Chapter 73
Practice-Oriented Solution Methods for the Integrated Dial-a-Ride Problem

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Abstract The concept of demand-responsive transport is seen as a promising solution for the necessary changes in the mobility sector in the context of sustainable urban transport. In this work, a model for the Dial-a-Ride problem with public transport integration is presented. The model aims to minimise the sum of waiting times for all passengers, focusing on suburban stations with realistic route data. The model is optimally solvable for small instances, but a heuristic approach based on simulated annealing is proposed for larger instances. The research results suggest that the model is well suited to address the problem discussed and the heuristic approach provides reliable results in a reasonable computational time and is therefore suitable for practical applications.

Keywords Integrated Dial-a-Ride problem · Simulated annealing · Public transport integration · Sustainable urban transport

Introduction

Dial-a-Ride services (DARS) or ride pooling services are an important concept in modern mobility services, as shared rides allow to reduce the traffic and the number of required vehicles (resources) while improving the access to mobility services especially in rural and suburban areas. In the literature, the underlying problem is known as the Dial-a-Ride Problem (DARP; cf. review articles [2, 4, 6]). In the master's thesis [10], a comparison was conducted between the scientific literature and practical applications of DARS. The study found that only a limited number of papers (e.g. [3, 8, 11]) focus on the application aspects of ride pooling services; in particular, the integration of ride-pooling services into public transport has rarely been investigated in the scientific literature (e.g. [7]), although it is one of the main

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characteristics of practical applications. Therefore, the focus of this article is on a DARS as a feeder to a public transport network.

Firstly, a formal problem description and a mixed-integer programming formulation are presented (Sect. 73.2). Afterwards, a simulated annealing (SA)-based heuristic is outlined (Sect. 73.3), and a computational study is presented (Sect. 73.4). The paper closes with a conclusion (Sect. 73.5).

Problem Description and Mathematical Model

The DARP is the problem of assigning a given set of n customer requests to a given number of vehicles K with capacity Q and determining the vehicles' routes ensuring that the pickup and delivery locations of each customer are visited in the correct sequence by a single vehicle. The set $\mathcal{I} = \{0, 1, \ldots, 2n, 2n+1\}$ contains all locations, with $i = 0$ and $i = 2n+1$ representing the depot and locations in $\mathcal{P} = \{1, \ldots, n\}$ correspond to the pickup locations of the n requests, whereas locations in $\mathcal{D} = \{n+1, \ldots, 2n\}$ represent delivery locations ($\mathcal{P}, \mathcal{D} \subset \mathcal{I}$). For each request i, the pickup location indexed by i corresponds to the delivery location $j = n + i$. For the integrated DARP (I-DARP), one of the two customer locations (pickup or delivery) is fixed, since one is a location at which the transfer to conventional public transport takes place, i.e., it represents a train station. Requested trips must either start or end at the station, where customers transfer to or from trains $z \in \mathcal{Z}$ of several lines/destinations (c_z^{tl}), taking into account departure/arrival times. For the I-DARP, there are two customer types, namely customers collected in the service area going to the station ($c_i^{\mathrm{t}} = 0$) and customers travelling vice versa ($c_i^{\mathrm{t}} = 1$). Requests $i \in \mathcal{P}$ contain the trip type c_i^{t}, the stated pickup or delivery location (coordinates), the desired connection line at the station c_i^{l}, the number of persons to be transported q_i, and the desired pickup/delivery time at the station l_i.

The mixed-integer programming formulation is based on the model presented in [9]. Thereby, for set S, $S \subseteq \mathcal{I}$ holds and $\mathcal{S}$ is the set of all sets S for which $0 \in S$ holds and that at least one delivery location $n+i$ of a customer i is in S without the pickup location i being in S, i.e., $\mathcal{S} = \{S : 0 \in S \wedge \exists\, i : (i \notin S \wedge n+i \in S)\}$.

$$\min \quad \Psi = \sum_{i \in \mathcal{P} \cup \mathcal{D}} t_i^w \cdot |q_i| \tag{73.1}$$

s.t.

$$\sum_{j \in \mathcal{I} \setminus \{0\}} X_{i,j} = 1 \qquad \forall i \in \mathcal{P} \tag{73.2}$$

$$\sum_{j \in \mathcal{I} \setminus \{0\}} X_{0,j} = \sum_{i \in \mathcal{I} \setminus \{2n+1\}} X_{i,2n+1} = K \tag{73.3}$$

$$\sum_{j\in\mathcal{I}} X_{j,i} - \sum_{j\in\mathcal{I}} X_{i,j} = 0 \qquad \forall i \in \mathcal{P} \cup \mathcal{D} \tag{73.4}$$

$$(B_i + t_{i,j}) \cdot X_{i,j} \leq B_j \qquad \forall i, j \in \mathcal{I} \tag{73.5}$$

$$(Q_i + q_j) \cdot X_{i,j} \leq Q_j \qquad \forall i, j \in \mathcal{I} \tag{73.6}$$

$$B_{n+i} \geq B_i \qquad \forall i \in \mathcal{P} \tag{73.7}$$

$$t_i^w \leq t^{w,\max} \qquad \forall i \in \mathcal{P} \cup \mathcal{D} \tag{73.8}$$

$$B_{n+i} - B_i \leq t_{i,n+i} + t_{\max}^{r+} \qquad \forall i \in \mathcal{P} \tag{73.9}$$

$$\sum_{z\in\mathcal{Z}} d_z \cdot Y_{i,z} \geq B_i + t_{\mathrm{c}} \qquad \forall i \in \mathcal{D}|c_i^t = 0 \tag{73.10}$$

$$\sum_{z\in\mathcal{Z}} Y_{i,z} = 1 \qquad \forall i \in \mathcal{D}|c_i^t = 0 \tag{73.11}$$

$$\sum_{z\in\mathcal{Z}} c_z^{\mathrm{tl}} \cdot Y_{i,z} = c_i^{\mathrm{l}} \qquad \forall i \in \mathcal{D}|c_i^t = 0 \tag{73.12}$$

$$\sum_{i,j\in S} X_{i,j} \leq |S| - 2 \qquad \forall S \in \mathcal{S} \tag{73.13}$$

$$B_{n+i} - B_i - t_{i,n+i} \leq t_i^w \qquad \forall i \in \mathcal{P}|c_i^t = 0 \tag{73.14}$$

$$B_i - l_i \leq t_i^w \qquad \forall i \in \mathcal{P}|c_i^t = 1 \tag{73.15}$$

$$\sum_{z\in\mathcal{Z}} d_z \cdot Y_{i,z} - l_i - t_{\mathrm{c}} \leq t_i^w \qquad \forall i \in \mathcal{D}|c_i^t = 0 \tag{73.16}$$

$$l_i - B_i \leq t_i^w \qquad \forall i \in \mathcal{D}|c_i^t = 0 \tag{73.17}$$

$$B_i - B_{i-n} - t_{i-n,i} \leq t_i^w \qquad \forall i \in \mathcal{D}|c_i^t = 1 \tag{73.18}$$

$$B_i \geq l_i \qquad \forall i \in \mathcal{P}|c_i^t = 1 \tag{73.19}$$

$$t_i^w,\ B_i,\ Q_i \geq 0 \qquad \forall i \in \mathcal{I} \tag{73.20}$$

$$X_{i,j} \in \{0;\ 1\} \qquad \forall i, j \in \mathcal{I} \tag{73.21}$$

$$Y_{i,z} \in \{0;\ 1\} \qquad \forall i \in \mathcal{I},\ z \in \mathcal{Z} \tag{73.22}$$

The objective function (73.1) is set as the sum Ψ of waiting times t_i^w experienced by q_i passengers of request i which is to be minimised, representing a key quality aspect of public transport integrated DARS. Waiting times include time spent waiting for transport as well as additional travel time due to detours. It must hold that each customer $i \in \mathcal{P}$ is assigned to the route of one vehicle, since trip requests may not be rejected (73.2). The binary variable $X_{i,j}$ is assigned a value of 1 if customer location j is visited directly after location i. Each of the K routes must start and end at the depot (73.3), which is denoted by $i = 0$ and $j = 2n + 1$, respectively. The restriction block (73.4) ensures that each customer location $i \in \mathcal{P} \cup \mathcal{D}$ has a predecessor and a successor, that is, the location is both visited and left. With restrictions (73.5), the arrival time at customer location j is set to the arrival time at the predecessor node B_i plus the travel time $t_{i,j}$ from i to j, if this relation is chosen, i.e., $X_{i,j} = 1$. Analogously, the number of people in the vehicle Q_j at a location is set by restrictions

(73.6) depending on the number of people boarding/alighting at j denoted by q_j. Both of these restriction blocks (73.5) and (73.6) need to be linearised. Restrictions (73.7) ensure the correct order of pickup (i) and delivery locations ($n+i$) for each request i. To limit the dissatisfaction of individual passengers, the waiting times t_i^w are capped at a permitted maximum $t^{w,\max}$ (73.8). Likewise, with (73.9) a maximum permissible detour $t_{\max}^{r+}$ between pickup and delivery location for each request is introduced. For travellers towards the station, it is allowed to miss the train specified with their request, so a matching binary variable $Y_{i,z}$ is introduced and assigned a value of 1, if request i is assigned to train z, and 0 otherwise. The assigned train has to have a departure time d_z after the passengers have arrived at the station (B_i) and have enough time to change (t_c), see (73.10). At the same time, the line number of the assigned train c_z^tl and the desired line c_i^l have to match and only one train can be assigned to each request, see (73.11) and (73.12). To prohibit routes containing the delivery location $n+i$ of a trip request but not the corresponding pickup location i, ergo the related locations would be visited by different vehicles, restriction block (73.13) is introduced. Both passenger types experience two kinds of waiting times. These include excess ride times that occur due to operational detours in comparison to taxi service, as well as waiting times at the station (either before pickup or after delivery). The difference between actual ride time and the time needed for a service along the direct route ($t_{i,n+i}$) yields the excess ride time. This value is assigned to the location that is not the station, so for customers of type $c_i^\text{t} = 0$ to their pickup location by (73.14) and for customers travelling vice versa ($c_i^\text{t} = 1$) to their delivery location by (73.18). The waiting time at the station in restrictions (73.16) for passengers travelling to the station is calculated as the difference between the departure time d_z of the assigned train, indicated by $Y_{i,z} = 1$, and the desired arrival time at the station l_i, minus the time required for changing t_c in case the passengers arrive at the station later than the specified time. If they arrive early, restrictions (73.17) set the waiting time to the difference between desired and actual arrival time at the station. Similarly, with (73.18) the time spent waiting for the DARS vehicle at the station is set for the requests with $c_i^\text{t} = 1$. Inequalities (73.19) ensure that these customers, who wish to be picked up at the station, are collected after their train has arrived. Restrictions (73.20)–(73.22) define the domains of the variables.

The mixed-integer program (MIP) is solved using CPLEX by adding the sets S of infeasible routes dynamically to the set $\mathcal{S}$ (cf. [9]).

Heuristic Approach Based on Simulated Annealing

Since the DARP is an NP-hard problem (cf. [1]), a heuristic approach is developed to provide good results for the problem in a short time. First, a feasible initial solution is created by generating tours that correspond to a taxi operation. This only takes precedence and capacity constraints into account, not time constraints. These are not explicitly respected, but hold as well as possible due to the objective function. The starting temperature is set to $T_0 = -\frac{\Psi^0}{\ln(P_A)}$ with $P_A = 30\%$ being the probability of accepting a solution with twice the objective value at the beginning. Tests showed that a damping rate of $r = T_0^{-1/m_T}$ and a total number of $n_{\text{iter}} = 22500$ iterations distributed over $m_T = 75$ temperature steps with $n_T = 300$ iterations each are suitable for this problem, as they provide a good compromise between computational time and exploration of the solution space. A new solution is created by randomly selecting a trip request and deleting its pickup and delivery location from the current route and inserting it at a randomly selected position of a randomly chosen trip. Initially, the delivery location is inserted directly after the pickup location and the modified route is checked for feasibility. If this solution is not feasible in terms of capacity, the solution is rejected and the next iteration is performed. Otherwise, all solutions are generated that result from moving the delivery location step by step until the vehicle capacity is exceeded or the last position of the vehicle route is reached. The weighted random solution selection procedure applied afterwards is roulette wheel selection. For this, the squared reciprocals of the objective function values for the different variants with respect to the delivery location positions are used to determine the probability of choosing the respective solution. If the chosen solution shows a superior objective function value than the best solution found so far, the solution is accepted. If not, the Metropolis criterion is applied.

Computational Study

For the computational study, exemplary instances were generated representing realistic scenarios for a potential service area in which the first/last mile problem occurs. For each of 14 parameter combinations of (K, n), 30 instances were randomly created. Both approaches were used to solve these instances with a maximum computation time of 1000 s. Table 73.1 shows an excerpt of the results obtained from the computational study, aggregated for each group of instances. The columns contain the mean $\overline{(\cdot)}$ and standard deviation values $s(\cdot)$ of computation times t_{com}, average waiting time per person t_p^w, and maximum waiting time t_μ^w based on the solved instances. Using MIP, only few instances could be solved within 1000 s, so the number of solved instances #sol is shown, too.

Using the MIP approach, the computation times were already quite high for small instances and varied considerably between instances with the same parameter combination as well as between combinations. Moreover, solutions could only be found for instances with a small number of vehicles within the specified maximum computation time. The SA method, on the other hand, provided solutions for each instance within reasonable computation times and low variability ($\leq 15\%$). The computational results of this simple SA approach are comparable to those obtained in [5]. Examining the average waiting times, it is observed that the SA values are slightly worse than those of the MIP (for comparable instances). Mean average waiting times of about three minutes per passenger for almost all parameter combinations are still considered appropriate for practice, as the definition is rather strict here. In addition,

Table 73.1 Results of computational study (excerpt)

	MIP							SA					
Inst.	#sol	$\frac{\bar{t}_{com}}{s}$	$\frac{s(t_{com})}{s}$	$\frac{\bar{t}_p^w}{min}$	$\frac{s\left(t_p^w\right)}{min}$	$\frac{\bar{t}_\mu^w}{min}$	$\frac{s(t_\mu^w)}{min}$	$\frac{\bar{t}_{com}}{s}$	$\frac{s(t_{com})}{s}$	$\frac{\bar{t}_p^w}{min}$	$\frac{s\left(t_p^w\right)}{min}$	$\frac{\bar{t}_\mu^w}{min}$	$\frac{s(t_\mu^w)}{min}$
(1, 5)	30	1	0.3	1.38	1.23	6.35	3.84	30.53	3.84	3.50	1.88	11.50	5.95
(1, 10)	8	256	247	2.35	0.57	11.43	3.94	90.50	11.38	3.79	0.90	13.75	4.32
(1, 15)	0							141.49	20.95	5.77	1.91	22.04	7.22
(2, 8)	18	30	25	0.59	0.47	4.98	4.04	60.60	3.65	2.25	1.00	9.79	3.70
(2, 10)	5	125	106	0.45	0.32	3.59	3.73	81.87	8.91	2.43	0.87	11.61	4.52
(2, 15)	0							137.29	14.46	2.55	0.72	12.41	3.85
(3, 10)	9	109	113	0.03	0.06	0.35	0.66	75.79	6.08	2.72	1.36	11.75	5.17
(3, 12)	3	314	180	0.00	0.00	0.00	0.00	98.75	7.46	1.99	0.96	12.90	7.32
(3, 15)	0							139.28	13.08	2.47	1.13	14.51	7.35
(3, 20)	0							189.49	18.22	2.51	0.76	18.97	8.55
(3, 25)	0							246.78	20.10	3.16	0.93	16.18	5.49
(4, 15)	0							129.26	9.96	2.53	1.34	16.05	8.29
(4, 20)	0							179.07	17.78	2.73	1.05	21.34	8.17
(4, 25)	0							228.85	18.82	3.00	1.05	19.27	7.10

regarding the mean maximum waiting time, it can be noted that the relatively low average waiting times do not occur at the expense of individual requests' waiting times. Besides the presented values, the pooling rate was also computed as the ratio between the mean of the number of pooled trip requests and the total number of requests. The results indicate that the heuristic solutions yield a substantially higher rate compared to the MIP solutions, highlighting their operational suitability.

Conclusion

A mathematical model for the I-DARP along with two solution approaches were presented. The heuristic approach yields good results consistently within a quasi-predictable computation time for each dataset. In comparison, solutions using MIP are less reliable and require significantly more computing time and capacity. Given the effort and benefits of each approach, it is recommended to employ a heuristic method such as this or another for the practical operation of a DARS, as it can produce results quickly and reliably for planning purposes. Future research should investigate advancements in both the model formulation and the development of sophisticated heuristic approaches for improving results.

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