x) e^{3x}$

$$PI = \frac{1}{D^2 - 6D + 9} e^{3x}$$

$$= \frac{x}{2D - 6} e^{3x}$$

$$= \frac{x^2}{2} e^{3x}$$

$\therefore$ Complete soln $= (C_1 + C_2 x) e^{3x} + \frac{x^2}{2} e^{3x}$

9) $\frac{d^3y}{dx^2} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = e^{-x}$

$= D^3 - 3D^2 + 3D + 1 = e^{-x}$

Aux Soln : $m^3 + 3m^2 + 3m + 1 = 0$

$m = -1, -1, -1$

$CF = (C_1 + C_2 x + C_3 x^2) e^{-x}$

$$PI = \frac{1}{D^3 + 3D^2 + 3D + 1} e^{-x}$$

$$= \frac{x}{3D^2 + 6D + 3} e^{-x}$$

$$= \frac{x^2}{6D + 6} e^{-x}$$

$$= \frac{x^3}{6} e^{-x}$$

Complete soln $= (C_1 + C_2 x + C_3 x^2) e^{-x}$

$D-1)^3 y = 16e^{3x}$

soln :

$(m-1)^3 = 0$

$m_1 = m_2 = m_3 = 1$

$(c_1 + c_2 + c_3 x^2) e^x$

$\frac{1}{(D-1)^3} 16 e^{3x}$

$16 e^{3x}$

e^{0x}

$y = (c_1 + c_2 + c_3 x^2) e^x + 2e^{3x}$

Exercise 3.21

1) $(D^2 + 5D + 4)y = 3 - 2x$

Aux soln $\Rightarrow e^{mx}(m^2 + 5m + 4) = 0$

$\because e^{mx}$ cannot be 0 :

$m^2 + 5m + 4 = 0$

$\therefore m_1 = -4, \; m_2 = -1$

$y_c = c_1 e^{-4x} + c_2 e^{-x}$

$y_p = \frac{1}{D^2 + 5D + 4} 3 - 2x$

$= \frac{1}{4\left[1 + \left(\frac{D^2 + 5D}{4}\right)\right]} 3 - 2x$

$= \frac{1}{4}\left[1 + \left(\frac{D^2 + 5D}{4}\right)\right]^{-1} (3 - 2x)$

$= \frac{1}{4}\left[1 - \left(\frac{D^2 + 5D}{4}\right) + \left(\frac{D^2 + 5D}{4}\right)^2 - \cdots\right](3 - 2x)$

$= \frac{1}{4}\left[1 - \frac{D^2}{4} - \frac{5D}{4} + \frac{D^4 + 10D^3 + 25D^2}{16} \cdots\right](3 - 2x)$

$= \frac{1}{4}\left[(3 - 2x) + \frac{10}{4}\right] \cdots$

$= \frac{3-2x}{4} + \frac{5}{8}$

$= \frac{6-4x+5}{8}$

$= \frac{11-4x}{8}$

$= \frac{1}{8}(11-4x)$

$\therefore$ complete solution;

$y = c_1 e^{-4x} + c_2 e^{-x} + \frac{1}{8}(11-4x)$

2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x$

$y(\cancel{D^2}D^2 + 2D + 1) = x$

Aux soln: $m^2 + 2m + 1 = 0$

$\Rightarrow m_1 = m_2 = -1$

$y_c = (c_1 + c_2 x) e^{-x}$

$y_p = \frac{1}{D^2+2D+1} x$

$= \frac{1}{1(1+2D+D)} x$

$= (1 + 2D + D^2)^{-1} x$

$= [1 - (2D+D^2) + (2D+D^2)^2 \cdots] x$

$= (1 - 2D + D^2) x$

$= x - 2$

complete solution.

$y = (c_1 + c_2 x)e^{-x} + x - 2$

3) $(2D^2 + 3D + 4) y = x^2 - 2x$

Aux soln: $2m^2 + 3m + 4 = 0$

$m_1 = -\frac{3}{4} + \frac{\sqrt{23}}{4} i$

$m_2 = -\frac{3}{4} - \frac{\sqrt{23}}{4} i$

From formula with $\alpha = -\frac{3}{4}$, $\beta = \frac{\sqrt{23}}{4}$

$y_c = e^{-3/4 x}\left(c_1 \cos\frac{\sqrt{23}}{4}x + c_2 \sin\frac{\sqrt{23}}{4}x\right)$

$y_p = \frac{1}{2D^2+3D+4}(x^2-2x)$

$= \frac{1}{4\left(1+\frac{3D+2D^2}{4}\right)}(x^2 - 2x)$

$= \frac{1}{4}\left[1+\left(\frac{3D+2D^2}{4}\right)\right]^{-1}(x^2-2x)$

$= \frac{1}{4}\left[1 - \left(\frac{3D+2D^2}{4}\right) + \left(\frac{3D+2D^2}{4}\right)^2 - \cdots\right.$

$= \frac{1}{4}\left[1 - \frac{3D}{4} - \frac{1}{2}D^2 + \frac{9D^2}{16} + \right](x^2 - 2x)$

$= \frac{1}{4}\left[(x^2-2x) - \frac{3}{4}(2x-2) - \frac{1}{2}(2) + \frac{9}{16}(\right.$

$$= \frac{1}{4}\left[(x^2-2x) + \frac{-6x+6}{4} - 1 + \frac{9}{8}\right]$$

$$= \frac{1}{4}\left[\frac{8x^2-16x-12-8+9}{8}\right]$$

$$= \frac{1}{4}\left[\frac{8x^2-28x+13}{8}\right]$$

$$= \frac{1}{32}(8x^2-28x+13)$$

The complete solution is

$$y = e^{-3/4x}\left(c_1 \cos\frac{\sqrt{23}}{4}x + c_2 \sin\frac{\sqrt{23}}{4}x\right) + \frac{1}{32}(8x^2-28x+13)$$

$$(D^2-4D+3)y = x^3$$

Aux Soln $= m^2-4m+3 = 0$

$m_1 = 1$, $m_2 = 3$

$$y_c = c_1 e^x + c_2 e^{3x}$$

$$y_p = \frac{1}{D^2-4D+3} x^3$$

$$= \frac{1}{3\left(1+\frac{D^2-4D}{3}\right)} x^3$$

$$= \frac{1}{3}\left(1+\frac{D^2-4D}{3}\right)^{-1} x^3$$

$$= \frac{1}{3}\left(1 - \frac{D^2-4D}{3} + \left(\frac{D^2-4D}{3}\right)^2 - \left(\frac{D^2-4D}{3}\right)^3 + \dots\right)(x^3)$$

$$= \frac{1}{3}\left[1 - \frac{D^2}{3} + \frac{4}{3}D + \frac{D^2-8D^3+16D^2}{9} + \frac{64}{27}D^3 \dots\right]$$

$$= \frac{1}{3}\left[x^3 - \frac{1}{3}(6x) + \frac{4}{3}(3x^2) - \frac{8}{9}(6) + \frac{16}{9} + \frac{64}{27}(6)\right]$$

$$= \frac{1}{3}\left[x^3 - 2x - 4x^2 - \frac{16}{3} + \frac{32x}{3} + \frac{128}{9}\right]$$

$$= \frac{1}{3}\left[\frac{9x^3 - 18x + 36x^2 - 48 + 96x + 128}{9}\right]$$

$$= \frac{1}{27}(9x^3 + 36x^2 + 78x + 80)$$

The complete solution,

$$y = c_1 e^{3x} + c_2 e^x + \frac{1}{27}(9x^3 + 36x^2 + 78x$$

B-22

1) $\frac{d^2y}{dx^2} + 6y = \sin 4x$

$(D^2 + 6)y = \sin 4x$

Aux Soln: $m^2 + 6 = 0$

$\Rightarrow m = \pm\sqrt{6}i$

$y_c = c_1 \cos\sqrt{6}x + c_2 \sin\sqrt{6}x$

$y_p = \frac{1}{D^2+6} \sin 4x$

$= \frac{\sin 4x}{-4+6}$

$y = c_1 \cos\sqrt{6}x + c_2 \sin\sqrt{6}x - \frac{1}{10} \sin 4x$

2) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$

$(D^2 + 2D + 3)x = \sin t$

Aux Soln: $m^2 + 2m + 3 = 0$

$\therefore m_1 = -1 + \sqrt{2}i$

$m_2 = -1 - \sqrt{2}i$

$\alpha = -1 \quad \beta = \sqrt{2}$

$y_c = e^{-x}(c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x)$

$I_P = \frac{1}{D^2+2D+3} \sin t$

$= \frac{1}{2D+2} \sin t$

$= \frac{\sin t \ (2D-2)}{(2D+2)(2D-2)}$

$= \frac{2D \sin t - 2\sin t}{4D^2 - 4}$

$= \frac{2\cos t - 2\sin t}{-8}$

$= \frac{\sin t - \cos t}{4}$

(5) $(D^3+1)y = 2\cos^2 x$

Ans Soln : $m^3+1=0$

$m_1 = -1, \; m_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$m_3 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

$y_c = c_1 e^{-x} + e^{\frac{1}{2}x} \left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x\right)$

$IP = \frac{1}{D^3+1} \, 2\cos^2 x$

$= \frac{1}{DD^2+1} \cdot 2 \, \frac{1+\cos 2x}{2}$

$= \frac{1}{D^3+1} 1 + \cos 2x$

$= \frac{1}{D^3+1} 1 + \frac{1}{D^3+1} \cos 2x$

$= \frac{1}{D^3+1} e^0 + \frac{1}{D^3+1} \cos 2x$

$= 1 + \frac{1}{-4D+1} \cos 2x$

$= 1 + \frac{(1+4D) \cos 2x}{(1-4D)(1+4D)}$

$= 1 + \frac{1+4D}{1-16D^2} \cos 12x$

$= 1 + \left(\frac{1+4D}{1+64}\right) \cos 12x$

Exercise 3·23

1) $(D^2 - 5D + 6)y = e^x \sin x$

Aux Soln : $(m^2 - 5m + 6)e^{mx} = 0$

$\therefore m_1 = 2, m_2 = 3$

$y_c = c_1 e^{2x} + c_2 e^{3x}$

$y_p = \frac{1}{D^2 - 5D + 6} e^x \sin x$

$= e^x \frac{1}{(D+1)^2 - 5(D+1) + 6} \sin x$

$= e^x \frac{1}{D^2 + 2D + 1 - 5D - 5 + 6} \sin x$

$= e^x \frac{1}{D^2 - 3D + 2} \sin x$

$= e^x \frac{\sin x}{-1 - 3D + 2}$

$= e^x \frac{\sin x}{-3D + 1}$

$= e^x \frac{(-3D - 1)\sin x}{9D^2 - 1}$

$= e^x \frac{(-3\cos x - \sin x)}{9(-1) - 1} = e^x \frac{(3\cos x + \sin x)}{10}$

$y = c_1 e^{2x} + c_2 e^{3x} + \frac{e^x}{10}(3\cos x + \sin x)$

2) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = e^{2x}\sin x$

$\Rightarrow D^2 - 7D + 10 = e^{2x}\sin x$

Aux soln:

$m^2 - 7m + 10 = 0$

$\therefore m_1 = 2, \quad m_2 = 5$

$y_c = c_1e^{2x} + c_2e^{5x}$

$y_P = \frac{1}{D^2-7D+10} e^{2x}\sin x$

$= e^{2x}\frac{1}{(D+2)^2 - 7(D+2)+10}\sin x$

$= e^{2x}\frac{1}{D^2+4D+4-7D-14+10}\sin x$

$= e^{2x}\frac{1}{D^2-3D}\sin x$

$= e^{2x}\frac{1}{-1-3D}\sin x$

$= -e^{2x}\frac{1}{3D+1}\sin x$

$= -e^{2x}\frac{(3D-1)\sin x}{9D^2-1}$

$= -e^{2x}\frac{3\cos x - \sin x}{-10}$

$= \frac{e^{2x}}{10}(3\cos x - \sin x)$

$\therefore y = c_1e^{2x} + c_2e^{5x} + \frac{e^{2x}}{10}(3\cos x - \sin x)$

3) $\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = e^x\cos x$

$D^3 - 3D + 4 = e^x\cos x$

Aux soln is: $m^3 - 3m + 4 = 0$

$\therefore m_1 = -2, m_2 = 1+i, m_3 =$

$\therefore y_c = c_1e^{-2x} + e^x(c_2\cos x + c_3\sin x$

$y_P = \frac{1}{D^3-2D+4}e^x\cos x$

$$\Rightarrow e^{x}\frac{1}{(D+1)^3-2(D+1)+4}\cos x$$

$$\Rightarrow e^{x}\frac{1}{D^3+3D^2+3D+1-2D-2+4}\cos x$$

$$\Rightarrow e^{x}\frac{1}{D^3+3D^2+D+3}\cos x$$

$$\Rightarrow e^{x}\frac{1}{3D^2+6D+1}\cos x$$

$$\Rightarrow e^{x}\frac{x}{3(-1)+6D+1}\cos x$$

$$xe^{x}\frac{1}{6D-2}\cos x$$

$$xe^{x}\frac{(6D+2)}{36D^2-4}\cos x$$

$$xe^{x}\frac{(-6\sin x+2\cos x)}{-36-4}$$

$$e^{x}\frac{3\sin x-\cos x}{20}$$

$$e^{-2x}+e^{x}(c_2\cos x+c_3\sin x)+\frac{xe^{x}}{20}(3\sin x-\cos x)$$

4) $(D^2-4D+3)y=2xe^{3x}+3e^{3x}\cos 2x$

Ans Soln: $m^2-4m+3=0$

$\therefore m_1=1,\ m_2=3$

$y_c=c_1e^{x}+c_2e^{3x}$

$$y_p=\frac{1}{D^2-4D+3}2xe^{3x}+\frac{1}{D^2-4D+3}3e^{3x}\cos x$$

$$\Rightarrow 2e^{3x}\frac{1}{(D+3)^2-4(D+3)+3}+3e^{3x}\frac{1}{(D+3)^2-4(D+3)+3}\cos 2x$$

$$\Rightarrow 2e^{3x}\frac{1}{D^2+6D+9-4D-12+3}x+3e^{3x}\frac{1}{D^2+6D+9-4D-12+3}$$

$$\Rightarrow 2e^{3x}\frac{1}{D^2+2D}x+3e^{3x}\frac{1}{D^2+2D}\cos 2x$$

$$\Rightarrow 2e^{3x}\frac{1}{2D(1+\frac{D}{2})}x+3e^{3x}\frac{1}{(-4+2D)}\cos 2x$$

$$\Rightarrow 2e^{3x}\frac{1}{2D}\left(1+\frac{D}{2}\right)^{-1}x+3e^{3x}\frac{1}{2D-4}\cos 2x$$

$$=e^{3x}\frac{1}{D}\left(x-\frac{D}{2}+\cdots\right)x+3e^{3x}\left(\frac{2D+4}{4D^2-}\right.$$

15-04-202

LapLACE TRANSFORMA

"DF forerarm"

let, $F(t)$ be function of t specified for $t>0$. Then the Laplace transformation of $f(t)$ denoted by $\mathcal{L}\{f(t)\}$ is defined by $\mathcal{L}\{F(t)\} = \int_0^{\infty} e^{-st} F(t)\,dt$ Where assume the Parameter s is real.

$\mathcal{L}\{F(t)\} = f(s)$

$F(t)$		$\mathcal{L}\{F(t)\} = f(s)$
01) c	$\longrightarrow$	$\frac{c}{s}$
02) t^n	$\longrightarrow$	$\frac{n!}{s^{n+1}}$
03) e^{at}	$\longrightarrow$	$\frac{1}{s-a}$
04) $\sin at$	$\longrightarrow$	$\frac{a}{a^2+s^2}$
05) $\cos(at)$	$\longrightarrow$	$\frac{s}{s^2+a^2}$

for hyperbolic function

06) $\sin h(at) \rightarrow \frac{a}{s^2-a^2}$

07) $\cos h(at) \rightarrow \frac{s}{s^2-a^2}$

$e^{-\infty} = 0$

$e^{0} = 1$

$e^{\infty} = \infty$

01) Prove that $\mathcal{L}\{c\} = \frac{c}{s}$

Sl^n: By defination of laplace transformation.

$$\mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\,dt$$

Here,

$F(t) = c$

$$\therefore \mathcal{L}\{c\} = \int_0^\infty e^{-st}\, c\, dt$$

$$= c\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= \frac{c}{s}\left[-e^{-s\infty} + e^0\right.$$ (costant s বাহিরে আসবে.

$$= \frac{c}{s}[0+1)$$

$$= \frac{c}{s}$$ Proved

02) Prove that,

$$\mathcal{L}(t) = \frac{1}{s^2}$$

By defination. of laplace transformation,

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} F(t)\, dt$$

$F(t) = \cancel{\frac{1}{s^2}}\; t$ $\qquad [F(t) = t$

So, $\mathcal{L}\{t\} = \int_0^\infty \underset{v}{e^{-st}} \underset{u}{F}(t)\, dt$

$$\boxed{\int uv\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right.}$$

Sign	D	I
+	t	
−	1	
+	0	

$$\therefore \mathcal{L}\{t\} = \left[-\frac{t\,e^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= \frac{-t\,e^{-\infty}}{s} - \frac{e^{-\infty}}{s^2} + \frac{0\cdot e^0}{s} + \frac{e^0}{s^2}$$

$$= \frac{1}{s^2}$$ (Proved)

* Prove that, $L(e^{at}) = \frac{1}{s-a}$

Soln: By defination of laplace transform,

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$$

Here, $F(t) = e^{at}$

$$\therefore L\{e^{at}\} = \int_0^\infty e^{-st} \cdot e^{at}\, dt$$

$$= \int_0^\infty e^{-t(s-a)}\, dt$$

$$= \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty$$

$$= \frac{e^{-\infty}}{-(s-a)} + \frac{e^{-0}}{s-a}$$

$$= \frac{1}{s-a}$$ Proved

* Prove that $L\{\sin at\} = \frac{a}{s^2+a^2}$

Soln: By defination of laplace Transformation.

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$$

Here, $F(t) = \sin(at)$ can be done with ILATE

$$\therefore L\{\sin(at)\} = \int_0^\infty e^{-st} \cdot \sin(at)\, dt$$

$$= \left[\frac{e^{-st}}{s^2+a^2}(-s \sin at - a \cos at)\right]_0^\infty$$

$$= \frac{1}{s^2+a^2}(0 - 0 + 0 + a)$$

* $\int e^{\alpha t} \cdot \sin(\beta t$ ...

$= \frac{e^{\alpha t}(\alpha \sin\beta t ...}{\alpha^2 + \beta^2}$ ...

HW Prove that $L\{\cos(at)\} = \frac{s}{a^2+s^2}$

By the defination of laplace

$$L\{F(t)\} = \int_0^{\infty} e^{-st} F(t)\, dt$$

$$\cosh(at) = \frac{e^{at}+e^{-at}}{2} \qquad \sinh(at) = \frac{e^{at}-e^{-at}}{2}$$

3) Prove that $L\{\cosh(at)\} = \frac{s}{s^2-a^2}$

Soln: By defination of laplace transformation

$$L\{F(t)\} = \int_0^{\infty} e^{-st} F(t).dt$$

Here, $F(t) = \cosh(at)$

$$L\{\cosh(at)\} = \int_0^{\infty} e^{-st} \cosh(at)\, dt$$

$$= \int_0^{\infty} \frac{e^{at} - e^{-at}}{2} e^{-st}\, dt$$

$$= \frac{1}{2} \int_0^{\infty} \left(e^{-t(s-a)} + e^{-t(s+a)}\right) dt$$

$$= \frac{1}{2} \left(\left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^{\infty} + \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^{\infty} \right)$$

$$= \frac{1}{2}\left(0 + \frac{1}{s-a} + 0 + \frac{1}{s+a}\right)$$

$$= \frac{1}{2} \frac{s-a+s+a}{(s-a)(s+a)}$$

$$= \frac{1}{2} \frac{2s}{s^2-a^2}$$

$$= \frac{s}{s^2-a^2} \quad \text{(Proved)}$$

H.W – Prove ...

Properties :

01 Linearity Property :

If c_1, c_2 are any constants, while $F_1(t)$, $F_2(t)$ are functions with laplace transforms $f_1(s)$, $f_2(s)$ respectively then

$$L\{c_1 F_1(t) + c_2 F_2(t)\} = c_1 L\{F_1(t)\} + c_2 L\{F_2(t)\}$$
$$= c_1 f_1(s) + c_2 f_2(s)$$

ধ্রুবক, value বাইরে আসবে।

ⓐ $L\{5e^{-t} + 2\cos 5t - 3\}$

$= 5L(e^{-t}) + 2L(\cos 5t) - L(3)$

$= 5\frac{1}{s+1} + 2\frac{s}{s^2+25} - \frac{3}{s}$ Δ

Ⓑ $L\{(t^2+1)^2\} = L\{t^4 + 2t^2 + 1\}$

$= \frac{4!}{s^5} + 2 \cdot \frac{2!}{s^3} + \frac{1}{s}$ Δ

Properties of laplace Transformation

02) First transformation / shifting Properties :

If, $L\{F(t)\} = f(s)$ then,

$L\{e^{at} F(t)\} = f(s-a)$

এর সাথে গুণ আকারে যা থাকবে তাই $F(t)$

ⓐ $L\{e^{-2t} \cos 5t\}$

Solve: we know, $L\{\cos 5t\} = \frac{s}{s^2+25}$

যেখানে যেখানে s আছে সেখানে মান চেঞ্জ করে

$\therefore L\{e^{-2t}\cos 5t\} = \frac{s-(-2)}{\{s-(-2)\}^2+25} = \frac{s+2}{(s+2)^2 + 25}$

(B) $L\{e^{5t}\sin t\}$

WKT, $L\{\sin t\} = \frac{1}{s^2+1}$

$\therefore L\{e^{5t}\sin t\} = \frac{1}{(s-5)^2+1}$

3) 2nd Translation / Shifting Property:

If, $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a) & t > a \\ 0, & t < a \end{cases}$ then

$L\{G(t)\} = e^{-as} f(s)$

1) If $G(t) = \begin{cases} (t-2)^3, & t > 2 \\ 0 & , t < 2 \end{cases}$ find $L\{G(t)\}$

Soln: $L\{G(t)\} = e^{-2s}\frac{3!}{s^4}$

02) $\begin{cases} \sin(t-\frac{\pi}{2}), & t > \frac{\pi}{2} \\ 0, & t < \frac{\pi}{2} \end{cases}$ find $L\{G(t)\}$

Soln: $L\{G(t)\} = e^{-\pi/2 \cdot s}\frac{1}{s^2+1}$ Ans

$\boxed{\text{04 multiplication by } t^n\text{:}}$

if $L\{F(t)\} = f(s)$ then,

$$L\{t^n F(t)\} = (-1)^n \cdot \frac{d^n}{ds^n} f(s)$$

(Q1) $L\{t^2 \cos at\}$

Soln: $L\{\cos at\} = \frac{s}{s^2+a^2}$

$L\{t^2 \cos at\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{s}{s^2+a^2}\right)$

$= \frac{d}{ds}\left\{\frac{(s^2+a^2)\frac{d}{ds}(s) - s\frac{d}{ds}(s^2+a^2)}{(s^2+a^2)^2}\right\}$

$= \frac{d}{ds}\left\{\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}\right\}$

$= \frac{d}{ds}\frac{a^2-s^2}{(s^2+a^2)^2}$

$= \frac{(s^2+a^2)^2\frac{d}{ds}(a^2-s^2) - (a^2-s^2)\frac{d}{ds}(s^2+a^2)}{(s^2+a^2)^4}$

$$= \frac{-2s\,(s^2+a^2)^2 - (a^2-s^2)\cdot 2(s^2+a^2)\,2s}{(s^2+a^2)^4}$$

$$= \frac{-2s(s^2+a^2) - 4s(a^2-s^2)}{(s^2+a^2)^3}$$

$$= \frac{-2s^3 - 2a^2s - 4a^2s + 4s^3}{(s^2+a^2)^3}$$

$$= \frac{2s^3 - 6a^2s}{(s^2+a^2)^3} \quad \text{A}$$

2) $L\{t^{50}e^{2t}\}$

Soln: We know, $L\{t^{50}\} = \frac{50!}{s^{51}}$

$$\therefore L\{t^{50}e^{2t}\} = \frac{50!}{(s-2)^{51}} \quad \text{A}$$

Laplace Periodic function

Let $F(t)$ is periodic function having period T

$$L\{F(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} F(t)\, dt$$

Q1) $F(t) = \begin{cases} 2t^2, & 0<t<1 \\ t, & 1<t<2 \end{cases}$ is periodic function of period 2 → Period function

Find (i) $L\{F(t)\}$

(ii) sketch $F(t)$

Soln (i) $L\{F(t)\} = \frac{1}{1-e^{-2s}} \int_0^2 e^{-st} F(t)\, dt$

$$\Rightarrow \frac{1}{1-e^{-2s}} \left[\int_0^1 e^{-st}\, 2t^2\, dt + \int_1^2 e^{-st}\, t\, dt \right.$$

$$\Rightarrow \frac{1}{1-e^{-2s}} \left[2\int_0^1 e^{-st}\, t^2\, dt + \int_1^2 e^{-st}\, t\, dt \right.$$

Sign

Exm 3: Find LP Transform of $\cos^2 t$.

$\boxed{\cos 2t = 2\cos^2 t - 1}$

$$\cos 2t = 2\cos^2 t - 1$$
$$\cos^2 t = \frac{1}{2}[\cos 2t + 1]$$

$$L(\cos^2 t) = L\left[\frac{1}{2}(\cos 2t + 1)\right]$$
$$= \frac{1}{2}[L(\cos 2t) + L(1)]$$
$$= \frac{1}{2}\left[\frac{s}{s^2+2^2} + \frac{1}{s}\right]$$
$$= \frac{1}{2}\left[\frac{s}{s^2+4} + \frac{1}{s}\right] \quad A$$

যদি n এর মান integer হয় $\frac{n!}{s^{n+1}}$

যদি ভগ্নাংশ/fractional হয় " $\quad$ গামা $n+1$

Exm 4: Find the laplace of $t^{-\frac{1}{2}}$

We know, $L(t^n) = \frac{\Gamma n+1}{s^{n+1}}$

Put $n = -\frac{1}{2}$, $L(t^{-1/2}) = \frac{\Gamma -\frac{1}{2}+1}{s^{-1/2+1}}$

$$= \frac{\Gamma \frac{1}{2}}{\sqrt{s}} = \frac{\sqrt{\pi}}{\sqrt{s}} \text{ where } \Gamma\tfrac{1}{2} = \sqrt{\pi}$$

laplace of the Derivative of $f(t)$

$$\boxed{L[f'(t)] = sL[f(t)] - f(0)}$$

where $L[F(t)] = F(s)$

Proof:

$$L[f'(t)] = \int_0^\infty e^{-st} f'(t)\, dt$$

integrating by parts we get,

$$L[f'(t)] = \left[e^{-st} \cdot f(t)\right]_0^\infty - \int_0^\infty (-se^{-st}) f(t)$$

$$\Rightarrow -f(0) + s\int_0^\infty e^{-st} f(t)\, dt \quad (e^{-st} f(t) = 0$$

Prime যত (s^{n}) এ n এর value তত

n এর মান যত f এর মান তার থেকে 1 কম

4) Find laplace transform of $t^{-\frac{1}{2}}$

we know, $L(t^n) = \frac{\sqrt{n+1}}{s^{n+1}}$

Put $n = -\frac{1}{2}$, $L(t^{-1/2}) = \frac{\overline{|-\frac{1}{2}+1}}{s^{-1/2+1}} = \frac{\overline{|\frac{1}{2}}}{\sqrt{s}}$ where, $\overline{|\frac{1}{2}} = \sqrt{\pi}$

integration এর notation operator থেকে সরাসরি যে f(t) দ্বারা হবে। ঐ f(t) কে integration করে f(s) দ্বারা হবে।
যখন দুটোর ওপর Laplace notation ব্যবহার তখন $\frac{f(s)}{s}$ দিতে হবে

laplace transformation of integral of f(t)

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s} f(s)$$

LT of t. f(t) (multiplication by t)

$L[f(t)] = F(s)$ then,

$$\boxed{L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)]}$$

Similarly, $L[t^2 f(t)] = (-1)^2 \frac{d^2}{ds^2} [F(s)]$

$L[t^3 f(t)] = (-1)^3 \frac{d^3}{ds^3} [F(s)]$

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)]$

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$

এই form এ কিছু থাকলে উল্টো করে $\left(\frac{1}{t}\right)$ প্রতিস্থাপক $f(t)$ $\left(\frac{1}{t}\right)$ t এর উপরে বা থাকলে তবে $f(t)$

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LT of $\boxed{\frac{1}{t}} f(t)$ (division by t)

if $L[f(t)] = F(s)$ then $L\left[\frac{1}{t} f(t)\right] = \int_0^\infty e^{-st} f(t)\, dt$

integrating s,

$$\int_s^\infty F(s)\, ds = \int_s^\infty \left[\int_0^\infty e^{-st} f(t)\, dt\right] ds$$

$$= \int_0^\infty \left[\int_s^\infty e^{-st} f(t)\, ds\right] dt = \int_0^\infty \left[\frac{e^{-st} f(t)}{-t}\right]_s^\infty dt$$

26/04/2024 Inverse laplace

Inverse laplase Transforms:

if $F(s)$ is $f(t)$, $L[f(t)] = F(s)$

then $\boxed{F(t) = L^{-1} f(s)}$

Formula:

① $L^{-1}\left(\frac{1}{s}\right) = 1$

② $L^{-1}\frac{1}{s^n} = \frac{t^{n-1}}{(n-1)!}$

③ $L^{-1}\frac{1}{s-a} = e^{at}$

④ $L^{-1}\frac{s}{s^2-a^2} = \cosh at$

⑤ $L^{-1}\frac{1}{s^2-a^2} = \cos at$

⑧ $L^{-1}F(s-a) = e^{at} f(t)$

③ $\frac{1}{s^{n+1}} = \frac{t^n}{n!}$

④ $\frac{1}{s^2+a^2} = \frac{\sin(at)}{a}$

⑥ $\frac{1}{s^2-a^2} = \frac{\sinh(at)}{a}$

⑦ $\frac{1}{s^n} = \frac{t^{n-1}}{(n-1)!}$

Inverse laplace Transform

First translation / shifting Property:

if $L^{-1}\{f(s)\} = F(t)$, then

$$L^{-1}\{f(s-a)\} = e^{at}F(t)$$

Q1) $L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$

$= L^{-1}\left\{\frac{6s-4}{(s-2)^2+16}\right\}$

$\Rightarrow L^{-1}\left\{\frac{6(s-2)+8}{(s-2)^2+16}\right\} + 8L^{-1}\left\{\frac{1}{(s-2)^2+16}\right\}$

$= 6\cos 4t\, e^{2t} + 8\frac{\sin 4t}{4} e^{2t}$

$L^{-1}\left\{\frac{1}{s^2-2s+5}\right\}$

$= L^{-1}\left\{\frac{1}{(s-1)^2+4}\right\}$

$= \frac{\sin 2t}{2} e^{t}$

Heaviside Expansion Formula:

let $P(s)$, $Q(s)$ be polynomials where $P(s)$ has degree less that of $Q(s)$ supposd that $Q(s)$ has 'n' distinct zeros, a_k, $k=1,2,3\ldots n$.

then,

$$L^{-1}\left\{\frac{P(s)}{Q(s)}\right\} = \sum_{k=1}^{n} \frac{P(a_k)}{Q'(a_k)} e^{a_k t}$$

$$= \frac{P(a_1)}{Q'(a_1)} e^{a_1 t} + \frac{P(a_2)}{Q'(a_2)} \cdots + \frac{P(a_n)}{Q''(a_n)} e^{a_n}$$

Q1) $L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

Soln: $P(s) = 2s^2 - 4$

$Q(s) = (s+1)(s-2)(s-3)$

$= s^3 - 4s^2 + s + 6$

$Q'(s) = 3s^2 - 8s + 1$

Here $Q(s)$ has 3 distinct zeros

13.6 Periodic function

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

$$\boxed{\text{Integration of } \int uv = u\int v\,dv - \int\left(du/dx \times \int v\,dv\right)dx}$$

$$\boxed{\int e^{ax} \sin bx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}}$$

$$\text{differentiation } \left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Partial Derivative এর রুল নিয়ম, $\frac{A}{P} + \frac{B}{P}$

$Q(s)$ যদি এ Normal থাকে তাহলে $\frac{A}{Pa}$ - - -

" " " s^2 " " $\frac{A}{} + \frac{Bs+c}{}$

" " $(s+a)^2$ " " $\frac{A}{s-} + \frac{B}{s+a} + \frac{c}{(s+a}$

যখন আর কোনো ভাবে A/B/C এর value 0/মান বসানো যাবে না তখন coefficient চেক করতে হবে

(i) equation -(i) গুণ করবো

(ii) s^2, s, constant আলাদা করে common নিবো

(iii)

$$\boxed{\frac{A}{(s+1)^2} \text{ or } \frac{AB+B}{s^2+1}}$$

এই term থাকলে Coefficient চেক করা mu