

VEHICLE ROUTING



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VEHICLE ROUTING

Problems, Methods, and Applications

Second Edition

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Preface to the Second Edition

The projects of reediting the Toth and Vigo book on vehicle routing and of editing a book on arc routing germinated during the ROUTE Conference in Sitges, Spain, in June 2011. The first edition of the vehicle routing book had been highly successful, and it was then felt that the evolution of the field over the past 10 years justified a significantly revamped reedition. This led Corberán and Laporte (while exploring the cellars of the Codorníu Winery during the conference excursion) to think up a proposal for a similar arc routing book that would be produced in parallel with the second edition of the vehicle routing book, with a similar structure and the same format. Again, the last major edited book on arc routing had been published more than 10 years before and the field had evolved considerably since then. Both proposals were presented to SIAM in the summer and were accepted. Today we are proud to offer to the research community two up-to-date collections of scientific contributions written by specialists in various areas of vehicle routing and arc routing. The two books are entitled “Vehicle Routing: Problems, Methods, and Applications, Second Edition”, Paolo Toth and Daniele Vigo, editors, and “Arc Routing: Problems, Methods, and Applications”, Ángel Corberán and Gilbert Laporte, editors, both published by SIAM.

The vehicle routing book contains 15 chapters. A few of these are amalgamations or significantly revised versions of chapters published in the first edition, while most of the others are entirely new. The first chapter offers an overview of the field of the Vehicle Routing Problem (VRP) and its main variants. The remainder of the book is made up of three parts: the capacitated VRP, important variants of the VRP, and applications. The first part contains two chapters on classical and new exact algorithms, as well as a chapter on heuristics. The second part surveys several variants: the VRP with time windows, pickup-and-delivery problems for goods or people transportation, stochastic VRPs, and miscellaneous variants. The third part is devoted to applications and covers the VRP with profits, real-time and dynamic VRPs, software and emerging technologies, ship routing, VRP applications in disaster relief, as well as green vehicle routing.

The arc routing book is new and contains 16 chapters. It opens with a chapter on historical perspectives, followed by three main parts: arc routing problems with a single vehicle, arc routing problems with several vehicles, and applications. The first part starts with a chapter on complexity, which is followed by four chapters on the Chinese Postman Problem and on the Rural Postman Problem. The second part contains four chapters on the Capacitated Arc Routing Problem and two on arc routing problems with min-max and profit maximization objectives. The last part covers some of the most important arc routing applications, including meter reading, salt spreading, snow removal, garbage collection, and newspaper delivery.

We thank all authors for the quality of their contributions, as well as all referees who carefully reviewed the chapters, and Claudio Gambella for his help in editing the final manuscript of the VRP book. Thanks are also due to Dr. Thomas Liebling, Ms. Elizabeth Greenspan, Ms. Ann Manning Allen, and Ms. Sara J. Murphy from SIAM for their support and encouragement.

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May 2014

Preface to the First Edition

The Vehicle Routing Problem (VRP) calls for the determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers, and it is one of the most important, and studied, combinatorial optimization problems.

More than 40 years have elapsed since Dantzig and Ramser introduced the problem in 1959. They described a real-world application concerning the delivery of gasoline to service stations and proposed the first mathematical programming formulation and algorithmic approach. A few years later, in 1964, Clarke and Wright proposed an effective greedy heuristic that improved on the Dantzig–Ramser approach. Following these two seminal papers, hundreds of models and algorithms were proposed for the optimal and approximate solution of the different versions of the VRP. Dozens of packages for the solution of various real-world VRPs are now available on the market. This interest in VRP is motivated by both its practical relevance and its considerable difficulty: the largest VRP instances that can be consistently solved by the most effective exact algorithms proposed so far contain about 50 customers, whereas larger instances may be solved to optimality only in particular cases.

This book covers the state of the art of both exact and heuristic methods developed in the last decades for the VRP and some of its main variants. Moreover, a considerable part of the book is devoted to the discussion of practical issues.

The realization of this project would have been impossible for us alone to accomplish. We thus involved an enthusiastic group of very well known experts, whose contributions form a large part of the recent history of the VRP (as well as that of Mathematical Programming and Combinatorial Optimization). As editors, we constantly devoted our efforts to reducing as much as possible the overlap between chapters and to preserving coherence and ensuring uniformity of the notation and terminology.

Although focused on a specific family of problems, this book offers a complete overview of the effective use of the most important techniques proposed for the solution of hard combinatorial problems. We, however, assume that readers have a basic knowledge of the main methods for the solution of combinatorial optimization problems (complexity theory, branch-and-bound, branch-and-cut, relaxations, heuristics, metaheuristics, local search, etc.).

The book is divided into three parts, preceded by an introductory chapter in which we present an overview of the VRP family, define the most important variants of the problem, and introduce the main mathematical models. The first part covers the basic and extensively studied version of the VRP, known as capacitated VRP. Three chapters examine the main exact approaches (branch-and-bound, branch-and-cut, and set-covering-based methods), while two other chapters review traditional heuristic approaches and metaheuristics, respectively. For all methods extensive computational results are analyzed. The second part covers three main variants of the VRP: the VRP with time windows, the VRP with backhauls, and the VRP with pickup and delivery. In each chapter, both

exact and heuristic methods are examined. Finally, in the third part, the issues arising in real-world VRP applications, as the presence of dynamic and stochastic components, are discussed by analyzing relevant case studies and presenting software packages.

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