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Enhancing urban mobility: Integrating ride-sharing and public transit

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ABSTRACT

Seamless integration of ride-sharing and public transit may offer fast, reliable, and affordable transfer to and from transit stations in suburban areas thereby enhancing mobility of residents. We investigate the potential benefits of such a system, as well as the ride-matching technology required to support it, by means of an extensive computational study. Our study shows that the integration of a ride-sharing system and a public transit system can significantly enhance mobility and increase the use of public transport.

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1. Introduction

People all around the world use private cars to travel to work. Most of these commuter trips are single-occupant vehicle trips. In the U.S., for example, single-occupant trips represent approximately 77% of all commuter trips (Polzin and Pisarski, 2013); similar percentages are found in Europe (European Environment Agency, 2010). The low vehicle occupancy rates combined with the high number of trips during peak-hours often leads to severe traffic congestion in urban areas. The resulting stress and air pollution, caused by vehicle emissions, can have serious negative health effects.

To reduce the negative externalities of car travel, local governments encourage the use of public transit. Unfortunately, many suburban and rural areas are not adequately served as they lack the population density to justify public transit, i.e., the public transit is not economically viable. In cities with sprawling suburban areas, the utilization of public transit to commute to work is often low, e.g., less than 5% in metropolitan areas like Houston and Atlanta (McKenzie, 2010).

To attract more riders from suburban areas to public transit, transportation agencies must find adequate solutions to accommodate the first and last mile from the riders' home to and from the transit stations. Possible solutions for a transportation agency include operating a fleet of demand-responsive feeder vehicles and collaborating with local taxi service providers. In the U.S., for example, public transport providers have started collaborating with Uber and Lyft to better coordinate their service offering (Murphy, 2016). While the services provided by Uber and Lyft are convenient for the riders, they are often (too) costly for the transportation agency and/or the riders.

A cheaper and more environmentally sustainable alternative is to use existing private car trips as a feeder for public transit. A recent TRB report highlights ride-sharing as an important opportunity for transportation agencies to address the "last mile problem" (Murray et al., 2012). Deutsche Bahn (German railways) is running several pilots to synchronize bus and ride-shares to provide convenient door-to-door transportation to their travelers (Deutsche Bahn, 2015). The Flinc ride-sharing smartphone app integrates train timetables and regional public transport schedules across Germany. Research projects like SocialCar are seeking to incorporate ride-sharing into existing mobility systems (http://socialcar-project.eu/).

In this paper, we examine the potential benefits of integrating ride-sharing and public transit. Ride-sharing and public transit can, in fact, complement each other. On the one hand, ride-sharing can serve as a feeder system that connects less densely populated areas to public transit. On the other hand, the public transit system can extend the reach of ride-sharing and reduce the detours of the drivers. As such, it may help to overcome incompatibilities in the itineraries of drivers and riders and facilitate the matching process.

Consider the example depicted in Fig. 1 with driver d_1 and rider r_1 and two stations s_1 and s_2, where circles indicate origins and squares indicate destinations, where the number above an arc rep-

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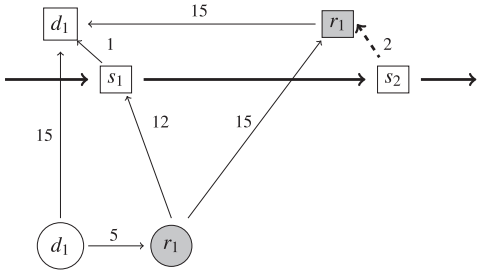

Fig. 1. Rider (grey) and driver (white) traveling from origin (circle) to destination (square) via public transit.

resents the time it takes to travel between the nodes, and where the driver is willing to accept an increase in trip time of at most five minutes. Without the use of public transit, a match between d_1 and r_1 is not feasible because the required increase in trip time (20 min) exceeds the driver's limit. If, however, the rider is willing to take the transit line from s_1 to s_2 and walk 2 min to his final destination, a feasible match between d_1 and r_1 is possible, because d_1 has to make a smaller detour. (We have to make sure that the rider can reach his final destination in time based on the timetable of the transit line.)

We consider a centralized system that automatically establishes matches between drivers and riders. A driver can either move a rider from his origin directly to his destination or to a transit station so that he can take the train to his final destination. Whereas most journey planners take an individual perspective and seek an optimal path for a single individual (without taking system effects into account), we take a system perspective and seek an optimal set of journeys for multiple individuals (simultaneously) in order to achieve a system objective. We assume that there is some flexibility in the itinerary and the time schedule of participating drivers, which we exploit to consider multiple alternative routings (visiting different transit stations), which, in turn, allows the system to consider and choose among a variety of (transfer) options for the riders. The transit system operates a fixed time table and drivers and riders announce their itineraries and time schedules on short notice. In this paper, we assume the system creates matches in a way that maximizes the number of matched riders. As a secondary objective, the system minimizes the additional driving distance of the matched drivers. As such, this hierarchy aims to maximize rider mobility while minimizing the inconvenience of the matched drivers and, at the same time, the negative externalities of car travel.

When there is a park and ride facility at the station, a driver may even decide to park his car after taking a rider to the station and take public transit to his final destination himself. Drivers provide the ride in exchange for a small fee that covers the costs of operating the vehicle and a small remuneration for the time that has been "lost" (due to pick-up and drop-off as well as the detour). The public transit provider may choose to cooperate with other stakeholders and offer additional benefits to drivers that are willing to accommodate riders. These might come in the form of a free park-and-ride ticket, a toll waiver, an HOV lane permit, or priority parking in the city center. As such, this service can be offered at a relatively low cost to the rider. This may help to increase rider-ship on public transit (which will lead to an increase in public transportation revenues) and decrease the number of personal vehicles on the road.

The main contributions of our research are that (i) we introduce a new and relevant problem that considers the integration and synchronization of ride-sharing and scheduled public transit, (ii) we present a solution approach to optimally create single or multi-modal ride-share matches, and (iii) we conduct an extensive numerical study on artificial instances that capture the main characteristics of many real-world transit settings and quantify the benefits of integrating ride-sharing and public transit.

The remainder of the paper is organized as follows. In the next section, we discuss relevant and related literature. In Sections 3 and 4, we formally define the problem and introduce a solution approach. In Section 5, we discuss the details of our numerical experiments and present the main results. We close the paper with a summary of our findings and recommendations for practitioners.

2. Literature

The use of demand-responsive transportation services to enhance the performance of public transportation systems has been promoted for some time and a wide variety of such services is offered in practice, often focusing on, or reserved for, people with disabilities and subsidized by the government.

Cayford and Yim (2004) present a basic demand responsive feeder system for the city of Milbrae, California. In low-demand periods, buses can deviate from their "fixed" routes as well as from their scheduled departure times based on actual demand. The route deviation is limited to skipping portions of the fixed route when there is no demand for drop-offs on those portions on a particular trip. Buses operate according to the fixed schedule in high-demand periods.

One of the common integration options between a fixed-schedule system and an on-demand feeder system is the so-called Demand Responsive Connector (DRC). Koffman (2004) present examples of DRCs in several US cities and observes that it is one of the most popular types of flexible transit services. Such systems typically operate within a service area and move passengers to and from a transfer point that connects to a major fixed-route transit network.

A critical question in this context is under what conditions is it better, in terms of service and cost, to operate the feeder system using a fixed route policy and under what conditions is it better to operate the feeder system as a demand-responsive service. Quadrifoglio and Xiugang (2009) and Li and Quadrifoglio (2010) develop analytical and simulation models for this purpose and find that the switching point between a demand responsive and fixed route policy is in the range from 10 to 50 customers/mi^2/h.

A DRC is typically deployed within a specified zone, where the zone has one transfer point and where one transportation company provides the service. Li and Quadrifoglio (2011) consider the optimal zone design problem and develop an analytical model based on continuous approximations. Lee and Savelsbergh (2014) study a setting in which a zone has multiple transfer stations and a passenger can be dropped off at any of these transfer stations as long as the passenger's desired service is met. They demonstrate that a more flexible system can offer substantial cost savings, especially when transit services are frequent and/or transit stations are relatively close together.

In this paper we consider the use of ride-sharing as a feeder system for scheduled transit. Several recent papers focus on algorithmic approaches to optimally match riders and drivers in the ride-sharing context. The work in this area typically considers door-to-door trips in which a driver moves one or more riders from their origin to their destination (see for example Agatz et al., 2011; Lee and Savelsbergh, 2015; Wang et al., 2016, 2017; Stiglic et al., 2016). Furuhata et al. (2013) and Agatz et al. (2012) provide a detailed overview of this line of research and the links to other modes of transportation.

In Stiglic et al. (2015), we do not require door-to-door transportation but allow riders to walk to and from meeting points to

facilitate more convenient rides for the drivers. Here, we extend this work by also allowing transfers to a transit service with a fixed schedule. While there are only a few meeting points within walking distance of a specific rider, many more transit stations are within reach by using scheduled transit. The synchronization of the ride-share trips with the transit service links our work to multi-hop ride-sharing which involves the temporal synchronization of different ride-share trips (see Herbawi and Weber, 2011; Coltin and Veloso, 2013; Drews and Luxen, 2013 for algorithmic approaches to these types of problems).

3. Problem definition

We consider a transit service provider that receives a sequence of trip announcements A from participants. The set of announcements A can be partitioned into the set of trip announcements by the drivers, $D \subset A$, and the set of trip announcements by the riders, $R \subset A$. A trip announcement $a \in A$ has a submission time, σ_a, at which it becomes known in the system, has an origin location, o_a, and a destination location, d_a, as well as an earliest departure time, e_a, at the origin location and a latest arrival time, l_a, at the destination location. We assume that the departure times of the participants are somewhat flexible so that the difference $l_a - e_a$ is greater than the travel time from the origin to the destination. For each $a \in A$, there is also a maximum acceptable trip duration, T_a, a maximum acceptable walking distance from a transit station, M_a, and a maximum acceptable waiting time at a transit station, W_a. We denote the distance from location i to j by d_{ij}, the vehicle travel time from location i to j by t_{ij}, and the time it takes to walk from i to j by $\hat{t}_{ij}$. We assume that a service time τ is incurred when a driver picks up a rider.

We consider a connected public transit network with a fixed cyclic timetable. The timetable describes the departure and arrival times of each train at each station. (We refer to the transit vehicles as trains but these could also be buses, trams, or metros.) Let S denote the set of transit stations. Let $\bar{t}_{ij}$ be the (shortest) travel time by train between station $i \in S$ and $j \in S$, where the travel between two stations in the network can involve one train or multiple trains with transfers. In case the shortest route involves transfers then transfer and waiting times are included in the travel time. Note that for a cyclic timetable with identical trains the travel time $\bar{t}_{ij}$ is not dependent on the departure time from station i. (Modeling a more general setting in which the timetable is not cyclic will require a travel time that depends on a specific departure k, i.e., $\bar{t}^k_{ij}$.) We denote by $\theta^-_s(t)$ the last train departure at station $s \in S$ at or before time t and by $\theta^+_s(t)$ the first train departure at station $s \in S$ at or after time t.

For presentational convenience, we only consider trips into the city center, i.e., the morning commute. It is straightforward to extend the concepts and technology to accommodate more extensive travel patterns. Consequently, we assume for each station $s \in S$ an access time, τ_s, representing the time needed to walk from the drop-off location to the platform. Furthermore, we assume that a subset of the transit stations, $S_P \subseteq S$, have a park-and-ride facility and for each station $s \in S_P$ a park time, $\tilde{\tau}_s$, representing the time needed to park and walk from the parking facility to the platform. Finally, we assume that when a rider (or a driver) uses public transport, he will always travel to the station that is closest to his final destination. Let s_r be the closest station to the destination of rider r. Hence, using public transit is only feasible for rider r if s_r is within the maximum acceptable walking distance of his destination, i.e., when $d_{s_r d_r} \leq M_r$. For assessing the feasibility of matches involving public transport, it is convenient to calculate the latest time, l^s_r, at which a rider r can arrive at the platform at station $s \in S$ and reach his destination at or before l_r, i.e., $l^s_r = \theta^-_s(l_r - \hat{t}_{s_r d_r} - \bar{t}_{s s_r})$. The notation is summarized in Table 1.

We allow matches in which a driver picks up two riders for drop-off at a transit station (i.e., from two different pickup locations). However, to minimize the inconvenience for the driver, the two riders will be dropped off at the same transit station.

Thus, the transit service provider offers these types of matches:

- *A ride-share match:* a match between a rider and a driver in which the driver transports the rider from his origin to his destination;
- *A transit match:* a match between a rider and a driver in which the driver transports the rider to a transit station. Subsequently, the driver drives to his destination while the rider takes public transit to reach his final destination.
- *A park-and-ride match:* a transit match in which the driver parks his car and then uses public transport to reach his final destination

A ride-share match always involves one rider, but both a transit match and a park-and-ride match can involve one or two riders. At this stage, we do not consider matches with more than two rider pickups. The main reason is that it would increase the inconvenience for the driver (it will increase the driver's journey time and it will increase the risk of delays on the driver's journey). Note that we do allow multiple riders traveling together between the same origin and destination. A secondary reason is that it facilitates our presentation. Finally, we observe that, conceptually, it is straightforward to extend our algorithm to handle three or more rider pickups

3.1. Assessing the feasibility of a ride-share match

A ride-share match involves a driver $i \in D$ and a rider $j \in R$. The departure time e^j_i of a driver i matched with rider j depends on the announcement submission time of rider j, the earliest departure time of rider j, and the trip duration from o_i to o_j and is set as follows $e^j_i = \max(e_i, \sigma_j, e_j - t_{o_i o_j})$. This assures that driver i does not wait for rider j at o_j. The pickup time, e^i_j, of rider j is $e^j_i + t_{o_i o_j}$ and the arrival time, l^i_j, at the rider's destination is $e^i_j + \tau + t_{o_j d_j}$. Driver i will arrive at his destination at time $l^j_i = l^i_j + t_{d_j d_i}$. The ride-share match is feasible only if the trip time duration for driver i, i.e., $l^j_i - e^j_i$, is less than or equal to T_i. Also, driver i and rider j must arrive at their destinations before their latest arrival times l_i and l_j, respectively.

3.2. Assessing the feasibility of a transit match

A single-rider transit match involves a driver $i \in D$, a rider $j \in R$, and a transit station $s \in S$. The departure time of the driver, e^j_i, and the pickup time of the rider, e^i_j, are computed in the same way as for a ride-share match. The arrival time at station s is $e^i_j + \tau + t_{o_j s}$. The driver arrives at his destination at time $l^j_i = e^i_j + t_{o_j s} + \tau + t_{s d_i}$. The rider arrives at his destination at time $l^i_j = \theta^+(e^i_j + \tau + t_{o_j s} + \tau_s) + \bar{t}_{s s_r} + \hat{t}_{s_r d_r}$. The transit match is feasible if the rider arrives at the platform of the transit station in time, i.e., $e^i_j + \tau + t_{o_j s} + \tau_s \leq l^s_j$, if the driver arrives at his destination in time, i.e., $l^j_i \leq l_i$, and the trip durations of the rider, i.e., $l^i_j - e^i_j$, and the driver, i.e., $l^j_i - e^j_i$, are less than or equal to T_j and T_i, respectively.

A two-rider transit match involves a driver $i \in D$, riders j, $k \in R$ ($j \neq k$), and a transit station $s \in S$. Without loss of generality we assume that rider j is picked up before rider k. To avoid waiting at o_j and o_k, the driver departs at $e^{jk}_i = \max(e_i, a_j, e_j - t_{o_i o_j}, a_k, e_k - (t_{o_i o_j} + \tau + t_{o_j o_k}))$. The pickup time e^i_j of rider j is $e^{jk}_i + t_{o_i o_j}$ and the

Table 1
A summary of the notation used.

Sets	
D	Set of driver trip announcements
R	Set of rider trip announcements
A	Set of trip announcements, $A = D \cup R$
S	Set of transit stations
S_P	Set of transit stations with a park-and-ride facility, $S_P \subseteq S$
Locations	
o_a, d_a	Origin and destination location of trip announcement $a \in A$
s_r	The closest station to the destination of rider $r \in R$, $s_r \in S$
d_{ij}	Distance from location i to j
Time parameters	
σ_a	Announcement time of trip announcement $a \in A$
e_a	Earliest departure time from the origin of trip announcement $a \in A$
l_a	Latest arrival time at the destination of trip announcement $a \in A$
l_r^s	Latest time at which a rider r can arrive at the platform at station $s \in S$ and reach his destination at or before l_r
T_a	Maximum acceptable trip duration for trip announcement $a \in A$
M_a	Maximum acceptable walking distance from a transit station for trip announcement $a \in A$
W_a	Maximum acceptable waiting time at a transit station for trip announcement $a \in A$
t_{ij}	Travel time from location i to j by car
$\hat{t}_{ij}$	Travel time from location i to j by foot
$\tilde{t}_{ij}$	Shortest travel time between station $i \in S$ and $j \in S$ by train
τ	Service time at the pickup location of a rider
τ_s	Service time for the rider to walk from to drop-off location to the platform at station $s \in S$
$\tilde{\tau}_s$	Service time for the rider to park and walk from the parking facility to the platform at station $s \in S_P$
$\theta_s^+(t), \theta_s^-(t)$	First and last train departure at station $s \in S$ at or before time t

pickup time e_i^k of rider k is $e_j^i + \tau + t_{o_j o_k}$. The arrival time at station s is $e_k^i + \tau + t_{o_k s}$. The driver arrives at his destination at time $l_i^j = e_k^i + \tau + t_{o_j s} + t_{s d_i}$. The transit match is feasible only if both riders arrive at the platform at the transit station in time, if the driver arrives at his destination in time, and if the trip durations of the driver and the riders are less than the maximum acceptable trip durations.

Note that in the discussion above we have not explicitly considered the waiting time of a rider at the platform, e.g., $w_j^i = \theta^+(e_j^i + \tau + t_{o_j s} + \tau_s) - (e_j^i + \tau + t_{o_j s} + \tau_s)$ in the case of a single-rider transit match. However, it is easy to see that if the transit match is feasible, the waiting time at the transit station can be minimized by having the driver depart later. The delay in departure of the driver is bounded by $\min(w_j^i, l_i - l_j^i, l_j^s - e_j^i - (\tau + t_{o_j s} + \tau_s))$, i.e., the current waiting time, the remaining driver flexibility, and the remaining rider flexibility.

3.3. Assessing the feasibility of a park-and-ride match

Assessing the feasibility of a park-and-ride match is similar to assessing the feasibility of a transit match. We only have to account for the fact that after dropping off the rider(s) at the transit station, the driver continues using public transit rather than continues using his car. The first difference occurs in the calculation of the arrival time at the station platform: rather than using the access time τ_s, we use the park time $\tilde{\tau}_s$ to account for the fact that the driver parks his car and then driver and rider(s) walk to the platform from there. The second difference is calculating the time that the driver arrives at his destination, which now involves public transit.

3.4. The matching problem

As in Stiglic et al. (2015), we create a node for each driver $i \in D$ and each rider $j \in R$ and an edge connecting node i and j if there is a feasible match between driver i and rider j. We also introduce nodes that represent pairs of riders (j, k), where $j, k \in R$ and $j \neq k$. We add an edge connecting node i and (j, k) if there is a feasible match between driver i and rider pair (j, k). Each edge e has two weights: number of riders in the match, ν_e, and the additional driving distance for drivers, δ_e (note that this value may be negative in park-and-ride matches because the driver uses public transit to reach his destination from the park-and-ride station). Note that a transit match between a driver and a rider (or a driver and a pair of riders) may not be unique, because there may exist a number of feasible transit matches involving different stations and different departures. Similarly, a park-and-ride match between a driver and a rider (or a driver and a pair of riders) may not be unique. We only consider the match with the lowest driving distance for that combination of the driver and rider(s).

Let E represent the set of all edges in the bipartite graph and let the binary decision variable x_e for edge $e \in E$ indicate whether the edge is in an optimal matching ($x_e = 1$) or not ($x_e = 0$). Furthermore, let E_i and E_j represent the set of edges in E associated with driver i and rider j, respectively. Then, our ride-share matching problem with the objective of maximizing the number of matched riders can be formulated as the following integer program:

$$\max z_1 = \sum_{e \in E} \nu_e x_e \tag{1}$$

subject to

$$\sum_{e \in E_i} x_e \leq 1 \quad \forall i \in D, \tag{2}$$

$$\sum_{e \in E_j} x_e \leq 1 \quad \forall j \in R, \tag{3}$$

$$x_e \in \{0, 1\} \quad \forall e \in E. \tag{4}$$

Objective function (1) maximizes the number of matched riders. Constraints (2) and (3) assure that each driver and each rider is only included in at most one match in an optimal matching, respectively.

To obtain a matching that minimizes the total increase in driving distance for all drivers, the objective should be replaced by:

$$\min z_2 = \sum_{e \in E} \delta_e x_e. \tag{5}$$

Since both objectives, i.e., maximizing the number of matched riders and minimizing the total trip time increase for all partici-

pants, are relevant, we take both objectives into account in a hierarchical fashion, where we consider z_1 as the primary objective and z_2 as the secondary objective. We first solve (1) subject to (2)–(4). Let z_1^* be the number of matched riders. We then solve (5) subject to (2)–(4) plus the additional constraint $\sum_{e\in E} v_e x_e \geq z_1^*$.

4. Solution approach

Our ride-matching algorithm consist of a match identification phase and an optimization phase in which the optimal matching is determined (based on the set of feasible matches). Match identification is performed separately for each match type, i.e. ride-share matches, transit matches, and park-and-ride transit matches. To identify feasible matches with two riders, we take advantage of the property that a match between driver i and riders j and k, $j \neq k$, can be time feasible (but not necessarily) only if the matches between driver i and rider j and driver i and rider k are both time feasible.

4.1. Determining feasible ride-share matches

Single ride-share match identification is a relatively simple task. In this paper, we perform a straightforward enumeration of all possible driver-rider pairs and identify those that are feasible by checking the conditions outlined in Section 3.1.

4.2. Determining feasible transit matches

The identification of feasible transit matches starts with several preprocessing procedures. For each announcement $a \in A$, we determine the closest public transit station $s \in S$ to the destination location d_a. This is done by performing a query in a $k-d$ tree in which all transit station locations are stored. Let s_a denote the closest public transit station to the destination location for announcement a. The set of rider announcements $R_p := \left\{ j \in R \mid d_{s_j d_j} \leq M_j \right\}$ is considered in the determination of potential transit matches and the set of driver announcements $D_p := \left\{ i \in D \mid d_{s_i d_i} \leq M_i \right\}$ is considered in the determination of potential park-and-ride transit matches. Note that all driver announcements $d \in D$ are considered in the determination of potential transit matches.

The next preprocessing procedure serves to find for each rider $j \in R_p$ the set of feasible origin transit stations S_j and the set of feasible departures at each feasible origin station $s \in S_j$. For a rider $j \in R_p$, a train departure time t at origin station $s \in S_j$ is feasible if $e_j + \tau + t_{o_j s} \leq t$ and $t \leq l_j - \hat{t}_{s_j d_j} - \bar{t}_{ss_j}$, i.e., the earliest possible arrival time at the station is less than t and the earliest arrival at the destination departing at or after t is less than l_j. All feasible departures are stored for each individual rider separately.

In another preprocessing step, we find for each driver $i \in D$ the set of feasible transit stations S_d and the set of feasible departures at each feasible transit station $s \in S_d$ that he can visit. For a driver $i \in D$, a transit station s is feasible if he can visit this station on his way from his origin to his destination location without exceeding his maximum acceptable trip duration. A station departure t is feasible if the arrival of the driver occurs at or before $t - \tau_s$. To identify such departures, we build an implied time window $[e_s^i, l_s^i]$ at station s with earliest arrival time $e_s^i = e_i + t_{o_i s} + \tau$ and the latest departure time $l_s^i = l_i - t_{s d_i}$. All departures t for which $t - \tau_s \in [e_s^i, l_s^i]$ are feasible. A lookup table is constructed that enables one to query potentially feasible drivers based on origin station and departure. Feasible single rider transit matches are identified for each rider by inspecting all feasible stations and feasible train departures for that rider and retrieving all potentially compatible drivers. The feasibility of a particular match is checked by examining the conditions that are outlined in Section 3.2.

4.3. Determining feasible park-and-ride matches

To identify feasible park-and-ride matches, a slightly different preprocessing procedure for the drivers is performed in comparison to the one described in the previous section. The (only) difference is that the latest departure time l_s^i is calculated using transit trip duration from s to s_i, the destination station of the driver, i.e., $l_s^i = l_i - s_i \hat{d}_i - \bar{t}_{ss_i} - \tau_s^p$. Based on the preprocessed feasible departures of the riders and drivers, feasible single-rider park-and-ride matches are identified for each rider by inspecting all feasible stations and feasible train departures for that rider and retrieving all potentially compatible drivers. The feasibility of a particular park-and-ride match is checked by examining all conditions that are outlined in Section 3.3.

4.4. Determining feasible transit and park-and-ride matches with two riders

Once all feasible single-rider transit (or park-and-ride) matches are known, we group all matches of this type using a hash map on driver - station combinations. We examine each individual driver - station combination and construct all potential pairs of riders. We evaluate all possible combinations of departures for a driver - station - rider pair combination to identify and record the best feasible one according to total trip time increase for the participants in a match. Once all feasible transit and park-and-ride matches are known, we determine the best match for a particular driver - rider pair match.

It is straightforward to extend this logic to iteratively identify matches with more than two riders. To identify all feasible matches involving n riders one needs to examine each feasible grouping of a driver - station combination and a set of $n-1$ riders and construct and examine the feasibility of all possible groupings of the driver - station combination and n riders (where the initial $n-1$ riders form a subset of the n riders). The number of groupings that one needs to examine during this process is typically small because the earliest and latest arrival time constraints as well as maximum detour and maximum trip duration constraints rule out most of the possible groupings.

5. A computational study

In this section, we report the results of our computational study to assess the possible benefits and synergies that can arise as a result of integrating a ride-sharing system and a public transit system.

5.1. Generation of instances

We focus on a rectangular metropolitan area of 20 by 10 miles that features a circular urban center with a radius of 2.5 miles and a sprawling suburban area.

We consider a stylized transit network that captures the key features of a real-world commuter train network, such as the Bay Area Rapid Transit (BART) network in the San Francisco region. The radial route network consists of two commuter train lines and four urban rapid transit lines. Transfers between the different train lines are only possible at the transfer hub, which is located in the center of the urban area. The inter-station distance is approximately 2.25 mi on commuter lines and 0.75 mi on the urban transit lines. Fig. 2 illustrates the transit lines, stations, and the urban and suburban regions. The figure also indicates the eight park-and-ride stations.

The characteristics of the base case instances are summarized in Table 2. We generate n participant trips in which the probability of a driver or rider trip is equally likely. For each trip announce-

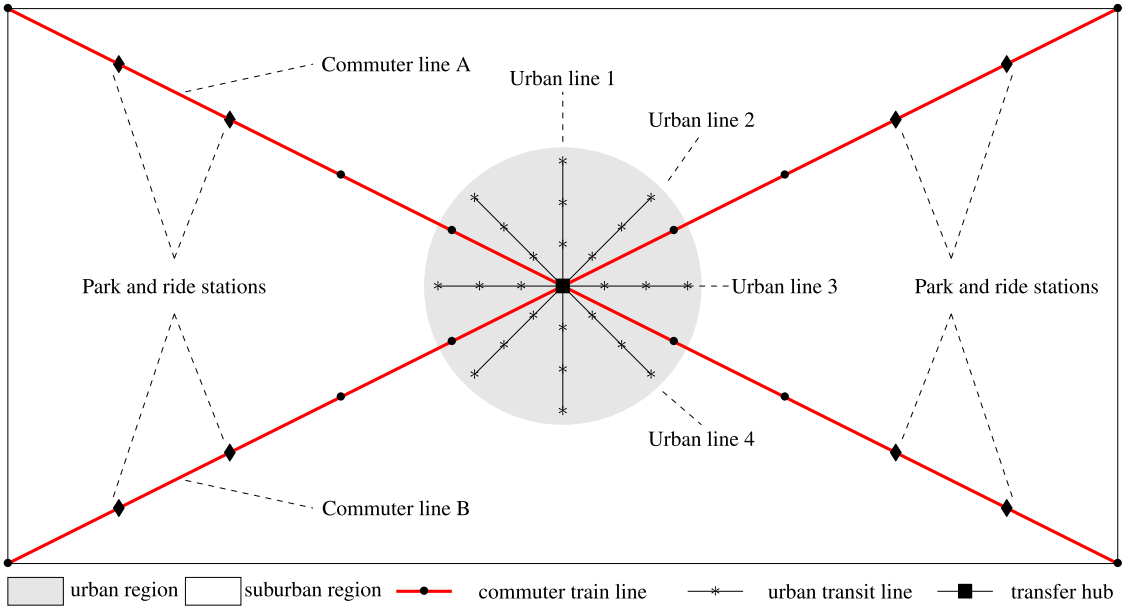

Fig. 2. Representation of public transit network and urban/suburban regions.

Table 2
Characteristics of the base case instances.

Trip pattern:	suburb to center
Avg. number of participants:	1000
Driver-rider ratio:	0.5
Departure time flexibility:	20 min
Submission lead-time:	15 min
Driver trip duration flexibility:	25%
Rider trip duration flexibility:	50%
Maximum number of driver stops:	3
Avg. driver trip distance:	8.0 mi
Avg. driver trip duration:	24.1 mi
Max. walk distance to transit station:	0.5 mi
Walk speed:	4 ft/s
Car speed:	20 mi/h
Suburban train speed:	40 mi/h
Urban train speed:	20 mi/h
Vehicle capacity:	2 seats
Pickup time per rider:	2 min
Transfer time street to platform:	2 min
Additional transfer time for park-and-ride:	2 min
Train dwell time:	1 min
Train hub dwell time:	3 min
Frequency of train departures:	15 min
Number of stations:	41
Number of park-and-ride stations:	8

ment, we generate an earliest departure time based on a truncated normal distribution with a standard deviation of 30 min. To create commuter trips, we randomly draw origins from the complete region and draw destinations from the urban center. Since we assume that it is unlikely that participants with very short trips will participate, we use rejection sampling to filter out the trips that are shorter than 1 mile.

With respect to the trip timing, we assume that all participants have a departure time flexibility, $l_a - e_a$, of 20 min and submit their trips to the system 15 min before their earliest departure time, i.e. $e_a - \sigma_a = 15$. Moreover, drivers are willing to accept trips that are up to 25% longer than their direct trip and riders are willing to accept trips that are up to 50% longer than their direct trip. These parameter values are similar to those used in other studies, e.g. Agatz et al. (2011).

Travel times on roads are calculated using Euclidean distances with a 30% uplift. We assume an average driving speed of 20 mph, which represents the travel speed in a congested urban area (Department for Transportation, 2016). We consider a pickup time of two minutes for each rider that is pickup by a driver. We assume a walking speed of 4 feet per second (LaPlante and Kaeser, 2004). The maximum walking distance for the rider to or from a transit station is 0.5 miles, which is a value that is commonly used in service planning to denote the catchment areas of railway stations (Daniels and Mulley, 2013; El-Geneidy et al., 2014).

We assume a cyclic schedule with the same departure frequency for all transit lines, with a default departure frequency of 15 min. The line schedules are synchronized at the hub location so that all transfers between lines take the same amount of time. The average speed of the commuter train is 40 mph and the average speed of the urban transit train is 20 mph. All trains have a dwell time of one minute for regular stations and three minutes at the hub station to allow for transfers. We assume that it takes two minutes to enter and leave a transit station which captures walking to and from the platform, with an additional two minute service time for all park-and-ride matches to account for the time that is needed to park the vehicle. Note that we do not consider parking costs as we expect parking to be free at a park-and-ride facility.

5.2. Computational results

The main aim of this research is to analyze and quantify the benefits of integrating ride-sharing with public transit and to determine what value this can create for different stakeholders, e.g., public transit agencies, ride-share providers, and system participants. We use the optimization results to compute and evaluate a number of metrics to gain insight into the system performance and the potential of ride-sharing to enhance mobility and to increase the use of public transport. In all experiments, we either use the base case setting or a setting in which one of the characteristics is varied in order to assess the sensitivity of the system performance to this characteristic.

Note that in our numerical experiments, we assume that all announcements are known in advance, which means that the matching rate obtained provides an upper bound on the matching rate

Table 3
Results for different ride-share settings (avg. over 10 random base case instances).

	RS	TRS1	TRS2	PTRS
Riders				
Matching rate (%):	*66.8*	*74.0*	*83.7*	*83.8*
Transit matches (%):	*0*	*32.4*	*37.2*	*33.9*
P+R matches (%):	*0*	*0*	*0*	*4.0*
Δ travel time (%):	*0*	*7.3*	*7.4*	*7.5*
Drivers				
Matching rate (%):	*68.3*	*75.5*	*73.8*	*74.2*
Transit matches (%):	*0*	*32.4*	*27.3*	*25.1*
P+R matches (%):	*0*	*0*	*0*	*3.3*
Δ travel time (%):	*19.1*	*17.1*	*21.6*	*21.9*
Δ distance (%):	*8.4*	*7.0*	*10.4*	*7.2*

that can be obtained in a dynamic setting where announcements arrive continuously over time. Previous research (e.g., Agatz et al., 2011; Wang et al., 2017) has shown that when trips are announced shortly before their earliest departure time, this upper bound is quite tight. This suggests that our results are likely to be quite similar to what would be found in a dynamic setting, and, therefore, that our findings are indicative of what would be seen in a dynamic setting.

The algorithm for generating feasible matches is implemented in C and the simulation framework is implemented in Python 3.4. CPLEX 12.6 is used for solving matching problems. All base case instances are solved within a few seconds, with solution times of up to a minute for the largest instances with 2000 participants.

5.3. Benefits of an integrated system

To evaluate the benefits of integrating ride-sharing and public transit, we analyze the matching rates for a number of different settings. As a benchmark, we consider a setting in which only door-to-door ride-share matches are generated, denoted by RS. Next, we consider settings in which both ride-share and transit matches are generated, where TRS1 denotes the setting in which only single-rider transit matches are generated and TRS2 denotes the setting in which transit matches with one or two riders are generated. Finally, PTRS denotes the setting in which, in addition to the matches considered in TRS2, a driver may opt to take public transport after dropping off riders at a transit station, i.e., park-and-ride matches.

The results for the base case can be found in Table 3, where we report averages over 10 randomly generated instances. When we report the percentage of transit and P+R matches, it is relative to the number of matches, i.e., if 80 out of a 100 riders are matched and 36 of them are dropped off at a transit station by drivers who then drive to their final destinations, and 4 of them were dropped off at a transit station by drivers who then take public transport to their destinations, the matching rate is 80%, the transit matching rate is 45%, and the P+R matching rate is 5%. Furthermore, when we report travel time increases for riders, it is relative to the travel time they would have experienced if they had driven themselves.

We observe that integrating ride-sharing and public transit can have significant benefits. The average number of matched riders (our primary objective) increases from 66.8% to 83.8%. Interestingly, the average length of the driver detour (our secondary objective) in an integrated system is smaller than in a ride-share only system, i.e., 7.2% compared to 8.4%. Thus, not only is mobility enhanced, but the negative externalities associated with car travel, such as emissions and congestion, are reduced. This reduction in the average length of the driver detour is due, in part, to drivers deciding to use public transport. Maybe as important is the fact that the average increase in travel time for riders is relatively small, i.e., 7.5%. This may encourage more people to consider public transport as a viable alternative to traveling by car.

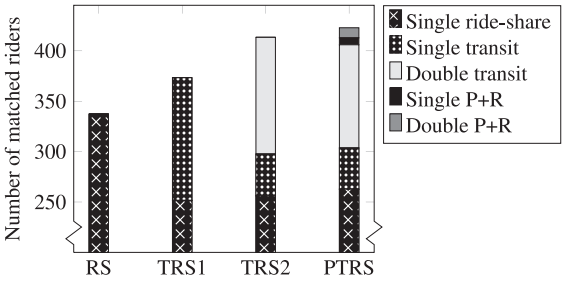

Fig. 3. Number of matched riders in different match types.

Another important observation is that to reach a high matching rate, it is critical that a driver is willing to pick up and drop-off more than one rider (in our setting at most two and dropped off at the same transit station). If drivers are only willing to pick up and drop off a single rider, the matching rate is 74% whereas if drivers are willing to pick up and drop off two riders the matching rate is 83.7%. Of course, this increase in matching rate comes at the expense of an increase in the length of the average driver detour (from 7.0% to 10.4%).

In Fig. 3, we provide more details on the matches found for riders by showing the number of riders in the different types of matches for the four settings. We see, again, that transit matches involving two riders are critical to achieving high matching rates in the settings that allow them (TRS2 and PTRS). We see too that the number of riders in park-and-ride matches is small (only 17 on average).

The fact that allowing park-and-ride matches results in an increase in matching rate may, at first, seem surprising as it mostly impacts drivers. However, some transit matches are time feasible only when the driver use public transport, because the driver can reach his destination faster using public transport than by car. Another factor contributing to the small number of drivers choosing to use public transport is the use of hierarchical optimization, i.e., maximizing the matching rate followed by minimizing (total) driver distance subject to the constraint that the matching rate cannot decrease. The constraint that the matching rate cannot decrease is restrictive and leaves little flexibility for reducing the detour distance. If we would be willing to accept slightly smaller matching rates, it is possible that we would see many more drivers opting for public transport. Finally, the location of the transit stations offering park-and-ride, which in our transit network are relatively far from the urban center, (negatively) impacts the benefits of the park-and-ride option.

To provide further insight into the opportunities offered by an integrated system, we look at the trips associated with the transit and park-and-ride matches generated for a single instance. Fig. 4 shows the paths from the riders' origins to their transit stations. For matches that involve two riders, the trip from the origin of the first rider to the origin of the second rider is represented by a dotted line and the trip from the second rider's origin to the transit station is represented by a solid line. Interestingly, we see that only about 25% of the riders is dropped off at the station that is closest to their origin. It is often more convenient for a driver to use a transit station closer to the urban center. As a consequence, the public transit trips for riders tend to be relatively short. This also suggests, that park-and-ride facilities at transit stations close to the urban center may significantly increase use of public transport by drivers.

To further analyze these trips, we break-down, in Fig. 5, the single-rider transit matches into: (1) waiting time at home (from

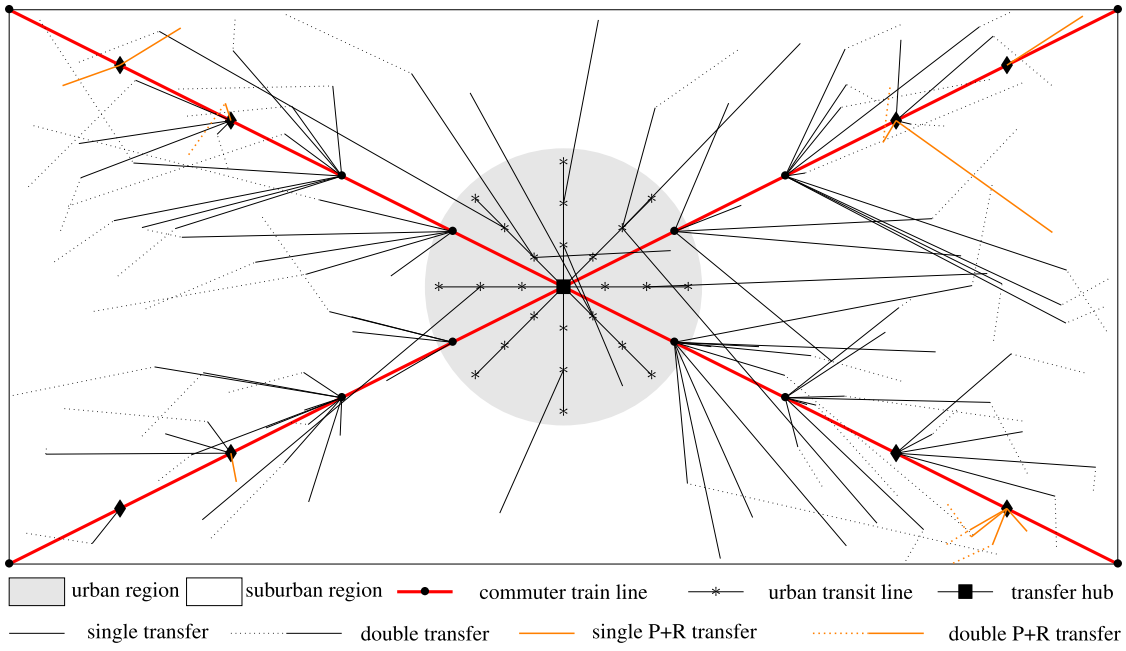

Fig. 4. Map of rider paths to transit stations for one of the base case instances for the PTRS setting.

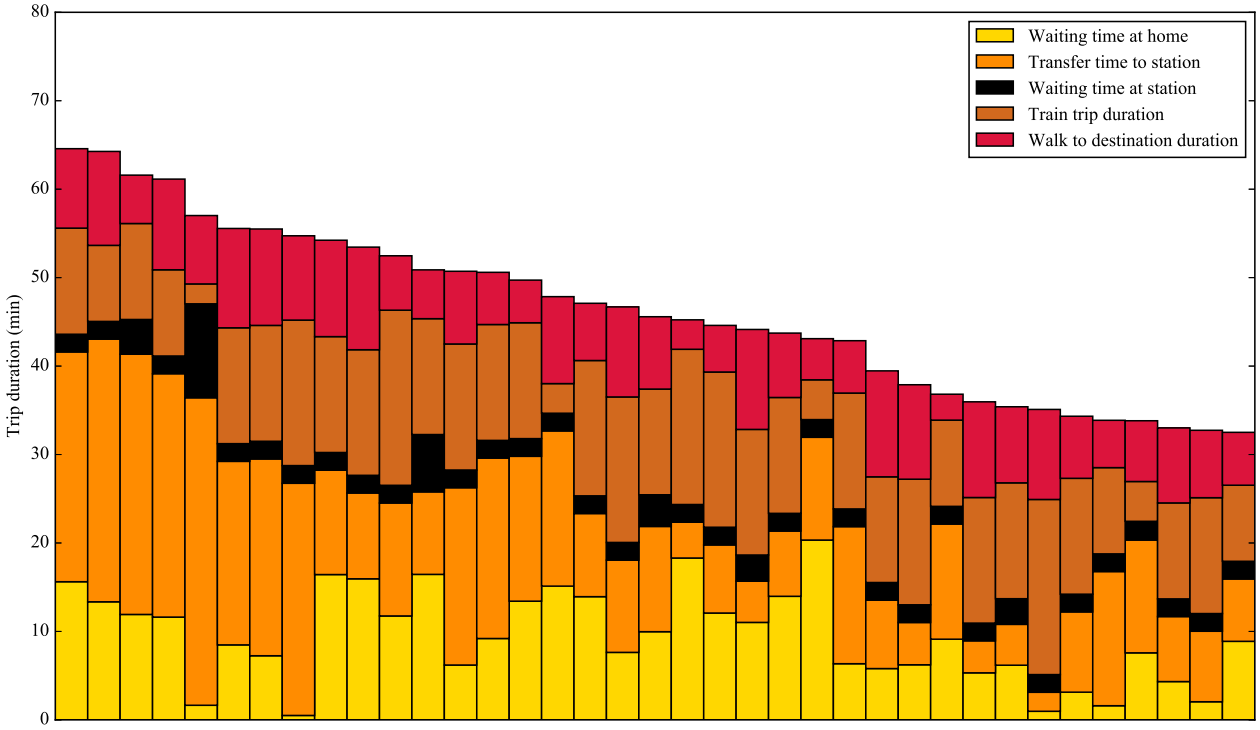

Fig. 5. Breakdown of trip durations and waiting at home for all riders matched in single transit matches in the optimal solution for one of the base case instances.

the earliest possible departure time), (2) transfer time to station, (3) waiting time at station, (4) transit trip duration, and (5) walking time from destination transit station to the destination. Note that the waiting time at a transit station includes the time to reach the platform. Similarly, the walking time from a transit station to the final destination includes the time to leave the platform. The waiting time at home is presented for completeness sake; it is, of course, not part of the rider's actual trip. We see that the average travel time of these riders on the transit lines is relatively small, i.e., 12–13 min, which is approximately 30% of the average total trip duration. This is in line with the earlier observation in Fig. 4 that most riders are dropped off at a transfer station closer to the urban center.

In the next subsections, we evaluate the impact of varying instance characteristics. We will report results only for the most and the least restricted ride-share settings, i.e., RS and PTRS.

5.4. Impact of driver matching flexibility

In this section, we evaluate the impact of the matching flexibility of drivers, i.e., their willingness to extend the latest arrival time at their destination. Table 4 shows, unsurprisingly, that a lower

Table 4
Results for different driver matching flexibilities (Avg. over 10 random instances).

	10 min		15 min		20 min	
	RS	PTRS	RS	PTRS	RS	PTRS
Riders						
Matching rate (%):	*64.9*	*78.8*	*66.8*	*83.8*	*66.3*	*84.0*
Transit matches (%):	*0*	*34.3*	*0*	*33.9*	*0*	*38.7*
Drivers						
Matching rate (%):	*65*	*70.8*	*68.3*	*74.2*	*66.7*	*73.7*
P+R matches (%):	*0*	*3.0*	*0*	*3.3*	*0*	*3.8*
Δ distance (%):	*8.1*	*6.4*	*8.4*	*7.2*	*8.5*	*7.0*

Table 5
Results for different numbers of participants in the system (Averaged over 10 random instances).

	500		1000		2000	
	RS	PTRS	RS	PTRS	RS	PTRS
Riders						
Matching rate (%):	53.5	67.9	66.8	83.8	76.0	93.7
Transit matches (%):	0	39.0	0	33.9	0	36.5
Drivers						
Matching rate (%):	56.3	63.6	68.3	74.2	74.1	78.7
P+R matches (%):	0	4.6	0	3.3	0	4.3
Δ distance (%):	7.1	4.7	8.4	7.2	8.8	6.7

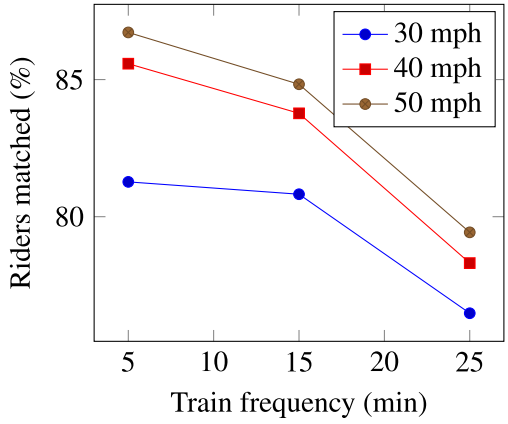

Fig. 6. Rider match rates for different transit line characteristics.

driver matching flexibility (i.e., 10 min) limits the value of integrating ride-sharing with public transit, because fewer two-rider transit matches are feasible (there is simply not enough time to accommodate more than one rider).

5.5. Impact of density

In this section, we investigate how the number of system participants affects the performance. Table 5 shows, as expected, that the matching rate increases significantly when the number of system participants increases; the net improvements in rider matching rate are 14.4 to 17.6 percentage points.

5.6. Impact of transit system parameters

In this section, we study the impact of the characteristics of the transit line on the performance of the system. We consider three speed scenarios for the suburban trains: 30 mph, 40 mph (base case) and 50 mph and three departure frequencies for all transit lines: every 5, 15 (base case) or 25 min. Fig. 6 shows the rider matching rates for the nine combinations. As expected, we see that more frequent departures and faster trains improve the performance of the system and create more rider matches. The results also suggest that the departure frequency is more important than speed, i.e. there is only a marginal benefit from increasing the suburban train speed from 40 mph to 50 mph.

6. Final remarks

Our study has shown that the integration of a ride-sharing system and a public transit system can potentially enhance mobility and increase the use of public transport. By reducing total system-wide vehicle miles it may also contribute to reducing the negative externalities associated with car travel.

We have found that driver willingness to pick up and drop off more than one rider is critical to the system performance. We have investigated only the simplest and most convenient (from a driver perspective) variant, i.e., driver willingness to pick up two riders and drop them off at the same transit station. Further investigation of more flexible variants is warranted. We have also observed the potential of park-and-ride facilities, but more experimentation is needed to fully understand the benefits, i.e., the location and number of transit stations with park-and-ride facilities as well as the use of different objective hierarchies when generating ride-share and transit matches.

In a real-world setting, riders and drivers will announce their trips over time and unexpected events can occur during the execution of planned trips. To deal with such dynamics, a rolling horizon strategy in which the static optimization model is solved each time new information becomes available or at fixed time points (e.g., every five minutes) may be appropriate. This is natural direction for further research. Other opportunities for further research include analyzing more complex transit systems, the use of meeting points in order to streamline the process of picking up riders, and the integration of (free) bike-sharing systems.

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