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Routing and charging optimization for electric bus operations

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A R T I C L E I N F O

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A B S T R A C T

The transition to alternative energy sources and the adoption of on-demand operating modes in urban bus systems are crucial steps towards reducing carbon footprints and improving public transit services. This paper presents a two-phase approach for the collaborative optimization of charging schedules and passenger services, aimed at enhancing the operation of on-demand electric bus systems. First, we propose a label-setting dynamic programming algorithm that enables the efficient generation of bus-trips for each bus line in response to passenger requests. Second, we introduce a time–space network optimization model that facilitates integrated multiple bus-trip planning for the transit network, involving multiple bus lines and charging spots. The model selects bus-trips from various time–space arcs, which represent passenger carrying, bus deployment, and bus charging activities. To validate the effectiveness of our approach, we conduct a case study using real-world data from bus lines in Beijing, China. Computational results demonstrate that our approach can handle on-demand electric bus operations within minutes of solution time, efficiently serving over 2,000 passengers. Practically, our approach achieves a notable reduction in average transit time and effectively reduces the waste of public transit resources. The proposed approach can serve as a beneficial tool for decision-makers and operators seeking to enhance the performance and environmental impact of their electric bus systems.

1. Introduction

On-demand bus services have emerged as a transformative and promising approach to revolutionize urban transportation, aiming to prioritize efficiency, flexibility, and reliability for customers. Departing from the limitations of conventional fixed routes and timetables, on-demand services have embraced modern technologies and real-time communication platforms to collect and analyze actual passenger requests, allowing for dynamic and adaptive scheduling (Liu and Ceder, 2015; Peled et al., 2021; Ali-Eldin and Elmroth, 2021). This innovative approach seeks to overcome the long-standing obstacle of relying on historical data to design routes and schedules (Schöbel, 2012), as it often leads to using the same routes during different periods, resulting in unnecessary empty runs and extended transit times for customers. Hence, operators of on-demand bus services have been actively exploring and implementing various on-demand bus strategies to establish cost-efficient and reliable systems (Khoat and Bernard, 2007). These strategies include adding reserve buses to fill gaps in demand and implementing deadheading to optimize vehicle utilization. Station control measures, such as holding and stop-skipping strategies, can also be deployed to mitigate delays and improve service reliability (Liu et al., 2013). With these innovative strategies, on-demand bus services can efficiently respond to changing passenger demands and provide a more seamless and enjoyable commuting experience.

Our research links to the stop-skipping, as an effective method to avoid unnecessary stops at stations with no transit demand. Fig. 1 presents the ratios of stops with boarding or alighting requests in 15-min intervals during both peak and off-peak hours,

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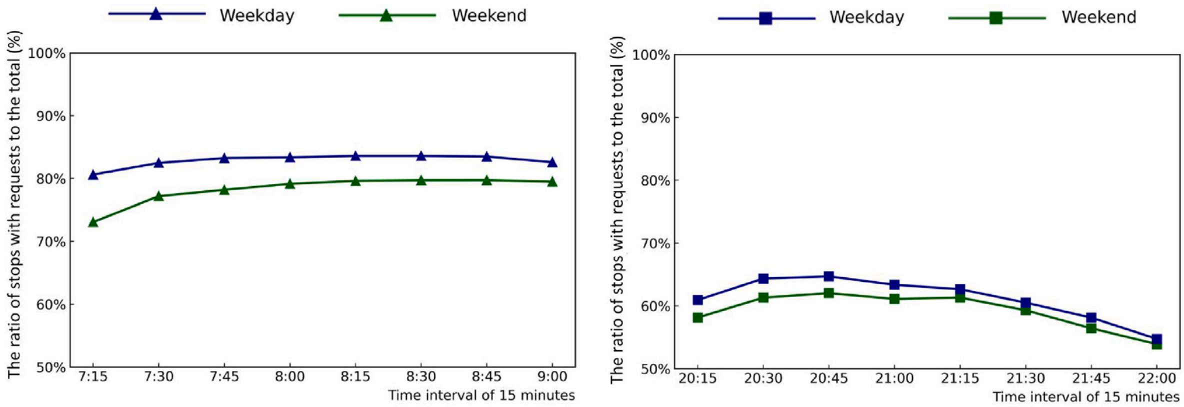


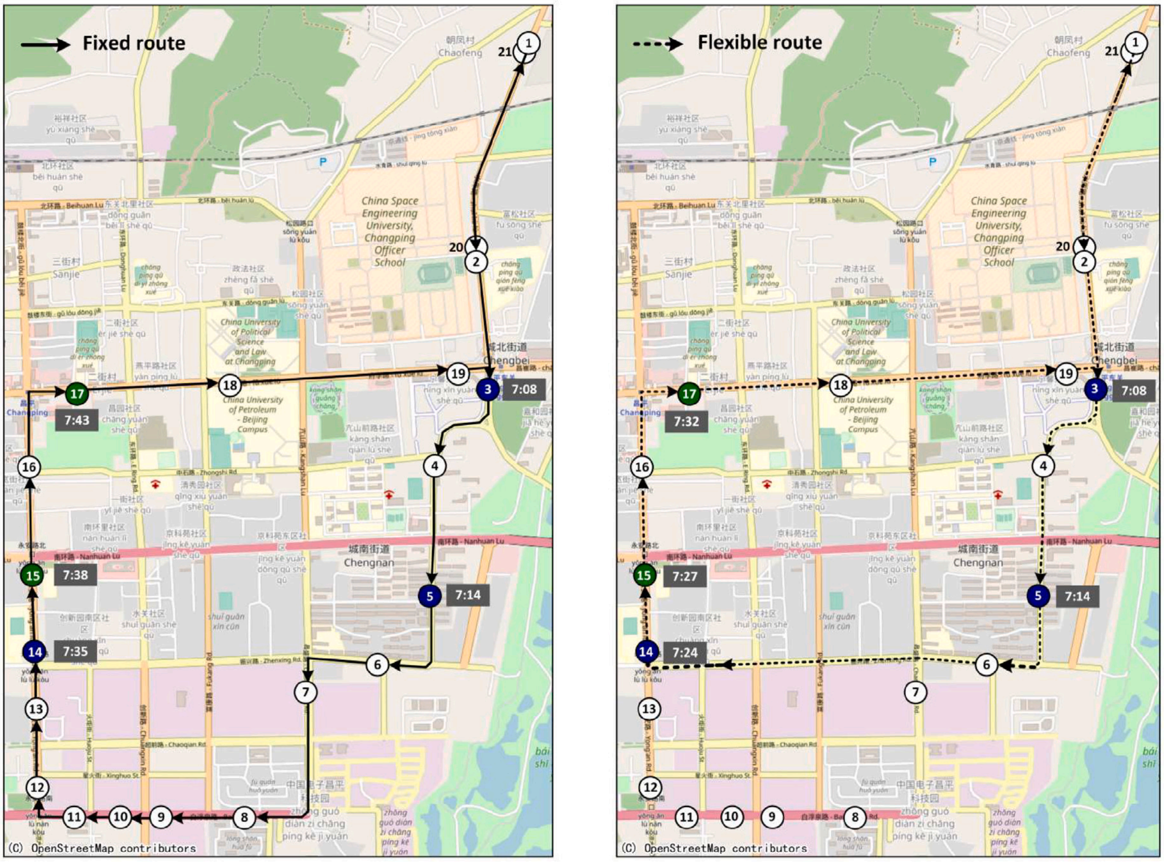
(a) During peak hours (b) During off-peak hours

Fig. 1. The ratios of the number of stops with boarding or alighting requests to the total number of stops in 15-min time intervals within the Beijing bus transit network. These ratios are calculated as averages from the data of passenger requests on each day of October 2019.

based on data from the Beijing urban bus transit network, encompassing 1217 bus lines. The data reveals that, even during peak hours, over 10% of stops encounter no passenger activity, while off-peak hours witness passenger demand at less than 70% of stops. These findings raise concerns as buses visit a significant number of stops without any passenger demand, leading to increased waiting and in-vehicle time, unnecessary detours, and ultimately diminishing the overall performance of urban bus systems. To address this issue, we introduce stop-skipping in our optimization approach for on-demand bus operations. Fig. 2 illustrates a specific example comparing bus routes with and without stop-skipping, revealing reductions in passenger waiting and in-vehicle time, respectively. Implementing stop-skipping based on actual passenger requests significantly shortens the average duration of this bus trip, constituting only half of the original overall transit time in this example. In this paper, our proposed optimization approach validates the effectiveness of the stop-skipping strategy, enhancing urban bus system efficiency and better catering to passenger needs.

Emission load, in addition to the aforementioned service quality, stands as another dominant characteristic of urban bus systems. The transportation sector has long been a significant contributor to urban air pollution, accounting for over 50% of the global carbon footprint (Sokhi, 2011). To address this environmental challenge, operators have increasingly turned their attention to the electrification of their bus fleets (Zhang et al., 2021b; Ji et al., 2022; He et al., 2022; Ruan and Lv, 2022). Over the past decade, electric buses have gained remarkable momentum and have secured a substantial share of the global market. In China, the electric bus fleet in the national urban transit system reached an impressive 466,000 units by the end of 2020, accounting for 66% of the total. Meanwhile, Europe has also taken significant strides in promoting energy transition, exemplified by the recently updated ambitious greenhouse gas emissions targets, as part of initiatives like REPowerEU (2022) and European Green Deal (2019), to drive the adoption of electric buses. Despite the allure of lower fuel costs and reduced carbon emissions, the limited driving range of electric buses poses a significant challenge to their widespread adoption. Studies have highlighted the concern that electric bus fleets may struggle to complete their required daily operations without mid-day charging (Zhang et al., 2021a). Studies suggest that, after a single charging session lasting around 30 min, an electric bus can only cover up to 30 km, which is less than twice the average length of regular bus routes (Guschinsky et al., 2021). Moreover, the actual driving range is influenced by various factors, including speed profiles, passenger loads, road conditions, and battery temperature (Zeng et al., 2022). Thus, the efficient management of charging schedules becomes essential to support the seamless operations of electric buses. To address this challenge, our research focuses on planning a fast charging strategy, wherein electric buses can complete a quick charging process within minutes during their idle time while in operation. The charging strategies are generally classified into three categories as battery swapping, regular charging, and fast charging (Li, 2016). Diverging from regular charging strategies that require buses to be charged for up to seven hours overnight, our approach aims to optimize fast charging schedules, enhancing the utilization of electric buses without compromising their operational efficiency. By devising efficient charging solutions, we aspire to overcome the driving range limitations, accelerate the adoption of electric buses and pave the way towards greener urban transportation networks.

Additionally, existing studies in the field of demand-responsive electric bus operations have primarily focused on single bus-trip planning and single-spot charging, neglecting the vital aspects of cross-line deployment and multi-spot charging (Liu et al., 2022; Zhou et al., 2022). To enhance practicability for real-world operations, this paper aims to develop a comprehensive and integrated framework for on-demand services tailored to electric buses operating within a transit network involving multiple bus lines and charging spots. In pursuit of this, we propose a novel approach that incorporates a dynamic programming algorithm seamlessly into a time–space network, facilitating the optimization of bus-trip planning across the entire network. Our approach is designed to work within the confines of existing static bus stations and station sequences on each line, while simultaneously accommodating the ability of electric buses to skip intermediate stations based on real customer demand. By utilizing the existing static bus stations and station sequences, our approach lessens the need for significant infrastructural changes or costly upgrades. This ensures a smooth



(a) (b)

Fig. 2. An example of bus services provided (a) in a fixed route and (b) with flexible routes considering stop-skipping strategies (blue nodes: stations with boarding demands; green nodes: stations with alighting demands; white nodes: stations without demands; gray rectangles: bus arrival time). The fixed route and stations are derived from a real bus line in Beijing urban bus system. The bus arrival times are real-world data collected from Route-planning Service provided by the Amap API. The passenger demands are simulated from a real scenario to compose the example. Stations except 3, 5, and 14 on this line have no transit demand over this route planning horizon. There are passenger requests from station 3 to 15 at 7:00, passenger requests from station 5 to 17 at 7:00, and passenger requests from station 14 to 17 at 7:20. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and cost-effective implementation, making it more feasible for transit agencies to adopt our proposed on-demand services for electric buses. Passengers can still rely on the established bus routes they are accustomed to, while electric buses can dynamically adjust their routes to meet real customer demand. This seamless coexistence allows for a more efficient and harmonious transition towards on-demand electric bus services. Ultimately, the advantage of our approach lies in its ability to seamlessly integrate with existing transit infrastructure while catering to changing customer demand.

The following is a condensed summary of the contributions of this paper: First, we present a dynamic programming algorithm for single bus-trip planning (in Section 4). Second, we propose a time–space network model that optimizes the bus-trips of a bus transit network comprising multiple bus lines and charging spots (in Section 5). This model is designed to handle the complex interactions between different lines and efficiently allocate resources. We define time–space nodes and arcs representing customer-serving, bus deploying, and bus charging trips. By selecting the most favorable arcs from the network, we aim to maximize the overall system profit. Finally, to validate the performance of our model, we conduct a comprehensive case study using real-world data (in Section 6). The results demonstrate the effectiveness of our solutions in significantly reducing the overall transit time and the waste of public transit resources. The case study reinforces the practicality and viability of our proposed approach in real-world transit operations.

2. Literature review

2.1. On-demand bus service

The concept of on-demand bus service traces back to its initial implementation during the 1970s. The pioneering work by Flusberg (1976) saw the successful operation of a point deviation bus system in Merrill, Wisconsin, proving the feasibility of on-demand service in lower-density areas. Subsequently, public transit services with similar concepts, such as customized buses (Cao and Wang, 2017) and personal rapid transit systems, have demonstrated steady growth. In conventional operating modes, urban bus system operators rely on historical and survey data to simulate future demand (Zhang et al., 2011). This approach leads to the design of services based on predicted demands within the same time horizon. However, the transition from conventional to on-demand bus services has dramatically changed the relationship between passenger demand and bus supply (Wu et al., 2019). Unlike long-term predetermined scheduling in conventional systems, on-demand bus scheduling can be dynamically re-optimized and updated in response to real-time passenger demand during each practical planning horizon.

A comprehensive survey on on-demand bus services conducted by Vansteenwegen et al. (2022) explores systems with different time resolutions. The time resolution refers to the length of each time interval in the scheduling plan, indicating the time horizon for collecting passenger demand in on-demand bus services. To ensure sufficient buffer time for gathering transit demand (Ma et al., 2013), the time resolution can vary from days to seconds, depending on the system's computational efficiency in re-optimizing bus scheduling. According to the survey, passenger demand typically needs to be stated at least 15 to 45 min before the operation begins (Vansteenwegen et al., 2022). For instance, Liu et al. (2019) propose a bus ride-sharing service where customers use an online hailing app to upload transit demand, and buses are assigned to customers after sufficient matched requests are integrated. Huang et al. (2020) develop a customized bus service, where customers make reservations for transit at least one hour before the operation. Ji-yang et al. (2020) investigate the "many-to-many" pattern, focusing on the key challenge of determining multi-target station allocation. In this context, customers specify their desired origin and destination stations along with their alighting deadlines beforehand. To design an efficient on-demand bus service, Huang et al. (2020) decompose the problem into two phases: the dynamic phase and the static phase. In the dynamic phase, passengers specify their requests, and the decision on whether to provide on-demand service is made. Subsequently, in the static phase, bus scheduling is solved based on the accepted requests. Their model is tested with passengers generated using a uniform distribution. In line with these advancements, the goal of this paper is to equip on-demand bus systems with the capability to efficiently respond to inhomogeneously distributed passenger requests, making them a practical and effective solution for real-world operations.

2.2. Flex-route transit planning

Researchers have explored various operation methods to enhance the performance of demand-oriented flex-route transit systems. Koffman (2004) characterized different flex-route transit strategies, such as line offsets, station offsets, section flexible buses, and regional flexible buses. According to Vansteenwegen et al. (2022), flex-route transit services can be classified as fully-flexible or semi-flexible, depending on whether a basic bus route and timetable are predetermined as a standard for different planning horizons. The study by Mishra and Mehran (2023) reveals the potential of semi-flexible transit to reduce costs for fixed-route users while providing a cost-effective and time-saving alternative for underlying flex-route users. In our research, we propose a semi-flexible operation mode in which buses respond to customer demand at static stations by traveling in a fixed sequence but can skip intermediate stations based on actual demand information.

Previous studies on flex-route transit have mainly focused on operation parameters, mode-improvement strategies, route planning, and integration with other travel modes. For instance, Li and Tang (2023) propose an assisted station strategy and use a simulation-based optimization method to identify suitable locations. Li et al. (2022) apply a hierarchical clustering algorithm to historical transit demand, classifying stations into static and dynamic categories. Introducing the concept of meeting points, Zheng et al. (2019) aim to enhance reliability under uncertain transit demand, allowing customers to board and alight at either their reserved stations or meeting points within walking distance. Lee et al. (2022) propose a zonal-based time–space network, characterizing routes in terms of zonal visits instead of stipulating time at specific nodes. Additionally, Lai et al. (2022) develop an origin–destination insertion algorithm, where vehicles decide whether to accept requests at certain stations based on calculated time loss. In our paper, we propose a novel approach that integrates vehicle capacities and time windows into the definition of trip-based variables for a time–space network. Time–space networks have been applied as solutions to multiple operational problems (An and Lo, 2014; Tong et al., 2017; Zhang et al., 2023). Unlike traditional dense time–space networks with numerous variables, our approach defines time–space nodes as empty-loaded points where buses carry no passengers and time–space arcs as trips performing different functions (passenger carrying, bus deploying, and bus charging), resulting in a sparser network. This innovative approach allows for more efficient and effective optimization of flex-route transit systems.

2.3. Personal rapid transit, customized buses and modular buses

A variety of on-demand transit systems have arisen in response to the growing need for flexible bus services, including personal rapid transit (PRT), customized buses (CB), and modular buses. The concept of PRT system was first defined in the early 1950s (Anderson et al., 1996), and PRT vehicles typically accommodate 2 to 6 passengers sharing the same origin and destination for a trip. Although implementations like the PRT system introduced at Heathrow Airport in 2011 and a similar one in Suncheon, South Korea in 2014 have been observed, PRT systems have not achieved widespread global adoption (Etgar et al., 2023). Their notable feature is their relatively low passenger capacity per operation, which hinders their economic efficiency compared to conventional buses (Mueller and Sgouridis, 2011). In contrast to PRT, customized buses have been rapidly emerging in China since 2013 (Shang et al., 2022). CB systems have gained popularity due to their adaptability to specific clientele, particularly commuters. They operate on a constantly evolving schedule to meet dynamic demand (Ma et al., 2017). The demand-responsive mode of operation gives CB systems an advantage in reducing urban congestion and improving traffic safety (Shang et al., 2022). Additionally, the development

Table 1
Personal rapid transit (PRT), customized buses (CB) and modular buses.

Type	Publication	Demand provision	Station setting	Route planning
Personal rapid transit (PRT)	Lees-Miller et al. (2010)	Historical data	Fixed stations	Fixed grid networks and flexible subpaths
	Mueller and Sgouridis (2011)	Historical data	Fixed stations	Fixed grid networks and flexible subpaths
	Fatnassi et al. (2015)	Pre-booked	Fixed stations	Fixed grid networks and flexible subpaths
	Elkamel et al. (2021)	Historical data	Fixed stations	Fixed serving routes and flexible recharging routes
	Etgar et al. (2023)	Pre-booked	Fixed stations	Fixed loops and single directions
Customized buses (CB)	Tong et al. (2017)	Pre-booked	Fixed stations	Flexible routes between two stations
	Guo et al. (2019)	Pre-booked	Fixed stations	Flexible routes with different number of stations
	Ma et al. (2020)	Real-time	Fixed stations (separate boarding and alighting)	Flexible routes between boarding and alighting stations
	Chen et al. (2021)	Pre-booked	Flexible stops generated on demand	Flexible serving areas based on demand
	Shang et al. (2022)	Pre-booked demand, real-time cancellation	Fixed stations	Flexible routes along central axis
Modular buses	Liu et al. (2021)	Pre-booked	Fixed checkpoints, flexible points generated on demand	Flexible routes covering demand points
	Chen and Li (2021)	Pre-booked	Fixed stations	Flexible routes between boarding and alighting stations
	Gong et al. (2021)	Pre-booked	Fixed stations	Flexible routes between boarding and alighting stations
	Tang et al. (2023)	Pre-booked	Fixed stations on base routes, dynamic nodes in deviation segments	Fixed base routes and flexible door-to-door subpaths

of new vehicle types has contributed to the flexibility of transit services. Modular buses represent an emerging demand-oriented transit solution that addresses the imbalance between public transport supply and demand in terms of bus capacity adjustment. These buses have the unique ability to dock and undock within a fleet, enabling some modular buses to detach and provide door-to-door services, offering flexibility and customization (Liu et al., 2021; Wu et al., 2021; Tang et al., 2023). Table 1 summarizes the key operational characteristics of these three transit systems. As depicted in the table, the primary demand provision is passenger pre-booking, though the time resolution (the length of time for a planning horizon) may vary. Regarding station settings, most studies build their operational schemes around fixed stations in existing bus networks. Some models introduce flexible nodes for boarding and alighting to accommodate passengers with longer walking distances. Furthermore, most of these studies dynamically adjust their routes based on fixed networks or static stations, with some allowing a portion of the bus fleet to deviate from the main route to offer door-to-door service.

2.4. Charging scheduling of electric buses

The implementation of policies in numerous nations has fueled a growing demand for electric buses, posing a challenge for operators to efficiently manage electric bus systems. The existing literature has predominantly focused on tackling the charging scheduling problem, which involves optimizing a bus dispatching and charging scheme while considering electric consumption during operation and adhering to state-of-charge constraints. Previous studies primarily aim to minimize operational costs and propose charging scheduling strategies based on the assumption of a single charging spot with charging infrastructure. For instance, Tang et al. (2019) present a robust optimal charging scheduling strategy, specifically designed for a single-charging-spot scenario. Zhou et al. (2020) decompose electric bus operation into two phases and propose a Bi-level planning model with charging scheme considerations based on a single charging spot. However, relying solely on a single charging spot can limit the flexibility and reliability of electric bus systems in meeting real-time charging demands. To address this limitation, researchers have explored scenarios involving multiple charging spots to enhance the level of electric bus operations. For example, Bie et al. (2021) devise a strategy where electric buses are charged during their idle time using charging piles at different spots. Wen et al. (2016) formulate an optimization model to minimize fleet size and total traveling range, incorporating a charging scheme with multiple charging spots. Additionally, Gkiotsalitis et al. (2023) propose a framework for multi-charging-spot deployment with time-windows, allowing electric buses to charge at any charging spot within the operation area. Despite the consideration of either single or multiple charging spots in charging scheduling studies, there is often insufficient emphasis on collaborative optimization that considers both bus charging scheduling and passenger service. In our research, we strategically position charging spots near the origins or destinations of bus lines, which serve as nodes in a time–space network. We then define time–space arcs, composed of different nodes, and select suitable arcs to create routes for each electric bus. Building on this approach, we solve the bus charging planning problem with multiple charging locations, while concurrently providing a cost-effective on-demand bus service. By integrating bus charging and passenger serving considerations, our proposed algorithm aims to optimize the overall performance of electric bus systems and enhance their operational efficiency and reliability.

3. Assumptions

The following is key assumptions of this paper:

– Effects of Traffic on Bus Operations: It is assumed that exclusive bus lanes can effectively mitigate the impact of highly congested traffic during peak hours. This assumption implies the constancy of key operational parameters such as average driving speed, travel time, dwell time, and others.
– Data Provision: Accurate data encompassing bus lines, station details, and travel times between stations at the road network level are presumed to be available prior to the commencement of bus operations. Additionally, it is assumed that travel demand data is gathered before each planning horizon through passenger pre-booking, supplying origin–destination (OD) information.
– Passenger Demand Occurrence: In each planning horizon, passengers are expected to pre-book their trips before the actual operation. Real-time cancellations of booked trips are not considered within this study. Any emergent travel demand during ongoing operations is treated as pre-booked demand for the subsequent planning horizon.
– Bus Traveling Policy: Buses are constrained to adhere to predetermined station sequences along a single line. However, they retain the ability to omit specific stops. Inter-line bus dispatch is exclusively permitted when a bus reaches the destination depot of one line and transitions to the origin depot of another line.
– Bus Dwell Time: It is assumed that dwell time, signifying the duration buses spend at stations, remains fixed and is not influenced by passenger demand.
– Bus Energy Consumption and Charging: The state of battery energy in electric buses is represented as battery duration time, indicating the duration a bus can travel with its current remaining battery power. Concerning charging, a constant charging coefficient is assumed, establishing a linear relationship between unit charging time and unit energy charged.

4. Single bus-trip planning

Given a bus line with a sequence of stations $\{0(\text{“fixed origin”}), 1, \dots, K, K+1(\text{“fixed destination”})\}$, a capacitated bus starts idle and available for dispatch at time $\overline{v}$ from a fixed origin station $k = 0$, then follows the sequence of stations, but has the flexibility to skip some intermediate stations, and finally reaches a fixed destination station $k = K + 1$. The travel time between any two bus stations is denoted as $t_{kk'}$, where $k' > k$ and $k, k' \in \{0, 1, \dots, K, K + 1\}$. The single bus trip planning determines the bus stations to be visited and the corresponding boarding/alighting times for passengers (referred to as *Bus timetabling* and *Passenger itinerary* in Section 4.1, respectively). To achieve this, we introduce a label-setting dynamic programming algorithm (described in Section 4.2) that generates feasible single bus-trip plans, considering different accepted passengers. These plans will later be incorporated into the multiple bus-trip planning in the subsequent section.

4.1. Problem background

Bus timetabling

The bus trip involves making stops at up to K bus stations, which are indexed as $\mathcal{I} = \{1, \dots, I\}$, where $I \le K$. We conventionally designate stop $i = 0$ as the fixed origin $k = 0$, and stop $i = I + 1$ as the fixed destination $k = K + 1$. The single bus-trip planning is responsible for determining the selected bus stations and their corresponding arrival times as follows:

s_i =the index of the bus station visited at stop $i \in \mathcal{I}$,

v_i = the time when the bus arrives at stop $i \in \mathcal{I} \cup \{0, I + 1\}$.

Subsequently, a bus timetable can be constructed as $\{(0, \overline{v}), (s_1, v_1), (s_2, v_2), \dots, (s_i, v_i), \dots, (K+1, v_{I+1})\}$, which can then be provided to a bus driver. We assume that each stop results in a bus dwell time ϵ, which remains independent of the number of boarding/alighting passengers. The time of arrival at each stop is calculated as $v_i = v_{i-1} + t_{s_{i-1},s_i} + \epsilon$, where $s_i > s_{i-1}$.

Passenger itinerary

We take as inputs n passenger requests $\mathcal{P} = \{1, \dots, n\}$ for the given bus line. For each $j \in \mathcal{P}$, q_j represents the number of passengers of this request traveling from a “boarding” bus station B_j to an “alighting” bus station A_j, along with the corresponding bus fare g_j. It is noted that $B_j \in \{1, \dots, K\}$ and $A_j \in \{B_j + 1, \dots, K\}$. A single-trip plan involves determining which passengers are to be served, along with their respective boarding and alighting times:

$$y_j = \begin{cases} 1 & \text{if passenger } j \in \mathcal{P} \text{ is served in this trip,} \\ 0 & \text{otherwise,} \end{cases}$$

v_j^b, v_j^a =the boarding time, alighting time of the served passenger $j \in \mathcal{P}$.

Each passenger has an earliest boarding time $\overline{E}_j \geq 0$ and a deadline (latest alighting time) $\overline{T}_j \geq 0$:

$$\overline{E}_j \leq v_j^b \quad \text{and} \quad v_j^a \leq \overline{T}_j.$$

Consequently, passenger itineraries can be defined as $\{(v_j^b, v_j^a), j \in \{\mathcal{P} : y_j = 1\}\}$, which are then sent to the accepted passengers.

If passenger j is accepted, the corresponding boarding station B_j and alighting station A_j must be visited by the bus. The boarding time v_j^b and alighting time v_j^a are determined when the bus arrives at bus station B_j and A_j, respectively. A valid bus-trip plan must satisfy the bus capacity limitation Q. Each passenger request indicates the number of passengers q_j to board. As a convention, each alighting has a negative number of passengers $-q_j$.

Ultimately, each bus-trip plan consists of accepted passenger itineraries $\{(v_j^b, v_j^a), j \in \{\mathcal{P} : y_j = 1\}\}$, along with a valid bus timetable $\{(0, \overline{v}), (s_1, v_1), (s_2, v_2), \ldots, (s_i, v_i), \ldots, (s_I, v_I), (K+1, v_{I+1})\}$. Following the bus-trip plan, a bus departs from the origin at time $\overline{v}$, sequentially stops at bus station s_1 at time v_1, then at bus station s_2 at time v_2, and so on, until it stops at bus station s_I at time v_I, before finally arriving at the destination at time v_{I+1}. The passengers $\{\mathcal{P} : y_j = 1\}$ are served during this bus trip, with each passenger $j \in \{\mathcal{P} : y_j = 1\}$ boarding at time v_j^b from the requested boarding station B_j and alighting at time v_j^b at the requested alighting station A_j.

4.2. Label-setting dynamic programming algorithm

In this section, we introduce the label-setting dynamic programming algorithm, a fundamental component of our approach to generating efficient bus-trip plans.

4.2.1. Algorithm inputs

Our algorithm tackles the following inputs:

(i) A Bus Line: The bus line is defined by a sequence of bus stations $\{0(\text{"origin"}), 1, \ldots, K, K+1(\text{"destination"})\}$, including the origin (station 0) and the final destination (station K+1).
(ii) Initial Bus Status: At a given time $\overline{v}$, we have an empty bus stationed at the origin (station 0), ready to embark on its journey.
(iii) Passenger Requests: A set of passenger requests $\mathcal{P} = \{1, \ldots, n\}$, each characterized by:

  – The number of passengers (q_j) needing to board at bus station B_j.
  – An earliest boarding time ($\overline{E}_j$), which ensures passengers are not left waiting excessively.
  – A deadline ($\overline{T}_j$) for alighting at bus station A_j, ensuring passengers reach their destinations on time.

4.2.2. Algorithm overview

Our algorithm systematically extends states to manage passengers following the sequence of bus stations $\{1, \ldots, K+1\}$. The process involves several steps: For bus station $k \in \{1, \ldots, K+1\}$:

Step (i): If there are alighting passengers at the current station k, the bus must visit this station k, leading to Step (ii). Alternatively, two streams of possibilities are considered:

– Stream 1: The bus visits station k, and the algorithm proceeds to Step (iii).
– Stream 2: The bus bypasses station k and moves directly to the next bus station $k+1$.

Step (ii): All on-board passengers $j \in \{\mathcal{P} : A_j = k, y_j = 1\}$ who need to alight at station k must be dropped off. A new state is created to represent this alighting process (see *State transitions for alighting*). Afterward, the algorithm updates the remaining bus capacity and proceeds to Step (iii).

Step (iii): The algorithm evaluates whether passenger $j \in \{\mathcal{P} : B_j = k\}$ boards at current station k. Multiple states are generated to represent all possible boarding scenarios (see *State transitions for boarding*). The process then moves to the next bus station $k+1$.

4.2.3. State representation

We define $(i^l, k^l, Q^l, \mathbb{S}^l, \mathbb{K}^l)$ as a state in our label-setting dynamic programming algorithm, which indicates the current status of a bus: i^l tracks the "current" stop, k^l tracks the "current" bus station, Q^l tracks the number of passengers on the bus, $\mathbb{S}^l \subseteq \mathcal{P}$ tracks the served passengers, and $\mathbb{K}^l \subseteq \{1, \ldots, K\}$ tracks the subsequent un-visited bus stations that need to be visited (i.e., have alighting passengers). For every state, we keep track of the time as $v(i^l)$. The initial state is $(i^0 = 0, k^0 = 0, Q^0 = 0, \mathbb{S}^0 = \emptyset, \mathbb{K}^0 = \emptyset)$, with $v(i^0) = \overline{v}$. Notably, from the state $(i^l, k^l, Q^l, \mathbb{S}^l, \mathbb{K}^l)$, we can extract bus timetabling information:

– $s_{i^l} = k^l$: the index of the bus station that is visited at stop i^l;
– $v_{i^l} = v(i^l)$: the time when the bus arrives at stop i^l;

and also extract passenger itineraries information:

– $y_j = 1$ for $j \in \mathbb{S}^l$: the passengers served in this trip;
– $v_j^b = v(i^l)$ for $j \in \{\mathbb{S}^l : B_j = k^l\}$: the boarding times of the passengers who need to board at this station;
– $v_j^a = v(i^l)$ for $j \in \{\mathbb{S}^l : A_j = k^l\}$: the alighting times of the passengers who need to alight at this station.

4.2.4. State transitions

State transitions for alighting. Here we present the process of dropping off all on-board passengers who need to alight at this bus station. For each bus station $k' \in \{k^l, \dots, K+1\}$, the state is updated to $(i^{l'}, k^{l'}, Q^{l'}, \mathbb{S}^{l'}, \mathbb{K}^{l'})$, such that:

- $i^{l'} = i^l + 1$ if $k' > k^l$ and $i^{l'} = i^l$ if $k' = k^l$: the sequence number of the stop is updated only when the bus moves to a subsequent different bus station;
- $k^{l'} = k'$;
- $Q^{l'} = Q^l - \sum_{j' \in \{\mathbb{S}^l : A_{j'} = k'\}} q_{j'}$: to drop off all the on-board passengers who need to alight at this bus station.
- $\mathbb{S}^{l'} = \mathbb{S}^l$;
- $\mathbb{K}^{l'} = \mathbb{K}^l \setminus \{k'\}$.

This transition is admissible if:

- $k' \in \mathbb{K}^l$: there exist on-board passengers who need to alight at the current bus station k';
- $\{1, \dots, k'-1\} \notin \mathbb{K}^l$: all passengers who need to alight before the current bus station k' (if they exist) have been dropped off.

State transitions for boarding. Here we present the process of picking up one of those passengers who need to board at this bus station. For each bus station $k' \in \{k^l, \dots, K+1\}$, for each passenger $j' \in \{\mathcal{P} : B_{j'} = k'\}$, the state is updated to $(i^{l'}, k^{l'}, Q^{l'}, \mathbb{S}^{l'}, \mathbb{K}^{l'})$, such that:

- $i^{l'} = i^l + 1$ if $k' > k^l$ and $i^{l'} = i^l$ if $k' = k^l$: the sequence number of the stop is updated only when the bus moves to a subsequent different bus station;
- $k^{l'} = k'$;
- $Q^{l'} = Q^l + q_{j'}$;
- $\mathbb{S}^{l'} = \mathbb{S}^l \cup \{j'\}$: update passenger j' as a served passenger;
- $\mathbb{K}^{l'} = \mathbb{K}^l \cup \{A_{j'}\}$ if $A_{j'} \notin \mathbb{K}^l$: the bus has to stop at the passenger's alighting bus station $A_{j'}$.

This transition is admissible if:

- $k' \notin \mathbb{K}^l$: all alighting passengers (if they exist) have been dropped off at bus station k' (i.e., alighting is performed before boarding, so that the remaining capacity of the bus can be realized);
- $\{1, \dots, k'-1\} \notin \mathbb{K}^l$: all on-board passengers who need to alight before the current bus station k' (if they exist) have been dropped off;
- $Q^l + q_{j'} \le Q$: the bus has sufficient remaining capacity;
- $j' \notin \mathbb{S}^l$: the bus can only serve passenger request j' if they have not been served yet.

4.2.5. Enforcing the earliest boarding time and deadline

Whenever we extend a boarding passenger j^l, we perform a check to ensure that the earliest boarding time constraint is satisfied: $v(i^l) \ge \overline{E}_{j^l}$. If this constraint is violated, state l is considered infeasible and can be deleted. Additionally, we conduct a multi-step forward feasibility check to enforce deadlines. For each must-visit and un-visited bus station $k \in \mathbb{K}^l$ (i.e., bus stations where alighting passengers must be dropped off in the future), we calculate the bus time at station k and further assess whether all the served passengers $j \in \{\mathbb{S}^l : A_j = k\}$ can alight by their respective deadlines $\overline{T}_j$. If any of the served passengers cannot alight within their specified deadlines, state l is deemed infeasible and can be deleted.

4.2.6. Acceleration strategy

As previously mentioned, extending state for "each" boarding passenger generates "all" possible bus-trip plans, which can be time-consuming. To address this, we introduce an acceleration strategy after *State transitions for boarding* to efficiently pick up "as many as possible" of the passengers that need to board at this bus station and alight at must-visit and un-visited bus stations. For state $(i^{l'}, k^{l'}, Q^{l'}, \mathbb{S}^{l'}, \mathbb{K}^{l'})$, we define the set of passengers as $\mathcal{J}^{l'} = \{j \in \mathcal{P} : B_j = k^{l'}, A_j \in \mathbb{K}^{l'}, j \notin \mathbb{S}^{l'}\}$ (i.e., indicating that no additional stop needs to be created for these passengers $\mathcal{J}^{l'}$). We then identify the subset of passengers, denoted as $\overline{\mathcal{J}}^{l'} \subseteq \mathcal{J}^{l'}$, that satisfy the earliest boarding time and deadlines, as well as the capacity limitation. If the capacity is the limiting factor, $\overline{\mathcal{J}}^{l'}$ comprises the set of passengers that can generate the highest fare income while sacrificing the others. Subsequently, we update the bus load as $Q^{l'} \leftarrow Q^{l'} + \sum_{j \in \overline{\mathcal{J}}^{l'}} q_j$ and include the served passengers in $\mathbb{S}^{l'} \leftarrow \mathbb{S}^{l'} \cup \overline{\mathcal{J}}^{l'}$. By incorporating this acceleration strategy, we generate only "some" promising bus-trip plans instead of "all" possible ones, leading to significant efficiency improvements.

5. Multiple bus-trip planning

In the context of a bus transit network with multiple bus lines $\mathcal{L}$, the multiple bus-trip planning encompasses several critical aspects. First, it determines which single bus-trip plan, generated by Section 4, should be selected for buses to efficiently serve passengers. Second, it addresses the flexible deployment of buses within the bus transit network, allowing for re-deployment of buses from one bus line to another. Lastly, it optimizes the scheduling of charging activities for electric buses, considering a set of charging spots C. For each bus line $l \in \mathcal{L}$, let O^l be the origin and D^l be the destination. Each charging spot $c \in C$ is strategically located at either the origin or destination of a specific bus line $k^c \in \{O_l, l \in \mathcal{L}\} \cup \{D_l, l \in \mathcal{L}\}$. Furthermore, we are provided with a fleet of electric buses denoted by $\mathcal{V}$, where each bus $r \in \mathcal{V}$ is available for operation at time v_r, located at its corresponding bus origin O_r, and equipped with a residual battery power duration denoted by F_r. In this section, we introduce the concept of time–space arcs to effectively describe bus flows and present our time–space network model, which serves as a powerful tool to optimize multiple bus-trip plans.

5.1. Time–space nodes

Let $\mathcal{S}$ represent a set of time–space nodes, indexed by s and characterized by their respective time $t(s)$ and location $L(s)$. The locations $L(s)$ can indicate either the origin or destination of a bus line or a specific charging spot. We divide the planning horizon $[0, \overline{T}]$ into discrete time intervals of length κ. The set of time–space nodes $\mathcal{S}$ is the union of five subsets:

- Initialization time–space nodes $\overline{\mathcal{S}}$: these time–space nodes correspond to the bus stations O_r where the buses $r \in \mathcal{V}$ are available at times v_r. It is the collection, across all buses $r \in \mathcal{V}$, of the nodes defined by $L(s) = O_r$ and $t(s) = \lceil v_r/\kappa \rceil \kappa$.
- Finalization time–space node $\hat{s}$: this time–space node indicates the buses' (dummy) final location and final time $t(\hat{s}) = \lceil \overline{T}/\kappa \rceil \kappa$.
- Bus-trip origin time–space nodes $\{\mathcal{S}_O^l,\ l \in \mathcal{L}\}$: for each origin O^l of a bus line $l \in \mathcal{L}$, we define a node s with $L(s) = O^l$ and $t(s)$ taking values from $\left\{0, \kappa, 2\kappa, \dots, \lceil \overline{T}/\kappa \rceil \kappa\right\}$.
- Bus-trip destination time–space nodes $\{\mathcal{S}_D^l,\ l \in \mathcal{L}\}$: for each destination D^l of a bus line $l \in \mathcal{L}$, we define a node s with $L(s) = D^l$ and $t(s)$ taking values from $\left\{0, \kappa, 2\kappa, \dots, \lceil \overline{T}/\kappa \rceil \kappa\right\}$.
- Charging time–space nodes $\mathcal{S}_c$: for each charging spot $c \in \mathcal{C}$, we define a node s with $L(s) = c$ and $t(s)$ taking values from $\left\{0, \kappa, 2\kappa, \dots, \lceil \overline{T}/\kappa \rceil \kappa\right\}$.

5.2. Time–space arcs

Based on the defined time–space nodes, we establish the time–space arcs as follows:

Passenger carrying

For each bus line $l \in \mathcal{L}$, we define the set $\mathcal{U}^l_{passenger}$ of time–space arcs for passenger carrying. Each arc starts at a bus-trip origin time–space node $s \in \mathcal{S}_O^l$ and ends at a bus-trip destination time–space node $s' \in \mathcal{S}_D^l$. These arcs represent buses starting empty at location $L(s) = O^l$ and time $t(s)$, serving passengers by stopping at selected bus stations, and returning to location $L(s') = D^l$ at time $t(s')$. These arcs are generated using the label-setting dynamic programming algorithm described in Section 4. The algorithm starts from each bus-trip origin time–space node $s \in \mathcal{S}_O^l$, indicating an empty bus available for dispatch at time $\overline{v} = t(s)$.

Bus deployment

Let $\mathcal{U}_{deployment}$ be the set of time–space arcs for all possible bus deployment plans, which includes the following cases: (i) Arcs starting at bus-trip destination time–space nodes $s \in \{\mathcal{S}_D^l, l \in \mathcal{L}\}$ and ending at bus-trip origin time–space nodes $s' \in \{\mathcal{S}_O^l, l \in \mathcal{L}\}$: These arcs represent empty buses being re-deployed from the destination of a bus line to the origin of this or another bus line. (ii) Arcs starting at initialization time–space nodes $s \in \overline{\mathcal{S}}$ and ending at bus-trip origin time–space nodes $s' \in \{\mathcal{S}_O^l, l \in \mathcal{L}\}$: These arcs represent buses being dispatched from their initial locations to the origin of a bus line. (iii) Arcs starting at bus-trip destination time–space nodes $s \in \{\mathcal{S}_D^l, l \in \mathcal{L}\}$ and ending at the finalization time–space node $\hat{s}$: These arcs indicate buses finishing their trips. For all these arcs, $t(s') - t(s)$ is equal to the travel time between the two locations $L(s)$ and $L(s')$.

Bus charging

Let $\mathcal{U}_{charging}$ denote the set of time–space arcs for bus charging. Each arc connects two charging time–space nodes $s \in \mathcal{S}_c$ and $s' \in \mathcal{S}_c$, with $L(s) = L(s')$ to indicate the same charging spot. The time difference is $t(s) = t(s') - 1$ to indicate that a bus performs one unit of charging from time $t(s)$ to time $t(s')$. Additionally, let $\overline{\mathcal{U}}_{charging}$ represent a set of time–space arcs for dummy trips, which involves buses being repositioned to/from charging spots. These arcs connect charging time–space nodes $s \in \{\mathcal{S}_c, l \in \mathcal{L}\}$ and bus-trip origin/destination time–space nodes $s' \in \{\mathcal{S}_O^l, l \in \mathcal{L}\} \cup \{\mathcal{S}_D^l, l \in \mathcal{L}\}$ where the charging spot is located. Here, $L(s') = k^{L(s)}$ and $t(s') = t(s)$ to represent a dummy trip "leaving" the bus charging spot after charging (from s to s') or "heading to" the bus charging spot before charging (from s' to s). These arcs facilitate proper charging and positioning of buses in the network.

5.3. Time–space network model

Let $\mathcal{U} = \{\mathcal{U}^l_{passenger}, l \in \mathcal{L}\} \cup \mathcal{U}_{deployment} \cup \mathcal{U}_{charging} \cup \overline{\mathcal{U}}_{charging}$. We use $\mathcal{U}_s^+$ and $\mathcal{U}_s^-$ to store the time–space arcs starting from and ending at time–space node $s \in \mathcal{S}$, respectively. For each $u \in \mathcal{U}$, $b(u)$ represents the starting time–space node, and $e(u)$ represents the ending time–space node. Let $\hat{\mathcal{P}}$ be the set of all passenger requests for multiple bus lines $\mathcal{L}$ under consideration. For each $u \in \mathcal{U}$, we use a^{uj} as a binary indicator equal to 1 if customer j is served by arc $u \in \mathcal{U}$, and 0 otherwise. The fare income for customer j is denoted as g_j. The charging coefficient, indicating that one unit of charging period can supply $\boldsymbol{\eta}$ units of traveling time consumption, is denoted as $\boldsymbol{\eta}$. The maximum battery power duration is represented by $\overline{F}$, and c is the cost for a unit of travel time. The decision variables for our time–space network model are as follows:

$$z^u = \begin{cases} 1 & \text{if one of the buses traverses time–space arc } u \in \mathcal{U} \\ 0 & \text{otherwise} \end{cases}$$

$$x_j = \begin{cases} 1 & \text{if passenger request } j \in \hat{\mathcal{P}} \text{ is accepted} \\ 0 & \text{otherwise} \end{cases}$$

Λ =the number of buses used to serve passengers

f_s =the residual battery power duration of a bus upon arrival at node $s \in \mathcal{S}$.

For $s \in \overline{\mathcal{S}}$, if s is defined by bus r, we denote $\hat{f}_s$ as the residual battery power duration F_r at the time when bus r is available for dispatch. Our time–space network model is formulated as follows:

$$\max \sum_{j\in\hat{\mathcal{P}}} g_j x_j - \sum_{u\in\mathcal{U}\setminus\mathcal{U}_{charging}} c(t(e(u)) - t(b(u)))z^u, \tag{1}$$

$$\text{s.t.} \sum_{u\in\mathcal{U}_s^+} z^u - \sum_{u\in\mathcal{U}_s^-} z^u = 0,\ \forall s \in \{\mathcal{S}_O^l, l\in\mathcal{L}\} \cup \{\mathcal{S}_D^l, l\in\mathcal{L}\} \cup \mathcal{S}_c, \tag{2}$$

$$\sum_{u\in\mathcal{U}_s^+} z^u - \sum_{u\in\mathcal{U}_s^-} z^u \le 1,\ \forall s \in \overline{\mathcal{S}}, \tag{3}$$

$$\sum_{s\in\overline{\mathcal{S}}} \left(\sum_{u\in\mathcal{U}_s^+} z^u - \sum_{u\in\mathcal{U}_s^-} z^u \right) = \Lambda, \tag{4}$$

$$\sum_{u\in\mathcal{U}_s^+} z^u - \sum_{u\in\mathcal{U}_s^-} z^u = -\Lambda,\ \forall s \in \{\hat{s}\}, \tag{5}$$

$$\sum_{u\in\{\mathcal{U}_{passenger}^l, l\in\mathcal{L}\}} a^{uj} z^u \ge x_j,\ \forall j \in \hat{\mathcal{P}}, \tag{6}$$

$$f_{b(u)} - (t(e(u)) - t(b(u)))z^u + \overline{F}(1 - z^u) \ge f_{e(u)}, \forall u \in \{\mathcal{U} \setminus \mathcal{U}_{charging} : e(u) \ne \hat{s}\}, \tag{7}$$

$$f_{b(u)} - (t(e(u)) - t(b(u)))z^u - \overline{F}(1 - z^u) \le f_{e(u)}, \forall u \in \{\mathcal{U} \setminus \mathcal{U}_{charging} : e(u) \ne \hat{s}\}, \tag{8}$$

$$f_{b(u)} + \boldsymbol{\eta}(t(e(u)) - t(b(u)))z^u + \overline{F}(1 - z^u) \ge f_{e(u)}, \forall u \in \mathcal{U}_{charging}, \tag{9}$$

$$f_{b(u)} + \boldsymbol{\eta}(t(e(u)) - t(b(u)))z^u - \overline{F}(1 - z^u) \le f_{e(u)}, \forall u \in \mathcal{U}_{charging}, \tag{10}$$

$$\sum_{u\in\mathcal{U}_s^-} z^u \le 1, \forall s \in \mathcal{S} \setminus \{\hat{s}\}, \tag{11}$$

$$f_s = \hat{f}_s, \forall s \in \overline{\mathcal{S}}, \tag{12}$$

$$0 \le f_s \le \overline{F}, \forall s \in \mathcal{S}, \tag{13}$$

$$0 \le \Lambda \le |\mathcal{V}|, \tag{14}$$

$$z^u \in \{0, 1\},\ \forall u \in \mathcal{U}, \tag{15}$$

$$x_j \in \{0, 1\},\ \forall j \in \hat{\mathcal{P}}. \tag{16}$$

Eq. (1) maximizes the profit. Eqs. (2)–(5) represent the flow balance constraints. Eq. (6) determines whether a passenger is served or not. Eqs. (7)–(10) ensure that the residual battery power duration is sufficient for any selected time–space arc. Eq. (11) guarantees that any time–space node (except the dummy finalization time–space node) can only be selected once. Eq. (12) specifies the buses' residual battery power duration at the beginning of dispatch. Eqs. (13)–(16) enforce the decision variables.

6. Case study

6.1. Experimental setup

We conducted our study using real-world data from 74 bus lines in Beijing, known as special-lines (Fig. 3). Special-lines exclusively serve micro-circulation bus routes, covering specific communities that regular bus lines cannot access. The total length of each line ranges from 2 km to 8 km, and each line consists of 4 to 10 bus stations. To construct bus driving times, we utilized the Route-planning Service provided by the Amap API, assuming an average bus speed of 20 km/h. Additionally, we used demand data from a dataset containing public transport trip-chains in Beijing for October 2019. The data was collected and provided by Beijing Public Transport. Notably, the Beijing bus system required customers to swipe their cards when both boarding and alighting, ensuring the accuracy of trip chains. The dataset contains records for each passenger, including their user ID, trip mode, boarding and alighting times, boarding and alighting station locations (specified in terms of latitude and longitude), as well as the names of each line and station involved.

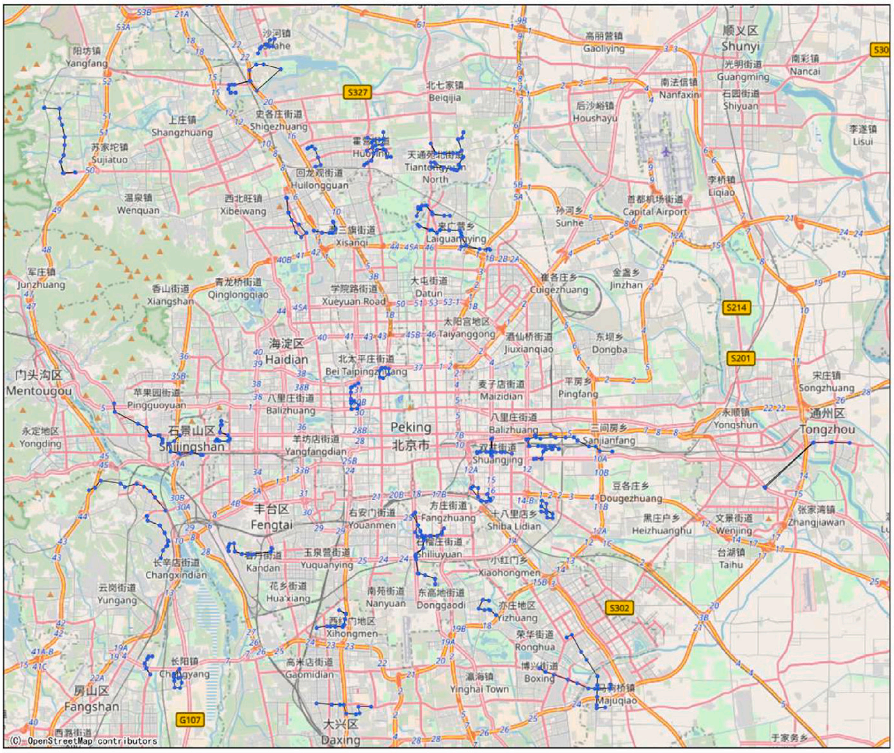


Fig. 3. 74 Beijing special-lines under consideration (with stations marked as blue circles). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Our analysis focused on trips originating between 7:10 AM and 7:40 AM on October 14th, 2019, resulting in 2364 customer requests over the 30-min horizon. Fig. 4 highlights the inhomogeneous distribution of transit demands. Among the 569 stations considered, only 12 received more than 50 customer visits, and these stations were concentrated on 9 particular lines. Most stations received less than 20 passenger requests in total. As a result, operating all 74 lines in the conventional fixed-route mode would lead to a high percentage of empty buses and unnecessary increases in overall transit time for customers. Compared to regular bus lines with more evenly distributed demand, the transition to an on-demand operating mode is an urgent need for special-lines. To illustrate the need of on-demand operation, we examined Line Longjinyuan-Huoying as an example (Fig. 5). This line is among the most visited, with a total cross-sectional passenger flow of 151. In the conventional mode, the bus follows a fixed route (black line) and visits all static stations (blue circles). However, boarding and alighting demands are concentrated at the fourth and ninth stations, while intermediate stations experience relatively sparse passenger flow. For this line, the block pattern road network allows significant reductions in travel time between distant stations by skipping intermediate stops.

To create realistic instances for our study, we manipulated available passenger data to reflect practical scenarios. We set each passenger's earliest boarding time to be 10 min earlier than their actual boarding time from the data. This adjustment effectively simulates a 10-min waiting time at the bus station. This simulated waiting time was introduced as a practical approximation, given the data available to us. The deadline $\overline{T}_j$ for alighting was set as the real alighting time from the data. We followed the Beijing Public Transport Pricing Scheme, setting the fare accordingly (2 RMB for the first ten kilometers and 1 RMB for each additional five kilometers). As the available information is for each passenger, we have $q_j = 1$. We considered different scenarios with varying numbers of buses (100, 200, 300, and 500), each with a capacity of 30 passengers. Without loss of generality, the availability of buses at the beginning of the planning was set to 50%, with the remaining 50% becoming available later on (v_r uniformly sampled between 7:00 AM to 7:30 AM) at a destination bus station. The bus dwell time ϵ was set at 90 s for each stop-off. We assumed an electric bus cost of 15 RMB per hour, with two charging spots at each origin and destination bus station for each line and a charging coefficient $\boldsymbol{\eta}$ of 3. The maximum battery time $\overline{F}$ was set at 5 h,[1] and each bus's initial battery time F_r was uniformly sampled between 0 and 5 h when it became available.

[1] In general, an electric bus can achieve a driving range of approximately 75 km to 100 km on a full charge (Schücking et al., 2017; Liu et al., 2023; Zeng and Qu, 2023). In our case study, with speed of 20 km/h, $\overline{F}$ was set at 5 h.

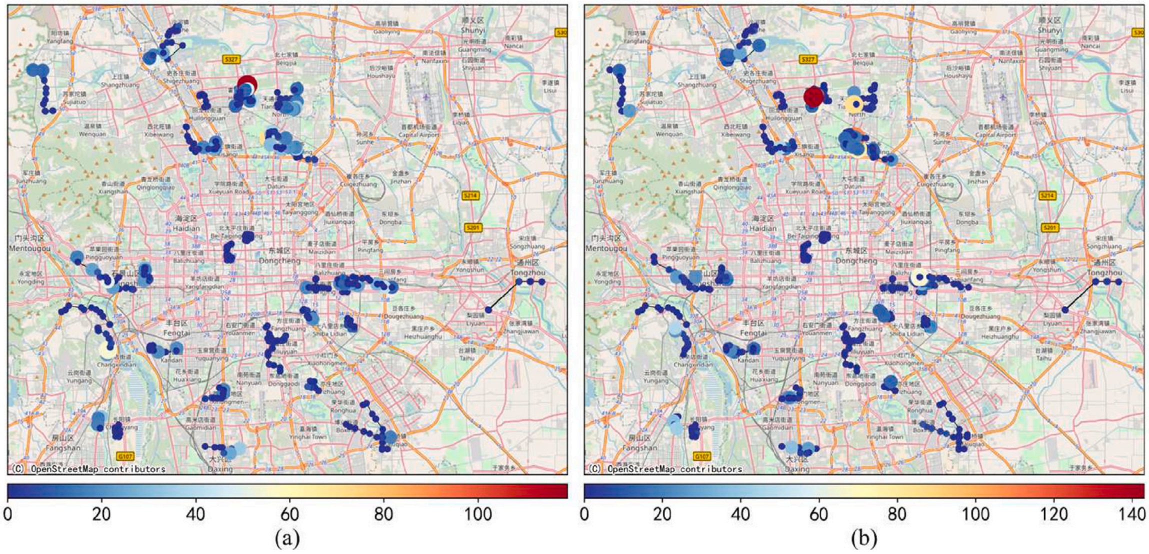


Fig. 4. Distribution of (a) boarding and (b) alighting demand for each station over the 30-min horizon. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

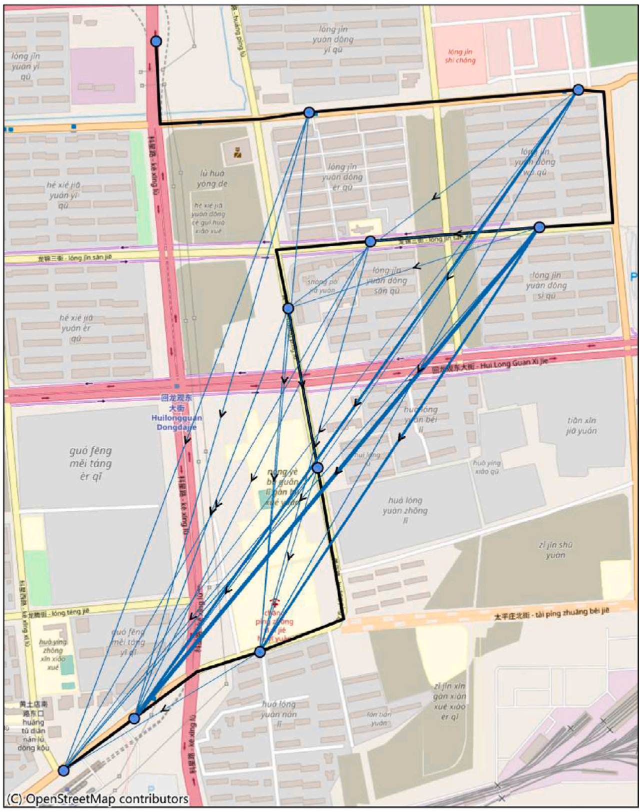


Fig. 5. Fixed-route for Line Longjinyuan-Huoying and trips between stations (the width of lines connecting two stations represents the number of passenger requests from boarding stations to alighting stations). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Computational results with 100, 200, 300 and 500 buses.

Number of buses	100	200	300	500
(I) Profit and customer acceptance				
Total customers	2364	2364	2364	2364
Accepted customers	1652	1905	2075	2132
Acceptance rate	70%	81%	88%	90%
Profit	2840	3243	3548	3630
Revenue	3485	4002	4344	4459
Cost	645	759	796	829
(II) Bus route and bus stop visits				
In-service line	47	50	50	51
Not-in-service line	27	24	24	23
In-service line (%)	64%	68%	68%	69%
Avg number of visits of an in-service line	1.91	2.38	2.60	2.69
In-service stop	256	292	304	318
Not-in-service stop	313	277	265	251
In-service stop (%)	45%	51%	53%	56%
Avg number of visits per in-service stop	1.58	1.81	1.93	1.96
Avg number of in-service stops per in-service line	4.50	4.44	4.52	4.54
Standard deviation of headway per in-service line (Avg)	1058	1120	1123	1109
(III) Customer boarding/alighting, customer in-vehicle time, waiting time and total transit time				
Avg number of boarding/alighting for a line visit	38.22	34.20	33.66	33.52
Avg number of boarding/alighting for a stop visit	8.48	7.70	7.41	7.37
Original in-vehicle time	1279	1267	1261	1261
Optimized in-vehicle time	576	572	573	590
In-vehicle time reduction	703	695	688	671
In-vehicle time reduction (%)	55%	55%	55%	53%
Optimized waiting time	431	409	380	350
Waiting time reduction	169	191	220	250
Waiting time reduction (%)	28%	32%	37%	42%
Original overall transit time	1879	1867	1861	1861
Optimized overall transit time	1007	981	953	939
Overall transit time reduction	873	886	908	922
Overall transit time reduction (%)	46%	47%	49%	50%
(IV) Computational efficiency				
Number of time–space node	12,089	12,189	12,289	12,489
Number of time–space arc (Passenger carrying)	67,265	67,265	67,265	67,265
Number of time–space arc (Bus deployment)	44,712	47,155	49,622	54,585
Number of time–space arc (Bus charging)	23,680	23,680	23,680	23,680
Number of time–space arc (All)	135,657	138,100	140,567	145,530
Computational time (s) (Dynamic programming)	4	4	4	4
Computational time (s) (Time–space network model)	194	188	201	382
Computational time (s) (All)	198	192	205	386

6.2. Profit and customer acceptance

Table 2 presents the performance metrics of our optimized solutions, showcasing profits, bus flex-routes, customer transit time, and computational efficiency. The primary observation is that as the number of total buses increases, both the customer acceptance rate and profit rise. This increase is attributed to serving more customers, resulting in higher revenue generation. Additionally, our demand-oriented optimized solutions significantly reduce empty-loaded running time through flex-routes, leading to a substantial reduction in the cost per transit. Notably, the acceptance rate is a critical metric in the performance evaluation of on-demand bus systems. However, it is essential to recognize that achieving a 100% acceptance rate, similar to conventional bus systems, may not always be the most efficient approach. On-demand bus systems, as considered in our research, introduce time windows for passenger boarding and alighting, a feature absent in conventional systems. This leads to optimized service provision with resource-efficient operations. In our results, we achieve an acceptance rate ranging from 70% to 90%, which represents the percentage of passenger requests that can be accommodated within their respective time windows and other operational constraints. The acceptance rate in on-demand bus systems is often a trade-off between providing efficient service and optimizing resource utilization. As more buses are added to the system, the acceptance rate typically increases, but it may reach a point of diminishing returns.

6.2.1. Reduction of wasted bus resources

We have observed that increasing the number of buses does not significantly affect the proportion of in-service lines or in-service stops. However, it leads to a substantial rise in the customer acceptance rate (see Fig. 6). This finding highlights significant shortcomings in the fixed-route operating mode: (i) stations with 90% of transit demands only constitute about 50% of all stations, and (ii) stations with demands are distributed among less than 70% of the total lines. The optimized solutions obtained from our

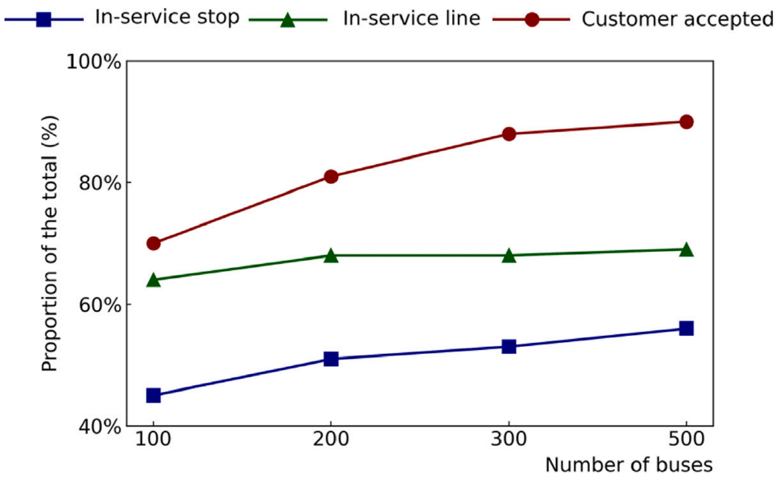


Fig. 6. Usage rate of stops, lines and acceptance rate of customers.

model align with our previous analysis in Fig. 4. Approximately 30% of all 74 lines require very few bus operations over the time horizon. In practice, Beijing's special-lines are typically served by only 2 to 6 buses in the conventional mode, leading to a potential waste of bus resources and unnecessary emissions. Furthermore, while the fixed timetable for Beijing special-lines maintains uniform headways of 10 min (Beijing Public Transport, 2023), our solutions exhibit a higher standard deviation of headways. This reaffirms that rather than rigidly assigning buses to follow fixed routes, adjusting departure times based on real-time demands in the transit network is a more appropriate and cost-effective approach.

6.2.2. Reduction of transit time

We decompose the overall transit time into in-vehicle time versus waiting time to gain insights into the reduction achieved in the flex-route mode: (i) By selectively skipping stations with no or fewer boarding customers and prioritizing those with higher boarding demands, the average waiting time for customers is reduced. (ii) If the bus's capacity is insufficient or subsequent stations have low demand, the bus skips stations to head directly towards stations with alighting demands, resulting in a decrease in the average in-vehicle time for customers. This advantage is particularly pronounced for Beijing special-lines that connect residential areas and subway stations, where boarding and alighting stations are relatively concentrated.

Our approach has achieved notable results in reducing overall transit time by up to 50%. As the number of buses increases, the average waiting time steadily decreases, while the reduction rate of in-vehicle time remains relatively constant. When there are more buses in the system, it becomes easier to match a customer's request with a nearby bus, resulting in shorter passenger waiting times for passengers. While the waiting time can be more directly impacted by the number of buses available, the in-vehicle time is affected by other factors beyond bus numbers, such as passenger demand distribution and transit network layout. Although more buses do not significantly decrease in-vehicle time further, they lead to higher profits and customer acceptance rates, making on-demand bus service more efficient and customer-oriented.

This reduction in overall transit time benefits both customers and operators. To illustrate, we analyze the optimized solution for Line Longjinyuan-Huoying (Fig. 5). When the number of buses is 100, the optimized service for Line Longjinyuan-Huoying involves 3 time–space arcs indexed as (a), (b), and (c). Fig. 7 demonstrates the detailed schedules, with the number of customers served in arc (a), (b), and (c) being 27, 25, and 26, respectively. The average service time for Line Longjinyuan-Huoying is 19.3 min per bus, compared to 28.8 min in the fixed-route mode. According to real-world data, two buses are practically required to run the full distance three times. In contrast, our optimized solution reduces the cost for Line Longjinyuan-Huoying by 33%. This exemplifies the significant cost-effectiveness and efficiency improvement achieved by our approach.

6.2.3. Fast charging during operation

Our solutions enable fast charging at any candidate charging spot. As shown in Fig. 8, the average charging duration for one vehicle is around 10 min, with a maximum of approximately 30 min. Moreover, we observe that buses getting charged mostly have a relatively low battery power duration. Some of these buses complete a short-duration charging at the beginning of the operation or after being repositioned, before serving any customers. On the other hand, buses with a relatively high initial battery power duration primarily head straight to serve customers, with a few opting for charging during their idle time after dropping off all passengers. Our approach enables the seamless and flexible implementation of fast charging during operation, leading to a significant enhancement in the reliability of electric bus systems. By integrating fast charging capabilities directly into the operational framework, our method ensures that electric buses can efficiently recharge their batteries within minutes, shortening downtime and enlarging their availability for passenger service.

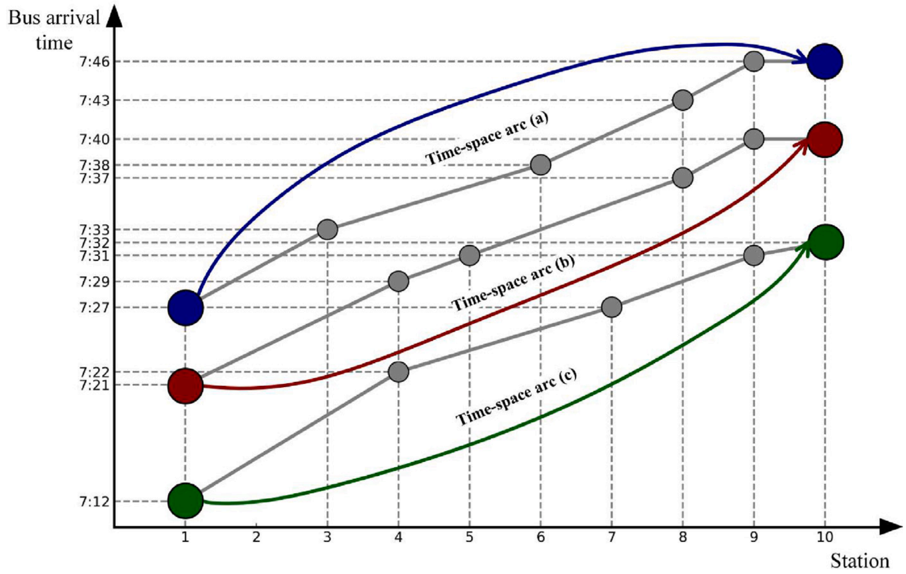


Fig. 7. Optimized solution for Line Longjinyuan-Huoying with a total of 100 buses (the nodes at either end of time–space arcs indicate time–space nodes, while the gray nodes represent intermediate stations visited by a bus).

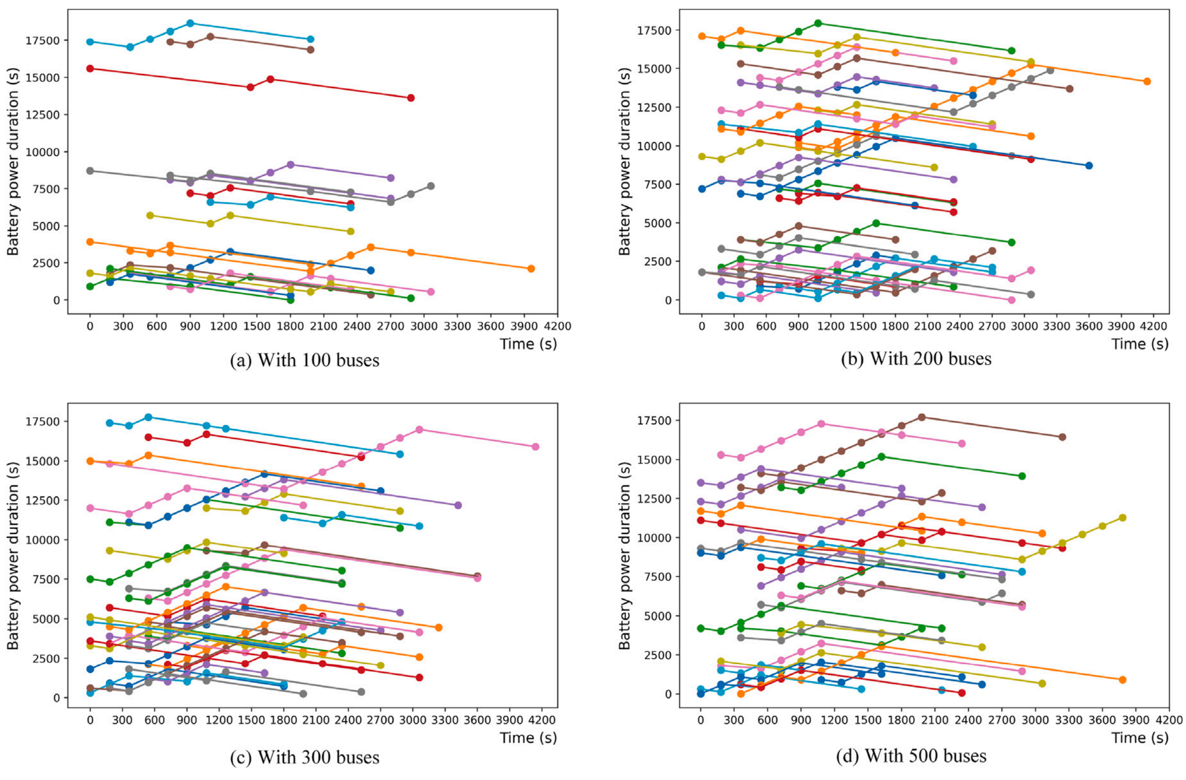


Fig. 8. The battery state variation curves of buses, with each curve indicating a bus charged during operation, given a total of 100, 200, 300 and 500 available buses.

6.2.4. Solution time

Table 2 offers a granular view of critical computational aspects, including the number of time–space nodes, various arc types, and the computational time associated with distinct components of our method. As depicted in Table 2, our method has demonstrated scalability, capably managing instances involving over 2000 customers within minutes. The computational time is predominantly influenced by the time–space network model. This component efficiently optimizes bus routes and resource allocation. Notably, our dynamic programming algorithm executes within seconds. This rapid execution contributes to the practical viability of our approach. It allows for prompt decision-making and quick responses to passenger requests, which is vital for enhancing the reliability and convenience of the system. Moreover, one of the strengths of our method lies in its adaptability to different scenarios. For dealing with 100 to 500 buses, our method consistently delivers efficient and reliable results. This adaptability underscores its suitability for real-world electric bus operation.

7. Conclusion

Electrification and demand response play pivotal roles in the sustainable transformation of urban bus systems. The effective operation of on-demand electric bus systems remains a challenge, especially when it comes to coordinating bus charging and passenger serving. In response to this challenge, our research presents an optimization approach by enabling electric buses to respond to real-time passenger demands and undergo fast charging during operation.

The core of our proposed method consists of two main components: (i) a label-setting dynamic programming algorithm, which efficiently generates bus trips on individual bus lines, and (ii) a time–space network model, designed to optimize multiple bus trips within the entire bus transit network. In the time–space network, we represent time–space arcs as bus trips that encompass customer serving, bus deploying, and bus charging activities. By selecting arcs from this network, we successfully compose efficient electric bus operations. To validate the practicality and real-world applicability of our method, we conducted experiments using data from the Beijing urban bus transit network. The experimental results confirm that our optimization model can provide cost-effective solutions in mere minutes of computational time.

In this paper, we optimize electric bus services within the framework of pre-travel booking, where passengers book their trips in advance. This approach allows for efficient route and schedule optimization based on anticipated passenger demand. It is important to note that our scope does not encompass scenarios where passengers decide to travel with the bus at the last minute or request changes to their alighting stops while onboard. These dynamic passenger behaviors present interesting challenges for future research, such as the need for re-optimization strategies in real-time to accommodate such last-minute requests. Furthermore, while our model excels in optimizing on-demand electric bus systems under controlled conditions, the real-world bus operation can be influenced by various practical factors such as weather conditions, traffic congestion, and passenger behavior, which fall outside the primary scope of this study. Addressing these secondary factors may require additional considerations. Nevertheless, our work provides an efficient framework for enhancing the passenger experience and operational efficiency of on-demand electric bus systems.

From a practical standpoint, our optimization approach offers a comprehensive solution that addresses the limitations of conventional operating modes. The ability to skip intermediate stations allows our optimized solutions to create flexible routes within the existing bus transit network. This flexibility results in a significant reduction in transit time consumption and the waste of public transit resources during operation, leading to lower operating costs and higher transit efficiency when compared to fixed-route and timetable-based operating modes. Ultimately, our on-demand electric bus service scheme becomes a win–win solution for both operators and customers, offering economic benefits and an improved quality of service.

CRediT authorship contribution statement

Wei Zhang: Methodology, Writing – original draft. **Jiahui Liu:** Validation, Writing – original draft. **Kai Wang:** Methodology, Writing – review & editing. **Liang Wang:** Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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