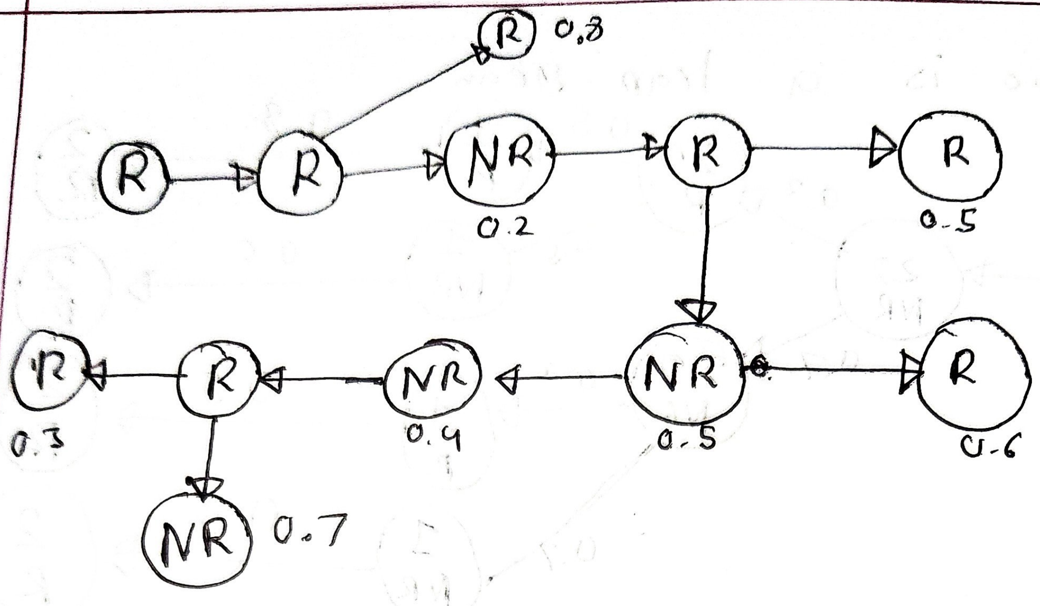
1)



P (Rain | thu) = 1 − 0.2 = 0.8

P (~~Ra~~ Not Rain | thu) = 0.2

P (friday | not rain)

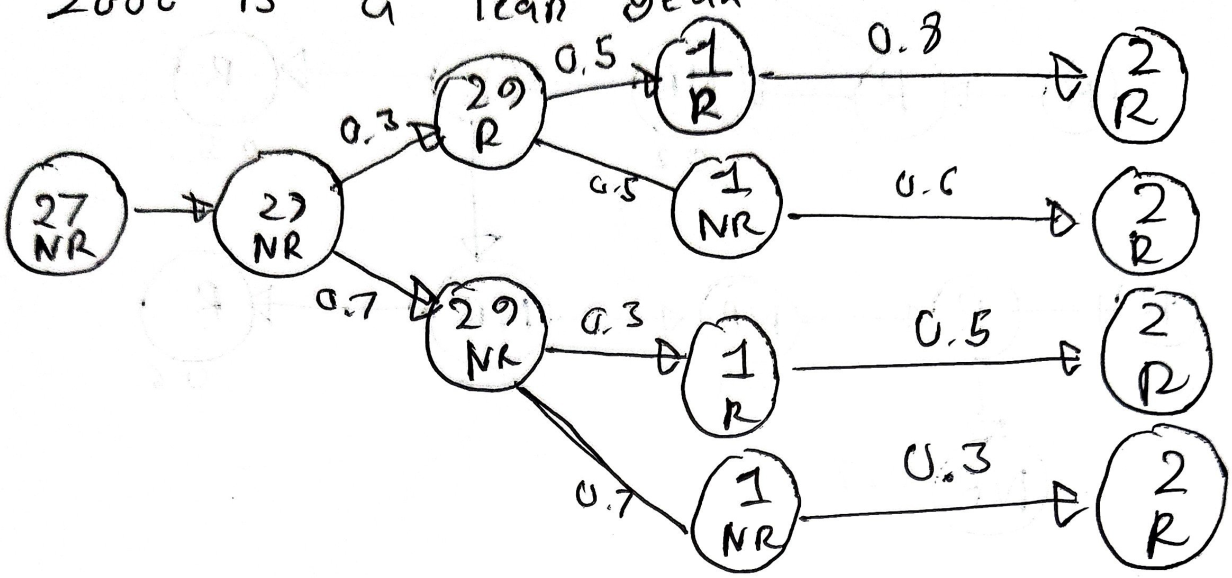
= P (Rain | thu) P (Not Rain | Fri) + P (Not Rain | thu) x P (Not Rain | Fri)

= (0.8) x (0.2) + (0.2) (0.4)

= 0.16 + 0.08

= 0.24

11) 2000 is a leap year



for 29 Based on 27, 28

P(29 | Rained) = 0.3

P(29 | NoT Rained = 1 - 0.3 = 0.7

for 1 Based on 28, 29

will rain at 1

P_1(Rained 29 but Not 28) = 0.5

P_2 (Not Rained 29 but Not 28) = 0.3

will not Rain at 1

$1 - P_1 = 1 - 0.5 = 0.5$

$1 - P_2 = 1 - 0.3 = 0.7$

So'

P (29 R and 1 R) = 0.3 X 0.5 = 0.15

P(29 R and 1 NR) = 0.3 X 0.5 = 0.15

P (29 NR and 1 R) = 0.7 X 0.3 = 0.21

P(29 NR and 1 NR) = 0.7 X 0.7 = 0.49

based on 29 and 1 for 2 maron

will rain in 2

~~P(Rain in 1 but not in 29) = 0.21~~

P (Rain 1 and rain 29) = 1 - 0.2 = 0.8

P (R2) = 0.8 X P (29 = R, 1 = R)

= 0.8 X 0.15 = $\boxed{0.12}$

P (Rain 1 but Not R 29) = 1 - 0.5 = 0.5

P (R2) = 0.5 X P (29 = NR, 1 = R)

= 0.5 X 0.21 = $\boxed{0.105}$

P(Rain 2 but Not Rain 1) $= 1 - 0.4 = 0.6$

P(R2) $= 0.4 \times 0.15 = \boxed{0.09}$

P(Not Rain 2 and Not Rain 1) $= 0.3$

P(ND2) $= 0.3 \times 0.49 =$ ~~0.49~~

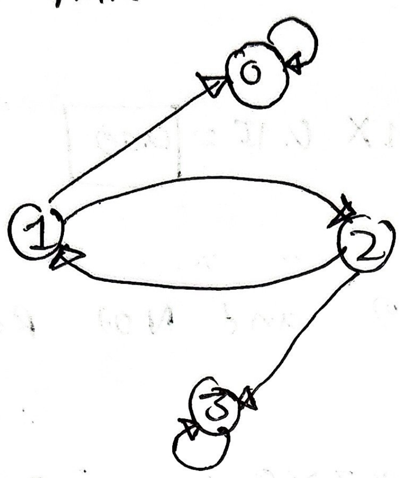
$= \boxed{0.147}$

total PR (Rain on 2)

$0.12 + 0.105 + 0.09 + 0.147$

$= 0.462$

2) Gambler's ruin



Setup

→ A gambler fortune after each play is an integer between 0 and 3

→ states 0 and 3 are absorbing ; means reaching 0 means ruin, reaching 3 meaning win

→ Let P_i be the probability of ~~early~~ eventually hitting 3 starting from fortune $i \in \{0, 1 \dots 3\}$

→ each player changes the fortune by ± 1 within their game.

$p = pr(\text{win}) = \frac{1}{2}$ and $q = pr(\text{lose}) = \frac{1}{2}$

Boundary condition

$P_0 = 0$ (already ruined)

$P_N = 1$ (already at target)

from state $i \in \{1 \ldots N-1\}$

$$P_i = \frac{1}{2} P_{i+1} + \frac{1}{2} P_{i-1}$$

multiply by 2

$$2P_i = P_{i+1} + P_{i-1}$$

$$P_{i+1} - 2P_i + P_{i-1} = 0$$

$$r^{i+1} - 2r^i + r^{i-1} = 0 \quad [P_i = r^i]$$

$$r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0$$

if $r = 1$

$$P_i = A + Bi$$

Apply Boundary condition

$$P_0 = A + B \cdot 0$$

$$P_0 = A$$

$$A = 0$$

$$P_N = BN = 1$$

$$B = \frac{1}{N}$$

$$P_i = \frac{i}{N} \quad \text{for } i = 0, 1, \ldots, N \text{ when}$$

$$p = q = \frac{1}{2}$$

Naive Bayes

Given the feature vector $x = (x_1 \text{ ----- } x_d)$

and finite set of classes C.

Start with Bayes' theorem

$$P(c|x) = \frac{P(x|c)\ P(c)}{P(x)}$$

$$P(x|c) = P(x_1 \text{ - - - -}, x_d | c) = \prod_{j=1}^{d} P(x_j|c)$$

Naive Bayes decision function

$$\hat{c} = \arg\max_{c \in C} \left[P(c) \prod_{j=1}^{d} P(x_j|c) \right]$$

Working in log space

$$\hat{c} = \arg\max_{c \in C} \left[\log P(c) + \sum_{j=1}^{d} \log P(x_j|c) \right].$$

3) **Memoryless property**

A nonnegative random variable T is memoryless if, for all $s, t \geq 0$

$$Pr(T > t + s \mid T > s) = Pr(T > t)$$

$$\Rightarrow \frac{Pr(T > t + s)}{PR(T > s)} = Pr(T > t)$$

The future tail probablity depends only on the time interval ahead, not on how long has ~~atha~~ already elapsed

$\Rightarrow$ time in bank exponetial with man 15 minutes.

let $T \sim Exp(\lambda)$ with mean $E(T) = 1/\lambda = 15$

$$\lambda = \frac{1}{15}$$

$$Pr(T>t) = e^{-\lambda t} = e^{-t/15}, \quad t>0$$

$\Rightarrow$ Customer spends more than 15 minute

$$Pr(T>15) = e^{-(15)/15} = e^{-1}$$

$$= 0.3678$$

$\Rightarrow$ Customer spend more than 1 hour given she is still there after 30 minutes.

$$Pr(T>60 \mid T>30) = Pr(T(60-30))) = Pr(T>30)$$

$$Pr(T>30) = e^{-(30)/15} = e^{-2} = 0.13533$$

4)

Quantity	Symbol	Value
Sensitivity	$P(+\mid D)$	0.8
false-positive	$P(+ D')$	0.05
Prevalance	$P(D)$	0.03
Helthiness	$P(D')$	$1-0.03=0.97$

total probabillity of ~~Posivi~~ Positive test

$$P(+) = P(+\mid D)\,P(D) + P(+D')\,P(D')$$

$$P(+) = 0.8(0.03) + 0.05(0.97)$$

$$= 0.0725$$

$$P(D\mid +) = \frac{P(+\mid D)\,P(D)}{P(+)}$$

$$= \frac{0.8 \times 0.03}{0.0725}$$

$$= 0.3310$$

$$= 33\%$$

4) ii) $P(A^+ \mid \text{computer}) = 0.85$

$P(A^+ \mid \text{chemistry}) = 0.25$

$P(\text{chem}) = \frac{2}{6} = \frac{1}{3}$ [5, 6 > 4]

$P(\text{comp}) = \frac{4}{6} = \frac{2}{3}$ [1, 2, 3, 4 ≤ 4]

$P(A^+ \text{ and chem}) = P(\text{chem}) \times P(A^+ \mid \text{chem})$

$= \frac{1}{3} \times 0.25$

$= 0.0833$

$= 8.3\%$