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A slack arrival strategy to promote flex-route transit services

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ABSTRACT

Flex-route transit, which combines the advantages of fixed-route transit and demand-responsive transit, is one of the most promising options in low-demand areas. This paper proposes a slack arrival strategy to reduce the number of rejected passengers and idle time at checkpoints resulting from uncertain travel demand. This strategy relaxes the departure time constraints of the checkpoints that do not function as transfer stations. A system cost function that includes the vehicle operation cost and customer cost is defined to measure system performance. Theoretical and simulation models are constructed to test the benefits of implementing the slack arrival strategy in flex-route transit under expected and unexpected demand levels. Experiments over a real-life flex-route transit service show that the proposed slack arrival strategy could improve the system performance by up to 40% with no additional operating cost. The results demonstrate that the proposed strategy can help transit operators provide more cost-efficient flex-route transit services in suburban and rural areas.

1. Introduction

Over the past few decades, an increasing number of suburban areas with low population density have been established as a consequence of economic growth and urban sprawl. This trend has led to new travel patterns that require transit services to be more flexible. Flex-route transit, which is also referred to as route deviation (Potts et al., 2010; Lu et al., 2015), mobility allowance shuttle transit (MAST) (Quadrifoglio et al., 2006) and demand adaptive transit system (DAS) (Crainic et al., 2012), has experienced dramatic growth in recent years. According to the investigation by Koffman (2004) and Potts et al. (2010), among the many types of flexible transit services, flex-route transit is by far the most widely used.

Traditional fixed-route transit is inconvenient in low-demand areas because of the lack of flexibility, as the locations of pick-up or drop-off stations do not correspond to the needs of individual riders. Pure demand-responsive transit could provide the flexibility desired by customers, but such a system tends to be considerably more expensive; therefore, it is largely limited to specialized operations such as paratransit for the elderly and disabled (Dikas and Minis, 2014). The flex-route transit service combines the low-cost operability of fixed-route transit with the flexibility of demand-responsive transit. It operates along the base route with mandatory checkpoints located in high-density demand zones with fixed departure times; it can also deviate from the base route to serve curb-to-curb requests, which make reservations in advance. Actual operational data from New Jersey have shown that flex-route transit is more cost-efficient than pure demand-responsive transit (Fittante and Lubin, 2015) and it is also considered more convenient than regular bus routes (Becker et al., 2013).

So far, relatively few studies have focused on flex-route transit. Quadrifoglio et al. (2006) evaluated the upper and lower bounds

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of the maximum longitudinal velocity of service vehicles and proposed scheduling algorithms of flex-route transit for both static and dynamic scenarios (Quadrifoglio et al., 2007, 2008). A sensitivity analysis was also performed over different service area shapes (Quadrifoglio and Dessouky, 2008). Zhao and Dessouky (2008) analyzed the relationship between the service cycle time and the length and width of the service area. Crainic et al. (2012) estimated the optimal cycle length using probabilistic approximations theory. Nourbakhsh and Ouyang (2012) developed a new structured flex-route transit system that performs better than fixed-route transit under a range of low-to-moderate demand levels. Alshalalfah and Shalaby (2012) investigated the feasibility and benefits of replacing fixed-route feeder transit with flex-route service in Toronto. Yang et al. (2016) presented a methodology to select the optimal route of a flex-route transit service based on the lowest operating cost per passenger. Qiu et al. (2014) proposed a dynamic station strategy to eliminate the rejection of curb-to-curb requests at unexpectedly high demand. Qiu et al. (2015a, 2015b) explored the feasibility of replacing fixed-route transit with flex-route operating policies and derived the switching demand densities between the two competing policies. Frei et al. (2017) conducted a stated-preference survey between regular bus transit, private cars and flex-route transit, the results of which can help transit planners identify potential users and guide the design of flex-route transit.

As an innovative operating policy, flex-route transit has considerable potential for shaping new travel patterns in low-demand areas. However, according to the investigation conducted by Potts et al. (2010), only a small percentage of transit agencies apply flex-route policies due primarily to the uncertain travel demand in suburban or rural areas (Velaga et al., 2012). In flex-route transit, the slack time allocated for deviation service in each route segment is predetermined based on the expected demand level, which is quite fragile in real-life operation because the actual demand may vary over time. Some of the requests have to be rejected when the predetermined slack time is used up. Moreover, the passengers on board may experience idle time at the checkpoints if the actual demand is lower than expected. Both scenarios would definitely degrade the service level of flex-route transit systems.

To utilize the unused slack time and improve the acceptance rate of the flex-route service, this study proposes a slack arrival strategy in which the fixed time constraints of some checkpoints are relaxed to some extent. In this strategy, the redistribution of slack time among segments enables more curb-to-curb customers to be served without affecting the overall service cycle of flex-route transit systems.

In this paper, we study the feasibility of applying the slack arrival strategy to promote the performance of flex-route service at both expected and unexpected demand levels. Theoretical and simulation models are developed to evaluate the system cost, including the vehicle operation cost and customer cost. The optimal slack time window is also investigated in the implementation of the slack arrival strategy in an actual flex-route transit service. This work provides a new strategy option for transit planners and could assist them in determining the suitable design and operation policy for flex-route services.

2. System description

The service area can be generally modelled as a rectangle of width W and length L. There are C checkpoints located at high-density demand areas or major connection points (see Fig. 1). The checkpoints can be identified as $c = 1,2...,C$. In each ride, the vehicle moves back and forth, starting from one terminal and ending at the other one after visiting all checkpoints one by one. The scheduled departure times at the checkpoints are fixed, which is generally regarded as an inviolable constraint in operation.

The demand outside the checkpoints is assumed to be uniformly distributed within the service area. We also assume that checkpoints locations and the travel demand among checkpoints are uniformly distributed along the base route. There are four main types of passengers in the service area with proportions of η_1, η_2, η_3, and η_4 ($\eta_1 + \eta_2 + \eta_3 + \eta_4 = 1$) as follows:

Type I: Pick up and drop off both at checkpoints.
Type II: Pick up at checkpoints, and drop off not at checkpoints.
Type III: Pick up not at checkpoints, and drop off at checkpoints.
Type IV: Pick up and drop off both not at checkpoints.

Type I passengers use the service as a regular fixed-route transit. Therefore, they simply show up at their pick-up checkpoint without booking. However, the other three types of passengers must make a reservation to schedule their non-checkpoint stops using smartphones or the Internet. The maximum allowable deviation width from the base route is $W/2$.

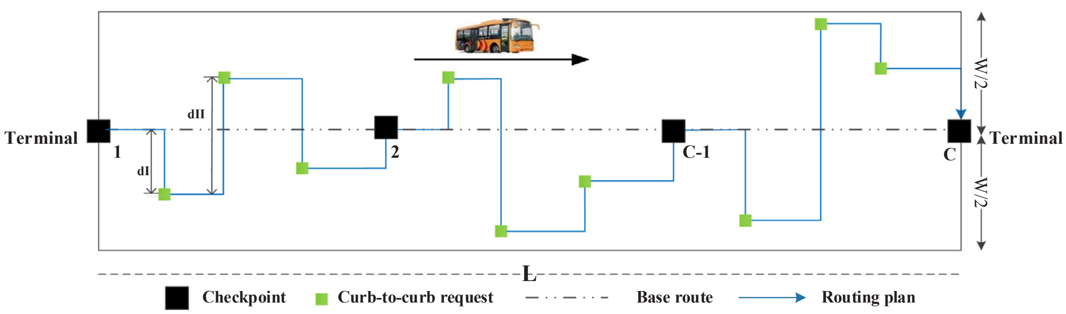

Fig. 1. Flex-route transit service.

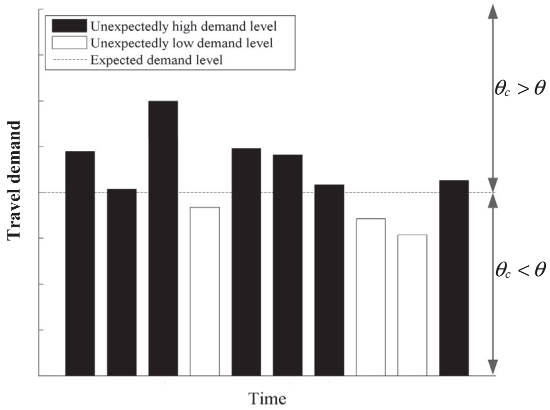

Fig. 2. Uncertainties in travel demand.

A certain amount of slack time is assigned to non-checkpoint stops in segments between checkpoints. Service vehicles are allowed to deviate from the base route to serve curb-to-curb requests within the slack time constraint. In real-life operation, the allocation of the slack time is critical for the flex-route system. If deviation service cannot use all the slack time, the vehicles will arrive earlier at the checkpoints and have to wait at the checkpoints (idle time) until the scheduled departure time. In contrast, if the slack time is not sufficient for the deviation service, the following curb-to-curb requests will be rejected.

3. Theoretical model

In this study, passengers are assumed to be transit-dependent. In practice, the actual travel demand θ_c frequently deviates from the expected demand level θ in the low-demand service areas due to uncertainties (see Fig. 2). To investigate the performance of flex-route transit, we measure the system performance at three demand levels, namely, the expected demand level, an unexpectedly high demand level and an unexpectedly low demand level. Real-time insertion is not considered here because the customers typically need to make advanced notice before the beginning of a ride. A no-backtracking constraint is applied to construct vehicle routing plans, which forces service vehicles to move only in the forward direction. Because the flex-route service is generally implemented in low-demand areas, the capacity constraint of service vehicles is not considered in our analysis.

3.1. Parameters and notation

We define the model parameters as follows:

K = expected walking time per passenger in the system (min);
A = expected waiting time per passenger in the system (min);
R = expected riding time per passenger in the system (including in-vehicle travel time and service time of the stops) (min);
I = expected idle time per passenger in the system (min);
ω_K = cost of walking time (\$/passenger/h);
ω_A = cost of waiting time before pick up (\$/passenger/h);
ω_R = cost of riding time (\$/passenger/h);
ω_I = cost of idle time at the checkpoint (\$/passenger/h);
O_p = operation cost per passenger (\$/passenger);
O_v = operation cost of the service vehicle (\$/vehicle/h);
T_r = single-trip time (min);
T_unit = time interval between the departure times of two consecutive checkpoints (min);
T_{unit}^c = actual time consumed between two consecutive checkpoints (min);
M = fleet size of the system;
δ = expected passengers served per ride per vehicle (passenger/ vehicle/ride);
δ_c = actual passengers served per ride per vehicle (passenger/ vehicle/ride);
θ = expected demand per hour in the service area (passenger/h);
θ_c = actual demand per hour in the service area (passenger/h);
V_b = average speed of the service vehicles (miles/h);
V_{wk} = average walking speed of the passengers (miles/h);
T_d^r = dwelling time at on-request stops (min);
T_d^f = dwelling time at checkpoints (min);
$\eta_1,\eta_2,\eta_3,\eta_4$ = proportions of type I, II, III, and IV passengers.

3.2. System performance measure

In our research, the performance measure of the transit system is defined as the system cost F, which represents the sum of the vehicle operation cost per customer and average customer cost. Infrastructure investments, such as bus stations, are not considered because they only account for a small fraction of the total cost in the long-term operation. As shown in Eq. (1), O_p represents the operation cost from the perspective of transit agencies, whereas the last four indicators reflect the service quality, which plays a significant role in affecting passengers' attitude toward the flex-route transit. It is obvious that a lower value of the system cost F indicates a better system performance.

$$F = O_p + \omega_K K + \omega_A A + \omega_R R + \omega_I I \tag{1}$$

3.3. At the expected demand level

At the expected demand level, we assume an ideal case in which all requests are accepted ($K = 0$) and there is no idle time at the checkpoints ($I = 0$). The expected vertical distance between the checkpoint and the first/last stop (see Fig. 1) is

$$d_{\mathrm{I}} = \frac{W}{4} \tag{2}$$

The expected vertical distance between each pair of stops is

$$d_{\mathrm{II}} = \frac{W}{3} \tag{3}$$

Thus the travel distance of a vehicle in one ride is

$$D = L + 2(C-1) \times d_{\mathrm{I}} + [\delta(\eta_2 + \eta_3 + 2\eta_4) - \mathrm{C} + 1)] \times d_{\mathrm{II}} = L + \frac{(C-1)W}{6} + \frac{\delta W(\eta_2 + \eta_3 + 2\eta_4)}{3} \tag{4}$$

Because there is no idle time at the checkpoints, the single-trip time of the flex-route transit can be calculated as

$$T_r = \frac{D}{V_b} + \delta(\eta_2 + \eta_3 + 2\eta_4)T_d^r + (C-1)T_d^f \tag{5}$$

The relationship between T_r and δ can be expressed as

$$\delta \times M = \theta \times T_r \tag{6}$$

Combining Eqs. (4)–(6), we can derive the single-trip time T_r as follows:

$$T_r = \frac{6ML + (C-1)MW + 6(C-1)MV_b T_d^f}{6MV_b - 2W\theta(\eta_2 + \eta_3 + 2\eta_4) - 6V_b T_d^r \theta(\eta_2 + \eta_3 + 2\eta_4)} \tag{7}$$

Thus, the time interval between the departure times of two consecutive checkpoints T_{unit} is set as

$$T_{\mathrm{unit}} = \frac{T_r}{C-1} \tag{8}$$

Because all requests are uniformly distributed, based on mathematical probability theory, the expected riding time of the four types of passengers can be expressed as [see details in Eqs. (9)–(11) in "Appendix A" (a), (b) and (c)]

$$R_{\mathrm{I}} = \frac{(C+1)T_{\mathrm{unit}}}{3} = \frac{(C+1)T_r}{3(C-1)} \tag{9}$$

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{(2\mathrm{C}-1)T_{\mathrm{unit}}}{6} = \frac{(2\mathrm{C}-1)T_r}{6(\mathrm{C}-1)} \tag{10}$$

$$R_{\mathrm{IV}} = \frac{(C-1)T_{\mathrm{unit}}}{3} = \frac{T_r}{3} \tag{11}$$

Thus, the expected riding time per passenger in the flex-route service is

$$R = \eta_1 R_{\mathrm{I}} + \eta_2 R_{\mathrm{II}} + \eta_3 R_{\mathrm{III}} + \eta_4 R_{\mathrm{IV}} = \frac{2(C+1)\eta_1 + (2C-1)(\eta_2 + \eta_3) + 2(C-1)\eta_4}{6(C-1)} T_r \tag{12}$$

In real-life operation, flex-route transit systems typically have minimum operating headways of 1 h or more (Potts et al., 2010). Passengers whose starting points are at checkpoints (types I and II) have a strong incentive to consult the schedule to eliminate unnecessary waiting time due to the long headway of flex-route transit. These passengers are considered schedule-dependent, and their waiting time can be ignored ($A_{\mathrm{I}} = A_{\mathrm{II}} = 0$).

Passengers whose starting points are not at the checkpoints (types III and IV) can receive the scheduled pick-up time from the scheduling system. They are more likely to spend their time in houses or indoor locations instead of waiting outside before the scheduled pick-up time. Therefore, their waiting time is defined as the interval between the scheduled pick-up time and actual pick-up time. For the a type III or type IV passenger j, waiting time occurs only when there are follow-up customers, who make a reservation later than j and result in additional stops before the vehicle picks up j. Based on mathematical probability theory, the expected value of waiting time can be expressed as

$$A_{\mathrm{III}} = A_{\mathrm{IV}} = \frac{\sum_{i=1}^{\frac{\delta(\eta_2+\eta_3+2\eta_4)}{C-1}-1} i}{\frac{\delta(\eta_2+\eta_3+2\eta_4)}{C-1}}\left(\frac{W}{3V_b}+T_d^r\right)\frac{1}{2} = \frac{W\delta(\eta_2+\eta_3+2\eta_4)}{12(C-1)V_b}+\frac{T_d^r\delta(\eta_2+\eta_3+2\eta_4)}{4(C-1)}-\frac{W}{12V_b}-\frac{T_d^r}{4} \tag{13}$$

Thus, the expected waiting time per passenger can be obtained as

$$\begin{aligned} A &= \eta_1 A_{\mathrm{I}} + \eta_2 A_{\mathrm{II}} + \eta_3 A_{\mathrm{III}} + \eta_4 A_{\mathrm{IV}} \\ &= \frac{(\eta_3+\eta_4)W\delta(\eta_2+\eta_3+2\eta_4)}{12(C-1)V_b} + \frac{(\eta_3+\eta_4)T_d^r\delta(\eta_2+\eta_3+2\eta_4)}{4(C-1)} - \frac{(\eta_3+\eta_4)W}{12V_b} - \frac{(\eta_3+\eta_4)T_d^r}{4} \end{aligned} \tag{14}$$

At the expected demand level, no requests are rejected by the system and no idle time occurs at the checkpoints. Therefore, the operation cost per passenger can be derived as

$$O_{\mathrm{p}} = \frac{O_{\mathrm{v}}M}{\theta} \tag{15}$$

3.4. At an unexpectedly low demand level

When $\theta_c < \theta$, all customers can be accepted and have no walking ($K = 0$). The actual number of customers in one trip per vehicle is $\delta_c = \theta_c T_r/M$ and the expected waiting time per passenger can be obtained by replacing δ with δ_c in Eq. (14)

$$\begin{aligned} A &= \eta_1 A_{\mathrm{I}} + \eta_2 A_{\mathrm{II}} + \eta_3 A_{\mathrm{III}} + \eta_4 A_{\mathrm{IV}} \\ &= \frac{(\eta_3+\eta_4)W\theta_c T_r(\eta_2+\eta_3+2\eta_4)}{12M(C-1)V_b} + \frac{(\eta_3+\eta_4)T_d^r\theta_c T_r(\eta_2+\eta_3+2\eta_4)}{4M(C-1)} - \frac{(\eta_3+\eta_4)W}{12V_b} - \frac{(\eta_3+\eta_4)T_d^r}{4} \end{aligned} \tag{16}$$

At unexpectedly low demand levels, there will be idle time at checkpoints due to the unused slack time. The expect travel distance between any consecutive pair of checkpoints can be estimated as follows:

$$D_{unit}^c = \frac{L}{C-1} + 2\times d_{\mathrm{I}} + \left[\frac{\delta_c(\eta_2+\eta_3+2\eta_4)}{C-1} - 1)\right]\times d_{\mathrm{II}} = \frac{L}{C-1} + \frac{W}{6} + \frac{\delta_c W(\eta_2+\eta_3+2\eta_4)}{3(C-1)} \tag{17}$$

The actual time consumed between two consecutive checkpoints can be calculated as

$$T_{unit}^c = \frac{D_{unit}^c}{V_b} + \frac{\delta_c(\eta_2+\eta_3+2\eta_4)}{C-1}T_d^r + T_d^f \tag{18}$$

Thus, the idle time between any consecutive pair of checkpoints is

$$T_{idle}^c = T_{\mathrm{unit}} - T_{unit}^c \tag{19}$$

Similar to Eqs. (9)–(11), we can derive the expect riding time of the passenger as

$$R_{\mathrm{I}} = \frac{(C+1)T_{unit}^c}{3} \tag{20}$$

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{(2\mathrm{C}-1)T_{unit}^c}{6} \tag{21}$$

$$R_{\mathrm{IV}} = \frac{(C-1)T_{unit}^c}{3} \tag{22}$$

Thus, the expected riding time per passenger in the flex-route service is

$$R = \eta_1 R_{\mathrm{I}} + \eta_2 R_{\mathrm{II}} + \eta_3 R_{\mathrm{III}} + \eta_4 R_{\mathrm{IV}} = \frac{2(C+1)\eta_1 + (2C-1)(\eta_2+\eta_3) + 2(C-1)\eta_4}{6}T_{unit}^c \tag{23}$$

The expected idle time of the four types of passengers can be expressed as

$$I_{\mathrm{I}} = I_{\mathrm{II}} = I_{\mathrm{III}} = \frac{(C-2)T_{idle}^c}{3} \tag{24}$$

$$I_{\mathrm{IV}} = \frac{C(C-2)}{3(C-1)}T_{idle}^c \tag{25}$$

Thus, the expected idle time per passenger can be obtained as

$$I=\eta_1 I_{\mathrm{I}}+\eta_2 I_{\mathrm{II}}+\eta_3 I_{\mathrm{III}}+\eta_4 I_{\mathrm{IV}}=\frac{(C-1)(C-2)(\eta_1+\eta_2+\eta_3)+C(C-2)\eta_4}{3(C-1)}T_{idle}^{c} \tag{26}$$

At unexpectedly low demand levels, the operation cost is higher than the expected level because fewer passengers are served. The operation cost per passenger O_{p} can be derived as

$$O_{\mathrm{p}}=\frac{T_{unit}^{c}O_{\mathrm{v}}M}{T_{\mathrm{unit}}\theta_c} \tag{27}$$

3.5. At unexpectedly high demand levels

The flex-route transit generally operates on a first-come, first-served basis. When the actual demand exceeds the service capacity of deviation services, some of the curb-to-curb customers have to be rejected due to the limited slack time. These rejected customers might have to wait more than one operating headway before being picked up. However, due to the great service cycle in flex-route services (Koffman, 2004), waiting for the next available vehicle makes it difficult for the rejected passengers to follow their schedules, and waiting cost might have a times greater value than the walking cost to the nearest checkpoint. Similar to Qiu et al. (2014, 2015a, 2015b, 2015c), we assume that rejected customers utilize the nearest checkpoint for desired transit service which could be regarded as the reasonable upper bound of the system cost.

When $\theta_c>\theta$, the slack time limit is met ($I=0$) by the first θ accepted passengers and the values of the passenger cost indicators are the same as the results in Section 3.3. For the last $\theta_c-\theta$ passengers, type I passengers can be accepted and the actual number of rejected requests is $(\theta_c-\theta)(\eta_2+\eta_3+\eta_4)$. For these $\theta_c-\theta$ passengers, $K_{\mathrm{I}}^{\theta_c-\theta}=0$, and the expected walking time of passenger types II and III can be expressed as follows:

$$K_{\mathrm{II}}^{\theta c-\theta}=K_{\mathrm{III}}^{\theta c-\theta}=\frac{L}{4(C-1)V_{wk}}+\frac{W}{4V_{wk}} \tag{28}$$

As proven by Qiu et al. (2015b), the expected value of walking time per type IV passenger can be described as

$$K_{\mathrm{IV}}^{\theta c-\theta}=\frac{1}{V_{wk}}\left[1-\frac{C-2}{(C-1)^2}-\frac{1}{2(C-1)^2}\right]\left[\frac{W}{2}+\frac{L}{2(C-1)}\right]+\frac{C-2}{3V_{wk}(C-1)^2}\left[W+\frac{L}{C-1}\right]+\frac{1}{6V_{wk}(C-1)^2}\left[W+\frac{L}{2(C-1)}\right] \tag{29}$$

Thus, the expected walking time of all passengers can be derived as

$$K=\frac{(\theta_c-\theta)}{\theta_c}\left[(\eta_2+\eta_3)\left(\frac{L}{4(C-1)V_{wk}}+\frac{W}{4V_{wk}}\right)+\eta_4 K_{\mathrm{IV}}^{\theta c-\theta}\right] \tag{30}$$

The rejected passengers (types II, III, and IV) either walk directly to their destinations or turn into type I passengers, who can receive the time schedule of the flex-route system. Hence, their waiting time can be ignored ($A^{\theta c-\theta}=0$). We define $\alpha=A$ in Eq. (14); then, the expected waiting time of all passengers is

$$A=\frac{\theta}{\theta_c}\alpha \tag{31}$$

Similarly, for the $\theta_c-\theta$ passengers, the expected riding time of type I passengers $R_{\mathrm{I}}^{\theta_c-\theta}$ is equal to the result of Eq. (9), and the expected riding time of type II, III, and IV passengers can be expressed as follows:

$$R_{\mathrm{II}}^{\theta c-\theta}=R_{\mathrm{III}}^{\theta c-\theta}=\frac{(C+1)T_{\mathrm{r}}}{3(C-1)}\left[1-\frac{1}{C}\right] \tag{32}$$

$$R_{\mathrm{IV}}^{\theta c-\theta}=\frac{(C+1)T_{\mathrm{r}}}{3(C-1)}\left[1-\frac{C-2}{(C-1)^2}-\frac{1}{2(C-1)^2}\right] \tag{33}$$

We define $\beta=R$ in Eq. (12); the expected riding time per passenger can be obtained as

$$R=\frac{\theta}{\theta_c}\beta+\frac{\theta_c-\theta}{\theta_c}\left[\eta_1+(\eta_2+\eta_3)\left(1-\frac{1}{C}\right)+\eta_4\left(1-\frac{C-2}{(C-1)^2}-\frac{1}{2(C-1)^2}\right)\right]\frac{(C+1)T_{\mathrm{r}}}{3(C-1)} \tag{34}$$

The operation cost per passenger O_{p} can be derived as

$$O_{\mathrm{p}}=\frac{O_{\mathrm{v}}M}{\theta+(\theta_c-\theta)\left[\eta_1+(\eta_2+\eta_3)\left(1-\frac{1}{C}\right)+\eta_4\left(1-\frac{C-2}{(C-1)^2}-\frac{1}{2(C-1)^2}\right)\right]} \tag{35}$$

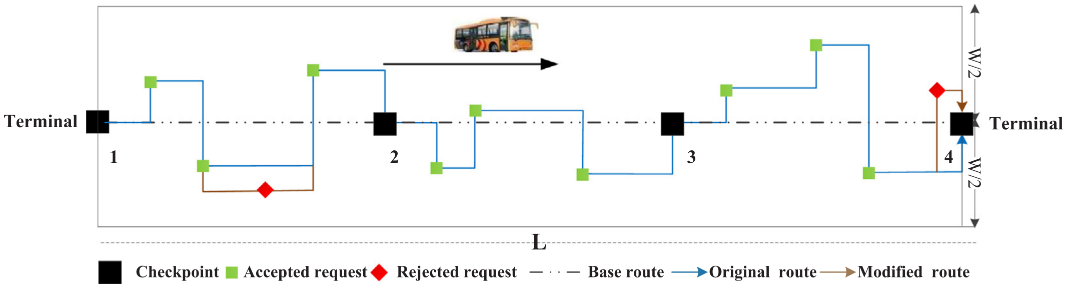

Fig. 3. Diagram of the slack arrival strategy.

4. Slack arrival strategy

In flex-route systems, the fixed slack time for deviation service is generally designed based on the expected demand level. However, in real-life operations, the actual travel demand may vary during different time periods, as discussed above, and even between adjacent route segments. The time needed to serve on-demand requests may also change depending on the location of these requests within the service area. Hence, it is quite difficult for deviation services to make full use of assigned slack time. The checkpoints in flex-route systems are generally built at locations with high travel demand or where customers can transfer to other transit lines. In practical use, a significant proportion of checkpoints are built only due to travel demand instead of connecting with other transit lines, since there is a much sparser public transit network in low-demand areas than urban areas.

In order to deal with the uncertain travel demand, in terms of reducing the amount of unused slack time at unexpectedly low demand levels and accepting more curb-to-curb requests at unexpectedly high demand levels, a slack arrival strategy is proposed in which transit vehicles are allowed to reach the checkpoints, which are not transfer stations, somewhat later than the scheduled departure time. Assume that the actual arrival time of at checkpoint i is Arr_i. The scheduled departure time is denoted by sd_i. The slack arrival strategy relaxes the constraint with a certain value of ε, which means that the vehicle can depart within the time window $[sd_i\, sd_i\ + \varepsilon]$. The actual departure time of the checkpoint Dep_i can be expressed as follows:

$$Dep_i = \begin{cases} sd_i & \text{if } Arr_i\ + T_d^f\ < sd_i \\ Arr_i + T_d^f & \text{if } sd_i \leqslant Arr_i + T_d^f \leqslant sd_i + \varepsilon \end{cases} \tag{36}$$

An overview of the procedure is provided based on an example (see Fig. 3). Two of the requests are rejected due to the inviolable departure time constraints of checkpoints 2 and 4. However, if the departure time constraints are relaxed in this new strategy, the two rejected requests could potentially be accepted which is shown as the modified route in Fig. 3. Because the departure time of checkpoint 2 is delayed, the idle time at checkpoint 3 is reduced accordingly. It should be noted that this slack arrival strategy does not affect the overall service cycle of flex-route services. It is likely for real-life cases, such as the PRTC Manassas OmniLink route (Potts et al., 2010), to have one or more checkpoints functioning as transfer stations where the arrival times follow a synchronized schedule with other transit lines. The slack arrival strategy will not be applied to these checkpoints because the delay might disrupt transfer passengers' plans to catch up with other transit lines.

The slack arrival strategy can utilize the unused slack time of flex-route transit by maximizing the number of accepted requests and minimizing the unnecessary idle time at checkpoints. The primary tactics of the strategy is to dynamically redistribute the unused slack time among segments to accept more on-demand requests without affecting the overall service cycle. Negative impacts of this strategy can be expected as: first, the passengers who starts their trips at checkpoints (types I, II), may wait longer because of the delayed departure time; second, it may also prolong the riding time of the passengers who disembark at checkpoints. However, in low-demand areas, customers are much more likely to endure a relatively small ε in low-frequency flex-route systems than general fixed-route systems, especially when they are informed about the relaxed departure time in advance. For instance, at a checkpoint with previously fixed departure time t, after $\varepsilon = 1.5$ min is assigned, on-board customers know that the vehicle will depart at this checkpoint earlier than $t + 1.5$, and customers waiting at this checkpoint know that the vehicle will depart within the time window $[t, t + 1.5]$ and they just need to arrive at this checkpoint earlier than t as usual to avoid missing the bus.

5. Simulation model

Simulation experiments for flex-route transit were performed using MATLAB R2017a software to validate the above theoretical models and test the influence of the slack arrival strategy. For each scenario, 50 replications of 5000 operation cycles were run to provide statistical estimates of system performance. An insertion heuristic algorithm was constructed in simulation to build a vehicle routing plan based on first-come, first-served policy and the no-backtracking constraint. The insertion algorithm is not real-time because current deviation services require passengers to make reservations in advance. Customers are generated randomly based on the demand rate and the proportion of passengers. Once an on-demand request is accepted, the system sends back a scheduled pick-up time to the customer. The rejected passengers either walk to the nearest checkpoint or walk directly to their destinations depending on the walking distance. Consecutive operation cycles are reproduced in the simulation experiments. The delay at the ending

Table 1
Parameter values.

Parameter	Value
L	10 miles
W	1 miles
M	1
C	3
θ	18 passenger/h
V_b	25 miles/h
V_{wk}	3 miles/h
ω_K	\$25/passenger/h
ω_A	\$15/passenger/h
ω_R	\$20/passenger/h
ω_I	\$30/passenger/h
O_p	\$60/vehicle/h
T_d^r	0.3 min
T_d^f	1 min
T_{unit}	20 min
T_r	40 min
$\eta_1/\eta_2/\eta_3/\eta_4$	0.1/0.4/0.4/0.1

terminal in the previous single trip, if this terminal does not belong to transfer stations, would affect the departure time of the next single trip, but the time windows of checkpoints in the next trip remain the same.

6. Analysis of the results

6.1. Parameter values

In this section, a case study is conducted based on Line 646 in Los Angeles County which operates under a flex-route policy during nighttime. Line 646 is a typical flex-route transit service and has been used as a testbed by Quadrifoglio et al. (2006, 2007, 2008), Quadrifoglio and Dessouky (2008) and Qiu et al. (2014, 2015b). Related parameter values for the analysis are provided in Table 1. All checkpoints are transfer stations. The operation cost of service vehicles and the values of the customer cost indicators are set based on the National Database of the US (2010) and Wardman (2004). Because early arrivals at checkpoints are always discouraged and have a negative impact on the level of service (Smith et al., 2003), the cost of idle time at checkpoints is assumed to be twice the cost of the waiting time. The system is designed with an expected demand rate of $\theta = 18$ passengers/h. Based on Eqs. (7) and (8), we can derive T_r and T_{unit} as 40 min and 20 min, respectively.

6.2. Comparison of theoretical and simulation models

The values of the customer cost indicators and the operation cost from the theoretical model and simulations are presented in Table 2. The results illustrate that at an unexpectedly low demand ($\theta_c = 8$ passengers/h) and unexpectedly high demand ($\theta_c = 28$ passengers/h), the simulation results correspond well with the outcomes from the theoretical modelling, which verifies the reliability of our simulation models.

However, at the expected demand level ($\theta_c = 18$ passengers/h), the results of the two models differ considerably. This discrepancy occurs because in theoretical modelling, we assume an ideal case in which all slack time is consumed by the deviation requests and no requests are rejected by the system. Hence, there is no idle time at the checkpoints, and the average walking time is equal to zero. However, in our simulation models, although the travel demand per hour is constant, the actual travel demand may vary in different

Table 2
The results from theoretical and simulation modeling.

Indicators	Demand (passengers/h)					
	8		18		28	
	Theoretical	Simulation	Theoretical	Simulation	Theoretical	Theoretical
K (min)	0	0.16	0	5.15	10.37	10.32
A (min)	0.23	0.23	0.68	0.32	0.44	0.38
R (min)	14.16	14.10	17.33	16.41	18.40	18.38
I (min)	1.28	1.29	0	0.48	0	0.23
O_p (\$)	6.12	6.06	3.33	3.25	2.34	2.32
F (\$)	11.54	11.53	9.28	11.18	12.91	12.95

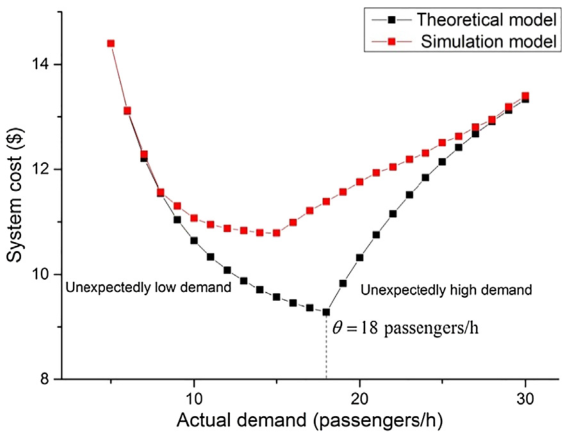

Fig. 4. System cost at different demand levels ($\theta = 18$ passengers/h).

route segments over a short period. The time needed to serve each request may also change depending on the location of the request within the service area. The uncertainties of the requests cause certain passengers to be rejected by the system and walk to the nearest checkpoint; meanwhile, on-board passengers have to bear unnecessary idle time at checkpoints. Table 2 illustrates that both K and I are greater than zero for simulation models. Because there are fewer curb-to-curb requests served in the simulation models than expected, the riding time and waiting time in the simulation models are shorter than those in the theoretical models. In general, the system cost F in the simulations is considerably greater than that in the modelling due to the longer walking time.

The trends of the system cost functions under the theoretical and simulation models at different demand levels are presented in Fig. 4. The curve of the theoretical model illustrates that system cost reaches its minimum value at the expected demand level ($\theta_c = \theta$). At unexpectedly low demand levels ($\theta_c < \theta$), system cost declines with increases in the actual demand, mainly due to the sharp decrease in the operation cost per passenger. However, when the actual demand exceeds the expected demand level ($\theta_c > \theta$), the system cost increases with the rise of the actual demand because of increased walking time.

The curve of the simulation model reproduces the theoretical model quite well when the actual demand is extremely low or high. However, due to the uncertainty of the request locations and short-term travel demand, the system cost in the simulation model is typically higher than that in the theoretical model near the expected demand level. Furthermore, the demand with the minimum system cost is lower than the expected demand. These results indicate that even if the slack time is precisely designed based on the expected demand in theoretical modelling, the flex-route services still cannot guarantee that all curb-to-curb requests can be served due to uncertainties.

6.3. Effect of the slack arrival strategy

To distinguish the effect of non-relaxed transfer stations constraints, the following 4 scenarios are defined for Line 646 service. Scenario1: none of the three checkpoints are transfer stations; Scenario 2: the intermediate checkpoint is a transfer station; Scenario 3: one of the terminal checkpoints is a transfer station; Scenario 4: the two terminal checkpoints are both transfer stations.

Simulations were firstly performed to study the influence of the slack arrival strategy at the expected demand level ($\theta = 18$ passengers/h) under Scenario 1. Table 3 illustrates that with increasing ε, the rejection rate of the system declines dramatically from 13.89% to 0.89% with a slow downward trend; the system cost can be reduced up to 11%. Meanwhile, the walking time K and idle time I are also considerably reduced. At the price of serving more curb-to-curb passengers, the waiting time A increases because the type I and type II passengers must wait until the delayed departure time and the increase of accepted requests also extends the

Table 3
Simulation results of the slack arrival strategy.

ε (min)	Reject rate	K (min)	A (min)	R (min)	I (min)	O_p ($)	F ($)
0	13.89%	5.15	0.32	16.41	0.48	3.25	11.19
1	10.41%	3.81	0.57	16.54	0.36	3.27	10.69
2	8.78%	2.76	0.93	16.63	0.29	3.27	10.34
3	5.45%	2.08	1.29	16.70	0.25	3.27	10.15
4	4.34%	1.56	1.74	16.60	0.22	3.27	10.00
5	2.91%	1.19	2.09	16.64	0.19	3.27	9.92
6	2.53%	0.92	2.44	16.67	0.18	3.27	9.91
7	1.59%	0.67	2.68	16.68	0.17	3.27	9.89
8	1.28%	0.55	2.95	16.70	0.16	3.27	9.88
9	1.14%	0.45	3.19	16.72	0.15	3.27	9.90
10	0.89%	0.32	3.40	16.73	0.14	3.27	9.90

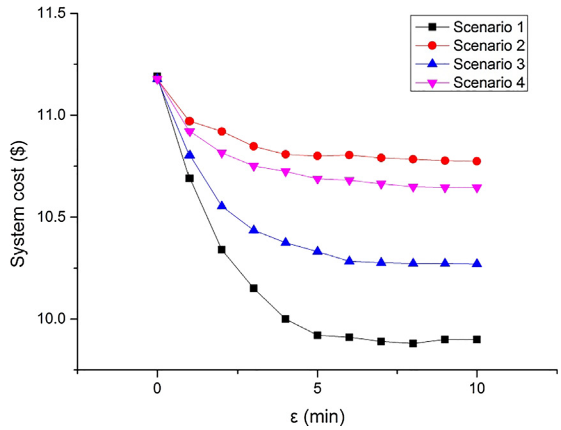

Fig. 5. System cost functions under different scenarios ($\theta = 18$ passengers/h).

waiting time of type III and type IV passengers. For the passengers whose destinations are at checkpoints (type I, III), their riding time is also increased due to the delayed arrival time. It is interesting to see that the system cost F experiences a gradual decrease when ε is less than 5 min and then stabilizes. This finding suggests that the value of ε should be carefully set to balance the tradeoff between flexibility and reliability.

The simulation results of the above 4 scenarios at the expected demand level ($\theta = 18$ passengers/h) with different ε are displayed in Fig. 5. The system costs are reduced in all 4 scenarios, which indicate that the slack arrival strategy is also effective even if it is only applied to some of the checkpoints. A larger ε generally means a lower system cost, but the values tend to be stable after ε exceeds 5 min. In Scenario 1, the system cost decreases more drastically than those in the other three scenarios because all checkpoint departure times are relaxed. It is also observed that the slack arrival strategy is more effective in Scenario 3 than that in Scenario 4. Interestingly, Scenario 2 has the smallest system performance improvement, which demonstrates that intermediate checkpoints are more sensitive to the slack arrival strategy than terminal checkpoints.

Simulations were also conducted at unexpected demand levels with different time windows ($\varepsilon = 1$, 3, and 5 min) under Scenario 1. Fig. 6 illustrates that the curve of the system cost functions in the simulations gradually approaches the theoretical curve with increasing ε. This trend indicates that the slack arrival strategy is also effective at improving the efficiency of the system at unexpected demand levels. The reduction in the system cost is small when the actual demand deviates considerably from the expected demand and matches well with the results from the theoretical modelling. When the actual demand is near the expected demand, the system performance improves greatly but still performs worse than the theoretical value due to the uncertainty of the requests. Fig. 6 also shows that the demand with the best system performance gradually shifts towards the expected demand with increasing ε. When ε = 5min, the best system performance is achieved when actual demand reach the expected demand level ($\theta_c = 18$ passengers/h).

The vehicle capacity constraint was not considered because flex-route transit is usually applied in low-demand areas. Some simulation experiments are conducted to analyze the maximum utilization of vehicle capacity with different time windows. The results showed that the slack arrival strategy generally has no impact on the maximum number of on-board passengers at the unexpected low demand level and expected demand level (see Fig. 7). However, when the actual demand is far beyond the expected demand (e.g., $\theta_c > 25$ passengers/h), a larger ε increases the maximum utilization of vehicle capacity. The maximum number of

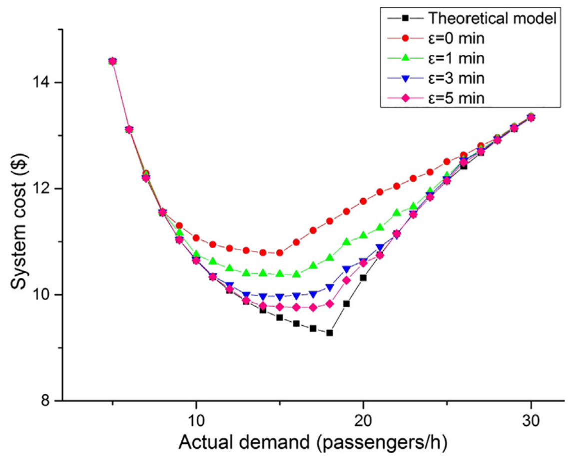

Fig. 6. Effect of the slack arrival strategy with different ε ($\theta = 18$ passengers/h).

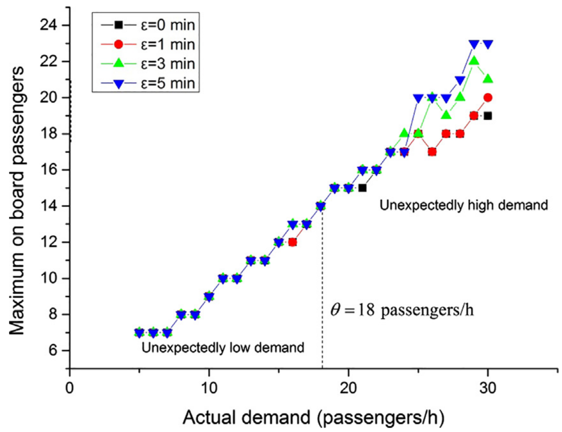

Fig. 7. Maximum on board passengers with different ε ($\theta = 18$ passengers/h).

passengers on board is 23 in all scenarios, which suggests that a service vehicle with 25 seats is confidently sufficient for the designed demand range. The vehicle capacity constraint should be considered if there is a higher travel demand is expected.

6.4. Sensitivity analysis

A sensitivity analysis of several system parameters was conducted for Scenario 1, and the results are presented in Fig. 8. In contrast to the previous analysis, we obtain the relationship between the system cost and the expected demand, which means that the single-trip time is varied at different expected demand levels. The analysis was performed to obtain the demand with the best performance in different cases and to test the effectiveness of the slack arrival strategy. The results from the base case (Fig. 8(a)) indicate that the system cost initially decreases and then increases with increasing demand. The demand with the best performance for this case is around 18 passengers/h. The simulation model without the slack arrival strategy differs considerably from with the theoretical models when the expected demand is extremely low. The rejection rate can be as high as 47.5%. The difference becomes smaller and the rejection rate decreases with increasing demand. The simulation model with the slack arrival strategy ($\varepsilon = 5$ min) reduces the rejection rate considerably and exhibits the same trend as that of the theoretical model.

To study the influence of the service area shape on system performance, another experiment with a smaller value of area width ($W = 0.5$ miles) was conducted (see Fig. 8(b)). In this case, the difference between the simulation model and theoretical model is more remarkable than that of the base case. After applying the slack arrival strategy ($\varepsilon = 5$ min), the difference is further reduced and corresponds well with the theoretical model. At extremely low demand ($\theta = 5$ passengers/h), the system performance could be promoted by up to 40%. The demand with the best performance is 25 passengers/h, which is higher than the base case. This finding suggests that a narrow service area can accommodate more passengers under the flex-route policy.

It is assumed the cost of the idle time at the checkpoint is twice the cost of the waiting time by default. One may argue that such an assumption gives an undue advantage to the slack arrival strategy and exaggerate the benefits. Here another case, where idle time at the checkpoints is regarded as part of the riding time of the passengers (i.e., $\omega_I = \omega_R$), is investigated. Fig. 8(c) reveals that the change of idle time cost has a very limited influence on the entire system performance and the slack arrival strategy still could reduce the system cost notably.

The number of checkpoints may be frequently updated in operation, and Fig. 8(d) illustrates another case with 5 checkpoints, with a shorter distance between checkpoints. Due to the reduced walking time of rejected customers, a decline of the system cost in the simulation model without the slack arrival strategy is observed. In this case, the slack arrival strategy still can greatly improve the system performance. This happens probably because more hard constraints, in terms of fixed departure time of checkpoints, make the entire service more inflexible. After applying the slack arrival strategy, the rejection rate can be reduced by up to 63% and the simulation results are closer to the theoretical results. This finding suggests that the slack arrival strategy becomes more efficient in cases with a larger number of checkpoints.

The proportions of passengers may vary over time, and in another case (Fig. 8(e)) the proportions are set as $\eta_1 = 0.5$ $\eta_2 = 0.2$ $\eta_3 = 0.2$ $\eta_4 = 0.1$. This case would occur when the demand between the checkpoints is extremely high. The demand with the best performance is 24 passengers/h. The results suggest that an area with more type I passengers should operate flex-route transit at relatively high demand.

Furthermore, a two-vehicle case ($M = 2$) was investigated, and the results are displayed in Fig. 8(f). The two vehicles run along the base route in opposite directions, and they leave the two terminals at the same time. The outcomes indicate that the single-vehicle operation performs better than the two-vehicle case at low-demand levels. However, the system cost of the two-vehicle operation decreases dramatically with increasing demand. As a result, the two operation models would have the same system cost at a switch point, which is also called critical demand in Fig. 8(f). In our case, the critical demand is 26 passengers/h. Transit planners should consider choosing two-vehicle operation policy when the demand is above the critical demand and vice versa. Furthermore, it is observed that the simulation model with the slack arrival strategy ($\varepsilon = 5$ min) has the same critical demand as the theoretical model.

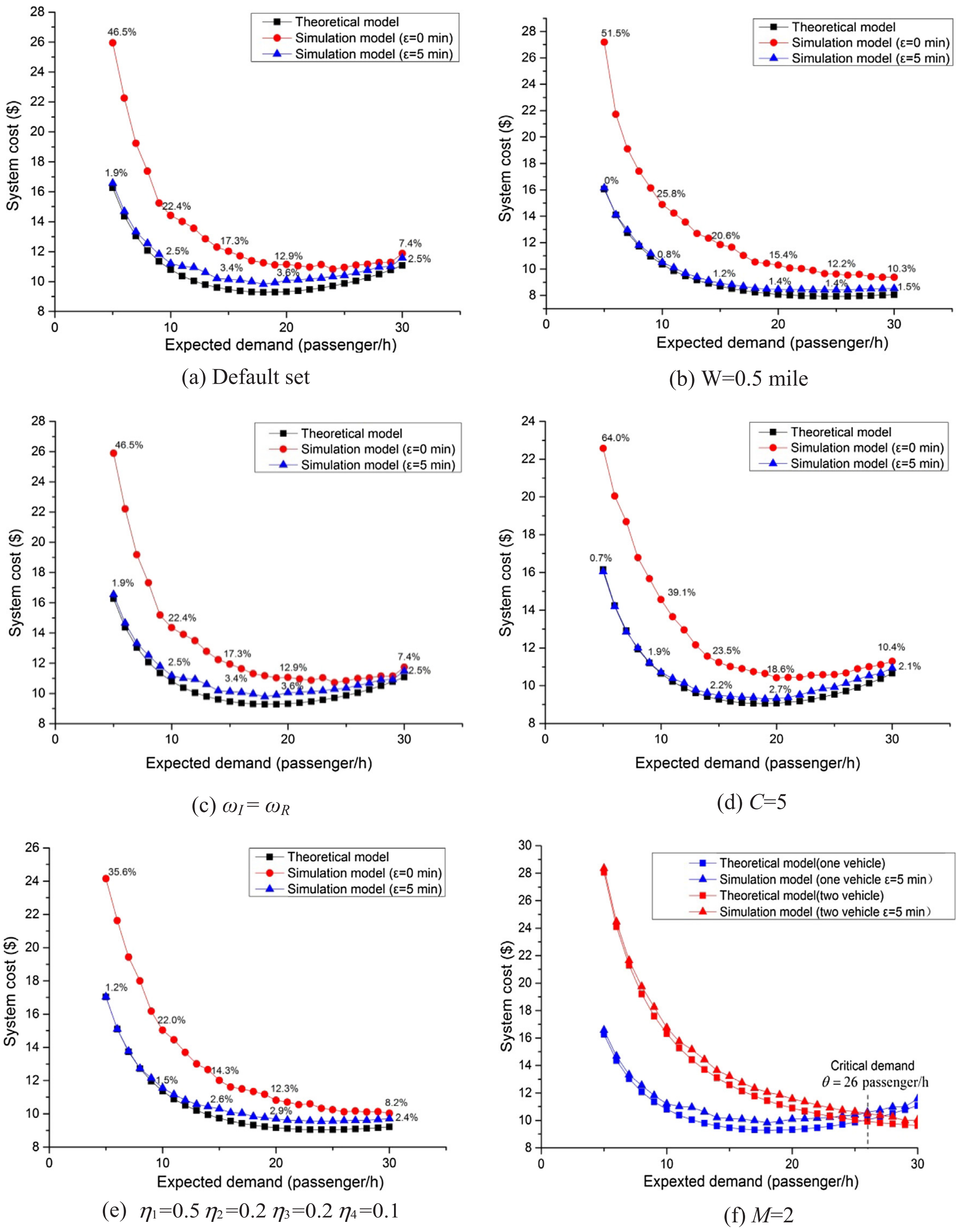

Fig. 8. Sensitivity analysis over (b) width; (c) cost of idle time; (d) number of checkpoints; (e) proportions of passengers; (f) fleet size.

The above analysis confirms that the proposed slack arrival strategy can significantly reduce the negative impacts of demand uncertainty and promote the overall performance of flex-route systems. This benefit is particularly notable when there is a relatively low demand in the service area. In the cases studied, a 5-min time window is able to keep the rejection rate below 4%.

7. Conclusion

Flex-route transit, which combines the low-cost operability of fixed-route transit with the flexibility of demand-responsive transit, is one of the most widely used flexible transit services. Current flex-route transit services have a fixed departure time schedule at all checkpoints. However, due to the uncertainty of travel demand, it is pretty difficult for the transit operator to design a deviation service with both reliability and efficiency.

In this paper, we propose a slack arrival strategy to diminish the negative effects of the uncertain travel demand. Theoretical and simulation models are developed under both expected demand and unexpected demand levels to evaluate the benefit of implementing the new strategy. The results indicate that the proposed slack arrival strategy could effectively reduce the rejection rate and idle time at checkpoints at the expense of very limited additional riding time and waiting time for passengers. This new strategy does not affect the synchronization with other transit lines at transfer stations and is also quite easy to implement in operation. The case study shows that it could reduce rejection rate and system cost by up to 63% and 40%, respectively, without any additional operating cost or changing the overall service cycle.

In practical use, planners can use the developed theoretical and simulation models to determine the cycle time, target demand, fleet size and appropriate width of the time window based on real parameters. The maximum delay time at the checkpoints should be carefully investigated before the implementation of the slack arrival strategy. Our study shows that there is an upper limit for the slack time window, usually several minutes, and a larger time window do not promote the system performance. Moreover, to guarantee the relaxed departure information is visible to passengers, a client side real-time information app should be developed to help customers easily adjust to the new strategy and enhance the reliability of the service. It is also considered that this novel strategy could be used in other kinds of flexible transit services. Future work will investigate the optimal slack time allocation with a specific time window and the implementation of the slack arrival strategy in cases with other flexible operating policies.

Acknowledgement

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Appendix A

(a) Derivation of Eq. (9)

The OD distribution of type I passengers of flex-route transit is displayed in Fig. 9. This distribution is uniform between any two checkpoints. The nature of the problem is to randomly choose two points from the C checkpoints and calculate the expected distance. The expected riding time of type I passengers can be derived as

$$R_{\mathrm{I}}=\frac{T_{\mathrm{unit}}\sum_{i=1}^{C-1}\sum_{j=1}^{i} j}{C_C^2}=\frac{T_{\mathrm{unit}}\sum_{i=1}^{C-1} i(i+1)/2}{C(C-1)/2}=\frac{T_{\mathrm{unit}}(C(C-1)/4+C(C-1)(2C-1)/12)}{C(C-1)/2}$$
$$=\frac{(C+1)T_{\mathrm{unit}}}{3}=\frac{(C+1)T_{\mathrm{r}}}{3(C-1)}$$

(b) Derivation of Eq. (10)

For a type II passenger, the pick-up point is at the checkpoint, and the drop-off point is at the segment between two consecutive checkpoints. It is equal to randomly choose two points, with one point at the checkpoints and another point at any position of the segment, and calculate the expected distance. The expected riding time of type II or type III passengers can be calculated as

$$R_{\mathrm{II}}=R_{\mathrm{III}}=\frac{T_{\mathrm{unit}}\sum_{i=1}^{C}\int_1^C |x-i|\,dx}{C(C-1)}=\frac{(2C-1)T_{\mathrm{unit}}}{6}=\frac{(2C-1)T_{\mathrm{r}}}{6(C-1)}$$

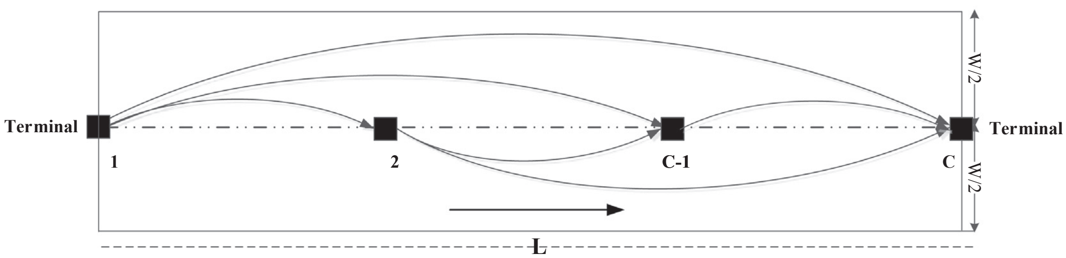

Fig. 9. Distribution of type I passengers.

(c) Derivation of Eq. (11)

For any type IV passenger, the pick-up time and drop-off time are randomly distributed within $[0,T_r]$. The problem is same as randomly choosing two points x and y within a segment and calculating the expected distance. The expected riding time of type IV passengers can be derived as

$$R_{\mathrm{IV}} = \frac{\int_0^{Tr}\int_0^{Tr}|x-y|dxdy}{T_{\mathrm{r}}\cdot T_{\mathrm{r}}} = \frac{T_{\mathrm{r}}}{3} = \frac{(C-1)\,T_{\mathrm{unit}}}{3}$$

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