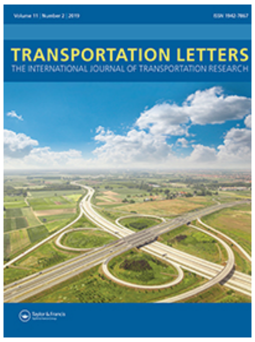

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The close–open mixed multi depot vehicle routing problem considering internal and external fleet of vehicles

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ABSTRACT
Distribution of finished goods from depots to predetermined customers and collection of goods and wastes are practical and challengeable optimization and logistics problems; therefore, appropriate management of transportation system can help companies decrease their costs and consequently earn more benefits. Several kinds of vehicle routing problems (VRPs) have been studied in the literature. In this paper, we will combine multi-depot vehicle routing problem (MDVRP) and close–open mixed vehicle routing problem (COMVRP), assuming that the fleet of vehicles is heterogeneous. Also, in this problem, contractors are used to meet a part of customers' requirements. The objective of the problem is to minimize the total cost of serving customers. To efficiently deal with the problem, a new mixed integer programming (MIP) model as well as a new hybrid metaheuristic is proposed. Moreover, analytic hierarchical process is utilized in hybridization of the genetic algorithm. The computational experiments are conducted to compare the results of HGA, GA and MIP solver, CPLEX.

KEYWORDS
Vehicle Routing Problem; multi-depot VRP; close–open mixed VRP; heterogeneous fleet of vehicles; hybrid genetic algorithm; analytic hierarchical process

Introduction

The last decades have seen an increasing attention to management of distribution system and transition of goods. Generally, finding the routes with the least cost is a critical operational level decision in distribution management of goods (Amiama et al. 2015). 'A large number of real-world applications, both in North America and in Europe, have shown that the use of optimization procedures for the distribution process planning produces savings (generally from 5 to 20%) in the global transportation costs' (Toth and Vigo 2002). As a result, it is possible to save some expenditures and achieve more profits by means of proper management of transportation system. Vehicle routing problem (VRP) is the distribution of commodity problem in which the objective is to deal with pick up or delivery activities (Wang, Dessouky, and Ordonez 2016). A typical VRP is the problem of designing least cost routes from one depot to a set of geographically scattered points such as cities, stores, warehouses, customers and so forth (Baykasoglu et al. 2011). Critical activity in this problem is to determine routes though which vehicles move from one or more depots to serve a given set of customers. VRP is one of the most important optimization problems. The typical problem is to determine the best routes with respect to the travelled distance or time. Operational decision is how the available fleet of vehicles or resources can be efficiently used to satisfy a given service demand according to a set of customers' requirements.

According to Lin et al. (2014), many kinds of VRPs exist in practice, such as VRP with time windows (Bräysy and Gendreau 2005a, 2005b; Kallehauge et al. 2005; Kallehauge 2008; Azi, Gendreau, and Potvin 2010) which is an extension of capacitated VRP (CVRP). Single depot VRP attracted so much attention in the early years, but it is not suitable for companies which have several depots in different areas. Another type of VRP is multi-depot VRP (MDVRP). In MDVRP, the decision-makers concentrate on three phases to solve the problem. The first phase is grouping in which customers are divided into clusters with respect to depots' capacity and proximity coefficient and all customers in a cluster are allocated to the same depot. The second phase is to solve the routing problem; i.e. customers which are served by the same depot, are divided into some routes. The third phase is dealing with scheduling problem. In this step, the order of serving the customers is determined. This classification of the problem was first proposed by Ho et al.

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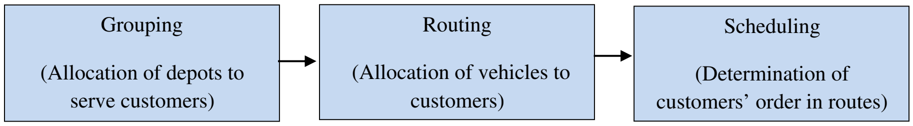

Figure 1. The three steps to solve MDVRP problem.

(2008). To better understand the approach, this classification is shown in Figure 1.

Earlier this century, researchers have been interested in a new kind of problem called open VRP, in which all of the routes are open; this means that vehicles do not have to come back to the depot and are free after serving the last customer. Open VRP was studied by Sariklis and Powell (2000) for the first time. This type of problem corresponds to distribution companies that do not own fleet of vehicles and delegate this process to a contractor or hire some drivers along with their own vehicles.

In this paper, we deal with a special type of problem called close–open multi-depot mixed vehicle routing problem (COMDMVRP) which is introduced for the first time. This type of problem can be seen as the combination of close–open mixed vehicle routing problem (COMVRP) and MDVRP considering heterogeneous fleet of vehicles which are different in capacity and maximum route length. In this case, some vehicles are not required to come back to the depot, but the rest of vehicles must return to the depot from which they have departed. Furthermore, a new hybrid GA algorithm is proposed in this paper which combines genetic algorithm with decision-making tools. COMDMVRP has a significant application in practice. For example, imagine a corporation that has its own vehicles, but the number of vehicles is limited. Therefore, the corporation cannot meet the customers' demands and thus, a part of servicing is performed by external fleet. In addition, corporation has multiple depots. Vehicles must depart from depots to serve customers according to the limitations and the objective is to minimize the total cost.

The remaining of the paper is organized as follows. A related literature of the VRP is summarized in Section 'Literature review'. Problem description and a new mathematical model are presented in Section 'Problem description'. Methodology, parameters tuning, numerical results and experiments as well as model validation are provided in Section 'Methodology'. Finally, concluding remarks and directions for future research are presented in Section 'Conclusions and future research'.

Literature review

The first work in this field dates back to almost 50 years ago when Dantzig and Ramser (1959) introduced this type of problem for gas distribution networks. They studied real-world application of VRP and considered the distribution of gasoline from a bulk terminal to a large number of gas stations and proposed the first mathematical model for this problem. Later, Clarke and Wright (1964) proposed a new heuristic algorithm and attempted to improve Dantzig and Ramser's approach to solve the problem. The classical VRP can be defined as follows: determination of the best routes to serve customers' demands. In classical cases, one depot is available (single-depot VRP), all of the routes are closed and the objective is to determine the routes with the least cost. Each vehicle located in a depot departs from the depot to serve customers and must return to the same depot. The travel time/distance and customers' demands are known and deterministic. Each customer should be served by a single vehicle and the total demand of customers in the same route must not exceed vehicle's capacity. There exists very rich scientific literature regarding VRP. Solving methods include exact algorithms, heuristics, and metaheuristics. In recent years, significant attention has been given to metaheuristics by researchers. Metaheuristics can carry out a broad search in the solution space and recombine solutions to make new ones.

In real world, VRP has extensive applications such as in solid waste, distribution of commodity, food, newspaper, and dairy. VRP includes extensive ranges of problems such as capacitated VRP (since 1959), time-dependent VRP (since 1966), multi-depot VRP (since 1969), periodic VRP (since 1974), VRP with time windows (since 1977), multi-echelon VRP (since 2009), close and open mixed VRP (since 2012). CVRP is defined as a problem in which vehicles depart from the depots and serve customers according to their limited capacity and go back to the depots. Several review papers have been published by now. Some instances of these reviews are by Laporte (2009), Marinakis and Migdalas (2007), and Toth and Vigo (2002). Bodin et al. (1983) studied the distribution of goods from company to customers. Küçükoğlu and Öztürk (2013) presented a differential evolution algorithm (DEA) to solve a VRP with backhauls and time windows (VRPBTW) which is an extension of VRP that involves capacity and time window constraints. They applied the algorithm to a catering firm. Zheng (2014) studied emergency evacuation policies that depend on public transit in which buses run continuously, rather than fixed route, based on the spatial and temporal information of evacuee requirements. They formulated an optimal bus operating strategy to minimize the exposed casualty time instead of operational cost, as a deterministic mixed-integer programming, and surveyed the solution algorithm. (Ai and Kachitvichyanukul 2009) provided a heuristic based on Particle Swarm Optimization (PSO) algorithm for solving VRP with time windows.

In open vehicle routing problem (OVRP), the vehicle leaving depot to serve the customers does not have to return to the depot and is free after serving the last customer. One of the earliest researches about the OVRP was conducted by Schrage (1981). In that paper, he tried to investigate VRP in practice and classify the features of this problem. Newspaper or mail delivery companies are examples of OVRP. Li, Golden, and Wasil (2007) reviewed OVRP algorithms, developed a variety of record-to-record travel algorithms for the standard VRP to handle open routes. Fleszar, Osman, and Hindi (2009) developed a variable neighborhood search (VNS) algorithm to solve OVRP. The neighborhoods are based on reversing segments of routes as well as exchanging segments between routes. A hybrid evolution strategy (ES) was presented by Repoussis et al. (2010) for solving OVRP. Li, Leung, and Tian (2012) studied a heterogeneous fixed fleet OVRP in which the number of vehicles was limited.

COMVRP is one of the most recent research topics in the literature. Before Liu and Jiang (2012), no attention was devoted to this problem. COMVRP has significant applications in transportation system. It can be seen as the combination of open and close VRPs in which some vehicles must return to the depot, while others do not have to do so. Liu and Jiang (2012) proposed a mix integer programming (MIP) model and an effective metaheuristic, i.e. memetic algorithm to solve the problem.

Multi-depot vehicle routing problem (MDVRP) is an extension to the classical VRP. In this problem, company has multiple depots from which vehicles depart to serve customers and each vehicle has to return to the depot it left. In this problem, products are stored in several depots in different areas and decision-makers should determine which depot serves each customer. VRP is an NP-hard problem; hence, multi-depot VRP is NP-hard as well; this means that there is no efficient algorithm to find the optimum solution for this problem. Therefore, heuristic and metaheuristic methods should be used to solve this problem. Sumichras and Markham (1995) formulated multi-depot problem for the first time. In their problem, the supplier had to transport raw material from several depots to the company. Ho et al. (2008) proposed two hybrid genetic algorithms (HGAs) to efficiently deal with MDVRP. The major difference between the HGAs was that the initial solutions were generated randomly in HGA1. But in HGA2, Clarke and Wright saving method and the nearest neighborhood heuristic were used for the initialization procedure. An improved *k*-means algorithm is proposed by Geetha, Poonthalir, and Vanathi (2013) for clustering that reduces the MDVRP to multiple VRP. They considered MDVRP with multiple objectives and proposed nested particle swarm optimization with genetic operators to solve each VRP. Salhi, Imran, and Wassan (2014) extended MDVRP to another application of this problem. They assumed that the fleet of vehicles is heterogeneous, formulated the problem and used VNS to solve it. A brief review of the related literature is presented in Table 1.

In this paper, we propose a problem with multiple depots in different areas, which is near to real-world applications. Also, we consider close and open routes for serving customers, simultaneously. Internal fleets of the vehicles must return and external ones are not obliged to do so. Exact solution for these problems is not reachable in many cases; thus, heuristic and metaheuristic approaches can support us to solve the problems. It is obvious that smaller gap between exact and approximated solutions is desired and can help us to make the best decision in different situations. In this paper, we try to prepare real-world situations and propose an efficient algorithm to solve this problem. It should be noted that the combination of Analytic Hierarchical Process (AHP) decision-making technique and metaheuristics has not been previously studied, and this paper proposes a new mathematical model based on these techniques to solve such problems.

Problem description

In this research, a fleet of heterogeneous vehicles with distance and capacity constraints is taken into consideration. These vehicles are located in depots. The objective is to deliver goods to the costumers, according to the constraints of vehicles' capacity and travel distance. These constraints are different for each vehicle. The current transportation system is not sufficient to meet costumers' needs. The corporation has decided to delegate a part of costumers' demands to contractors to fulfill. The number of internal vehicles is limited, while the number of external vehicles is unlimited. The type of the problem is close–open mixed VRP; i.e. internal vehicles should return to depots after serving the costumers, while external vehicles will be free after serving the last costumer. Internal vehicles have only variable cost, whereas external ones have hiring fixed cost, too. Two types of trip are shown in Figure 2. Internal vehicles correspond to closed routes and external vehicles associate with open routes. The multi-depot VRP is combined with close–open mixed

Table 1. Overview of the literature in the MDVRP and COMVRP.

Paper	Subject	Summary	Objective function	Methodology
Crevier, Cordeau, and Laporte (2007)	Multi-depot vehicle routing problem (MDVRP)	Deals with an extension of the MDVRP with inter-depot routes in which vehicles may be replenished at intermediate depots along their route	Minimizing travel duration of route	A heuristic combining the adaptative memory principle, a tabu search, and integer programming
Liu et al. (2010)	Close–open mixed vehicle routing problem (COMVRP)	Discusses COMVRP that can be applied to help identify routes when a carrier serves the customers by his private vehicles and vehicles hired from external carriers	Minimizing the fixed and variable costs for operating the private as well as the hired vehicles	Memetic algorithm
Polacek et al. (2004)	MDVRP	Proposes an algorithm based on the philosophy of the Variable Neighborhood Search (VNS) to solve MDVRP with Time Windows	Minimizing the total distance travelled by all vehicles	Variable Neighborhood Search (VNS)
Cordeau, Gendreau, and Laporte (1997)	MDVRP	Describes a new heuristic algorithm for the MDVRP with capacity and route length restrictions	Minimizing travel cost	Tabu search algorithm
Pichpibul and Kawtummachai (2012)	COMVRP	Aims at Solving COMVRP which simultaneously considers the constraints from both close and open versions	Minimizing the total transportation cost	A modified savings heuristic
Afshar-Nadjafi and Afshar-Nadjafi (2014)	MDVRP	Considers time-dependent MDVRP with time-windows and heterogeneous fleet	Minimizing the total heterogeneous fleet cost	A constructive heuristic algorithm
Carlsson et al. (2009)	MDVRP	Investigating the notion of Minimizing the maximal length of a tour in the problem	Minimizing the maximal length of a tour	Two heuristic algorithms
This paper	Close–open multi-depot mixed vehicle routing problem (COMDMVRP)	Combining the COMVRP and MDVRP assuming that the fleet of vehicles is heterogeneous	Minimizing the total cost	Hybrid genetic algorithm

VRP, considering that each model exists individually, but their combination is not available. The objective of the problem is to minimize travelling cost as well as to minimize the number of vehicles involved. A new mathematical model and two metaheuristics are proposed to tackle this problem.

Assumptions:

(1) The customers' demands are deterministic and known.
(2) Distances between nodes are deterministic and known.
(3) Each customer must be served by a single vehicle, but a vehicle can serve a set of customers unless the summation of the customers' demands exceeds the vehicle's capacity.
(4) Internal vehicles must return to the depots from which they have departed, while external vehicles do not have to do so.

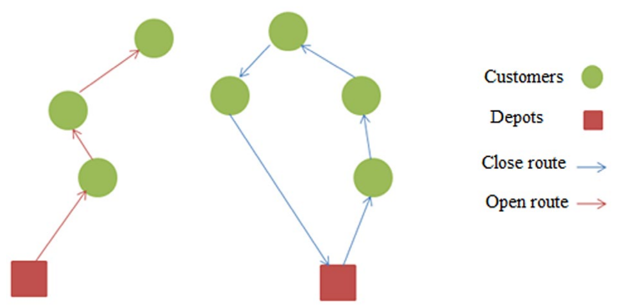

Figure 2. Two types of trip including open and close routes.

$$x(i,j,s,k) = \begin{cases} 1 \text{ if vehicle } k \text{ of fleet type } s \text{ travels from node } i \in C \cup D \text{ to node } j \in C \cup D, \\ 0 \text{ otherwise} \end{cases}$$

Sets:
$D = \{d|d = 1, 2, \ldots, R\}$; Set of depots,
$C = \{c|c = 1, 2, \ldots, N\}$; Set of customers,
$K = \{k|k = 1, 2, \ldots, P\}$; Set of vehicles,
$S = \{s|s = 0, 1\}$; Internal or external vehicles.
Parameters:
d_i = Demand of customer i,
Q_k = Capacity of vehicle type k,
H_k = Maximum allowable route length for vehicle k,
I = Maximum internal vehicles,
c_{ij} = Travel distance between node $i \in C \cup D$ and node $j \in C \cup D$ and $i \neq j$,
F_k = Fixed cost of using vehicle k,
V_k = Variable cost of vehicle k per unit of distance,
M = Great value.
Decision variables:

$$O(i,s,k) = \begin{cases} 1 \text{ if vehicle } k \text{ of fleet type } s \text{ departs from depot } i, \\ 0 \text{ otherwise} \end{cases}$$

$$y(i,s,k) = \begin{cases} 1 \text{ if vehicle } k \text{ of fleet type } s \text{ is allocated to customer } i, \\ 0 \text{ otherwise} \end{cases}$$

$U(i,s,k)$ = Continuous variable that represents the delivered load of vehicle k of type s after leaving the customer i,

$h(i,s,k)$ = Continuous variable that represents the traveled distance of vehicle k of fleet s from depot to the customer i.

Formulation:

The objective is to design the servicing routes from depots to the customers (i.e. determination of the sequence of customers in each route and allocation of the routes to depots). Customers' demands and travelling distances between the points are deterministic and known. The mathematical formulation is as follow:

$$Min\ Z = \sum_{k\in K} F_k \sum_{i\in D} O(i,1,k) + \sum_{k\in K} V_k \sum_{s\in S} \sum_{i\in D\cup C} \sum_{j\in D\cup C} c_{ij} x(i,j,s,k) - \sum_{k\in K} V_k \sum_{i\in C} \sum_{j\in D} c_{ij} x(i,j,1,k) \tag{1}$$

$$\sum_{s\in S} \sum_{k\in K} \sum_{i\in D\cup C} x(i,j,s,k) = 1 \quad \forall j \in C \tag{2}$$

$$\sum_{s\in S} \sum_{k\in K} \sum_{i\in D\cup C} x(j,i,s,k) = 1 \quad \forall j \in C \tag{3}$$

$$\sum_{i\in D\cup C} x(i,j,s,k) = \sum_{i\in D\cup C} x(j,i,s,k) \quad \forall j \in C, s \in S, k \in K \tag{4}$$

$$\sum_{i\in D} \sum_{j\in D} x(i,j,s,k) = 0 \quad \forall s \in S, k \in K \tag{5}$$

$$h(i,s,k) = 0 \quad \forall i \in D, s \in S, k \in K \tag{6}$$

$$h(i,s,k) + c_{ij} - M(1 - x(i,j,s,k)) \le h(j,s,k) \quad \forall i,j \in C, s \in S, k \in K \tag{7}$$

$$h(i,0,k) + c_{ij} - M(1 - x(i,j,0,k)) \le H_k \quad \forall j \in D, i \in C, k \in K \tag{8}$$

$$h(i,1,k) + c_{ij} - M(1 - x(i,j,1,k)) \le H_k \quad \forall i,j \in C, k \in K \tag{9}$$

$$0 \le h(i,s,k) \le \sum_{j\in D\cup C} x(j,i,s,k) H_k \quad \forall i \in C, s \in S, k \in K \tag{10}$$

$$\sum_{k\in K} \sum_{i\in D} \sum_{j\in C} x(i,j,0,k) \le I \tag{11}$$

$$\sum_{j\in D\cup C} x(j,i,s,k) = y(i,s,k) \quad \forall i \in C, s \in S, k \in K \tag{12}$$

$$\sum_{i\in C} d_i y(i,s,k) \le Q_k \quad \forall k \in K, s \in S \tag{13}$$

$$U(i,s,k) + d_j - M(1 - x(i,j,s,k)) \le U(j,s,) \quad \forall i,j \in C, s \in S, k \in K \tag{14}$$

$$d_i \le \sum_{s\in S} \sum_{k\in K} U(i,s,k) \le \sum_{s\in S} \sum_{k\in K} \sum_{j\in D\cup C} x(j,i,s,k) Q_k \quad \forall i \in C \tag{15}$$

$$\sum_{k\in K} \sum_{j\in D} \sum_{i\in C} x(i,j,1,k) = 0 \tag{16}$$

$$\sum_{i\in D} \sum_{j\in C} x(i,j,s,k) = O(i,s,k) \quad \forall k \in K, s \in S \tag{17}$$

$$\sum_{i\in C} x(i,j,0,k) = \sum_{i\in C} x(j,i,0,k) \quad \forall j \in D, s \in S, k \in K \tag{18}$$

$$x(i,j,s,k) = \{0,1\} \quad \forall i,j \in D \cup C, s \in S, \ k \in K \tag{19}$$

$$O(i,s,k) = \{0,1\} \quad \forall i \in D, s \in S, k \in K \tag{20}$$

$$y(i,s,k) = \{0,1\} \quad \forall i \in C, s \in S, k \in K \tag{21}$$

$$U(i,s,k) \ge 0 \quad \forall i,j \in D \cup C, s \in S, k \in K \tag{22}$$

$$h(i,s,k) \ge 0 \quad \forall i,j \in D \cup C, s \in S, k \in K \tag{23}$$

The objective function (1) minimizes the total cost of serving the customers considering customers' locations. Regarding the fact that only external vehicles have fixed cost, only external vehicles are considered in the first term of the objective function. In the second and third terms of the objective function, variable cost is considered. As is clear from the third term of the objective function, we have to reduce the returning cost for external vehicles from total, as is considered in the second term.

Constraints (2) and (3) indicate that each customer is assigned to a single route. Constraints (4) show that each vehicle that enters a customer's node, has to go out as well. Constraints (5) ban vehicles from moving between depots. Each vehicle has specific distance limitations; this means that travelled distance through routes must not exceed this limitation. Constraints (6)–(10) satisfy this distance restriction. Constraints (11) justify that the number of internal vehicles are limited. Constraints (12) define the new decision variables and specify relationship between variables. Constraints (13) ensure that delivered commodity through each route must not exceed allowable capacity. According to Liu and Jiang (2012) and Rabbani, Farrokhi-asl, and Rafiei (2016), the next two sets of constraints (14) and (15) are deformed subtour elimination constraints of Miller–Tucker–Zemlin (MTZ) for classical VRP, which were first proposed by Desrochers and Laporte (1991), and corrected and developed by Kara, Laporte, and Bektas (2004). In our problem, these constraints ensure subtour elimination. Constraints (16) ensure that no external vehicle will go back to the depots. Constraints (17) declare the relationship between two decision variables. Constraints (18) indicate that each internal vehicle must come back to the same depot it has left and this limitation is only performed on internal vehicles because there is no need for external vehicles return to depots. Constraints (19)–(23) determine the ranges of variables.

Methodology

As mentioned before, VRP is an NP-hard combinational optimization problem; thus, it is difficult to achieve optimal solution, especially in large-scale problems. Heuristic and metaheuristic algorithms can help reach optimal or near optimal solutions. One of the most commonly used metaheuristic algorithms in the literature is the genetic algorithm (GA) developed by John Holland in 1960, which is a powerful tool to obtain efficient solution. Metaheuristic algorithms have a significant role in solving routing problems (Quirion-Blais, Trépanier, and Langevin 2015). Similar to other artificial intelligence heuristics such as simulated annealing (SA) and tabu search (TS), GA can avoid being trapped in a local optimum by taking advantage of genetic operations called mutation.

Unlike conventional optimization approaches, GA searches for the population of solutions in despite a single point, and thus, this algorithm can provide multiple potential solutions and leave the selection of the final solution to the decision-maker (Akgüngör and Doğan 2009). The basic idea of GA is to provide an initial population and improve this population by searching the solution area. Initial population is the first generation in the evolutionary algorithms which can effect on quality of the best solution obtained by the algorithm. For this purpose, GA uses genetic operators called mutation and crossover. GA has shown successful results in a wide variety of optimization problems such as TSP (Goldberg 1989; Gen and Cheng 1997). It has a simple procedure as well as has a flexibility to be used in different optimization problems. Gendreau et al. (2008) have listed the bibliography of metaheuristics for solving VRP. Referring to the literature, it is realized that GA is a popular algorithm to solve VRPs; hence, this paper also applies GA to solve the problem. The algorithm is also improved by heuristic methods. In recent years, HGA has attracted much attention and proved to be one of the most efficient algorithms to solve optimization problems such as multi-stage supply chain networks (Gen and Syarif 2003).

In this paper, the problem is solved by GAMS software, simple genetic algorithm and HGA. Using experimental results, a comparison is conducted between the algorithms. The GA developed in this paper is hybridized by improvement methods such as AHP technique to achieve more efficient solution. In GA, the initial population is randomly generated and in each iteration the solution is improved using crossover and mutation operators. On the other hand, in HGA algorithm, nearest neighborhood (NNH) and AHP techniques are used to make an initial solutions. This solution is improved in each iteration using iterative swap procedure (ISP) and genetic operators. The flowchart of the proposed algorithms is shown in Figure 3.

As shown in Figure 3, unlike the algorithms in which grouping is performed initially, and followed by routing and scheduling, in this paper, scheduling of the vehicles and customers are the first step. The reason is that, vehicles are assumed to be heterogeneous with different capacities. The following steps show how the algorithm is applied to generate the initial population:

Step 1. Customers are randomly scheduled for both GA and HGA. Also, vehicles' order must be determined. In GA, vehicles are scheduled randomly, whereas in HGA, analytic hierarchy process (AHP) is used to specify vehicles priority to serve the customers. Pair-wise comparisons of criteria are specified by experts' judgments or by tuning the parameters using Taguchi method (Tsai, Chou, and Liu 2006).

Step 2. In this step, the vehicles are assigned to customers considering the customers' demands, customers' orders, and vehicles' capacity. It should be noted that summation of the customers' demands in each route must not exceed vehicles' capacities which are serving them. The customers are assigned to the first vehicle until the summation of their demand trespasses capacity of the vehicle. At this point, the assignment of customers to the second vehicle starts and so forth. Since the vehicles order is arranged by means of AHP, the next vehicle is not sent to service the customers until the previous vehicle is unloaded. The route length limitation is neglected at this step, so the penalty function is added to objective function, if this limitation is exceeded by vehicles. The amount of this penalty is proportional to the amount of exceeding the constraint.

Step 3. In this step, the depots are assigned to the routes. In GA algorithm, this is performed randomly, while in HGA, the nearest depot to the first customer is assigned to route regarding the number of vehicles in each depot.

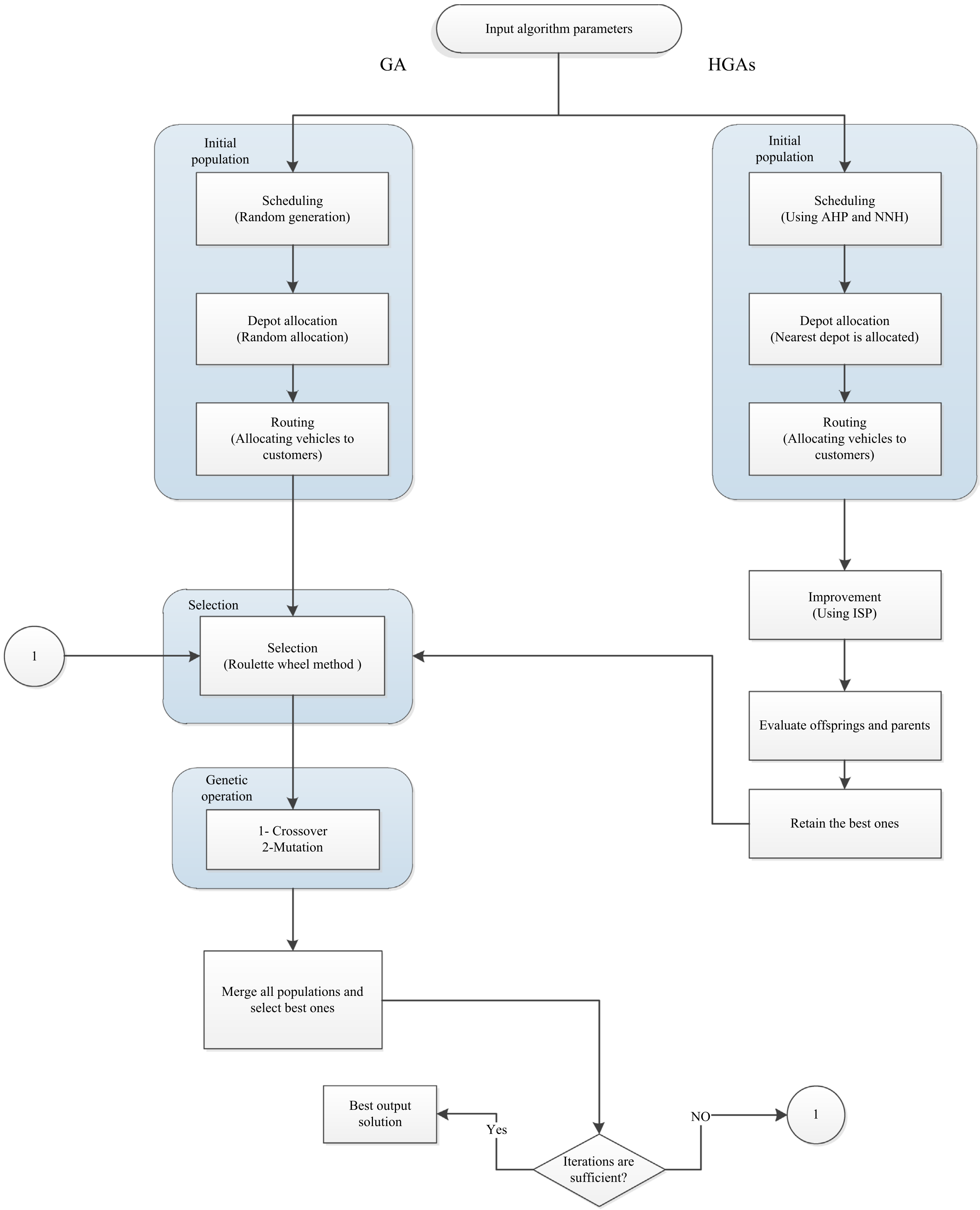

Figure 3. Flowchart of the HGA and GA.

Initialization

In both GA and HGA, an initial population must be generated, but the way in which these populations are generated are different with each other.

AHP is a decision-making technique proposed by Saaty (1977). This technique helps experts, managers and decision-makers to make accurate and appropriate decisions, organize, and analyze complex decisions according to the available criteria and alternatives. This technique has been refined by Saaty (2003). In this paper, we have heterogeneous fleet of vehicles. Each vehicle has unique characteristics such as capacity, maximum travelled distance, fixed and variable costs that differentiate each vehicle from others. As mentioned before, in the first step of the methodology, vehicles and customers must be scheduled; i.e. vehicles order to serve customers should be determined. Obviously, better vehicles should have more chance to be selected sooner than others. If there exists a vehicle with maximum capacity, maximum distance limitation, and minimum variable and fixed costs, it is better to select this vehicle at first. Since there is not a single vehicle that has all criteria in the best level, simultaneously, the best ones should be chosen with respect to their priority in terms of the criteria. AHP technique is applied to solve this problem. According to the problem, there are four criteria and multiple alternatives (types of vehicle). The vehicles are sorted by means of AHP technique. Figure 4 shows the hierarchy of criteria and alternatives as well as the final goal (i.e. vehicles scheduling), schematically.

For each pair of criteria, the decision-maker has to determine which one of these criteria is more important and specify index of relative importance. Relative importance for each pair of criteria is specified by experts or by DOE (design of experiments) methods. For each criterion, a table similar to Table 2 should be generated and all cells should be filled like Table 2. Cell in row i and column j shows the relative importance of vehicle i to vehicle j regarding that particular criterion. The weights are normalized and averaged to obtain a mean weight for each criterion.

In Table 2, w_{ij}^t denotes the importance of vehicle i compared to vehicle j with respect to criterion t.

$$W_{ij}^t = \begin{cases} c_i^t / c_j^t \text{ if criterion } t \text{ is } a \text{ positive criterion} \\ c_j^t / c_i^t \text{ if criterion } t \text{is } a \text{ negative criterion} \end{cases}$$

c_i^t denotes the performance value of vehicle i in terms of criterion t. The normalization process is conducted by dividing each cell by the summation of the column to which it belongs. The summation of all numbers in each row is the point of that row representing a certain alternative. These actions are done for all criteria and each alternative's weight is calculated with respect to all criteria. Therefore, each alternative's point can be calculated as follows:

$$p_i = \sum_t w_i^t \times W_t$$

where W_t is the weight of criterion t that indicates the importance of this criterion and can be determined based on experts' judgments or DOE methods. w_i^t which is obtained in the previous step, denotes the value of vehicle i in terms of criterion t. Vehicles points are calculated and then sorted in descending order.

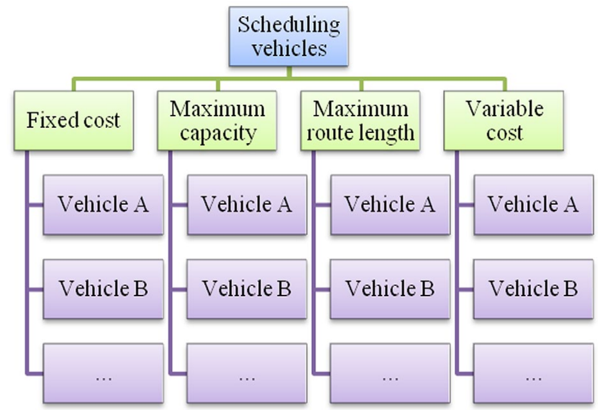

Figure 4. Criteria and alternatives in AHP technique.

Table 2. AHP technique pairwise comparision.

Fixed cost	Vehicle A	Vehicle B	Vehicle C	Vehicle D	Total
Vehicle A	1	w_{ab}^f	…	…	
Vehicle B	w_{ba}^f	1	…	…	
Vegicle C	…	…	1	…	
Vehicle D	…	…	…	1	
Total	$\sum_i w_{ia}^f$				

Improvement

2-Opt and 3-Opt improvement heuristic methods are usually used to improve the initial population solutions. In these methods, all two swaps for 2-Opt and all three swaps for 3-Opt methods are examined, and if a new solution generated by these methods is found to be better than its parent, this new solution will replace and change into new parent. Since all two and three swaps are checked in these methods, computational time increases. In this paper, we use ISP proposed by Ho and Ji (2003, 2004) to improve initial population solution. The principles of ISP are similar to that of 2-Opt and 3-Opt local search heuristics, except that in ISP, not all swaps are examined. The procedure of ISP is as follow:

Step 1. Generate a random integer number between 1 and 3. If the obtained number is 1, the Iterated Swap Procedure (ISP) is applied to customer order. If the obtained number is 2, the ISP is applied to vehicles order, and if the obtained number is 3, both customers and vehicles order must be changed.

Step2. Select two genes, randomly.

Step 3. Exchange the two selected genes.

Step 4. Swap the neighbors of the two genes to form four new offspring.

Step 5. Evaluate the offsprings and compare solutions with the parent. If the new solution is better than the parent, then exchange the offspring with the parent.

The iterated swap procedure is presented in Figure 5.

Parent	:	3	0	2	8	9	6	7	5	10	2
Offspring 1	:	3	0	2	8	5	6	7	9	10	2
Offspring 2	:	3	0	2	5	8	6	7	9	10	2
Offspring 3	:	3	0	2	8	6	5	7	9	10	2
Offspring 4	:	3	0	2	8	5	6	9	7	10	2
Offspring 5	:	3	0	2	8	5	6	7	10	9	2

Figure 5. Iterated swap procedure.

Parent 1	:	5	3	4	1	2	9	7	6	8
Parent 2	:	3	1	8	5	4	2	9	7	6
Child 1	:	8	3	4	1	2	9	5	7	6
Child 2	:	3	1	8	5	4	2	9	7	6

Figure 6. Example of crossover operation.

Parent	1	5	6	4	3	2	7	9	8
Child	1	5	7	2	3	4	6	9	8

Figure 7. Example of mutation operation.

Selection and crossover operations

Several crossover operators are available in the literature of genetic algorithm, such as linear order crossover (LOX) (Falkenauer and Bouffouix 1991), partial mapped crossover (PMX) (Goldberg and Lingle 1985), and order crossover (OX) (Oliver, Smith, and Holland 1987). In this paper, OX crossover operator is applied. The first step is to choose two parents from the solution pool to perform crossover operation. Roulette-wheel algorithm also known as fitness proportionate selection is selected in this paper, since it provides more chance to find a better solution. The parent is selected on the basis of objective function value. The probability of a parent to be selected increases as its objective function becomes better compared with that of its competitors. After selection of parents, crossover operator is performed. Crossover is a genetic operator used to alter the programming of chromosomes from one generation to the next. Crossover operator helps to move toward optimum solution. When the parent is determined to perform crossover operation, an integer number between 1 and 3 is randomly generated to specify the crossover operation deformed customers order chromosome, vehicles order chromosome or both of them. In crossover algorithm, two genes in chromosome are selected at random. The genes located between these two genes in parent 1 inherit to the first child identically. Each gene in this group is omitted from parent 2 and the remaining genes are transferred to the first child in the order in which they appear in the second Parent. These actions are performed for generating the second child identically. Generation of the child chromosomes from parents using OX is shown in Figure 6.

Mutation operation

Mutation is a genetic operator which is used to preserve diversity of GA from one generation of a population of chromosomes to the next. In this stage, a random number is generated to choose a parent from the population to do the mutation operation. When the parent is selected, a random integer number between 1 and 3 is generated to determine which customers order or vehicle order or both are selected to perform mutation operation.

Mutation operator helps to avoid being trapped in local optimum. A stream of genes is selected, this stream is reversed and the rest of genes remain unchanged. Figure 7 presents an example of mutation operation.

After performing crossover and mutation operations and generating a new population, the initial population is merged with the new population. Then, the population is sorted according to their objective functions. The best solutions stay in the new population and the rests are eliminated.

Parameters tuning

Hybrid and simple genetic algorithms are used to solve the problem. GA has several parameters. Accurate tuning of these parameters not only helps to gain better solutions, but also decreases the computational time. Moreover, AHP technique is used for vehicles scheduling. Degree of importance of each criterion is relative and experts' opinions can be different about pair-wise comparisons. AHP parameters are tuned to make the most appropriate decision about vehicles scheduling.

First, GA and HGA parameters are tuned, respectively. GA and HGA parameters include number of solutions in the initial population (*npop*), crossover parameters (*pc*), mutation parameters (*pm*), and roulette-wheel algorithm parameters (*beta*). Taguchi method is utilized for design of the experiments. The experiment's factors *are npop, pc, pm*, and *beta*. Three levels are considered for each factor. Minitab software is applied to design the experiment. For tuning the parameters, a medium-scale problem with 30 nodes is solved. The results of the experiments and analysis about GA parameters are presented in Table 3 and Figure 8 presents the analysis of Taguchi design.

This algorithm is repeated for HGA and new results are obtained. Table 4 shows the results of experiments and analysis of HGA parameters. Also, Figure 9 depicts the analysis of Taguchi design.

According to the experiments and analysis of the Taguchi design, it is decided to tune the parameters of GA and HGA algorithms as Table 5.

Numerical results

In this subsection, to validate the proposed mathematical model, GAMS software is used to solve a small-scale problem. Then the model is applied to solve several medium-scale problems. Finally, the results are compared with the solutions obtained by metaheuristics.

Model validation

In this section, to show the applicability of the model, the model is utilized to solve a small-scale example including

Table 3. Taguchi design and experiment results for GA.

Taguchi design	npop	pc	pm	Beta	Total cost
Experiment 1	75	0.3	0.2	2	440
Experiment 2	75	0.3	0.2	2	500
Experiment 3	75	0.3	0.2	2	440
Experiment 4	75	0.4	0.3	5	480
Experiment 5	75	0.4	0.3	5	460
Experiment 6	75	0.4	0.3	5	480
Experiment 7	75	0.5	0.4	10	430
Experiment 8	75	0.5	0.4	10	540
Experiment 9	75	0.5	0.4	10	440
Experiment 10	100	0.3	0.3	10	460
Experiment 11	100	0.3	0.3	10	370
Experiment 12	100	0.3	0.3	10	470
Experiment 13	100	0.4	0.4	2	470
Experiment 14	100	0.4	0.4	2	440
Experiment 15	100	0.4	0.4	2	400
Experiment 16	100	0.5	0.2	5	470
Experiment 17	100	0.5	0.2	5	420
Experiment 18	100	0.5	0.2	5	410
Experiment 19	150	0.3	0.4	5	440
Experiment 20	150	0.3	0.4	5	430
Experiment 21	150	0.3	0.4	5	480
Experiment 22	150	0.4	0.2	10	430
Experiment 23	150	0.4	0.2	10	490
Experiment 24	150	0.4	0.2	10	430
Experiment 25	150	0.5	0.3	2	440
Experiment 26	150	0.5	0.3	2	450
Experiment 27	150	0.5	0.3	2	570

Table 4. Taguchi design and experiment results for HGA.

Taguchi design	npop	pc	pm	Beta	Total cost
Experiment 1	75	0.3	0.2	2	460
Experiment 2	75	0.3	0.2	2	430
Experiment 3	75	0.3	0.2	2	450
Experiment 4	75	0.4	0.3	5	500
Experiment 5	75	0.4	0.3	5	430
Experiment 6	75	0.4	0.3	5	490
Experiment 7	75	0.5	0.4	10	470
Experiment 8	75	0.5	0.4	10	470
Experiment 9	75	0.5	0.4	10	440
Experiment 10	100	0.3	0.3	10	410
Experiment 11	100	0.3	0.3	10	430
Experiment 12	100	0.3	0.3	10	460
Experiment 13	100	0.4	0.4	2	390
Experiment 14	100	0.4	0.4	2	440
Experiment 15	100	0.4	0.4	2	420
Experiment 16	100	0.5	0.2	5	480
Experiment 17	100	0.5	0.2	5	450
Experiment 18	100	0.5	0.2	5	440
Experiment 19	150	0.3	0.4	5	400
Experiment 20	150	0.3	0.4	5	460
Experiment 21	150	0.3	0.4	5	390
Experiment 22	150	0.4	0.2	10	430
Experiment 23	150	0.4	0.2	10	510
Experiment 24	150	0.4	0.2	10	470
Experiment 25	150	0.5	0.3	2	440
Experiment 26	150	0.5	0.3	2	440
Experiment 27	150	0.5	0.3	2	470

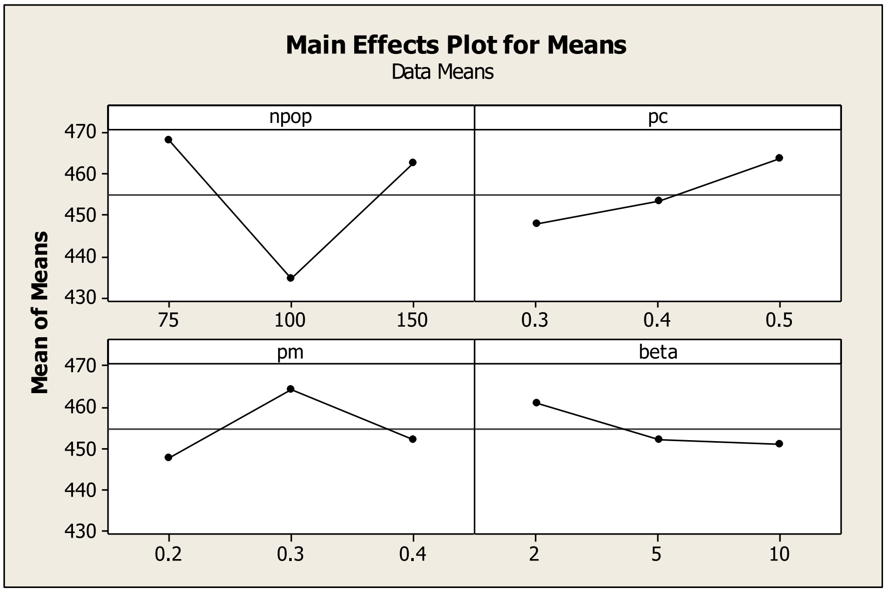

Figure 8. Analysis of Taguchi design.

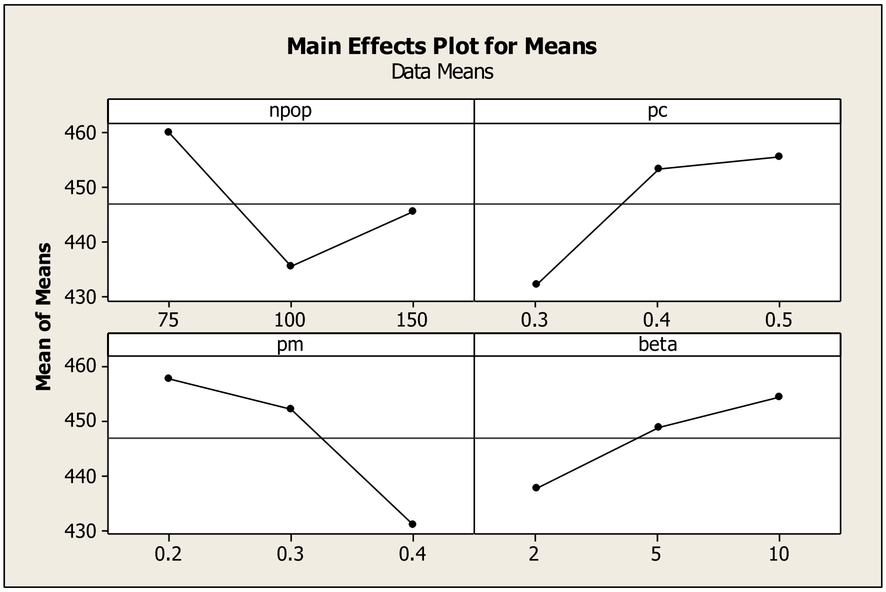

Figure 9. Analysis of Taguchi design.

Table 5. Parameter tuning.

	Parameter			
Algorithm	npop	pc	pm	Beta
GA	100	0.3	0.2	10
HGA	100	0.3	0.4	2

Table 6. Distance between nodes in small-sized problem (traveling time).

	Node numbers							
Node numbers	1	2	3	4	5	6	7	8
1	–	1	2	1	4	2	5	6
2	3	–	4	3	2	3	1	2
3	1	1	–	2	1	2	1	3
4	2	2	3	–	1	2	1	2
5	2	3	5	1	–	2	3	1
6	3	1	2	4	4	–	2	2
7	1	2	3	4	5	3	–	3
8	2	1	1	2	1	2	1	–

Table 7. Vehicles attributes.

	Vehicle			
Characteristic	1	2	3	4
Maximum capacity	450	300	380	300
Maximum route length	100	50	50	100
Fixed cost	100	100	120	50
Variable cost	10	20	30	15

Table 8. Customers' demand.

Node	1	2	3	4	5	6
Demand	150	250	140	50	100	50

Table 9. Relation between nodes in GAMS solution.

	Node number							
Node number	1	2	3	4	5	6	7	8
1	0	0	0	1	0	1	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0
4	0	0	0	0	1	0	0	0
5	0	0	0	0	0	0	0	1
6	0	0	1	0	0	0	0	0
7	1	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0

Table 10. Allocation of vehicles to nodes in the optimum solution.

	Vehicles type							
Node numbers	01	11	02	12	03	13	04	14
1	1	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0
4	1	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	1	0
8	1	0	0	0	0	0	0	0

8 nodes, 2 depots, 6 customers, and 4 types of vehicle including internal and external vehicles using GAMS software. Distances between nodes and vehicles attributes are illustrated in Tables 6 and 7, respectively. Table 8 presents customers' demands. Nodes 1 and 2 denote depots and nodes 3–8 represent customer node. The model is solved by GAMS software and the results are presented in Tables 9 and 10. The global optimum value of objective function is equal to 140 units.

The relations between nodes in the optimal solution are shown in Table 9, in which 1 means that the path is available between corresponding nodes.

Table 11. Computational results for GAMS and GA for small-\medium-sized problems.

	Problem.info				GA objective value			GA average	GAMS objective value			GAMS	GAMS objective value	
No	n	d	k	c	Min	Max	Average	time (s)	Obj	LB	Relative gap	time (s)	Gap^{mean}	Gap^{best}
1	8	2	4	6	140	140	140	56.4	140	140	0	15	0	0
2	10	2	3	8	222	228	224.4	78.2	222	222	0	23.4	0.01	0
3	12	4	5	8	208	213	211.4	101.2	208	206.2	0.008	32.5	0.025	0.008
4	20	2	5	18	413	444	419.2	237.2	1888.2	249.2	>1	1000	0.682	0.6573
5	30	5	5	25	598	612	604.4	382.4	2830	290	>1	1001	>1	>1

Table 12. Computational results for GAMS and HGA for small-\medium-sized problems.

	Problem.info				GA objective value			GA average	GAMS objective value			GAMS	GAP	
No	n	d	k	c	Min	Max	Average	time (s)	Obj	LB	Relative gap	time (s)	Gap^{mean}	Gap^{best}
1	8	2	4	6	140	140	140	66.2	140	140	0	15	0	0
2	10	2	3	8	222	222	222	85.3	222	222	0	23.4	0	0
3	12	4	5	8	208	208	208	124.6	208	206.2	0.008	32.5	0.008	0.008
4	20	2	5	18	373	398	388.2	237.2	1888.2	249.2	>1	1000	0.557	0.496
5	30	5	5	25	502	529	510.4	382.4	2830	290	>1	1001	0.7586	0.731

Table 13. Comparison of GA and HGA algorithms in large-scale problems.

	Problem info				HGA objective value			HGA average	GA objective value			GA average
No	n	d	k	c	Min	Max	Average	time (s)	Min	Max	Average	time (s)
6	75	10	4	65	1245	1710	1485.4	494	1680	2655	2150	374
7	100	5	5	90	2725	2780	2608	423	3035	3275	3178.2	322
8	125	25	10	100	1102	1585	1420	1390	1819	2002	1934.2	925
9	125	5	5	120	4620	5683	4815.4	328	5665	5762	5738	307
10	150	20	5	130	5745	7020	6385	944	6820	7900	7630	483
11	150	5	5	145	8550	9620	8870.2	329	9370	9640	9488	248
12	150	30	5	120	4655	5597	5195	603	4905	5750	5240	476
13	175	10	10	165	2386	2495	2408	623	3806	4105	3860.2	391
14	175	2	10	173	2565	3298	2767	381	4130	4505	4310	152
15	175	5	4	170	8270	9170	8905.2	245	8755	9899	93,445.4	134

The assignment of vehicles to nodes is provided in Table 10. The value of 1 indicates that the vehicle is allocated to the corresponding node. Each vehicle is represented by two indices. The first index shows that the vehicle is internal or external. 0 represents internal vehicles, while 1 represents external ones. The second index presents the type of vehicle.

Comparison of the two methods

As mentioned in previous sections, GA and HGA methods are used to solve the problem. GAMS software is also applied for small-scale and medium-scale problems; thus, a comparison can be made between these methods solutions. First, real small-/medium-sized problems are considered. Distances between nodes are uniformly distributed in range of 1 to 8. Vehicles' attributes are shown in Table 7. Also, demands of customers follow a uniform distribution between 50 and 300 units. This model is solved by GAMS software for five small-/medium-sized problems. Moreover, 10 large-scale problems are solved by the metaheuristics proposed in this paper. Lastly, all test problems have been solved by GA and HGA. These algorithms have been coded in MATLAB R2013a and implemented on an Intel Core i5 2.27 GHz personal computer with 4 GB RAM. In order to verify the performance of the metaheuristics, the results of GA and HGA for the small-/medium-sized problems are compared to the solutions obtained from GAMS 23.6. Numerical results for small-/medium-sized problems are shown in Tables 11 and 12. Table 13 presents the numerical results for the large-scale problems. All problems are run five times by each method and the minimum, maximum, and average results are recorded. In these tables, in problem information section n, d, k, c denote number of nodes, number of depots, number of vehicle types, and number of customers, respectively. Tables 11 and 12 also present the comparison of results obtained from metaheuristics with that of GAMS. Note that, gap^{mean} and gap^{best} indicate the gaps calculated for the average of solutions and the best solution, respectively. The gap percentage is calculated as follows:

$$Gap = \frac{\text{objective function of the algorithm - LB}}{\text{LB}} \times 100$$

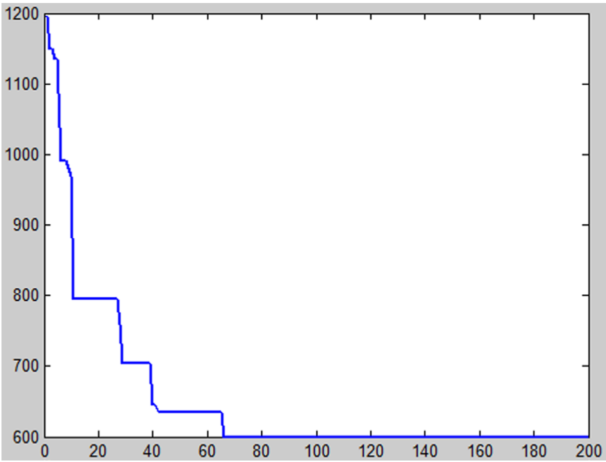

Figure 10. Operation of GA in a medium-sized problem (30 nodes).

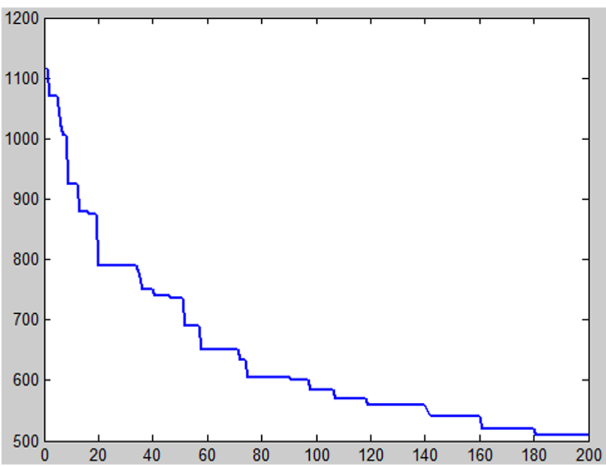

Figure 11. Operation of HGA in a medium-sized problem (30 nodes).

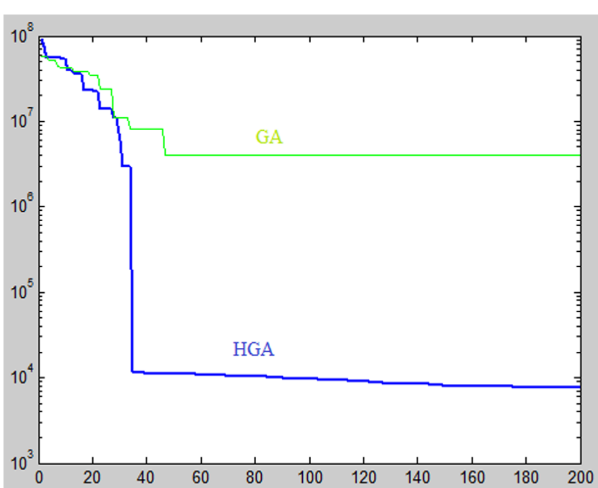

Figure 12. Comparison of two algorithms in large-scale problem (150 nodes).

Next, objective functions are calculated with the proposed algorithms. In some cases, the gap between the algorithms and CPLEX was 0%, indicating that these methods yield exact solutions. Since GAMS software is unable to solve large scale problems, only metaheuristics are applied to solve them. The results obtained by metaheuristics are compared with each other in Table 13.

All algorithms achieve reasonable solutions; however, it can be concluded from the tables that HGA yields more acceptable results compared with classical GA. HGA starts with a better initial population, uses a more efficient construction algorithm and finally achieves a better final solution. Figures 10 and 11 present a graphical comparison of GA and HGA algorithms for two test problems.

In large-scale problems, GAMS is unable to solve the problem and thus, GA and HGA algorithms are proposed in this paper. In a large-scale problem, 150 nodes including 10 depots, 140 customers, and 5 types of internal and external vehicles are considered. Ultimate objective functions associated with HGA and GA were equal to 3040 and 5480 units, respectively. The procedures of the two algorithms are depicted in Figure 12.

Conclusions and future research

In this paper, a mathematical model was proposed for COMDMVRP. The model was validated by means of conducting several experiments. According to the numerical results, both GA and HGA are found to have a good consequences; however, HGA usually gains better solutions. In this work, nearest neighborhood and AHP methods were used in the construction of initial population, and a heuristic algorithm, namely ISP (iterative swap procedure) was applied to improve the solutions. In the small-scale problems, numerical results demonstrated that the algorithms (especially HGA) can achieve an optimal solution for which the gap between objective values obtained from GAMS and algorithms are zeroes. In the medium-scale problems solved by GAMS software, as the number of nodes increased, the performance of the algorithm decreased and this software was unable to yield the optimal solution. The solutions achieved by metaheuristic algorithms were more suitable than GAMS solutions for medium-scale problems. In the large-scale problems, numerical results proved the efficiency of both of the metaheuristic algorithms; however, HGA was found to perform more efficiently in large-scale problems. The relationship between the number of depot nodes and vehicles and the running time and cost of the problems was shown in the problem information section in the experiments.

For the future research, we suggest considering time windows restrictions in the model in which serving time will be limited to time span; i.e. the vehicle can start its service in this period of time. Also, the time of the routes can be taken into account and several constraints can be applied to the travel time. Another interesting topic might be considering the application of these VRP models in various problems such as waste collection problem.

Disclosure statement

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