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A scalable non-myopic dynamic dial-a-ride and pricing problem for competitive on-demand mobility systems

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ABSTRACT

We propose a competitive on-demand mobility model using a multi-server queue system under infinite-horizon look-ahead. The proposed approach includes a novel dynamic optimization algorithm which employs a Markov decision process (MDP) and provides opportunities to revolutionize conventional transit services that are plagued by high cost, low ridership, and general inefficiency, particularly in disadvantaged communities and low-income areas. We use this model to study the implications it has for such services and investigate whether it has a distinct cost advantage and operational improvement. We develop a dynamic pricing scheme that utilizes a balking rule that incorporates socially efficient level and the revenue-maximizing price, and an equilibrium-joining threshold obtained by imposing a toll on the customers who join the system. Results of numerical simulations based on actual New York City taxicab data indicate that a competitive on-demand mobility system supported by the proposed model increases the social welfare by up to 37% on average compared to the single-server queuing system. The study offers a novel design scheme and supporting tools for more effective budget/resource allocation, planning, and operation management of flexible transit systems.

1. Introduction

The sharing economy provides opportunities for individuals in both the buying and sharing of products or services. Sharing-economy platforms allow people to find temporary employment, generate extra income, increase reciprocity, enhance social interaction, and access resources that are otherwise unattainable. Uber and Airbnb are among the world's leading businesses in transportation and hospitality, respectively. While many municipalities and regions have blocked these new forms of commerce, others have accepted change as inevitable and have been eager to provide new efficiencies for consumers. For example, Hall and Krueger (2016) argue that the availability of modern technology such as the Uber app provides numerous advantages, including lower consumer fares and potentially higher earnings for drivers, over the traditional taxicab dispatch system. Although the sharing economy is profitable, little is known about its use among unemployed and low-income individuals and families. Some reports suggest that this new platform of economic activity is highly insecure and that contingent employment leads to the exploitation of workers (Summers and Balls, 2015). Bernhardt (2014) signals a cautionary note about any claims of radical recent change being wrought across the U.S. economy and identifies data gaps that need to be closed and research questions that need to be answered in order to improve workplace and labor standards. For example, why is the traditional taxicab dispatch system unable to compete with or rival the new forms of commerce, and does the sharing economy really provide benefits? One key aspect of our smart on-demand transit approach is social efficiency, in terms of both the level of service and the computational effort needed to operate it.

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For a traditional taxicab dispatch system, the first–last mile problem is challenging because it calls for high flexibility under relatively low-density usage, which implies a high cost to implement. As an example, taxicab drivers in New York City (NYC) spent 39% of their total mileage cruising for passengers in 2005 (see Schaller Consulting, 2006). Such a system is often inefficient, because of the use of expensive vehicles, high fuel costs, heavily congested traffic, and a low ratio of busy to idle vehicles (see Powell et al., 2011). In addition to the costs, this is a major environmental concern, since the burning of fossil fuels in vehicles is a major contributor to global warming. If ridesharing systems and other forms of public transit increase in efficiency, fewer people will rely on personal vehicles to meet their transport needs. Thus a smart on-demand system could be a significant step in reducing costs and carbon emissions worldwide.

The state of a transit system can change rapidly, and static models lack the flexibility to adapt to changes in a timely manner. Because they ignore the elasticity of demand and price, such models can result in inefficiencies that stem from overestimation of improvements in the level of service—a limitation that could be reduced or eliminated if non-myopic considerations were taken into account. Thus what is needed is a smarter system that anticipates stochastic elements and dynamically adapts to them. Among other things, such a system should be able to incorporate high-speed-communication technologies in order to calculate opportunity costs, so that social benefits can be divided between customers and providers—a prospect which has been ignored in traditional transit systems.

Real-time information problems introduce a new dimension to decision-making in dynamic models. These models involve an iterative process: making decisions, then accessing information, then making more decisions and accessing more information, and so on. A Markov decision process (MDP) with discrete time intervals can be modeled using a Bellman equation (Powell, 2011), as follows:

$$\mathrm{V_t(S_t)} = \min_{\mathrm{x_t}}(\mathrm{C_t(S_t,x_t)} + \gamma \mathrm{E}[\mathrm{V_{t+1}(S_{t+1})}|(\mathrm{S_t,x_t})]), \tag{1}$$

where $\mathrm{V_t}$ is the value of the optimal dynamic policy, $\mathrm{C_t}$ is the immediate payoff of the decision x_t under state S_t (which is also typically driven by information on exogenous stochastic variables, and varies in size based on the underlying distribution of the variable(s)), and γ is a discount factor. The conditional expectation term depends on the future state while it depends on the state S_t which defines as location of vehicles and may also depend on x_t. It can cost time and money to visit a state, so we have to consider the future value of an action in terms of its effect on improving future decisions. As a result, we need to estimate the value function at a given state, which necessitates a tradeoff between visiting that state because we think it represents the best decision ("exploration") and estimation of downstream values to make what we think is the best possible decision ("exploitation"). As a result, the Bellman equation becomes one that depends on both past (through network effects) and future (through the expected value function). Sayarshad and Chow (2015) proposed an approximation method in which the network effects and the timing effects as illustrated with an intermediate value function (based on Chow and Regan, 2011; Chow and Sayarshad, 2015) from Eq. (1) are lumped together into one effect:

$$V_{t,i}(S_t) = \min_{\mathrm{x_{t,i}}}(C_t(S_t,x_{t,i}) + \Omega(V_{t,1},...,V_{t,i-1},V_{t,i+1},...,V_{t,K},E[V_{t+1}|S_t])), \tag{2}$$

where K is the number of link components for which decisions must be made, Ω is approximated by an expression derived from an M/M/s queue, and the whole expression is split into user costs and system costs. The state of all vehicles constitutes the state of the system at time t, denoted as S_t. The approach we propose in this study is not limited to only a single time-step look-ahead and instead exploits our estimates of the value function under an infinite-horizon look-ahead.

Autonomous vehicles (AVs) are potentially disruptive, both technologically and socially, with claimed benefits comprising increased safety, road utilization, energy savings, and driver productivity (Zhang and Pavone, 2016, Ma et al., 2017a). For example, it has been asserted that if five percent of new vehicles sold in 2030 (about 800,000 vehicles) were shifted to autonomous taxis, it would save about 7 million barrels of oil per year and reduce annual greenhouse gas emissions by 2.1–2.4 million metric tons of CO_2 per year, which is equal to the emissions savings from more than a thousand 2-megawatt wind turbines (Greenblatt and Saxena, 2015; Greenblatt and Shaheen, 2015). The technology which is needed to enable automation, particularly at levels 3–5 (based on the levels of AVs which have been defined by The National Highway Transportation Safety Administration), is extremely sophisticated and would have to make use of high-performance computational hardware, state-of-the-art online models, decision-making algorithms, and real-time information. Our proposed methodology could possibly be applied as a decision support tool for future urban mobility systems such as smart taxis and autonomous vehicle fleets. The smart system could also offer numerous transportation, infrastructure, land use, and environmental benefits (increased overall public transit and non-motorized modal use, including rail, walking, bicycling, and carpooling), as well as social benefits (access by college students; availability in rural, suburban, and disadvantaged communities; reduced stress on commuters; and often preferential parking and other incentives). As a result, the potential energy savings expected from AVs are much larger than the estimated worst-case growth in energy use and other factors (see Greenblatt and Shaheen, 2015). Smart on-demand mobility systems would therefore be a valuable alternative to personal vehicles; they would also be a help to communities with poor or nonexistent public transit, such as low-income communities, rural areas, and large suburbs.

The framework for our proposed system, where dynamic operations are driven by real-time information, is shown in Fig. 1, which lists the various technologies and data sources needed to ensure a viable "smart" transit system. For example, communications equipment is needed so that customers can send travel requests to the dispatch center. Intelligent vehicle-highways systems (IVHS) is an application of information and communications technologies (ICT) that could bring about better control of the flow of vehicles. Electronic data interchange (EDI) consisting of the electronic transfer from computer to computer could increase the speed of

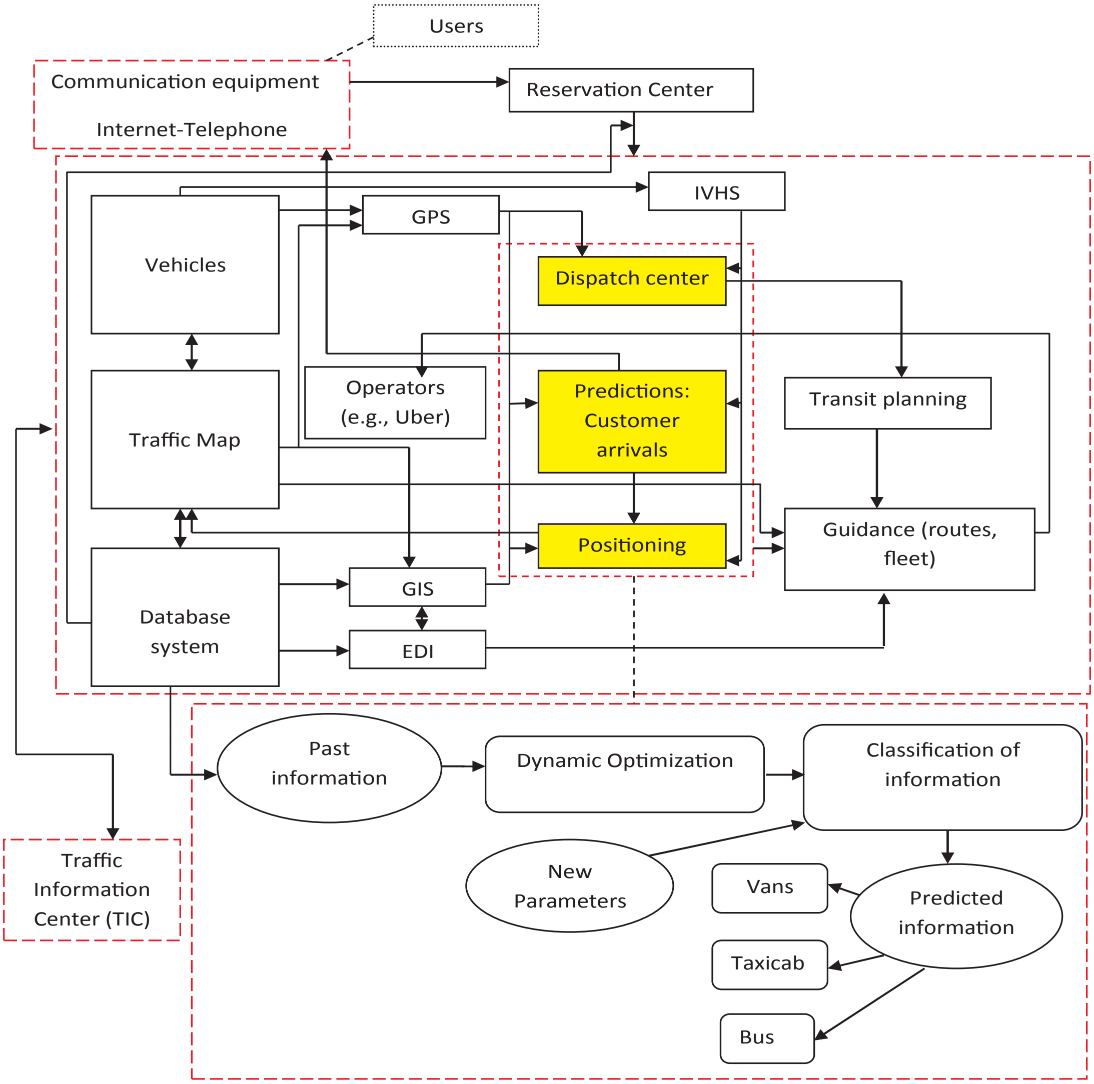


Fig. 1. Key components of "smart" mobility-on-demand service (adapted from: Sayarshad, 2015).

communication and control all aspects of the system using message data. Mobile-device communication systems are one example of a technology capable of meeting this need (Markovíc et al., 2015). The framework has three key functions: dispatch/vehicle routing, prediction of passenger travel demand (see Sayarshad and Chow, 2016), and repositioning of idle vehicles (see Sayarshad and Chow, 2017). The functions are tied together as key components of a "smart transit system" framework where dynamic operations are driven by real-time information. In this paper, we focus on the first function. To the best of our knowledge, our contributions in this work are the following:

- We propose a non-myopic dynamic dial-a-ride and pricing approach using a multi-server queue under infinite-horizon look-ahead which is able to estimate opportunity costs and future profits in relation to price and service quality.
- We present results of simulations which illustrate that our approach can perform better in terms of social welfare than existing MDP approaches which are based on a single-server queue.
- The simulations show that our proposed pricing policy is the socially efficient point with respect to the realized social welfare, as opposed to the revenue-maximizing price which is consistent with standard monopoly pricing that ignores user delay.
- We derive the customers' queue-joining threshold using a balking rule with respect to the socially efficient level and revenue maximization.

The remainder of the paper is organized as follows: The literature is reviewed in greater detail in Section 2. We provide a description of the proposed framework in Section 3. The dynamic ridesharing and pricing approach is formulated in Section 4, the results of the simulations we conducted to verify the model and algorithm are presented in Section 5, and also the effectiveness of the algorithm utilized in investigating the socially efficient level and the equilibrium-joining behavior in our simulation of actual NYC taxicab data is validated in Section 5.2.

2. Literature review

There are a couple of models proposed in the literature for the static DARP, as well as for the related static pickup and delivery problems. In addition, heuristics to solve DARPs have been proposed by Qu and Bard (2013), Parragh (2011), and Muelas et al. (2015). A comprehensive review of the static, stochastic, and dynamic dial-a-ride literature is given in Ho et al. (2018).

A smart mobility system framework takes real-time data and optimizes the dispatch/routing system to serve passengers, which can increase the effective capacity, manage demand, improve accessibility, and maximize efficiency and benefit. Combining advanced technologies such as mobile devices with scalable models such as dynamic ridesharing models can improve the ability to make routing decisions for a fleet of vehicles in real time (see Sayarshad, 2015).

One potential solution to the first–last mile problem is the concept of flexible transport (or transit) services (FTS). Most of the previous transit simulation studies extended the capabilities of traffic simulation models without representation of transit operations for specific applications. Cortés et al. (2005) presented a general microsimulation framework for evaluation of transit systems by using a commercial microsimulator. Mulley and Nelson (2009) introduced an operational model for providing transport to employment (T2E) services and implemented it in rural areas.

One of the most extensive frameworks for modeling of transit services is found in mathematical optimization models (see Psaraftis et al., 2016). For example, Quadrifoglio et al. (2007, 2008) and Quadrifoglio and Li (2009) introduced a mixed integer programming (MIP) formulation for mobility allowance shuttle transit (MAST) where vehicles may deviate from a fixed path (one that includes a few mandatory pick-up and/or drop-off points) to serve the demand within a given service area. Liu et al. (2015) formulated two static optimization models which simultaneously consider multiple trips, heterogeneous vehicles, multiple request types, and multiple vehicle capacities and used a branch-and-cut algorithm to solve the problem. Braekers and Kovacs (2016) presented a static multi-period dial-a-ride problem in order to ensure driver consistency, by bounding the maximum number of different drivers that transport a user over a given time horizon. Daganzo (2010) and Nourbakhsh and Ouyang (2011) provided a model where individual buses operate without fixed routes or predetermined stops, and instead travel freely within their own service regions to pick up or drop off passengers. Liaw et al. (1996) considered integration of paratransit dial-a-ride vehicles with fixed-route buses, in a system where transportation bookings are made in advance. Li and Quadrifoglio (2010) focused on the service performance of a demand-responsive operating policy for feeder transit services as opposed to a traditional fixed-route policy, and when to deterministically switch between them. Nair and Miller-Hooks (2010) and Sayarshad (2015) formulated an allocation–relocation model to address optimization of the movement of vehicles to balance the system and the movement of staff needed to drive them.

Several models have been created to optimize a flexible transit-operating design (see Agatz et al., 2012). Schofer et al. (2003) proposed a trip simulator based on queuing theory to assist planners of demand-responsive transit (DRT) services in developing a preliminary estimate of the number of vehicles required for a new or modified DRT service. Horn (2002) designed a software system for management of the deployment of a fleet of demand-responsive passenger vehicles such as taxis or variably routed buses. Lee et al. (2004) adopted a dispatch system that uses the Global Positioning System (GPS) and is based on the nearest-coordinate method (i.e., the taxi assigned to each booking is the one with the shortest direct straight-line distance to the customer's location). Lee et al. (2005) focused only on trips from homes to a metropolitan rapid transit (MRT) station (many-to-one) for a paratransit system. Daganzo (1978) proposed an approximate multi-server queue system for many-to-many demand-responsive transportation systems. Daganzo (1984) described a checkpoint demand-responsive system where pick-up and drop-off points are at centralized locations called checkpoints.

Countless models given in the literature consider customer demand when deciding to visit a future state. Cortés et al. (2009) formulated the problem as a hybrid predictive control problem using state-space models and a two-stage look-ahead and proposed a particle swarm optimization (PSO) heuristic to solve it. Mitrović-Minić et al. (2004) proposed a double-horizon-based heuristic for the dynamic pickup and delivery problem which considers the impact of the decision on both a short-term horizon and a long-term horizon when assigning a new request to a vehicle. Ichoua et al. (2006) defined a new strategy based on dummy customers (representing predicted requests) on vehicle routes to provide good coverage of future request arrivals. Ulmer et al. (2017) proposed an MDP for dynamic routing problems and showed that route-based MDPs are equivalent to the conventional MDP model. Thomas and White (2004) formulated an expected cost-to-go function for dynamic route selection as an MDP with look-ahead demand requests. Our proposed methodology is different from the existing non-myopic dynamic dial-a-ride problem (DARP) that allows for only a one- or two-stage look-ahead policy. Hyytiä et al. (2012) consider an approximate multi-server queue system comprised of individual M/M/1 queues that assign new customers to FTS vehicles to minimize both the user cost and the system cost.

Sayarshad and Chow (2015) introduced a new non-myopic pricing policy for a dial-a-ride problem based on MDPs. The model considers only a single-server queue for on-demand mobility services. But, a multi-server queuing approach could be more realistic. For instance, the previous model considers only a single-server queue that the parallel M/M/1 queues are independent of each other, while we consider an M/M/s queuing system that has s parallel servers with 1 queue. Thus, the proposed approach serves as a new integrated framework for future studies in urban transport informatics and design of types of urban mobility models such as for the bimodal dial-a-ride problem (Guo et al., 2017, Frei et al., 2017, Chen and Nie, 2017). Using the proposed queuing system, the customer delay and sojourn time are calculated based on the multi-server queue system where the optimal tours are optained by running a TSPPD algorithm. This is an insertion heuristic that takes a list of pickup and delivery nodes, and assigns a tour by first finding the best drop-off tour, and then inserting the pickup locations. Moreover, the demand function used in this study is calculated

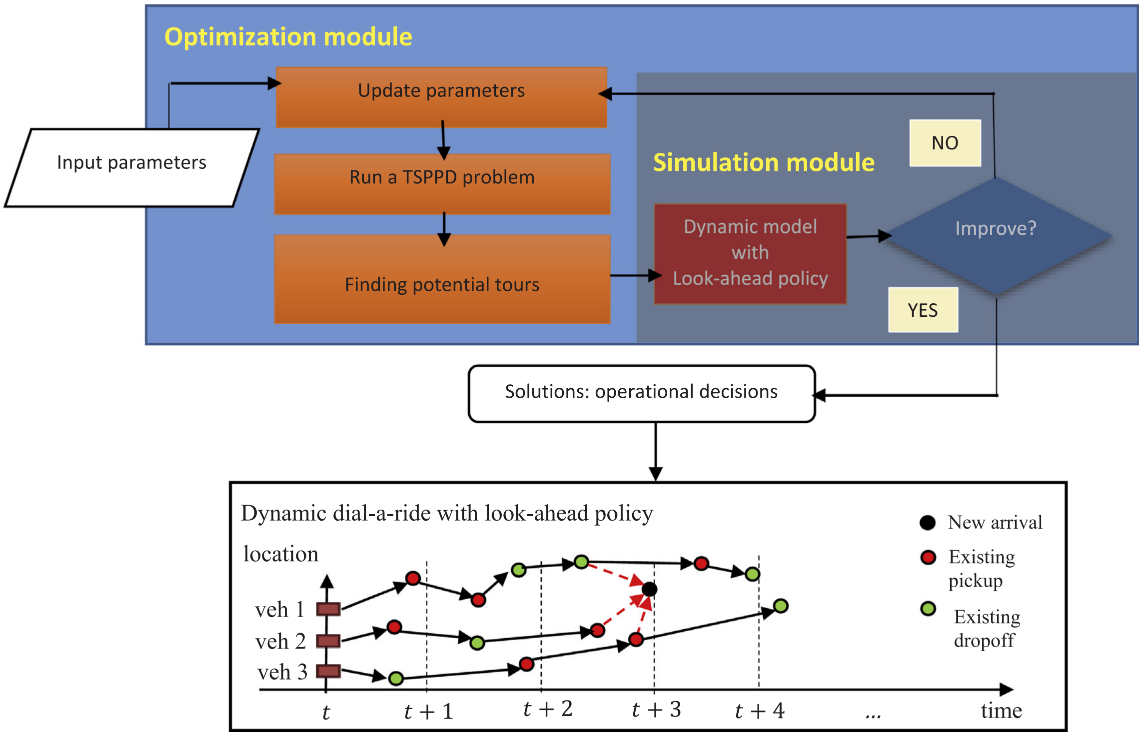


Fig. 2. Illustration of the non-myopic dynamic DARP as a multi-server queue system.

on the basis of a more realistic probability that a customer will join the service. The proposed pricing strategy to assign priority to customers based on their willingness to pay is considered to attain a social optimum. We also find the customers' queue-joining threshold using a balking rule with respect to the socially efficient level and the revenue maximization that it provides. The socially efficient level is providing a new approximation method into the dynamic vehicle routing and pricing problem for computing the conditional expected future cost portion, which is otherwise computationally difficult or impossible to calculate. Finally, actual New York City taxicab data were conducted to test the efficiency of the proposed approach.

This study investigates a tour-based ride-sharing/hailing model that captures (a) tour length per vehicle, (b) passenger waiting times, (c) dynamic price with elasticity of demand, and (d) customers' queue-joining threshold with respect to the socially efficient level. Compared to traditional flexible transit services such as dial-a-ride, ridesharing services based on this model could offer a distinct cost advantage and operational efficiency, and in the long run provide local residents with new employment opportunities to serve as self-driver systems.

3. Problem definition

We propose a new non-myopic policy that retains the core network structure, based on the premise of using queuing theory to approximate the future steady-state condition for non-myopic DARP. In doing so, we approximate the value function instead of approximating short time horizons, decision iteration functions, or states. It is possible to derive analytical expressions for the steady-state average delay costs, which is computationally cheap (Sayarshad, 2015). Consider the algorithm in Fig. 2, where the proposed new approximation queuing approach for the non-myopic dynamic DARP with look-ahead policy is illustrated. The algorithm consists of two modules: an optimization module and a simulation module. The optimization module attempts to find potential tours by running a TSPPD algorithm. The simulation module uses an approximation queuing approach to capture customer delays and system costs.

Customers send requests for travel from one location to another; as a result, dispatch, routing, and pricing decisions are made in real time by the dispatch center. The subgraph during each dispatch assignment consists of $G = \{O,D,P^+,P^-\}$, where O is a vector containing current locations of all vehicles, D is a pre-specified idle vehicle location, P^+ is the set of pickup nodes, and P^- is the set of drop-off nodes, where $|P^+| \leqslant |P^-|$ since a vehicle may be en route to dropping off an existing customer when a new customer arrives at that dropoff point (or an earlier one) along the route. In the numerical experiments in this study, D is set to be the fleet depot, but in future studies it can be set to some other location by Sayarshad and Chow (2017) model, which proposed a new queuing-based formulation of the online problem of relocating idle vehicles that entails calling an assignment routine to pre-position the idle vehicle.

4. Proposed methodology

In this section, we formulate a new parameterized heuristic control policy for vehicle and route selection in a demand-responsive mobility system. This multi-server queue policy aims to maximize social welfare (less a weighted sum of the distances traveled by the vehicles (per passenger)) and the mean passenger travel time, as described below.

Requests are assumed to be from single individuals arriving according to a Poisson process with intensity function λ. We consider time windows as being built into the elastic demand function. Since the objective includes customers' inconvenience, the pricing reflects late deliveries, which in turn feeds back to the demand for service. Based on the price quoted by the system, an arriving customer will join the queue if and only if the benefit is greater than the expected waiting cost. The service time, μ, is exponentially distributed and is approximated by a function $\hat{\mu}$ which is obtained from historical data and defined as the average number of served trip requests per time unit. Note that $\hat{\mu}$ depends on operator's dispatching, vehicles' positions, and routing policy, and arrival rate of customers. The service order can reorder the pick-ups and deliveries dynamically. For example, user 1 may be assigned to vehicle 1, but when user 2 arrives he/she might get picked up and dropped off by vehicle 1 if its intended route is along his/her way and the solution maximizes the value function. Each customer is assumed to receive the same reward R by using the transit service. But heterogeneous rewards may be possible and user behavior may be sensitive to them, which will be considered in future research (see Qiu et al., 2018). Additionally, a modification also may be called for if we were to assume that the reward obtained depends in some way on the effective traffic density.

To provide the optimal policy for the DARP, we model the incremental operational value functions using dynamic optimization as functions of demand ($D(p)$), system cost (C_s) and user delay(C_u). Let $(y_1;...y_n)$ denote the state of the queue with n tasks, and let $d = \sum_i^n y_i(v,\xi)$ denote the amount of unfinished work for each vehicle v (expressed in terms of the time it would take to finish it), where y_i denotes the service time (amount of work) for customer i, v is a vehicle, and ξ is a tour obtained for a traveling salesman problem (TSP) (see Larson and Odoni, 1981). Since the objective function includes future user costs, we have to calculate the relative value of state $(y_1;...y_n)$ during the time intervals $(0,d)$ and (d,∞) (Hyytiä et al., 2012). Then the social welfare function can be expressed as

$$SW = RD(p) - C_s - (C_u^{[0,d]} + C_u^{[d,\infty]}) \tag{3}$$

where customers earn a reward of R upon completion of service at the queuing station. Both the system cost and the customer delay are formulated in Section 4.1 and a demand function based on the probability that a customer will join the queue is proposed in Section 4.2.

4.1. Customer delay and tour length

In this section, we present a new social optimization problem for an on-demand mobility system with infinite-horizon look-ahead: to find an equilibrium point using a set of functions that considers demand, system costs, and user costs, which are incorporated into the socially efficient level as explained in Section 4.3. We solve ξ for each vehicle $v = 1,...,s$ for the new passenger under consideration, where $T(v,\xi)$ is the current tour length of the vehicle. We solve a traveling salesman problem with pickup and delivery (TSPPD) based on Mosheiov (1994) to find potential tours $T(v,\xi)$. The TSP tour construction of the drop-off nodes is solved using Christofides' (1976) algorithm. The parameter θ, $0 \leqslant \theta \leqslant 1$, is a weight that differentiates the system cost from the user delay. The relative value can be calculated as follows (see Appendix A):

$$c(v,\xi) = \theta T(v,\xi) + (1-\theta)\left[\sum_i y_i(v,\xi) + \frac{\lambda T(v,\xi)^2}{2} - \left[\lambda/\hat{\mu} + \left(\frac{(\lambda/\hat{\mu})^s \rho}{s!(1-\rho)^2}\right) P_0\right] T(v,\xi) + \beta E[V_A]\right], \tag{4.a}$$

where

$$\mathrm{P}_0 = \left[\frac{\lambda^s}{s!\hat{\mu}^s}\frac{s\hat{\mu}}{s\hat{\mu}-\lambda} + \sum_{n=0}^{s-1}\frac{\lambda^n}{n!\hat{\mu}^n}\right]^{-1} \tag{4.b}$$

$$E[V_A] = \begin{cases} w\sum_{j=0}^{s-1}\frac{1}{Erl(j,a)} + \frac{\frac{\lambda T(v,\xi)}{2}-1}{\hat{\mu}} & n \leqslant s \\ \frac{\left(\frac{\lambda T(v,\xi)}{2}-s\right)w}{Erl(s,a)} + \frac{\left(\frac{\lambda T(v,\xi)}{2}-1-s\right)\left(\frac{\lambda T(v,\xi)}{2}-s\right)}{2s\hat{\mu}(1-\rho)} + \frac{\frac{\lambda T(v,\xi)}{2}-s}{\hat{\mu}}, & n > s, \rho = \lambda/s\hat{\mu} \end{cases} \tag{4.c}$$

4.2. Demand function and pricing policy

In this section, we introduce tolling strategies to assign priority to customers based on their willingness to pay to attain a social optimum and the revenue-maximizing price. Sayarshad and Chow (2015) incorporate pricing into the non-myopic policy based on an M/M/1 queue by Mendelson and Whang (1990). Since our approximation is based on an M/M/s, so we need to examine the pricing policy based on a multi-server queue. Knudsen (1972) enforced a multi-server queue using the balking level $M/M/S/N$ and obtained the price by imposing a toll on the customers who join the system. In this particular system, the necessary toll will depend on the waiting-time function. Then, the toll p should be such that

$$R-\frac{n+1}{s\hat{\mu}}<p\leqslant R-\frac{n}{s\hat{\mu}} \tag{5}$$

By this logic, customers will adopt the following threshold strategy:

$$k(p)=(R-p)s\hat{\mu} \tag{6}$$

The customer joins the queue if $n\leqslant k(p)$ and leaves if $n>k(p)$. This sets up the framework to study pricing effects in queues. However, the toll p is not related to the customer arrival rate or the opportunity cost. Thus, we must take into account the effects of random fluctuations in demand on the price.

We now focus our attention on the relationship between prices and equilibrium arrival rates. The newly arrived customer is required to choose one of two scenarios: either (i) he joins the queue, incurs the losses associated with spending some of his time in it, and finally obtains a reward of R; or (ii) he refuses to join the queue, an action which does not bring about any gain or loss. The probability that the passenger eventually will receive a reward of R is the same as the ruin probability in the gambler's ruin problem when the initial asset is n, the goal is $n+1$ and the winning probability in each round is equal to $P=(\rho/1+\rho)$. Let $Q=1-P$, so the ruin probability is equal to $\frac{(\frac{Q}{P})^{n+1}-(\frac{Q}{P})^{n}}{\left(\frac{Q}{P}\right)^{n+1}-1}$ (Rao, 2009; Gross and Harris, 1998; Wolff, 1989). Corresponding to the price, p, customers join the queue only when the queue length is less than or equal to the threshold $k(p)$. For a maximum queue length of $k(p)$, the probability of observing $k(p)$ customers in the system is $\frac{\rho^{k(p)}}{\sum_{i=0}^{k(p)}\rho^{k(p)}}$, thus the probability that a customer will join the queue is given by the following equation (Rao, 2009; Gross and Harris, 1998; Wolff, 1989; Hassin and Haviv, 2006):

$$J(p)=\frac{1-\rho^{k(p)}}{1-\rho^{k(p)+1}},\rho=\frac{\lambda}{s\hat{\mu}} \tag{7}$$

In other words, the demand function is:

$$D(p)=\Lambda\frac{1-\rho^{k(p)}}{1-\rho^{k(p)+1}} \tag{8}$$

The aggregate arrival rate Λ is distinguished from an individual decision structure, $\lambda,0<\lambda\leqslant\Lambda$, for the decision of whether to join the system.

4.2.1. The socially efficient price

Knudsen (1972) enforced a multi-server queue using the balking level, $M/M/s/n_{max}$, and determined n_{max} by solving the equation $R-\frac{n_{max}+1}{s\mu}=0$. Then n_{max} is given by $n_{max}=Rs\mu-1$. Therefore, it will usually be possible to find the socially efficient price by enforcing a balking level $k(p)<n_{max}$, where $k(p)$ is a point that maximizes the function Z, and the price is then given by $p^{*}=R-\frac{k(p)+1}{s\hat{\mu}}$. To find the price that accounts for customer waiting times, we may write the following function (Hassin and Haviv, 2006; Rao, 2009):

$$\mathrm{Z}=\mathrm{RD(p)}-\mathrm{N(p)}=\Lambda\mathrm{R}\frac{1-\rho^{k(p)}}{1-\rho^{k(p)+1}}-\left(\frac{P_0^B a^s\rho}{s!(1-\rho)^2}[1-\rho^{k(p)-s+1}-(1-\rho)(k(p)-s+1)\rho^{k(p)-s}]\right), \tag{9.a}$$

where

$$P_0^B=\left[\frac{a^s}{s!}\left(\frac{1-\rho^{k(p)-s+1}}{1-\rho}\right)+\sum_{n=0}^{s-1}\frac{a^n}{n!}\right]^{-1}, \tag{9.b}$$

where $a=\lambda/\hat{\mu}$, and $\rho=a/s$. The first term is the rate at which reward is earned, and the second term is the rate at which waiting cost is incurred. The second term, $N(p)$, represents the average number of customers in the queue at any point in time and can be expressed in terms of $k(p)$ (Gross and Harris, 1998).

4.2.2. The revenue-maximizing price

In this scenario, the system should charge the price p^r that maximizes the expected revenue rate $p^rD(p^r)$. Thus we need to find the price p^r that maximizes the value of the revenue function which is consistent with the standard monopoly pricing model.

$$p^r=\text{arg}max\ pD(p)=\text{arg}max\left[p_n\Lambda\frac{1-\rho^{k(p)}}{1-\rho^{k(p)+1}}\right] \tag{10}$$

Two key computations shape our integration of non-myopic pricing into our approach: One is to find the price p^r that maximizes the expected revenue. The other is to calculate a positive price p^{*} that accounts for customer waiting times. This is due to the negative system and user costs that are present in our model under infinite-horizon look-ahead. In this situation, the second method is more useful in calculating non-myopic pricing, by considering a customer's decision to join or leave, which will influence future arrivals.

Apart from discovering this framework above, Naor (1969) also provides the following comparative:

$$k(p^r) < k(p^*) < k(p^0), \tag{11}$$

where $k(p^0)$ is the customers' queue-joining threshold strategy ($p = 0$) and

$$p^r > p^* > 0 \tag{12}$$

4.3. Social optimization

Now we can write down the social welfare function using the new pricing framework and the relative cost (which is calculated by Eq. (4)). The first term on the right-hand side is the rate at which reward is earned, the second term is the rate at which tour length is incurred, and the last term is the total customer delay. We also divide the costs by $T(v,\xi_n)$ to ensure that the units are the same as the benefits.

$$\max SW(p,v,\xi_n) - SW'(p,v,\xi_{n-1}), \tag{13.a}$$

where SW denotes the social welfare:

$$\mathrm{SW}(p,v,\xi_n) = \Lambda\mathrm{R}\frac{1-\rho^{k(p)}}{1-\rho^{k(p)+1}} - \theta\left[\frac{T(v,\xi_n)-p}{T(v,\xi_n)}\right] - \frac{(1-\theta)}{T(v,\xi_n)}\left[\sum_i y_i(v,\xi) + \frac{\lambda_n T(v,\xi_n)^2}{2} - \left[\lambda_n/\hat{\mu} + \left(\frac{(\lambda_n\hat{\mu})^s\rho}{s!(1-\rho)^2}\right)\mathrm{P}_0\right]T(v,\xi_n) + \beta E[V_A] + p\right] \tag{13.b}$$

$$k(p) \leqslant s\hat{\mu}(\mathrm{R}-p) \tag{13.c}$$

$$0 < \lambda_n \leqslant \Lambda, \tag{13.d}$$

where P_0 and $E[V_A]$ are calculated by Eq. (4.b) and Eq. (4.c), respectively. Once we have formulated the social optimization problem, we need to run the following algorithm in real-time operation:

Algorithm 1

Upon arrival of a new user n

1. Update the positions and service statuses of the tour ξ_{n-1} of every vehicle from the time of arrival of user $n-1$.
2. For each vehicle v:
   2.1. Solve a TSPPD algorithm to obtain a potential tour ξ_n
   For a given current vehicle location O, together with the set of pickup nodes P^+, the set of drop-off nodes P^-, the vehicle capacity K, and the distances between every node pair d_{lu}:
      2.1.1. Create a TSP tour S for $\{P^-,O\}$.
      2.1.2. For each $l \in P^+$, insert l into S such that
         a. The capacity constraint at that insertion point and all subsequent nodes is feasible.
         b. l is visited before its corresponding drop-off.
         c. The increased tour length is minimized.
      2.1.3. Update the capacities at each node in the tour from the insertion point, and go to step 3.
3. Determine $\mathrm{SW}(p,v,\xi_n)$
   3.A. Solve for the socially efficient price p_n^*
   3.B. Solve for the revenue-maximizing price p_n^r
4. Choose the vehicle that solves Eq. (13.a). Update ξ_n as the new tour for that vehicle while keeping the other vehicles' tours the same as before.

---

5. Experimental results

The experimental study is designed to address the following questions:

- Does the proposed methodology work with respect to the customer's inconvenience and the system's effort in order to improve transportation to workplaces? How much better does our policy work in terms of social welfare? In Section 5.1, we demonstrate the effectiveness of the proposed algorithm by comparing it to Sayarshad and Chow's approach, which is a single-server queue system.
- Does the proposed methodology work in large and smart cities? In Section 5.2, a simulation of real-time operation was conducted to test the efficiency of the proposed approach based on NYC taxicab data.
- Can we evaluate dynamic ridesharing using the revenue-maximizing price policy and the socially efficient price policy? We empirically demonstrate that we can find the socially efficient level by applying the proposed policy to the social optimization

Table 1
Output times under the single-server queue system.

User	Arrival time (min)	Pickup time (min)	Drop-off time (min)	Price
1	0.8189	83.5919	112.4041	6.9975
2	1.3042	4.8304	15.7659	9.6842
3	1.3224	10.6798	20.6063	7.9705
4	2.0778	3.9174	21.3826	9.1387
5	2.6647	12.2206	41.7767	9.6334
6	3.2175	29.9558	43.3387	8.2416
7	3.4932	9.0410	22.8385	9.4824
8	4.3065	59.6919	94.8464	9.0606
9	4.5217	103.5679	125.0394	9.4764
10	4.6119	136.9937	151.1590	9.1953
11	5.0672	31.8329	72.4274	4.3707
12	5.6507	118.3572	148.5094	9.2664
13	6.5001	16.7912	45.3020	9.5332

problem.

- What is the capability of this model in terms of tour length and social welfare using different pricing policies? In Section 5.2.2.1, we compare our policy with different pricing policies to provide a new framework for pricing, which contrasts sharply with traditional pricing policies that ignore customer convenience.
- How can we find the customers' queue-joining threshold based on revenue and social optimization? In Section 5.2.2.2, we demonstrate a trade-off between price and queue- joining threshold strategies which provides guidelines for agencies in allocating funds per passenger.

The simulations of the policies and arrivals were run in Matlab R2015b on a computer with an Intel Core i5-2450 CPU with 2.5 GHz and 8 GB RAM, running on a 64-bit Windows 7 operating system.

5.1. Illustrative example

In this section, we compare the performance of our proposed MDP with that of a single-server queuing system (Sayarshad and Chow, 2015). The following inputs were used for the non-myopic pricing policy: $\beta = 1$, $\Lambda = 2$, R= "$"10, $\lambda = 0.5$ $\hat{\mu} = 4$, $\theta = 0.5$, speed = 4/6 (km/min), vehicle capacity = 4, fleet size = 3. We evaluated tour length, average social welfare, and price under two types of queuing policies. The arrival times, pickup times, and drop-off times of each of the 13 arrivals are shown in Table 1. The resulting tours of the three vehicles under the single-server queue system are as follows: Vehicle 1 serves passengers 1, 7, 8, 9, 10, 11, 12, and 13; vehicle 2 serves passengers 2, and 3; and vehicle 3 serves passengers 4, 5, and 6. The tour lengths of vehicles 1, 2, and 3 are 154.336 min; 22.986 min, and 46.082 min, respectively.

Under the multi-server queue system, vehicle 1 serves passengers 1, 2, 3 and 6; vehicle 2 serves passengers 7, 8, 9, 10, 11, 12 and 13; and vehicle 3 serves passengers 4, and 5. The tour lengths of vehicles 1, 2, and 3 are 49.143, 112.007, and 34.252 min, respectively. The arrival times, pickup times, and drop-off times of each of the 13 arrivals under the multi-server queue system are shown in Table 2. This example demonstrates that the proposed policy can yield a different set of tours and can also improve upon the tour lengths of the vehicles.

Fig. 3 shows a comparison of the social welfare under the proposed policy to that under the single-server queue policy. After conducting 30 simulated runs, we found that the average social welfare for the proposed policy is 64.472, while the single-server queue system has an average social welfare of 40.906. Therefore, the proposed dynamic dial-a-ride problem increases the average

Table 2
Output times under the multi-server queue system.

User	Arrival time (min)	Pickup time (min)	Drop-off time (min)	Price
1	0.8189	6.3302	19.2681	6.9975
2	1.3042	34.2883	46.1175	9.6842
3	1.3224	23.2151	41.2770	7.9705
4	2.0778	3.9174	21.3826	8.9093
5	2.6647	12.2206	30.9071	8.9444
6	3.2175	9.0632	26.1677	8.2416
7	3.4932	32.5524	55.6793	9.4824
8	4.3065	92.5327	108.5516	5.4822
9	4.5217	25.6759	37.8125	9.4764
10	4.6119	13.8865	23.2598	9.1953
11	5.0672	64.6738	105.2682	4.3707
12	5.6507	19.4086	27.4336	9.2664
13	6.5001	49.6321	78.1428	7.7853

Fig. 3. Comparison of the social welfare for the proposed MDP to that for the policy of Sayarshad and Chow (2015).

social welfare by up to 37% compared to the single-server queuing system. Using the same input parameters as stated earlier, we were able to calculate and compare the average vehicle miles traveled (VMT) under the proposed dial-a-ride policy against that under the single-server policy. Our analysis demonstrates that the average VMT is 125.58 km under the proposed model, while it is 148.93 km under the single-server policy. The solution time to run the simulations is three seconds.

5.2. New York City taxicab simulation

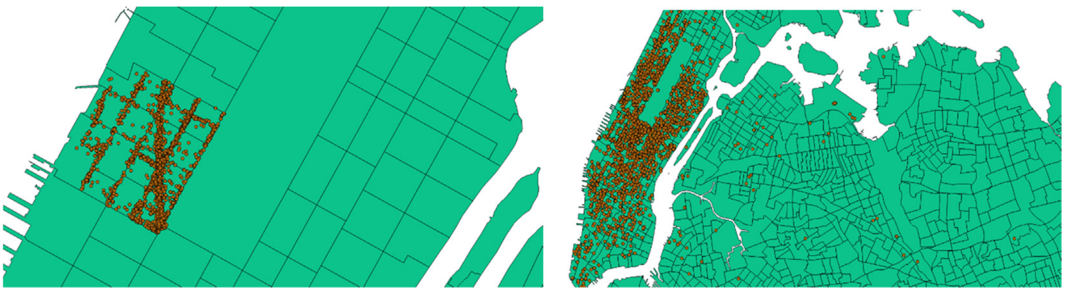
5.2.1. NYC taxicab data

The NYC taxicab data file contains about 13 million records, with comprehensive information on taxi pick-up and drop-off dates, times, locations (latitude/longitude), fares (including tolls, tips, and total fare amounts), and distances traveled. In this study, all the experiments were conducted using data from a single origin zone—Lincoln Square (zone ID 3806) in Manhattan. Lincoln Square is located at the intersection of Broadway and Columbus Avenue, between West 65th and West 66th streets. For our empirical study, we selected one zone with high congestion in Manhattan. Each file has about 1 million rows, and each zone has about 100,000 rows. 5 full weekdays of data are extracted from the file to estimate the parameters for the simulation scenario. Thus, we considered large samples for one zone to provide the parameter setting in our large case study. Pick-up and drop-off data are shown in Fig. 4. Fig. 5 shows the prices of the trips; the number of trips made on April 12, 2013 was 2224, and 1893 taxis were operated from Lincoln Square to all destinations.

5.2.2. The simulation

For this simulation, the experiments were conducted using data for 99 passenger arrivals between 8 AM and 9 AM. The following parameters were used for the flexible dispatch approach between 8 AM and 9 AM: speed = 4/6 km/min, $\Lambda = 4$, $\beta = 1$, vehicle capacity = 4, $\theta = 0.5, R = 15$, fleet size = 70, $\lambda = 1.65$, $\hat{\mu} = 5$, and 99 passengers in each of the 30 runs. Fig. 6 shows the resulting tours of 99 passenger arrivals between 8 AM and 9 AM.

5.2.2.1. Non-myopic pricing strategy and equilibrium. We evaluated the average performance of the proposed non-myopic policy under the revenue-maximizing price and the socially efficient price. In this scenario, we used the simulated tour results with 99 passenger arrivals between 8 AM and 9 AM. Fig. 7 shows the elasticity of the tour length for the two pricing policies. The manager may charge



a. b.

Fig. 4. (a) Pick-ups and (b) drop-offs of passengers in the Lincoln Square zone on April 12, 2013.

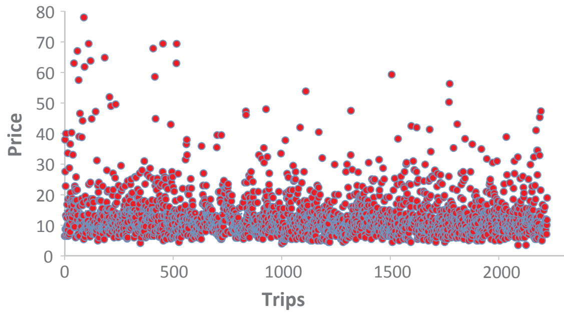


Fig. 5. Prices of trips from the Lincoln Square zone (April 12, 2013).

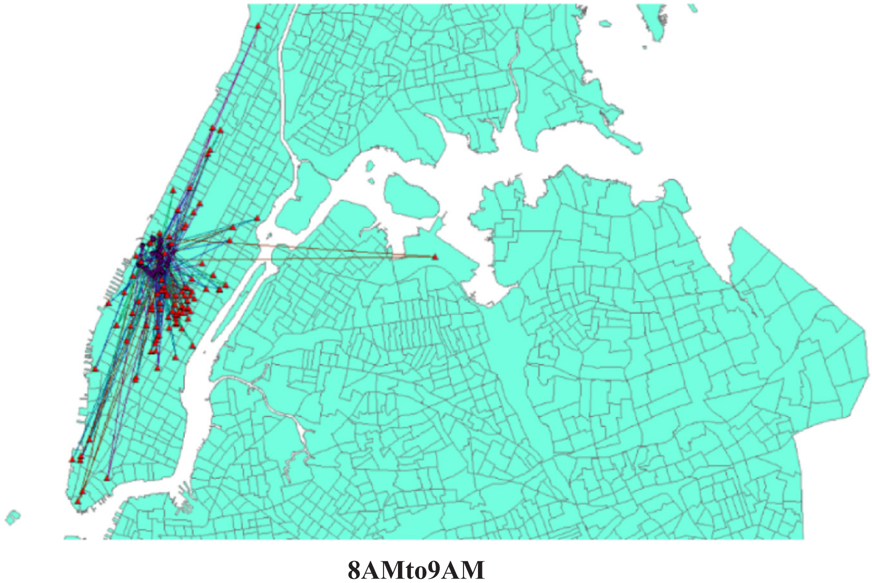


Fig. 6. The resulting tours under the non-myopic policy (from the Lincoln Square zone on April 12, 2013).

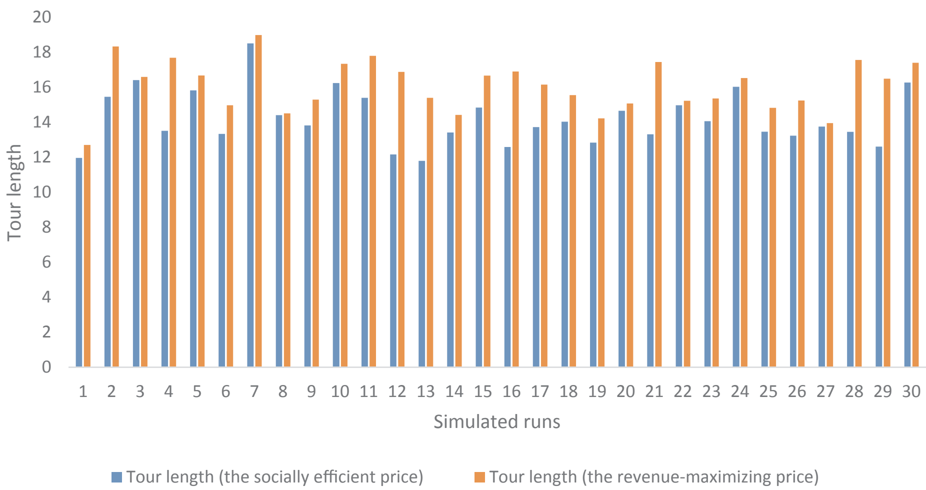


Fig. 7. Comparison of elasticity of tour length using under two pricing policies.

the price that maximizes the expected revenue p^r, while from the perspective of a social planner, we would charge the socially efficient price p^*. The first is consistent with standard monopoly pricing models, but the second is not. This is due to the negative congestion externality that is present in queuing models. The average solution time to run the simulations is 30 s.

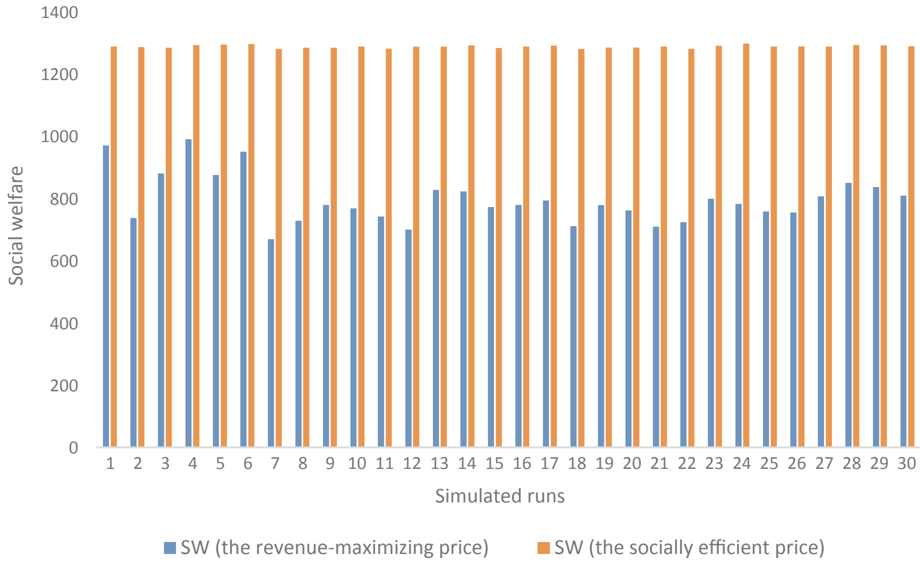


Fig. 8. Comparison of elasticity of social welfare using different pricing policies.

Fig. 8 represents elasticity of the social welfare under the two different pricing policies. This illustrates that the socially efficient level can have the greater effect: The minimum, average, and maximum social welfare for that pricing policy are 1287.573, 1279.577, and 1297.252, respectively; the corresponding values when the price is based on the maximum revenue are 795.254, 668.744, and 990.636, respectively. The proposed policy using the socially efficient price performed better than using the standard monopoly pricing mechanism, with an increase in social welfare of 47% (Min), 38% (Ave), and 23% (Max) over the 30 runs. In addition, our analysis demonstrates that the average VMT is 460.53 km under the socially efficient pricing policy, and 483.53 km under the revenue-maximizing pricing policy. Customers make their joining decisions out of self-interest, but their decisions influence the well-being of future arrivals. In this situation, pricing can be used to address such externalities and increase social welfare.

5.2.2.2. Equilibrium behavior evaluation. We surveyed a sample of passengers and prices to provide an analysis of price and queue-joining threshold strategies for two different time periods: one with high congestion and one with low congestion. For simulated runs between 12 midnight and 1 AM, we used the following parameters: $\Lambda = 4$, speed $= 4/6$ km/min, $\lambda = 0.44$, $\hat{\mu} = 6$, vehicle capacity $= 4$, $\theta = 0.5, R = 20$, $\beta = 1$, fleet size $= 25$, and 32 passengers in each of the 30 runs.

Based on actual New York data, the prices of those trips from the Lincoln Square zone to all destinations are shown in Fig. 9. Prices depend on the operator's dispatching policy, vehicles' positions, routing policy, service rate, value of time, and arrival rate of customers. Figure (a) shows that the average price between 12 midnight and 1 AM is $20, but Figure (b) shows that the average price between 8 AM and 9 AM is $15. For this study, we assume that the customer reward R does not depend on traffic density, but the possibility that the reward obtained depends in some way on the effective traffic congestion could be considered in future research. Note that the high prices (as shown in Fig. 9) are related to serving passengers from Lincoln Square to New York airports. So, it will be necessary in future studies to better understand how shared taxis are impacting airport access demand and consumer surplus (Ma et al., 2017b). Hence, a dynamic dispatch as well as updating price to evaluate the welfare effects of the range of shared taxi matching and fare allocation policies for airport access will be considered in future research.

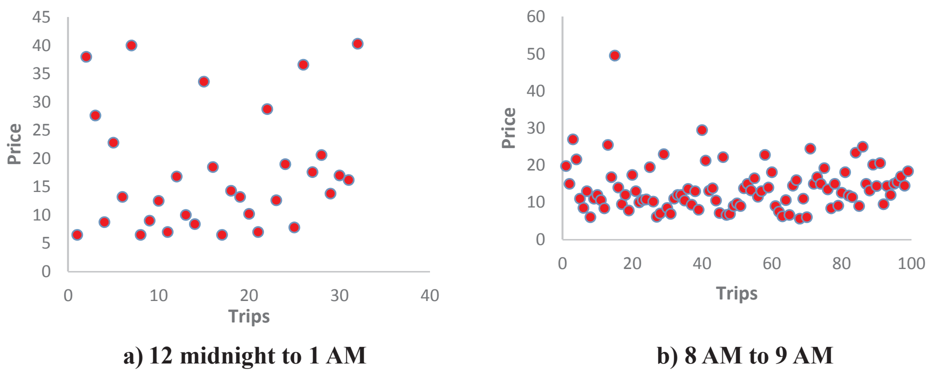


Fig. 9. Prices of trips from the Lincoln Square zone on April 12, 2013.

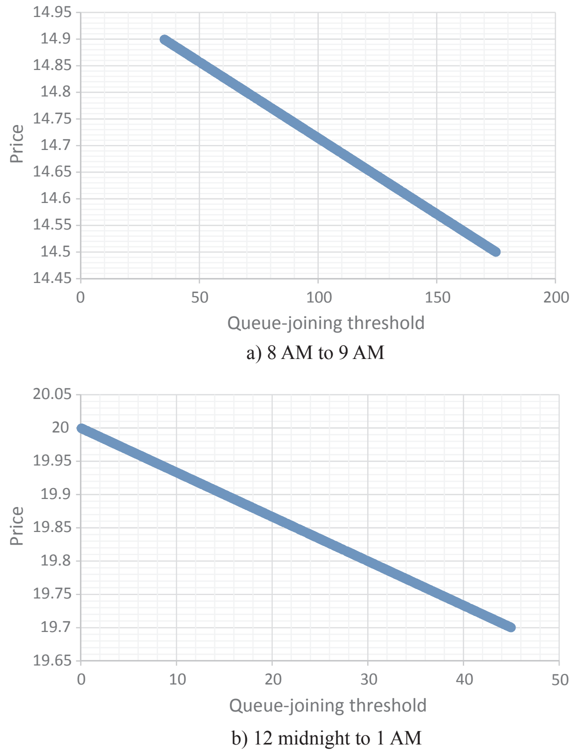


Fig. 10. The trade-off between price and queue-joining threshold strategies.

Fig. 10 shows the trade-off between the price and queue-joining threshold strategies which were calculated using the proposed policy for two different time periods. For instance, we found that the revenue-maximizing prices for the last customer in each period (customer 32 in the low-congestion period, and customer 99 in the high-congestion period) are p_{32}^{r} = "\$"20 and p_{99}^{r} = "\$"14.9, while the socially efficient prices are p_{32}^{*} = "\$"19.7 and p_{99}^{*} = "\$"14.5. Equivalently, the socially efficient queue-joining and revenue-maximizing queue-joining thresholds are $k(p_{32}^{*}) = 44$ and $k(p_{32}^{r}) = 0$, respectively, for customer 32, and $k(p_{99}^{*}) = 174$ and $k(p_{99}^{r}) = 35$, respectively, for customer 99. At the social efficient price "\$"19.7, customers are interested in joining the system, but they are not joining when the price is "\$"20. This relatively high price in the case means the supply falls short of the potential demand. This result shows that why the traditional taxicab dispatch system is unable to compete with or rival the new forms of commerce.

As for revenue-maximizing price, recall that p_{32}^{r} = "\$"20 where the demand rate become 0. Note that $k(p^{r}) = 0$ for the revenue-maximizing price scenario and represents the customers who are not interested in joining the system, because there is no available socially efficient level with which to calculate prices and thresholds. For the individual passenger, the optimal strategy is therefore to join the queue if and only if the number of passengers already present in the queue at the instant of his arrival is more than the threshold $k(p^{r})$. Hence, one advantage of our simulations is that they introduce a new approximation method into the dynamic vehicle routing and pricing problem for computing the conditional expected future cost portion using the queue-joining threshold, which is otherwise computationally difficult or impossible to calculate. This motivates agencies to allocate budget and resources per passenger in order to provide service using queue-joining threshold strategies. Based on this key insight, the proposed framework is able to consider equilibrium-joining behavior together with socially optimal behavior under a dynamic pricing policy. The results show that the queue-joining threshold which uses the revenue-maximizing price is less than that which uses the socially efficient price; this agrees with the results of Naor (1969) and Knudsen (1972).

6. Conclusion

Our on-demand mobility framework provides not only direct competition for traditional taxi services but also opportunities to revolutionize conventional transit services that are plagued by high costs and low ridership, particularly in poor communities, rural areas, or sprawling suburbs. We proposed a competitive on-demand mobility system with a new framework pricing policy under infinite-horizon look-ahead. We demonstrated that the proposed approach works better using a multi-server queue system, and it increases the average social welfare by up to 37%.

Operational considerations often influence the optimal price and provide more realistic equilibrium outcomes in competitive settings. We considered the equilibrium-joining behavior for the socially optimal problem under different pricing policies. The proposed policy using the socially efficient price increases the social welfare by up to 40% compared to the standard monopoly pricing policy. As a result, we expect the proposed pricing framework to serve as a definitive research reference for future studies in dynamic ridesharing and the design of new types of urban mobility systems such as carsharing, ridesharing, demand-responsive transit, smart taxis, and autonomous vehicle fleets, all of which require consideration of willingness-to-pay policies.

Simulations of our dynamic ridesharing problem were conducted using actual NYC taxicab data. We presented an analysis of the average performance of the proposed non-myopic policy using the revenue-maximizing price and the socially efficient price. Our proposed policy is able to calculate the equilibrium-joining behavior of each region during the time horizon and to easily compute the conditional expected future cost portion, which could be very useful for future studies on non-myopic dynamic optimization policies.

The ridesharing concept could be integrated with other modes of transportation such as public transit. We suggest proceeding in this area of research with a dynamic multimodal transit system under a non-myopic pricing policy which can evaluate the customer delay for different modes of transportation (Pender et al., 2017); this could be considered in a future study by adding the equilibration of supply and demand using a Poisson game (Myerson, 1998). The most immediate extension of the proposed research could involve integrating a bimodal dial-a-ride problem and consider the impact of fleet design and vehicle miles traveled on carbon emissions. An integrated system of this nature has the potential to greatly improve the efficiency and scalability of urban transportation while simultaneously reducing negative externalities such as congestion and pollution. Provision of a dynamic optimization model for switching between fixed and flexible transit services using non-myopic dynamic dispatch would be highly valuable. For example, such a system could guide smart vehicles in a service area that requires many-to-many first–last-mile transit service, either for conventional shared vehicles that provide fixed-route transit service or for vehicles that provide flexible-route on-demand mobility.

Acknowledgements

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Appendix A. Tour length and customer delay

Let $(y_1;...y_n)$ denote the state of the queue with n tasks, and let $d = \sum_i^n y_i(v,\xi)$ denote the amount of unfinished work (backlog), expressed in terms of the time it would take to finish it, where y_i denotes the service time (amount of work) for customer i, v is a vehicle, and ξ is a tour obtained for a traveling salesman problem (TSP). Since the objective includes user costs, we begin by formulating the unfinished work for two time intervals, $(0,d)$ and (d,∞), where d is the time that corresponds to the current state (see Sayarshad and Gao, 2017; Hyytiä et al., 2012). The second term on the right-hand side of Eq. (A.1) denotes the extra cost during the time interval $(0,d)$less the mean cost during that interval$(0,d)$. From the customer's perspective (sojourn time), the relative value of state $(y_1,...,y_n)$ in a multi-server queuing system is

$$V_{(y_1,...,y_n)} - V_0 = \sum_i y_i(v,\xi) + \frac{\lambda \mathrm{d}^2}{2} - \mathrm{L_q} * \mathrm{d} + \mathrm{E}[\mathrm{V_A}], \tag{A.1}$$

where $\mathrm{E}[\mathrm{V_A}]$ is the relative value during time interval (d,∞).

In order to calculate the second term on the right-hand side, we can use a property of the Poisson process (see Hyytiä et al., 2012; Ross, 1970). The number of new customers arriving during the time interval $(0,d)$, which we denote by A, obeys a Poisson distribution with parameter λd: $A\sim$ Poisson(λd). Let Z denote the cost accrued by time d by the A customers that arrive during $(0,d)$ Given that $A = i$ customers have arrived, their arrival times are uniformly distributed in $(0,d)$, which gives $[Z|A] = A\frac{d}{2}$, hence

$$E[Z] = E[E[Z|A]] = E\left[A\frac{d}{2}\right] = \frac{\lambda \mathrm{d}^2}{2}$$

The third term on the right-hand side of Eq. (A.1) is due to the mean cost rate during the time interval $(0,d)$(see Gross and Harris, 1998). Thus, the contribution to the relative value during the time interval $(0,d)$ is

$$\sum_i y_i(v,\xi) + \frac{\lambda \mathrm{d}^2}{2} - [\lambda/\hat{\mu} + \left(\frac{(\lambda/\hat{\mu})^s \rho}{s!(1-\rho)^2}\right)\mathrm{P_0}] * \mathrm{d}, \tag{A.2}$$

where

$$\mathrm{P_0} = \left[\frac{\lambda^s}{s!\hat{\mu}^s}\frac{s\hat{\mu}}{s\hat{\mu}-\lambda} + \sum_{n=0}^{s-1}\frac{\lambda^n}{n!\hat{\mu}^n}\right]^{-1} \tag{A.3}$$

Next, consider the time interval (d,∞). At time d, the initial n customers have departed and the number of customers in the queue is A. Utilizing the memoryless property (a property of a Poisson process), the contribution to the relative value during (d,∞) is

$$E[V_A] = \sum_{i=0}^{\infty} P\{A = i\}V_i$$

Thus, we need to provide the relative value function using the Howard equation in state n (Howard, 1971):

$$n-\left[a + (\frac{a}{s-a})C(s,a)\right] + \lambda(V_{n+1}-V_n) + \hat{\mu}1_{n>0}(V_n-V_{n-1}) = 0, \tag{A.4}$$

where $a = \lambda/\hat{\mu}$, n corresponds to the cost rate with n customers, $\left[a + \left(\frac{a}{s-a}\right)C(s,a)\right]$ is the mean cost rate, and $C(s,a)$ is the probability of a customer being delayed, which is called *Erlang's C formula* (see Gross and Harris, 1998):

$$C(s,a) = \frac{\frac{a^s}{(s-1)!(s-a)}}{\sum_{n=0}^{s-1}\frac{a^n}{n!} + \frac{a^s}{(s-1)!(s-a)}}$$

Krishnan (1987,1990) showed that $V_{n+1}-V_n$ satisfies Eq (A.4) and that the value function in an $M/M/s$ is given by:

$$V_{n+1}-V_n = \begin{cases} \frac{w}{Erl(n,a)} + \frac{1}{\hat{\mu}} & 0 \leqslant n \leqslant s \\ \frac{w}{Erl(s,a)} + \frac{n-s}{s\hat{\mu}(1-\rho)} + \frac{1}{\hat{\mu}}, & n > s, \rho = \lambda/s\hat{\mu}, \end{cases} \tag{A.5}$$

where $Erl(s,a)$ is the Erlang's formula with s servers and the offered load of $a = \lambda/\hat{\mu}$ and w is the average delay for service which gives $w = \frac{C(s,a)}{s\hat{\mu}(1-\rho)}$. Since relative costs alone are significant, we set V_0 to 0 and obtain

$$V_n = \begin{cases} \frac{w}{\sum_{j=0}^{n-1} Erl(j,a)} + \frac{n-1}{\hat{\mu}} & 0 \leqslant n \leqslant s \\ \frac{(n-s)w}{Erl(s,a)} + \sum_{j=n-1} \frac{j-s}{s\hat{\mu}(1-\rho)} + \frac{n-s}{\hat{\mu}}, & n > s \end{cases} = \begin{cases} \frac{w}{\sum_{j=0}^{n-1} Erl(j,a)} + \frac{n-1}{\hat{\mu}} & 0 \leqslant n \leqslant s \\ \frac{(n-s)w}{Erl(s,a)} + \frac{(n-1-s)(n-s)}{2s\hat{\mu}(1-\rho)} + \frac{n-s}{\hat{\mu}}, & n > s \end{cases} \tag{A.6}$$

Thus, the contribution to the relative value during the time interval (d,∞) is

$$\begin{aligned} E[V_A] &= \sum_{n=0}^{\infty} P\{A = n\}V_n \\ &= \begin{cases} w\sum_{j=0}^{s-1}\frac{1}{Erl(j,a)} + \frac{E[A]-1}{\hat{\mu}} & n \leqslant s \\ \frac{(E[A]-s)w}{Erl(s,a)} + \frac{(E[A]-1-s)(E[A]-s)}{2s\hat{\mu}(1-\rho)} + \frac{E[A]-s}{\hat{\mu}}, & n > s \end{cases} \\ &= \begin{cases} w\sum_{j=0}^{s-1}\frac{1}{Erl(j,a)} + \frac{\frac{\lambda d}{2}-1}{\hat{\mu}} & n \leqslant s \\ \frac{(\frac{\lambda d}{2}-s)w}{Erl(s,a)} + \frac{(\frac{\lambda d}{2}-1-s)(\frac{\lambda d}{2}-s)}{2s\hat{\mu}(1-\rho)} + \frac{\frac{\lambda d}{2}-s}{\hat{\mu}}, & n > s, \rho = \lambda/s\hat{\mu} \end{cases} \end{aligned} \tag{A.7}$$

The free policy parameter θ, $0 \leqslant \theta \leqslant 1$, is a weight that differentiates the user cost from the system cost, and β is the degree of look-ahead that's needed to accommodate the difference between the myopic policy and the non-myopic policy. Thus, the relative cost is:

$$\hat{V}_{(y_1,\ldots,y_n)} = \theta d + (1-\theta)\left[\sum_i y_i(v,\xi) + \frac{\lambda d^2}{2} - \left[\lambda/\hat{\mu} + \left(\frac{(\lambda/\hat{\mu})^s\rho}{s!(1-\rho)^2}\right)P_0\right]*d + \beta E[V_A]\right]$$

We need only solve ξ for each vehicle v for the new passenger under consideration and select the minimum value decrease where $T(v,\xi)$ is the current tour length of the vehicle.

$$c(v,\xi) = \theta T(v,\xi) + (1-\theta)\left[\sum_i y_i(v,\xi) + \frac{\lambda T(v,\xi)^2}{2} - \left[\lambda/\hat{\mu} + \left(\frac{(\lambda/\hat{\mu})^s\rho}{s!(1-\rho)^2}\right)P_0\right]*T(v,\xi) + \beta E[V_A]\right] \tag{A.8}$$

Appendix B. Glossary of symbols

See Table A1.

Table A1
Nomenclature.

Symbol	Description
λ	customer arrival rate
$\hat{\mu}$	service rate
$(y_1;...y_n)$	state of the queue with n tasks
ξ	a tour
ξ_{n-1}	the previous tour updated to the time of the time of arrival of the previous user, $n-1$
R	reward of service
$D(p)$	demand function
C_s	system cost
C_u	user cost
v	a vehicle
θ	a weight that differentiates the system cost from the user delay
$T(v,\xi)$	tour length of a vehicle–route pair
$c(v,\xi)$	relative cost of a vehicle–route pair
w	average delay for service
s	number of vehicles
n	number of customers
p	fair price
p^r	revenue-maximizing price
p^*	socially efficient price
$k(p)$	customers' queue-joining threshold
Λ	aggregate arrival rate
V_A	relative value
β	degree of look-ahead
P^+	set of pickup nodes
P^-	set of drop-off nodes
D	set to the fleet depot
O	current vehicle location
Q	probability of winning in a gambler's ruin problem
P	probability of losing in a gambler's ruin problem
K	a vehicle capacity
d_{lu}	distance between a pair of nodes

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