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Optimizing Bus Bridging Services in Response to Disruptions of Urban Transit Rail Networks

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With growing dependence of many cities on urban mass transit, even limited disruptions of public transportation networks can lead to widespread confusion and significant productivity losses. A need exists for systematic approaches to developing efficient responses to minimize such negative impacts. We present an optimization-based approach that responds to degradations of urban transit rail networks by introducing smartly designed bus bridging services that take into consideration commuter travel demand at the time of the disruption. The approach consists of three fundamental steps, namely, (1) a column generation procedure to dynamically generate demand-responsive candidate bus routes, (2) a path-based multicommodity network flow model to identify the most effective combination of these candidate bus routes, and (3) another optimization-based procedure to determine simultaneously the optimal allocation of available vehicle resources among the selected routes and corresponding headways. The approach is applied to two case studies defined using actual data. The results show that the proposed approach can be carried out efficiently and that adding nonintuitive bus routes to the standard bus bridging services can significantly reduce the average travel delay. Moreover, the approach distributes delay more equitably. Many realistic operating constraints can also be handled.

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1. Introduction

Large cities worldwide are becoming increasingly dependent on their public transport systems. These systems can carry passengers more efficiently than private transport, thus reducing per-unit transportation costs, as well as environmental impacts. For example, in Singapore, a city with a population of five million, approximately four million public transport trips (on buses and on the transit rail system) are currently made on any given day. In London, the Underground (Tube) system alone carries more than four million passengers on a typical weekday.

The heavy reliance on public transport means that even limited service disruptions can easily lead to unacceptable outcomes affecting large numbers of commuters. This is especially true in the case of intensively utilized components of the overall system, such as transit rail tracks, stations, and vehicles. Major disruptions (and the inadequate responses to them) not only cause travel delays and significant productivity loss on a city scale but also have longer-term effects such as confidence and loss of ridership using the public transport system and increased reliance on private vehicles. A simple event involving a runaway maintenance locomotive shut down the entire Northern Line of the London Underground in the early morning of August 13, 2010, creating chaotic conditions for commuters the entire morning. A five-hour breakdown of Singapore's urban transit rail system affecting 11 stations on December 15, 2011 seriously inconvenienced hundreds of thousands of commuters, and led to a far-reaching public inquiry.

An effective response capability is critical to minimizing the severity of potential negative impacts of network disruptions. This paper addresses one of the most critical components of disruption responses, namely, the design of temporary bus services and routes that restore connectivity among the undisrupted parts of *transit rail networks* in an optimal way. This type of temporary network is often referred to as a "bus bridging service." Our contribution consists of the following: (a) developing for this purpose a novel mathematical optimization approach that not only designs the temporary service network but also allocates available bus resources among the network's parts; and (b) demonstrating through two case studies that the approach is practicable and may yield significant benefits for disrupted urban travelers.

With respect to (a), the proposed approach consists of three fundamental steps. First, a column generation procedure is used to dynamically generate demand-responsive candidate bus routes, possibly including nonintuitive routes that may connect spatially dispersed stops or provide direct links between different transit rail lines. Second, a path-based multicommodity network flow model identifies the most effective combination of these candidate bus routes. Last, another optimization-based procedure determines simultaneously the allocation of available vehicle resources among the selected routes and the corresponding headways on each route.

With regard to (b), the results obtained for two representative case studies suggest the following: (1) the proposed approach can be carried out in a computationally efficient way, even in contexts that require a quick response plan; (2) the addition of nonintuitive bus routes to the standard bridging services (on which transit rail operators have typically relied) can lead to significant reductions in the expected travel time delay of system users; (3) the distribution of travel delay can also be made much more equitable through an optimized response; and (4) realistic operational constraints can be handled. Moreover, as opposed to the bus bridging services, which attempts to recover the network topology, the proposed approach is demand responsive, i.e., sensitive to the time-of-day travel requirements of the system's users, which addresses the real challenge more directly.

The same approach may also be used to design temporary bus routes that are put in place in response to *scheduled* local suspensions of transit rail services. Such suspensions occur commonly in urban environments in order to permit the performance of maintenance, repairs, or upgrading of transit rail infrastructure, such as tunnels, rails, switches, etc. These activities require an interruption of service in parts of the network (e.g., on a segment of a subway line covering two or more consecutive stops) and may last for as little as a few hours or as long as several months. Such scheduled service suspensions are typically announced to the public well in advance and are always accompanied by information on the temporary replacement services (almost invariably temporary bus lines) that urban travelers can use until the original service is restored.

The most obvious practical obstacle to implementing our approach is the difficulty of communicating to disrupted passengers the actions they should take when it comes to using the more complex, nonintuitive bus bridging routes. (Note that this is not an issue when it comes to *scheduled* suspensions of service, because adequate time exists to provide information to the traveling public.) However, the rapidly growing use of smartphones and "apps" specifically tailored to urban commuters may soon make it possible to, at least partially, overcome this problem. One can even conceive of smartphone applications that would advise *individual* travelers on which disruption-service bus route to use (and where to board and alight) depending on each traveler's current location and eventual destination.

The remainder of the paper is organized as follows: §2 reviews briefly relevant papers in the literature. Section 3 presents the disruption response problem and describes in detail the various parts of our three-step optimization approach. The results of applying the method to two case studies, which are based on actual data, are presented and discussed in §4. Section 5 summarizes our conclusions and discusses some implementation issues.

2. Literature Review

Traditional planning for scheduled urban public transport services (bus, rail, tram, etc.) requires addressing a sequence of decision problems ranging from strategic to operational. This process is often viewed as consisting of three general steps: network design, line planning, and timetabling. Various models and methods based on optimization and operations research techniques have been developed in the literature to support these steps. Readers may refer to Odoni, Rousseau, and Wilson (1994); Bussieck, Winter, and Zimmermann (1997); and Schöbel (2012) for comprehensive reviews. The *network design* step deals with the configuration of a transportation system to achieve some specified objectives. Relevant papers include Ceder and Wilson (1986); Melkote and Daskin (2001); and Mauttone and Urquhart (2009). Once a network structure has been determined, *line planning* involves the design of line routes and the associated allocation of vehicles and frequencies, given an overall distribution of travel demand (Schöbel and Scholl 2006; Borndörfer, Grötschel, and Pfetsch 2007). Finally, *timetabling*, the last step in the process, establishes the detailed time schedules and frequency of service on each line route, with the objective of minimizing passenger waiting times both at stops and at transfer points (Liebchen 2008; Kaspi and Raviv 2013).

In contrast to the extensive literature on strategic planning and daily operations under normal conditions, only a limited amount of research had been published until recently on preventing disruptions and responding to them. These topics, however, are now receiving increasing attention. Some *preventive planning* concepts and models have been proposed for designing public transport services that are robust to disturbances—see Cicerone et al. (2009); Liebchen et al. (2009); Fischetti and Monaci (2009); and Chen, Cohn, and Pinar (2011). On the other hand, the work directly relevant to this paper deals with the *design of responses to disruptions*, once they have occurred, and is complementary to preventive planning. Its focus is on alleviating the consequences of disruptions. Compared to strategic and operations planning, whose objectives is to ensure good service quality and low operating costs in the long term, disruption responses aim at minimizing impacts over a much shorter time horizon, usually no longer than several hours. Moreover, certain bus routes (see §4), which might be reasonable in the context of temporary disruption responses, would not be admissible as normal bus routes. Consequently, the strategic and operations models and techniques of the previous paragraph are generally not applicable when planning for disruption responses.

Jespersen-Groth et al. (2009) discuss three subproblems related to disruption response management for railway systems—timetable adjustment, rolling stock rescheduling, and crew rescheduling. Kroon and Huisman (2011) summarize the disruption response models and algorithms developed for the Netherlands Railways company. In an early paper, Meyer and Belobaba (1982) examine in a qualitative way the contingency planning processes relevant to transit rail systems. Darmanin, Lim, and Gan (2010) propose disruption recovery strategies for the specific case of the existing bus routes of the Melbourne metro system. Pender et al. (2012) survey the disruption response management practices adopted by 48 international passenger transit rail organizations. Measures that are commonly employed include deploying bus bridging services, transferring passengers to other rail lines, and tracking crossovers. Cadarso, Marín, and Maróti (2013) develop an integrated model of timetabling and rolling stock that gives specific consideration to disruption response. The main objective is to minimize the recovery period, i.e., returning to the original timetable as soon as possible, after dealing with the temporary increase in passenger demand requirements resulting from a disruption. Wang et al. (2014) analyze the queueing and waiting behaviors in the context of bus bridging response to rail disruptions. Jin et al. (2014) introduce a localized rail-bus integration approach aimed at enhancing the urban transit rail networks resilience to disruptions. Kepaptsoglou and Karlaftis (2009) propose a methodological framework for planning and designing an efficient bus bridging network that consists of two key steps: designing of bus routes and allocation of bus resources among the routes. We have also used the same framework, but with a more integrated approach and an entirely different methodology. Whereas the bus bridging routes in Kepaptsoglou and Karlaftis (2009) are generated using a shortest path algorithm and subsequently modified through a heuristic approach, we employ a column generation algorithm to efficiently identify all potentially beneficial and demand-responsive bus routes. Whereas the selection of bus routes and allocation of bus resources are performed sequentially in their paper, we model explicitly the trade-offs involved in making these two decisions and develop an integrated optimization model to carry out both of these steps simultaneously.

3. Network Disruption Response Planning Problem

In this section, we first introduce the disruption response planning problem in the context of urban transit rail networks. Then, we show how the problem can be handled by (1) developing a network flow representation of the integrated rail and interim bus service network, (2) finding the candidate set of bus bridging routes using a column generation procedure, and (3) selecting the optimal subset of bus bridging routes using a path-based multicommodity network flow model.

3.1. Problem Description

Consider the scenario where a disruption event causes a temporary closure of a portion of the urban transit rail network, affecting one or a few links between stations. As illustrated in Figure 1, links

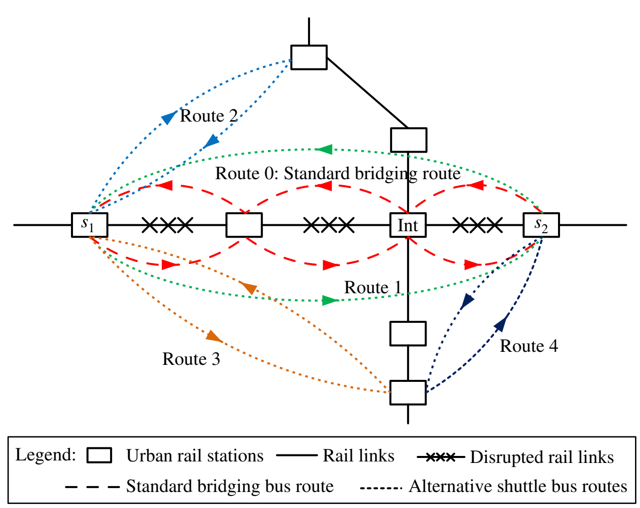

Figure 1 (Color online) Illustrative Example of Transit Rail Network Disruption and Responsive Bus Bridging Routes

between two stations s_1 and s_2 are down. Consequently, some train services at a number of the affected stations will have to be suspended. In a densely connected service network, commuters may still be able to find alternative ways to get to their destinations. However, in many instances and absent a proper intervention by the network's operator, a large number of commuters may be left stranded at various parts of the network or may have to follow extremely circuitous routes to their destinations.

A common response of transit rail system operators to such disruptions is to replicate the lost train service by running bus bridging services in parallel to the disrupted section of the network, such as route 0 (denoted as "standard bridging route") in Figure 1. However, this alone is often not the best and sufficient response because the pattern of commuter travel demand has not been considered. For example, in Figure 1, if a large number of commuters are traveling between stations s_1 and s_2, it may be wise to divert some resources to provide a direct bus bridging service (route 1) between these two stations. Moreover, since trains run on dedicated tracks and buses do not, what may be a good route for trains may be an inefficient one for bus services. It is therefore conceivable that the response may be improved substantially by complementing routes 0 and 1 with additional routes, such as routes 2 to 4 shown in Figure 1, which provide direct bus connections between high-demand stations. In fact, some commuters may in this way reach their destinations earlier, in spite of the disruption.

In planning for responses to network disruptions, the challenge is to identify good candidate routes for a given disruption scenario and anticipated commuter travel demand, and then find the optimal combination of these routes to deploy bus resources so as to minimize the overall travel delay. Formally, this is a design problem: construct a temporary integrated rail-bus service network with the objective of minimizing the total travel time delay of all commuters, given a disrupted urban transit rail network and the commuters' travel demand patterns. Because of the temporary nature of the problem, disruption response faces additional challenges in communication to both commuters and drivers, and limited possibilities to improve infrastructures, such as adding pick-up bays and shortening transfer distances. Consequently, it deviates from the standard network design problem in the following aspects: (1) limited total number of routes, (2) limited routes per station, (3) limited fleet resources, and (4) the need to consider transfer times.

3.2. Network Representation

The integrated rail and bus service network is represented by a directed graph $G(\mathcal{N}, \mathcal{A})$, an example of which, containing two noninterchange rail stations and one interchange station joining two rail lines, is shown in Figure 2. The node set $\mathcal{N}$ is a union of disjoint subsets $\mathcal{N}_M$ and $\mathcal{N}_B$, which contain rail and bus nodes, respectively. Note that noninterchange stations correspond to one rail node and one bus node (e.g., stations A and C in Figure 2), whereas interchange stations are associated with one bus node and multiple rail nodes (e.g., station B). Each node $i \in \mathcal{N}$ is defined as a $(\rho^+(i), \rho^-(i))$ tuple, where $\rho^+(i)$ indicates the station name and $\rho^-(i)$ corresponds to the index of bus or rail lines. Similarly, the arc set $\mathcal{A}$ consists of disjoint sets $\mathcal{A}_M$, $\mathcal{A}_B$, and $\mathcal{A}_T$, where $\mathcal{A}_M$ contains directional arcs between two rail nodes, and $\mathcal{A}_B$ follows the same convention but refers to bus nodes. Set $\mathcal{A}_T$ contains transfer arcs between either (1) a rail node and its adjoining bus node or (2) two different rail nodes within an interchange.

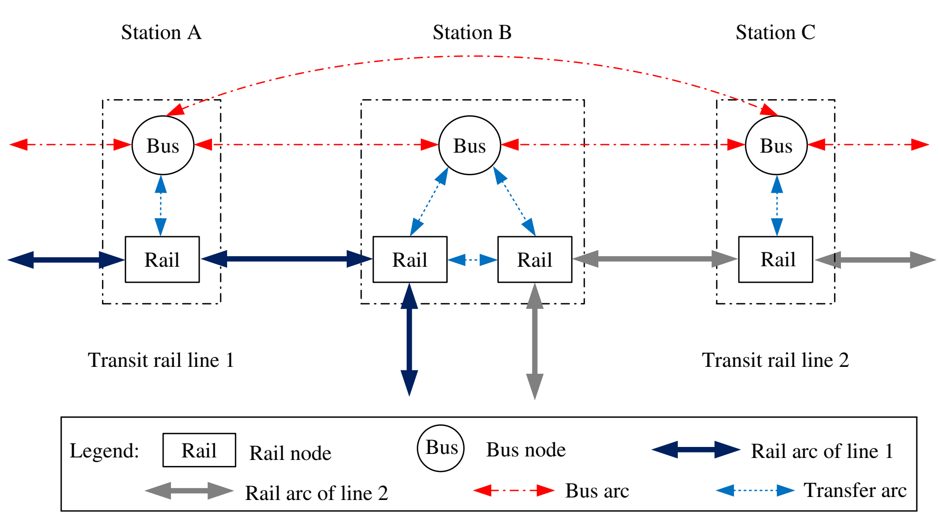

Figure 2 (Color online) Network Representation of Integrated Rail-Bus Service Network

3.3. Finding the Set of Candidate Bus Bridging Routes

For a major disruption scenario, it may be impossible to enumerate all potential bus bridging routes since their number is of the order of $n!$, where n is the number of rail stations that can be reached through some combination of bus services. We therefore develop a column generation algorithm to identify the set of good candidate bus bridging routes, taking into consideration commuter demand. The set of route candidates consists of both intuitive and nonintuitive routes (Figure 3). Without loss of generality, we assume that candidate bus bridging routes are shuttle routes, i.e., begin from a bus node at the end of a disrupted rail segment and terminate at that same bus node. Figure 3 shows some examples of such bus bridging routes starting from station s_1: express bus service route A running in parallel to the disrupted rail section; direct bus service route B linking station s_1 with another neighboring station; and two more complex service routes C and D passing through two stations and returning back to s_1. Some decision makers might want to carry out this step manually. Good intuitive routes such as those shown in Figure 3(a) can indeed be found this way. However, with more complicated scenarios, good routes such as the nonsymmetric ones shown in Figure 3(b) cannot be found easily. These nonintuitive routes can be seen as standard ones, but travel demands justify the skipping of some stops for better overall performance. Although nonintuitive routes might be difficult to remember and inadmissible for routine line planning for everyday operations, that is less of a concern under disruption scenarios.

Before introducing the column generation algorithm to find the candidate routes, we define the following notations:

$\mathcal{R}$: set of candidate bus bridging routes, $\mathcal{R}$ starts with the standard bus bridging route r_0.

$\mathcal{K}$: union of disjoint commuter sets k grouped by their origin and destination.

α_{ij}^r: $\in \{0, 1\}$, 1 if bus link $(i, j) \in \mathcal{A}$ is covered by route r, and 0 otherwise.

o_k: origin rail node of commuter group $k \in \mathcal{K}$.

d_k: destination rail node of commuter group $k \in \mathcal{K}$.

Q_k: number of commuters in group $k \in \mathcal{K}$ during the disrupted period.

c_{ij}: travel time associated with arc $(i, j) \in \mathcal{A}$.

c_k^0: journey time of a single commuter in group $k \in \mathcal{K}$ when there is no disruption.

Note that commuter travel demand is represented by parameters o_k, d_k, and Q_k, which can be estimated from historical data. With implementation of automated fare collection systems in some cities, e.g., Singapore, such information can be easily obtained. Link travel time parameter c_{ij} should be precalibrated for all rail, bus, and transfer arcs. The original travel times for commuters without disruption can be obtained by finding the shortest paths in the original transit rail network.

Our formulation allows the inclusion of practical planning constraints such as the following:

- Routes shall only visit selected stations: a planner may want to consider only routes visiting selected stations, possibly those that are close to the disrupted segment by distance or travel time. Because a functioning urban transit rail network is more efficient than the temporary shuttle service, the latter should primarily be used to deliver stranded commuters to the unaffected rail station nearby.
- Shuttle shall provide loop service originating from selected stations: a planner may want to consider only routes originating from stations with more stranded commuters, such as s_1, s_2, and Int in Figure 1.
- Routes shall be limited by traveling time and number of stops: An efficient route should neither

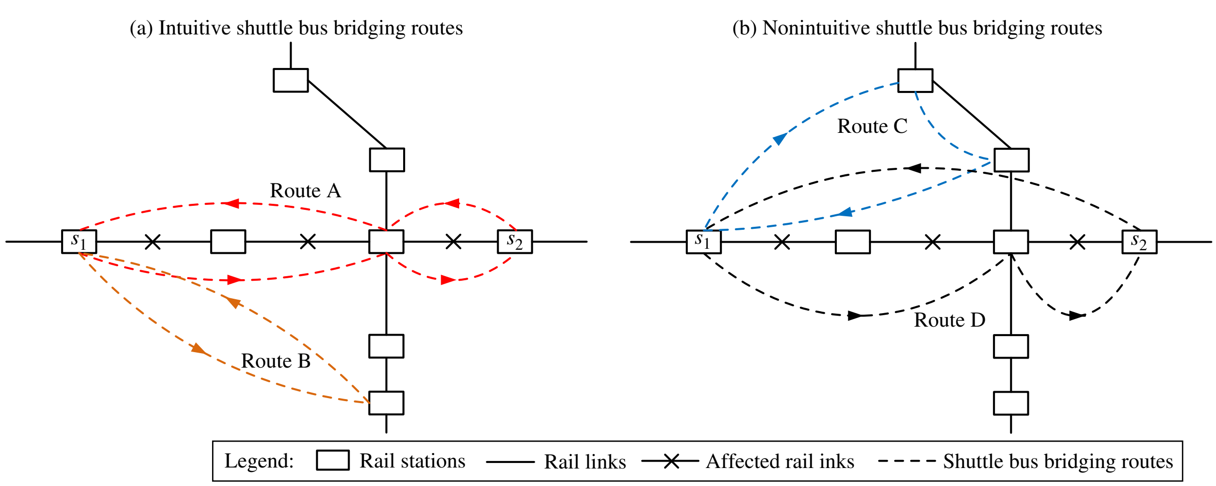

Figure 3 (Color online) Examples of Bus Bridging Routes

take a long time to complete, nor visit too many stations. Note that all of these constraints can be relaxed without any implications to the method developed.

Under the column generation procedure, a restricted master problem (RMP) and a pricing subproblem (PSP) are solved iteratively until no beneficial shuttle routes can be generated. We shall now proceed to describe the RMP and the PSP.

3.3.1. Restricted Master Problem. The RMP employs the following decision variables:

x_{ij}^k: $\in [0,1]$, $\forall (i,j) \in \mathcal{A}$, $\forall k \in \mathcal{K}$, fraction of commuter group k that uses link (i,j).

y_r: $\in \{0,1\}$, $\forall r \in \mathcal{R}$. 1 if bus bridging route r is employed; and 0 otherwise.

Then, the RMP is formulated as follows:

$$\text{[RMP]} \quad \text{minimize} \sum_{k\in\mathcal{K}} Q_k \left(\sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij}^k - c_k^0 \right) \tag{1}$$

subject to

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k = 1 \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{N}_M \,|\, \rho^+(i) = o_k, \tag{2}$$

$$\sum_{i\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k = 0 \quad \forall k \in \mathcal{K}, \quad \forall j \in \mathcal{N}_M \,|\, \rho^+(j) = o_k, \tag{3}$$

$$\sum_{i\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k = 1 \quad \forall k \in \mathcal{K}, \quad \forall j \in \mathcal{N} \,|\, \rho^+(j) = d_k, \tag{4}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k = 0 \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{N} \,|\, \rho^+(i) = d_k, \tag{5}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k - \sum_{j\in\mathcal{N}|(j,i)\in\mathcal{A}} x_{ji}^k = 0 \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{N}_B \,|\, \rho^+(i) = o_k, \tag{6}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k - \sum_{j\in\mathcal{N}|(j,i)\in\mathcal{A}} x_{ji}^k = 0 \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{N} \,|\, \rho^+(i) \notin \{o_k, d_k\}, \tag{7}$$

$$x_{ij}^k - \sum_{r\in\mathcal{R}} \alpha_{ij}^r y_r \le 0 \quad \forall (i,j) \in \mathcal{A}_B, \quad \forall k \in \mathcal{K}, \tag{8}$$

$$0 \le x_{ij}^k \le 1 \quad \forall (i,j) \in \mathcal{A}, \forall k \in \mathcal{K}, \tag{9}$$

$$0 \le y_r \le 1 \quad \forall r \in \mathcal{R}. \tag{10}$$

Objective function (1) minimizes the total increase in journey time (rail and bus riding time plus transfer time) of all commuter groups over the entire disrupted period. Note that the waiting times, when transferring to buses or rail, are not considered in RMP, which focuses on the generation of efficient candidate bus bridging routes. Waiting times will, however, be considered in the subsequent steps (§3.4). Constraints (2)–(7) are flow conservation constraints for each commuter group at the source, sink, and other nodes. Constraints (2) ensure that there is a unit flow starting at the origin node corresponding to the origin rail station for each commuter group. Constraints (4) ensure that, by contrast to the constraints defined with respect to the origin nodes, commuter trips may terminate at any node (bus node or rail node) corresponding to their destination. Constraints (3) and (5) are imposed to prevent any flow going back to the origin and destination nodes, respectively. Flow conservation restrictions at other nodes are defined by Constraints (6) and (7). Constraints (8) guarantee that bus arcs that are not covered by employed bus bridging services cannot be used. Constraints (9) and (10) define the domain of the decision variables. Note that the binary decision variable y_r is relaxed to be continuous in RMP. Normally, for a general column generation model with integer decision variables, solving the linear relaxation of the RMP may miss generating columns important to the integer solution. However, based on the case studies reported in §4, we find that this is not an issue in the problems addressed here. We believe that this is mainly because there is no resource constraint imposed on the decision variable y_r in the RMP. All potentially beneficial routes that are fully or partially used in any iteration of the column generation procedure can be identified and added into the bus route set. Therefore, solving the linear relaxation of the formulation is adequate for generating the set of beneficial candidate bus bridging routes.

3.3.2. Pricing Subproblem. Let $\pi_{ij}^k \le 0, \forall (i,j) \in \mathcal{A}_B$, $\forall k \in \mathcal{K}$ be the dual variables associated with Constraints (8) of the RMP. Therefore, the reduced cost of bus bridging route r is

$$\tilde{c}_r = \sum_{k\in\mathcal{K}} \sum_{(i,j)\in\mathcal{A}_B} \pi_{ij}^k \alpha_{ij}^r. \tag{11}$$

Should the reduced cost of a route $r \notin \mathcal{R}$ be negative, it is a route that can reduce the objective of RMP if added to $\mathcal{R}$. Therefore, we define the PSP to identify bus bridging routes with minimum $\tilde{c}_r$. If the minimum reduced cost is negative, the bus bridging route should be added to set $\mathcal{R}$. We introduce the decision variable z_{ij}, $\forall (i,j) \in \mathcal{A}_2$, which takes a value of 1 if the bus link (i,j) is covered by the new generated bus bridging route in the PSP and 0 otherwise. Therefore, the reduced cost could be updated as

$$\tilde{c}_r = \sum_{k\in\mathcal{K}} \sum_{(i,j)\in\mathcal{A}_B} \pi_{ij}^k z_{ij}. \tag{12}$$

Then, the PSP could be formulated as follows:

$$\text{[PSP]} \quad \text{minimize} \sum_{k\in\mathcal{K}} \sum_{(i,j)\in\mathcal{A}_B} \pi_{ij}^k z_{ij} \tag{13}$$

subject to

$$\sum_{(i,j)\in\mathcal{A}_B} z_{ij} = 1 \quad \rho^+(i) = s_1, \text{ or } s_2, \tag{14}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}_B} z_{ij} - \sum_{j\in\mathcal{N}|(j,i)\in\mathcal{A}_B} z_{ji} = 0, \quad \forall\, i \in \mathcal{N}_B, \tag{15}$$

$$\sum_{(i,j)\in\mathcal{A}_B} c_{ij} z_{ij} \le L^1_{\max}, \tag{16}$$

$$\sum_{(i,j)\in\mathcal{A}_B} z_{ij} \le L^2_{\max}, \tag{17}$$

$$z_{ij} \in \{0, 1\} \quad \forall\,(i,j) \in \mathcal{A}_B. \tag{18}$$

The objective function (13) minimizes the reduced cost of the generated bus bridging route. Constraint (14) ensures that the bus bridging route should start from selected stations, s_1 or s_2 in this case. Constraints (15) ensure that flow conservation is observed at all nodes. In addition, two operational restrictions commonly required by operators are also imposed on the generated bus bridging route: Constraint (16) ensures that the total travel time of the route should not exceed $L^1_{\max}$, and Constraint (17) ensures that the generated bus bridging route consists of no more than $L^2_{\max}$ legs. The above two constraints are introduced for the sake of operational efficiency, since bus bridging routes with a long travel distance or too many legs may not be efficient for disruption response.

Note that the PSP is a shortest path problem with negative link costs that make the commonly used label setting algorithms inapplicable. The negative link costs also introduce a challenge in that subtours may exist in the solution obtained by the PSP integer program. Hence, a subtour elimination procedure is applied such that, if any subtour exists, the following cuts are added before the PSP is resolved and until no subtours exist:

$$\sum_{(i,j)\in S^t} (1 - z_{ij}) \ge 1, \tag{19}$$

where S^t is the set of bus links comprising the subtours in the tth run.

If the optimal objective value of the PSP is negative ($\tilde{c}_{r^*} < 0$), we add the bus bridging route r^* to set $\mathcal{R}$ and rerun the RMP. The optimal solution $\{z_{ij}\}$ can be used to initialize the parameter $\{\alpha^r_{ij}\}$ for the new generated bus bridging route r^*. Otherwise, no beneficial bus bridging route can be found.

3.3.3. Column Generation Procedure. The above RMP and PSP are solved iteratively in the column generation procedure until no bus bridging routes with negative reduced cost can be generated. Note that the column generation procedure should be conducted for both s_1 and s_2. By solving the PSP, we observe that Constraint (17) is tight in most of the cases. In other words, the PSP tends to generate bus bridging routes serving as many stations as possible. Based on this observation, we run the column generation procedure with a set of values $[2, \bar{L}^2_{\max}]$ for the parameter $L^2_{\max}$ aiming at introducing diversity in terms of the number of legs that the generated bus bridging routes serve. Since the column generation procedure is employed to generate bus bridging routes for each departing station and each value of parameter $L^2_{\max}$, identical bus bridging routes may exist in the bus route set $\mathcal{R}$. Therefore, we employ a post-checking step to eliminate any identical routes generated in different runs. The overall column generation procedure for bus bridging route generation is summarized by Algorithm 1.

Algorithm 1 (Column generation procedure of bus bridging route generation**)**

1: **Input**: $G(\mathcal{N}, \mathcal{A})$ and all parameters
2: **Output**: a set of candidate bus bridging routes $\mathcal{R}$
3: Initialize $\mathcal{R} = \{r_0\}$, $\boldsymbol{\pi} \leftarrow \mathbf{0}$;
4: **For** s_1 and s_2
5:     **For** $L^2_{\max} = 2$ to $\bar{L}^2_{\max}$
6:         solve the restricted master problem; [RMP]
7:         update dual variables $\boldsymbol{\pi}$;
8:         solve the pricing subproblem **[PSP]**;
9:         **If** subtour exists
10:             add new constraint (19);
11:             **go to** Line 8;
12:         **End if**
13:         **If** the objective function value of **[PSP]** is negative
14:             obtain the new bus bridging route r^*;
15:             $\mathcal{R} \leftarrow \mathcal{R} \cup \{r^*\}$;
16:         **Else**
17:             **break**;
18:         **End if**
19:     **End for**
20: **End for**
21: Eliminate identical bus bridging routes in $\mathcal{R}$;

3.4. Route Selection and Bus Deployment

With the set of candidate bus bridging routes $\mathcal{R}$, designing the temporary bus service network includes (1) selecting some bus bridging routes from set $\mathcal{R}$, and (2) allocating bus resources to serve the selected routes. Given a limited number of buses, numerous trade-offs have to be addressed, including more routes and less buses to serve each route on average, and having more buses to serve a route and hence less on other routes.

To best address these trade-offs, a path-based multicommodity network flow model is developed. In the model, we specifically introduce the time dimension and the detailed headway of buses running on selected routes in order to model commuter waiting time when transferring from rail to bus. The objective is to minimize the total travel time (consisting of riding time on bus and rail, transfer time, and waiting time) of all commuters. Note that the waiting time suffered when commuters transfer back to rail is not considered since train headways are short typically and commuters can, in most cases, get on the first train that arrives.

To model the time dimension, we develop a time-space network in the following way. For each commuter group k, the travel demand Q_k for the entire disrupted period is discretized into $\bar{u}$ time periods each of which corresponds to one train and is associated with demand $q_{(k,u)}$ satisfying $\sum_u q_{(k,u)} = Q_k$. For each bus bridging route r, we introduce a set of service slots $\mathcal{B}_r := \{(r, v),\ \forall\, v = 1, 2, \ldots, \bar{v}\}$, where each service slot represents a potential bus service in a particular time point and $\bar{v}$ is the total number of bus time slots during the disrupted period. Let $\mathcal{B} = \bigcup_r \mathcal{B}_r$. Figure 4 shows an illustrative example with one bus bridging route and one commuter group. As can be seen, the bus deployment plan on the route is to run buses on every other service slot, as indicated by the shaded nodes. The travel itinerary is shown by the bold lines. Consider, for example, the commuters leaving at the second time period. They first take a train to the transfer station and walk to the bus stop, and then wait for half of the bus headway and board the bus. Finally, they transfer back to rail and continue their journeys to their destination. Note that the bus arcs are defined with the bus capacities, and commuters will have to wait for the next bus when the first is full. With the constructed time-space

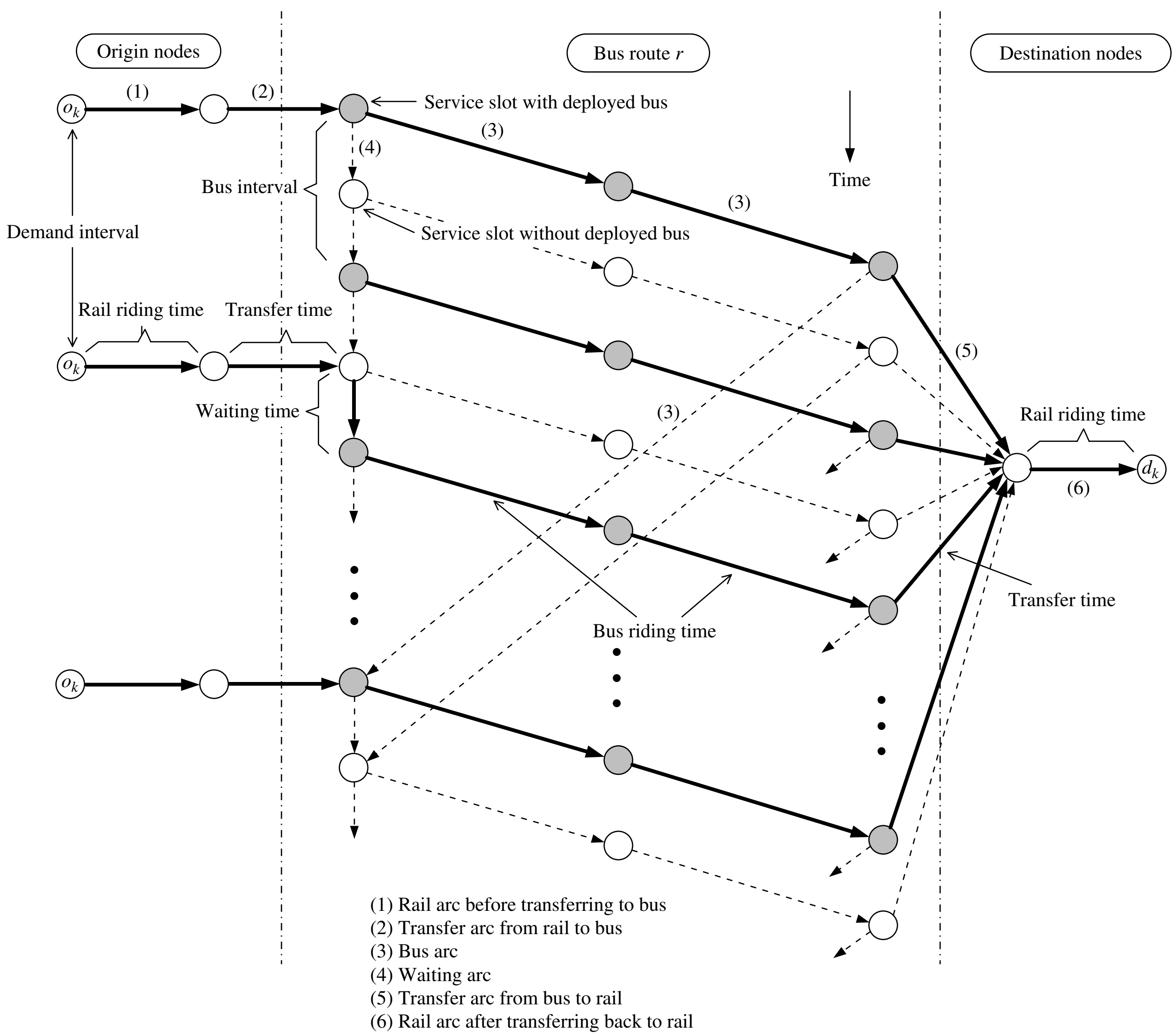

Figure 4 **An Illustrative Time-Space Network with One Bus Bridging Route and One Commuter Group**

network, we are able to capture reasonably accurately the waiting time of all commuters.

We further define

$$\mathcal{P} := \{(r, h),\ \forall\, r \in \mathcal{R},\ \forall\, h_r^{\min} \le h \le h_r^{\max}\}$$

as the set of bus deployment plans where each plan (r, h) is characterized by route index r and the bus headway h, whereas $h_r^{\min}$ and $h_r^{\max}$ denote the minimum and maximum allowed headway for the bus bridging route, respectively. Let binary coefficient $\beta_{((r,h),(r,v))}$ be 1 if the bus deployment plan $(r, h) \in \mathcal{P}$ covers bus service slot $(r, v) \in \mathcal{B}$, and 0 otherwise. Let $n_{(r,h)}$ be the number of buses that bus plan (r, h) requires in order to maintain the service's headway.

The path traveling time of commuter group k from their origin to destination consists of two terms: (1) journey time c_{kr} by taking bus bridging route r that is spent on riding rail and bus as well as transferring from rail to bus, and (2) waiting time $w_{((k,u),(r,v))}$ of commuter group (k, u) by taking bus service slot (r, v). We have

$$w_{((k,u),(r,v))} = \max\{0, t_{(k,(r,v))} - \bar{t}_{(k,u)} - \tilde{t}_{(k,u)}\}, \tag{20}$$

where $t_{(k,(r,v))}$ is the time when bus service slot (r, v) arrives at the transfer station of commuter group k, $\bar{t}_{(k,u)}$ is the time when commuter group (k, u) boards the origin station o_k, and $\tilde{t}_{(k,u)}$ is the travel time (rail riding time plus transfer time) of commuter group (k, u) from origin station to the transferring bus stop.

The following decision variables are defined:

- $\delta_{(r,h)} \in \{0, 1\}$: $\forall\, (r, h) \in P_r,\ \forall\, r \in \mathcal{R}$. 1 if bus deployment plan (r, h) is employed; and 0 otherwise.
- $\lambda_{((k,u),(r,v))} \ge 0$: the number of commuters in group (k, u) who take bus service slot $(r, v) \in \mathcal{B}_r$.

To reduce the number of decision variables $\{\lambda_{((k,u),(r,v))}\}$, we introduce set Ω and only define the decision variable over the set

$$\Omega := \{((k, u), (r, v)) \mid (k, u), (r, v) \in B_r \mid t_{(k,(r,v))} - \bar{t}_{(k,u)} - \tilde{t}_{(k,u)} \ge 0 \text{ and } w_{((k,u),(r,v))} \le \bar{w}\}. \tag{21}$$

Set Ω excludes those combinations of $((k, u), (r, v))$ for which transfer is not possible (i.e., bus leaves before the arrival of commuters) or when the waiting time exceeds a limit $\bar{w}$. Considering the case in which travel demand may exceed the bus service supply, we further define decision variable $\lambda_{(k,u)}$ as the number of commuters in group (k, u) who are unable to get on any bus by the waiting time limit $\bar{w}$.

Then, the route selection and bus deployment subproblem could be formulated as follows:

[SDP]

$$\text{minimize} \sum_{((k,u),(r,v))\in\Omega} (c_{kr} + w_{((k,u),(r,v))} - c_k^0)\lambda_{((k,u),(r,v))} + \theta \sum_{(k,u)} \lambda_{(k,u)} \tag{22}$$

subject to

$$\sum_{(r,v)\in\mathcal{B}} \lambda_{((k,u),(r,v))} + \lambda_{(k,u)} = q_{(k,u)}, \quad \forall\, (k, u), \tag{23}$$

$$\sum_{(k,u)} \gamma_{(k,(r,l))}\lambda_{((k,u),(r,v))} \le \sum_{h|(r,h)\in\mathcal{P}} \beta_{((r,h),(r,v))}\delta_{(r,h)}Q_0, \quad \forall\, (r, v) \in \mathcal{B}_r,\ \forall\, (r, l) \in L_r,\ \forall\, r \in \mathcal{R}, \tag{24}$$

$$\sum_{h|(r,h)\in\mathcal{P}} \delta_{(r,h)} \le 1, \quad \forall\, r \in \mathcal{R}, \tag{25}$$

$$\sum_{h|(r,h)\in\mathcal{P}} \delta_{(r,h)} = 1, \quad r = 0, \tag{26}$$

$$\sum_{(r,h)\in\mathcal{P}} n_{(r,h)}\delta_{(r,h)} \le F_1^{\max}, \tag{27}$$

$$\sum_{(r,h)\in\mathcal{P}|r\ne0} \mu_{r1}\delta_{(r,h)} \le F_2^{\max}, \tag{28}$$

$$\sum_{(r,h)\in\mathcal{P}|r\ne0} \mu_{r2}\delta_{(r,h)} \le F_2^{\max}, \tag{29}$$

$$\delta_{(r,h)} \in \{0, 1\}, \quad \forall\, (r, h) \in \mathcal{P}, \tag{30}$$

$$\lambda_{((k,u),(r,v))} \ge 0, \quad \forall\, ((k, u), (r, v)) \in \Omega, \tag{31}$$

$$\lambda_{(k,u)} \ge 0, \quad \forall\, (k, u). \tag{32}$$

The objective function (22) of the route selection and bus deployment model [SDP] is to minimize (1) the total increase in travel time for all commuters taking shuttle buses, and (2) the number of commuters who cannot board a bus and weighted by the time parameter θ. The commuter flow conservation constraint (23) ensures that the total number of commuters taking bus bridging services and those that cannot board by the time limit equals the travel demand. Constraint (24) guarantees that, on each route leg $l \in L_r$ of bus service slot (r, v), the number of commuters traveling on the leg observes the bus capacity restriction Q_0 if the service slot is covered by a certain bus plan $(r, h) \in B_r$, where L_r denotes the set of legs on bus bridging route r and binary parameter $\gamma_{(k,(r,l))}$ takes value 1 only if leg l is used by commuter group k when they take bus bridging route r. Constraint (25) imposes the restriction that at most one bus deployment plan can be adopted on each route, and Constraint (26) ensures that the standard bus bridging route must be introduced. The total number of buses deployed on the selected bus bridging routes should not exceed the number of available buses $F_1^{\max}$, as imposed by the bus resource capacity constraint (27). Operational constraints (28) and (29) further restrict the additional number of employed bus bridging routes starting from station s_1 or s_2 to be less than or equal to a limit $F_2^{\max}$, where parameters μ_{r1} and μ_{r2} are set to 1 if bus bridging route r passes by stations s_1 and s_2, respectively. These operational constraints are required because (1) there may be limited bus parking bays beside the stations, and (2) commuters may be confused by

Table 1 **Parameters of Disruption Cases**

	Case 1 (minor)	Case 2 (major)
Disruption		
No. of stations	1	7
No. of links	4	14
Period	10 A.M.~11 A.M.	10 A.M.~11 A.M.
Network		
$\lvert\mathcal{N}_M\rvert$	122	122
$\lvert\mathcal{N}_B\rvert$	11	25
$\lvert\mathcal{A}_M\rvert$	232	222
$\lvert\mathcal{A}_B\rvert$	110	600
$\lvert\mathcal{A}_T\rvert$	58	88
Commuter		
$\lvert\mathcal{K}\rvert$	26	24
Train headway	5 min	5 min
Demand per interval	906	1,466

excessive route choices. The domains of the decision variables $\delta_{(r,h)}$, $\lambda_{((k,u),(r,v))}$, and $\lambda_{(k,u)}$ are defined by Constraints (30)–(32).

4. Case Studies

In this section we test the performance of the proposed approach by studying two disruption cases based on a major city's transit rail network. In the second, more severe case, about 125,000 commuters were disrupted, seven trains stalled between stations for a few hours and considerable political fallout ensued.

The mixed-integer linear programs associated with the column generation procedure and time-space network model were solved with CPLEX 12.1 running on a 3.4 GHz Intel Core i7 PC with 16 GB RAM.

4.1. Disruption Cases

Table 1 shows the details of the two disruption cases: number of disrupted stations and links, disruption period, size of the graph representing the integrated rail-bus network, and size of commuter travel demand. The original rail network consists of four rail lines and three *light rapid transit* (LRT) lines with 109 stations and 236 directed links. Case 1 concerns a relatively minor disruption event in which only one rail station (A2 in Figure 5) and the four directed links that connect it to the two neighboring stations are disrupted. Case 2 involves a major disruption in which seven stations along a rail line and the links that connect them are disrupted (see Figure 6 for partial network representation). Note that the number of bus nodes $\lvert\mathcal{N}_B\rvert$ is less than the number of transit rail stations (109) since we only define bus nodes for the stations that are located near the disrupted region. Choosing a subset of stations to plan the temporary services is also a realistic operational consideration. Our methodology can still be applied if all of the stations are included. Travel demand in both cases are derived from historical fare card data.

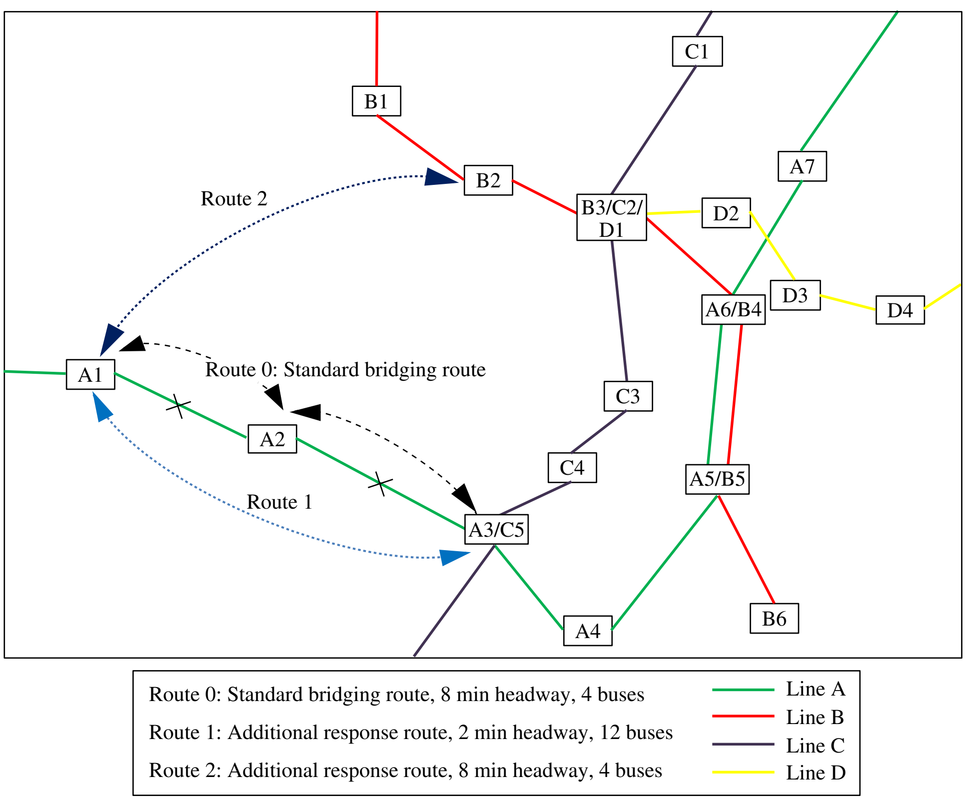

Figure 5 **(Color online) Optimal Bus Bridging Services of Case 1**

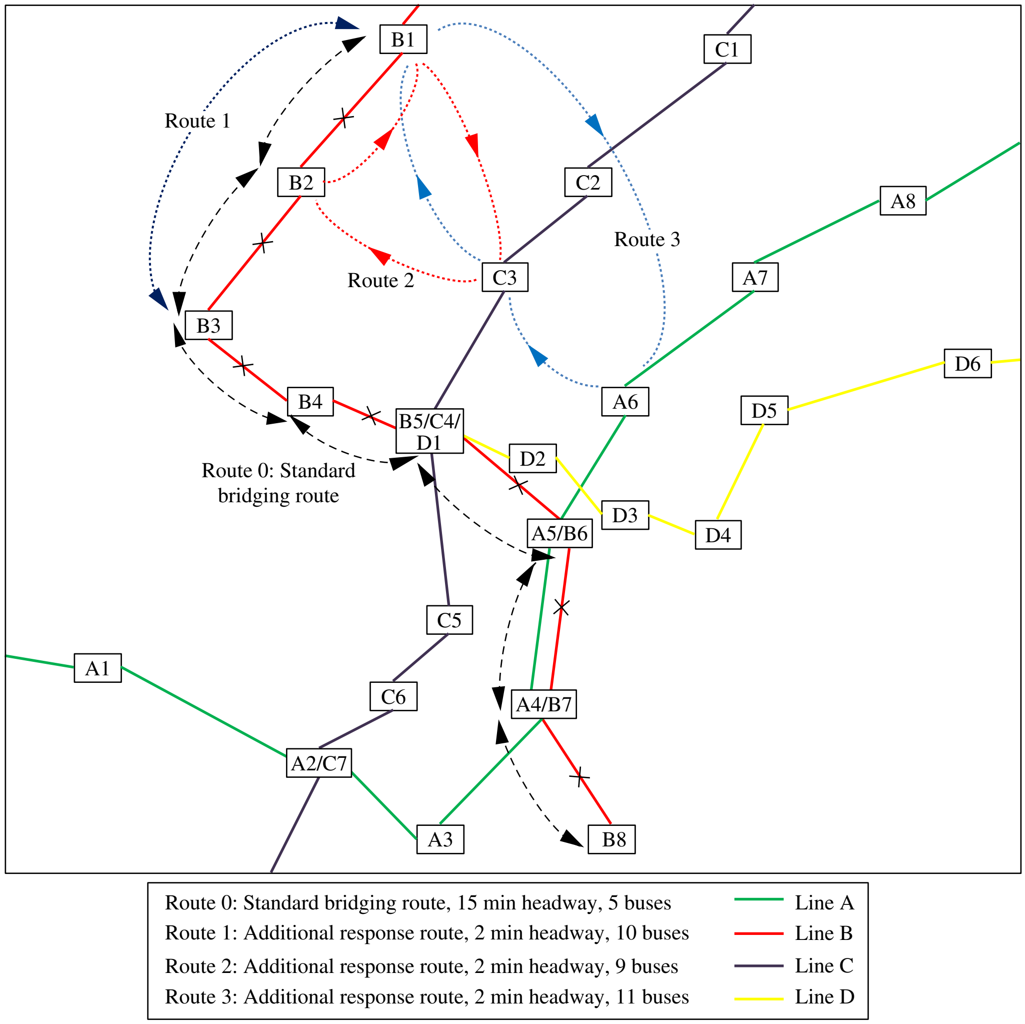

Figure 6 (Color online) Optimal Bus Bridging Services of Case 2

The parameters related to the operational restrictions of bus bridging routes are set as follows:

- Maximum travel time of bus bridging routes: $L^1_{\max} = 35$ min.
- Upper limit of legs of generated bus bridging routes: $L^2_{\max} = 3$.
- Bus capacity: $Q_0 = 140$.
- Minimum and maximum headways of bus deployment plans: $h_r^{\min} = 1$ min, $h_r^{\max} = 15$ min.
- Limit on commuter waiting times: $\bar{w} = 30$ min.
- Penalty for commuters who cannot board a bus by the time limit: $\theta = 50$ min.

4.2. Effectiveness of Disruption Response Planning with a Fixed Bus Fleet Size

The standard response of transit agencies to such disruptions is to operate bus bridging services between disrupted stations that run entirely parallel to the disrupted sections of the rail network. Pender et al. (2012) performed an extensive survey of 63 transit agencies and report that 86% relied on this type of bus bridging service to respond to rail line blockages. In the case of the transit agency involved in our two case studies, provision of bus-bridging services parallel to the disrupted lines is the only response considered. To assess the effectiveness of our proposed response planning approach, we compare the characteristics of our solutions against such standard bus-bridging service responses. The size of the bus fleet is assumed to be fixed ($F_1^{\max} = 20$ for case 1 and $F_1^{\max} = 35$ for case 2). Figure 7 summarizes the sensitivity of the response's effectiveness (delays to commuters, percentage not served) to the number of additional bus bridging routes, with 0 denoting the solution that consists solely of standard bus bridging services. The introduction of additional bus bridging routes, even when keeping the total number of buses constant, clearly yields a significant decrease in the average travel delay of all commuters: from 28.7 to 20.1 minutes (or 30%) for case 1, and from 43.2 to 18.1 minutes (58%) for case 2. Similarly, the percentage of

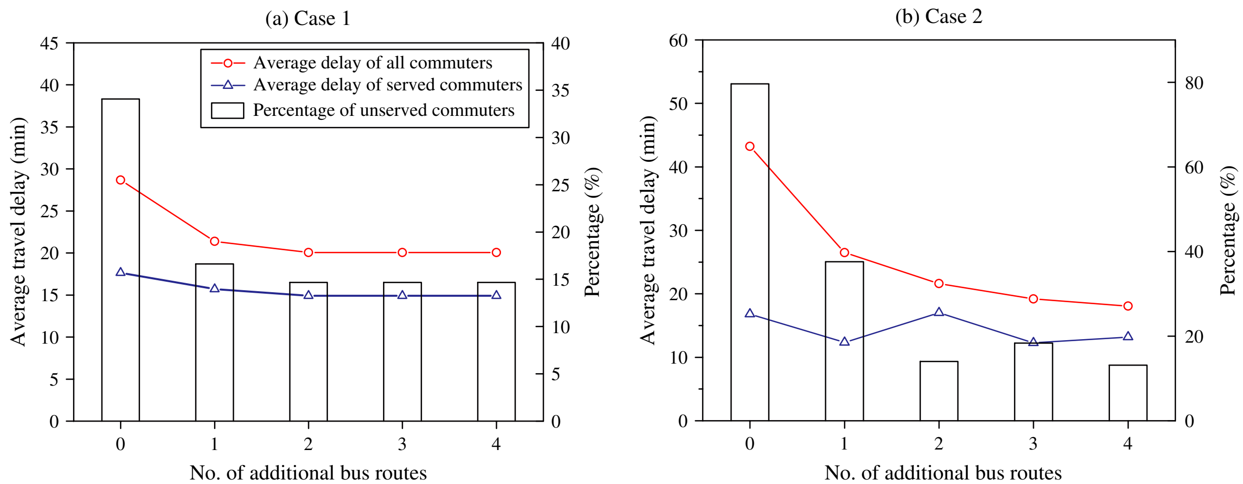

Figure 7 (Color online) Sensitivity Analysis of Number of Additional Bus Bridging Service Routes

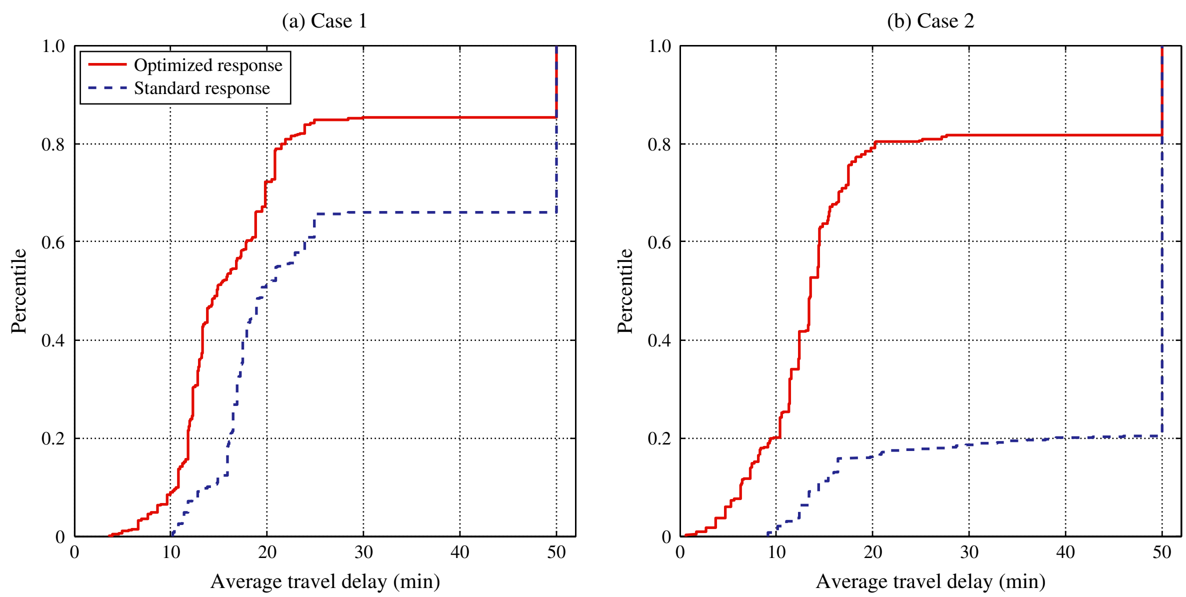

Figure 8 (Color online) Cumulative Distribution of Travel Delay With and Without Additional Bus Bridging Services

commuters who cannot board shuttle buses within the time limit $\bar{w}$ also decreases significantly: from 34.1% to 14.7% for case 1 and from 79.6% to 13.2% for case 2. Note, however, that the average travel delay of the served commuters is lower than 20 minutes for all scenarios and does not vary significantly as more bus service routes are introduced. Therefore, the improvements in the overall travel delay can be mostly attributed to the reduction of the number of nonserved commuters. On the basis of Figure 7, we set the upper limit on the number of additional bus bridging routes to three ($F^2_{\max} = 3$) in the numerical experiments reported next, since the contribution from more than three bus bridging routes is marginal.

We now investigate the distribution of travel delay over all of the commuters. We compare the distribution under the standard bridging response with the distribution resulting from our proposed response approach. This is shown through the cumulative distribution functions depicted in Figure 8. As can be seen, the introduction of three additional bus bridging routes greatly improves the distribution of travel delay. For case 1, the overall travel delay is reduced from 2.93×10^5 to 2.05×10^5 passenger minutes. In addition, about 51% of the commuters suffer fewer than 15 minutes of travel delay under the optimized response situation, whereas only 12% have fewer than 15 minutes of delay under the standard response. The improvement for the major disruption case (case 2) is even more significant. The overall travel delay decreases from 7.03×10^5 to 3.12×10^5 passenger minutes, and about 16% of the commuters

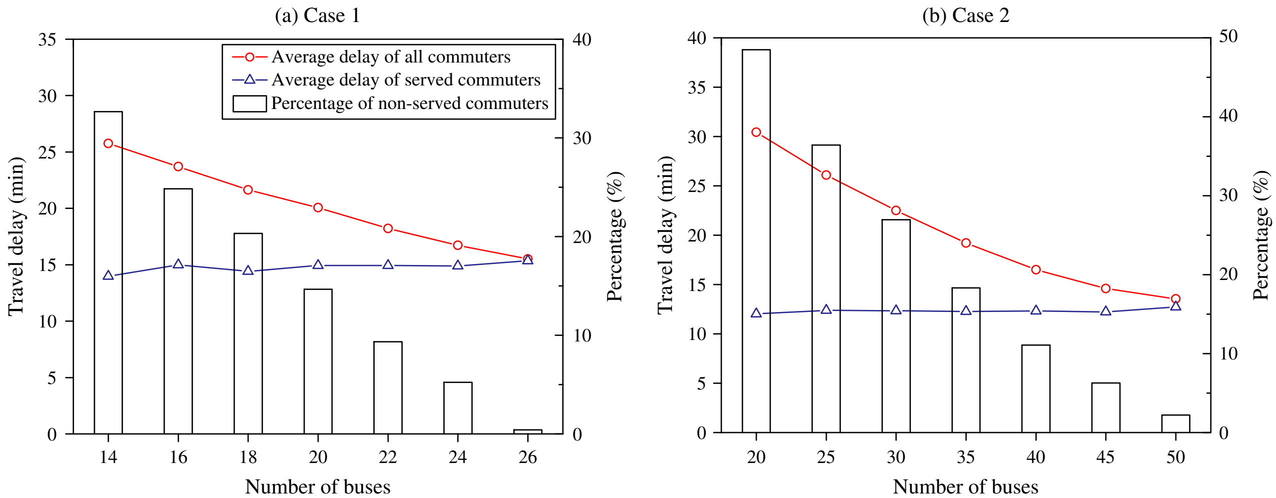

Figure 9 (Color online) Results of Performance Measures Under Different Sizes of Bus Fleet

suffer fewer than 20 minutes of travel delay under the standard response condition, whereas running additional bus bridging services improves this percentage to about 79%.

The optimal bus bridging routes for the two test cases are shown in Figures 5 and 6, respectively. For case 1, the selected additional bus bridging services include an express service (route 1) linking stations A1 and A3/C5, and a second express service (route 2) providing a shortcut between lines A and B. For case 2, the standard bus bridging route as well as three additional bus bridging routes are employed: route 1 connecting station B1 with the most popular station B3 (with about 30% of commuters traveling directly between the two stations); route 2 providing interline service between lines B and C by connecting B1 with the nearest station C3 on Line C; and route 3 linking lines A and C. Note that routes 2 and 3 belong to the class of "nonintuitive bus bridging routes" illustrated in Figure 3. This demonstrates the significance of employing nonintuitive bus bridging routes, and the necessity of the column generation approach in finding such routes and the network flow approach to evaluate all of the potentially good routes concurrently. An insight that can be drawn from the two cases is that promising additional bus bridging routes that should be considered are (a) those that provide direct connections between stations with heavy travel demand, and (b) those that quickly transfer commuters from disrupted lines to other nondisrupted lines.

4.3. Determining Bus Fleet Size

Another type of sensitivity analysis explores the trade-off between commuter travel delay and the number of buses to be deployed. Figure 9 reports the delay-related performance achieved with various bus fleet sizes for cases 1 and 2. As can be seen, increasing the number of buses reduces almost linearly the average travel delay of all commuters, and also generally reduces the percentage of commuters who cannot obtain bus bridging service within the 30-minute time limit. However, the average travel delay of served commuters does not exhibit a significant correlation with the bus fleet size. More buses can simply serve more commuters, thus reducing the average travel delay of all commuters; however, once a commuter is served by a bus, the commuter's average travel delay will be more or less the same as that of the other commuters who had been served with a smaller fleet size. Using the results of this sensitivity analysis, operators can determine the bus fleet size required to achieve a certain level of response effectiveness, such as an overall average travel delay or a desired percentage of served commuters.

4.4. Sensitivity to Travel Demand

To investigate the impacts of the size of travel demand on performance, we test the disruption planning model with the demand levels present in three other intervals during the day (8 A.M.∼9 A.M., 9 A.M.∼10 A.M., 11 A.M.∼12 P.M.), in addition to the base scenario of 10 A.M.∼11 A.M. Figures 10 and 11 report the results for the two disruption cases. Overall, given a fixed number of buses (20 for case 1 and 35 for case 2), the average waiting time of all commuters increases with the number of commuters. However, the average waiting time of commuters who can board buses within the time limit (30 minutes) does not vary significantly with the travel demand, as it is less than 20 minutes for all test scenarios. It can be seen that the percentage of nonserved commuters is the performance measure most sensitive to travel demand.

4.5. Computational Performance

Table 2 shows the statistics concerning the computational performance of the solution approach. As can

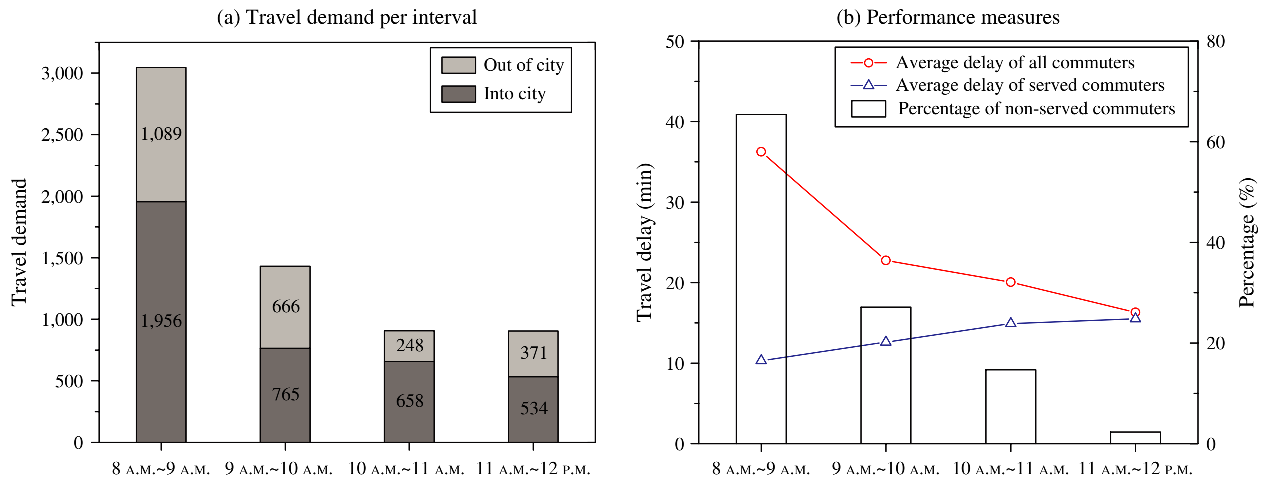

Figure 10 (Color online) Results of Performance Measures Under Different Travel Demand for Case 1

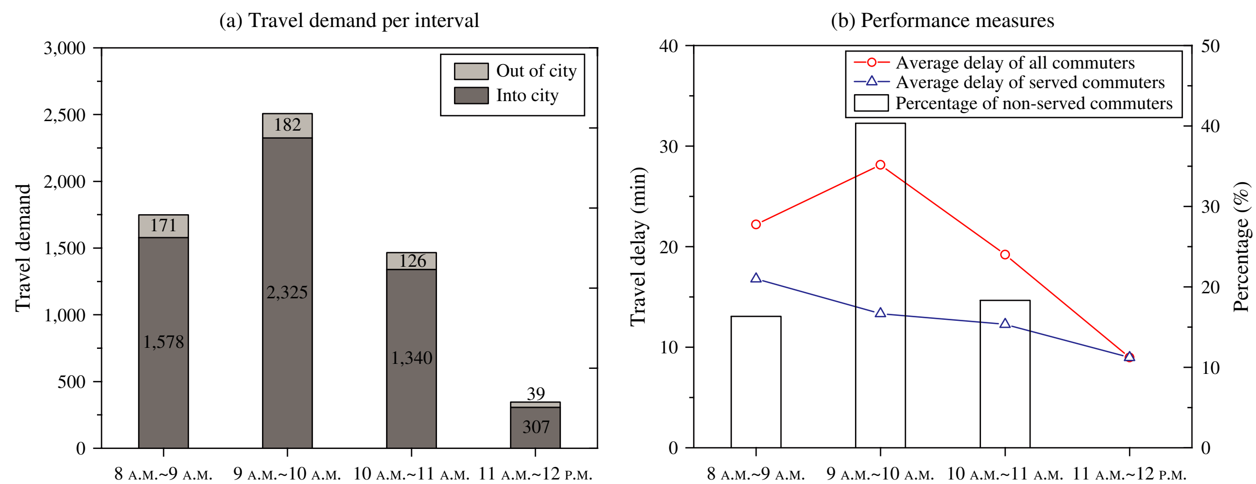

Figure 11 (Color online) Results of Performance Measures Under Different Travel Demand for Case 2

be seen, both test cases can be solved very efficiently (within a few minutes). The column generation procedure successfully identifies 25 (out of 190) and 56 (out of 1,128) candidate bus bridging routes for cases 1 and 2, respectively, demonstrating its superiority to enumeration methods for generating the bus route set. Most of the computational effort, reflected in the distribution of running times, is devoted to solving the bus deployment and route selection model [SDP]. Another observation is that the number of generated bus bridging routes and the running time are sensitive to the size of the set of bus node $\mathcal{N}_B$ and set of bus arcs $\mathcal{A}_B$. This confirms the necessity of defining bus nodes for only the rail stations that are located near the disrupted region instead of for all rail stations.

Table 2 Computational Results of the Solution Approach

	Case 1 (minor)	Case 2 (major)
Running time (sec):		
Column generation for generating bus routes	0.4	3.5
Parameter initialization for route selection	2.6	7.4
Bus deployment and route selection	17.7	232.6
Total	20.7	243.5
No. of generated bus routes:	25	56

5. Conclusion

This paper presented an optimization-based approach to the design of a network of temporary bus services in response to local disruptions of urban transit rail systems. Although the approach is motivated by unexpected disruptions that require an immediate response capability, it can be applied equally well to routine and scheduled suspensions of service, which are planned well in advance to allow for preventive maintenance, repairs, and upgrades of transit networks.

The proposed approach consists of three fundamental steps: a column generation procedure to dynamically generate demand-responsive candidate bus bridging routes; a path-based multicommodity network flow model to identify the most effective combination of these candidate bus bridging routes; and another optimization-based procedure to determine simultaneously the optimal allocation of available vehicle resources among the selected routes and the corresponding headways.

The results obtained for two representative case studies suggest the following practical observations: (1) the proposed approach can be carried out in a computationally efficient way, even in the context of unexpected disruptions that require a quick response plan; (2) the addition of nonintuitive bus bridging routes to the standard bridging services on which responses have typically relied can lead to significant reductions in the expected travel time delay of system users; (3) the distribution of travel delay can also be made much more equitable through an optimized response; and (4) realistic operational constraints, such as limits on the number of stops on a route or on the number of bus services that may visit a station, can be handled. Moreover, the proposed approach is demand responsive, i.e., sensitive to the time-of-day travel requirements of the system's users.

Finally, a promising direction of future research is to integrate into the network considered by our models, other major transportation nodes (e.g., bus interchanges) and services (bus trunk lines) in addition to the urban transit rail network. Given this expanded service network that includes both transit rail and trunk bus services, commuters could be directed with more flexibility during disruptions. This offers the possibility of further improving the performance of response measures.

Acknowledgments

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