# Joint Optimization of Running Route and Scheduling for the Mixed Demand Responsive Feeder Transit With Time-Dependent Travel Times

Zhengwu Wang, Jie Yu<sup>®</sup>, Wei Hao<sup>®</sup>, Member, IEEE, and Jian Xiang

Abstract—As an emerging urban public transport mode, responsive feeder transit system is flexible and can offer door-todoor services between new districts at margins with low urban transit coverage and trunk bus station. In this study, a joint optimization of running route and scheduling for responsive feeder transit under mixed demand (i.e., reservation and real-time demands) of the time-dependent road network was investigated. A two-stage optimization method was designed together with considering the mixed demands. At the first stage, the initial running route and scheduling were determined according to all reservation demands. At the second stage, the running route and scheduling were continuously optimized based on the real-time demands. The real-time demand responsive strategy, which is built up by using quantitative batch treatment rather than immediate treatment and dynamic route updating strategy for global optimization, were designed by utilizing the submission order of real-time demands. A joint optimization model of running route and scheduling was constructed based on the quantitative batch decision points in the time-dependent road network together with combination of the actual road network. In this model, the minimum total system cost was used, which is composed of the vehicle running costs and passengers' traveling time costs with constraints including vehicle capacity, passengers' time window, and vehicle running time. A solving algorithm based on the adaptive genetic algorithm was designed by considering the characteristics of the joint optimization model.

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#### I. INTRODUCTION

ITH the continuous expansion of urban area, high-capacity trunk bus, including subway, light rail and bus rapid transit (BRT), are extensively applied. Although these high-capacity trunk buses can solve traveling among regions effectively, certain regions are beyond the coverage of public transportation services because of the large distances for trunk bus stations, especially in new districts in urban fringes. This situation decreases the scope of trunk bus services and lowers the traveling efficiency of passengers to a certain extent. Demand responsive transit (DRT) successively emerges in various big cities because of the rapid development of the Internet and mobile communication technology [1], [2]. Responsive feeder transit (RFT) is an important operation pattern of DRT. RFT generally serves for the residents living around trunk bus stations, and it can arrange vehicles and routes according to the scheduled boarding time and location of passengers. This measure is effective in solving the "first/last mile" problem of traveling of residents. There is a consensus among the research community which RFT shows better maneuverability and flexibility, it is more flexible to low-density bus traveling region or for disadvantaged groups, including the aged, children, and the disabled, compared with the fixed route transit (FRT). However, operators seem to agree that their widespread applications have been quite underwhelming mainly related to the contradiction between operating costs and service quality.

The motivation of this article is to improve the efficiency and performance of RFT according to designing a joint optimization model of running route and scheduling using a two-stage method. Most of the current relevant studies focused on DRT system with time-independent travel times, including the topic of the influence factor and willingness [3]–[5], service performance and market adaptability [6]–[11], coordinated application of DRT and FRT [12]–[15], critical demand density and critical link [16]–[18], station location [19], ticket price [20], the vehicle routing problem [21]–[29], optimization of scheduling parameters including vehicle departure time, total number of vehicles, etc. [30]–[36]. However, limit studies have been conducted to study the joint optimization of running

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route and scheduling of DRT, although some studies have determined scheduling plan according to the routes required to reservation demands. Braekers et al. [37] proposed a branch-and-cut algorithm to solve the dial-a-ride problem with heterogeneous users, vehicles and multiple depots. Wang et al. [38] discussed the coordinated optimization problem of vehicle routes and departure times for responsive feeder transit under the simultaneous pick-up and delivery mode. Tong et al. [39] developed a mixed inter programming model to jointly determine stop locations, running routes, timetables, and passengerto-vehicle assignments for customized bus. Considering the operator cost, passenger cost and departure time of the rail line and fixed-route feeder service, Lu et al. [40] also designed a mixed integer programming model to decide route planning and scheduling for hybrid flexible feeder service. Until now, several research topics of dynamic DRT system, such as dynamic demand [41]–[45], dynamic route [46]–[60], dynamic scheduling [61], [62], dynamics witching in fixed and flexible transit services [63], dynamic station [64], are concerned. The travel time of dynamic vehicle routing problem (DVRP) is time dependent. According to the composition of demand, dynamic vehicle routing problem can be divided into two categories: the first category is only reservation demand; the second category is both reservation demand and real-time demand. The DVRP with simultaneous reservation demand and real-time demand can be formulated as a sequence of static VRP or dynamic VRP models. In the former, when there are new real-time requirements, the static vehicle routing model is used to re optimize the unfinished journey to complete all the reservation requirements and real-time requirements that need to be completed, while in the latter, the dynamic vehicle routing model is used.

After a review of the relevant literature, some gaps and limitations were found in those existing studies:

- a) Limited consideration is given during coordinating running route and departure time. In fact, vehicle running route and departure time exhibit mutual influence. Running route decides which passengers can be picked up, the time window requirements of these passengers and real-time road impedance determine the departure time of vehicles, whereas the departure time of vehicles and real-time road impedance determine the arrival time at the demanding point and which passengers' demands can be met, thereby determining the vehicle running route.
- b) Existing studies generally set the rules of service regions and hypothesize that developed road networks (direct access from different nodes to the demanding points) are sufficient. However, most practical service regions are irregular, and the regional road network density has a relatively small value. For example, the road network density in big cities is 3–4 km/km² in accordance to the suggestions of the National Standards of the People's Republic of China GB50220-95.
- c) Most relevant studies hypothesize that the vehicle speed in the system is constant, and few of them have considered the time-dependent characteristics of traveling speed of the feeder transit. In the practical running process of vehicles, the traveling speed of vehicles varies with time because of the influences of various factors, such as traffic flows

and management. If time-varying conditions are ignored, then the passenger's time window requirements during the transit services will not be met, thereby decreasing service quality.

d) Existing response strategies is that real-time demands shall be immediately treated once new passenger demands are added in RFT system. However, the running routes frequently change when the real-time demands are high. Vehicle running route and scheduling plan are adjusted mainly on the basis of the insertion method, which judges whether the system can respond to the real-time applications of passengers. The real-time demands that are declined either have no responses or viewed as the reservation demand that the next bus trip must respond to. This mechanism is known as local adjustment strategy.

With respect to these disadvantages and in combination with the practical road network, a joint optimization method composed of the "initial optimization stage of reservation demand + dynamic optimization stage of real-time demand" was proposed by using the RFT system with time-dependent road network as the research object. This method considers both the reservation and real-time demands in the system. Compare with the aforementioned studies, the contributions of this study are as follows:

- a) The joint optimization of running route and scheduling of RFT was implemented on the basis of the predicted real-time traveling time of road section by considering the time-dependent characteristics of road networks.
- b) This study hypothesized that only the applications before the starting time of the dispatching cycle are considered reservation demands, whereas the remaining ones are real-time demands. Through re-definition, the initial running route and scheduling plan are optimized and determined at the starting time of the dispatching cycle. The initial running route and scheduling plan of vehicles before each bus trip need not be re-optimized, thereby decreasing the demands for computing power. The changes of reservation demands between the new and the old definitions are considered real-time demands. Such changes can slightly influence the final running route and scheduling plan because the system adopts the adjustment strategy of global optimization in the second step.
- c) The response strategy of the immediate processing of real-time demands is changed into quantitative batch processing. During the running process of the feeder transit, the running route and scheduling plan have to be re-optimized only when the newly increased real-time demands of passengers reach the preset threshold to decrease the time of optimization and update the frequencies of routes.
- d) The local adjustment strategy is changed into adjustment strategy of global optimization. This notion indicates that after the newly added real-time demands of passengers reach the preset threshold, the remaining route of the current running bus trips and running route and departure time of the subsequent bus trips were simultaneously optimized according to the newly added real-time demands and unfinished reservation demand. The adjustment strategy of the global optimization was applied, which considers the running route and scheduling of all unfinished bus trips before adjustment.

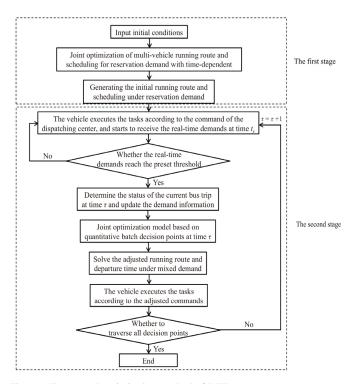


Fig. 1. Two-staged optimization method of RFT.

The remainder of this article is organized as follows: Section II describes the problem and constructs the two-stage joint optimization model. Section III designs the solving algorithm of the model. Section IV displays a case to illustrate the proposed model and algorithm. Section V is the conclusion of the paper.

## II. METHODOLOGY

## A. Description and Hypothesis of Problems

This study selected the RFT system under mixed demands of the time-dependent road network as the research object. In addition, this study hypothesized that a certain number of passengers near the transit station of the trunk bus plan to travel via trunk bus, and they can release their travel information, which involve the boarding location, boarding time, through a mobile application or the Internet to public transportation enterprises. After the public transportation enterprises receive the travel information of passengers, they determine the running routes and scheduling plan according to the personalized demand of passengers and the minimum total system costs while meeting constraints, such as passengers' demands and carrying capacity of vehicles. Finally, passengers are delivered from different demand points to transit stations. The two-stage joint optimization problem of running routes and scheduling for multiple vehicles can be described as follows (Fig. 1). In the first stage, the dispatching center determines the initial scheme of the running routes and scheduling after the deadline of receiving reservation demand. This approach is performed according to the reservation applications of passengers and the joint optimization model for the running route and scheduling of multiple vehicles under the reservation demand of the time-dependent road network while meeting constraints of passengers' time

window and demands. In the second stage, when vehicles run along the initial routes, the remaining routes of the current running bus trip and the running route and scheduling of the subsequent ones are determined through global optimization. This approach is performed by repeatedly using the joint optimization model based on the quantitative batch processing response strategy according to the continuously generated real-time demands. The number of subsequent bus trips might increase accordingly with the total demands.

Some hypotheses were proposed during modeling by considering the relevant regulations in existing studies according to the characteristics of the research problems:

- a) The system has only one transit station and all vehicles which depart from and return to the transit station after finishing the feeder task which has the fixed scope of service.
- b) The transit station in this study is trunk bus station in low-density traveling district at urban edge. The road network faces repetitive traffic jams because of morning and afternoon rush hours. Hence, this study hypothesized that the traveling speed of vehicles varies in different time intervals, but it can be predicted in advance. Vehicles run along the shortest route between the demand point and nodes (shortest traveling time).
- c) The passengers at the same point who have the same time window requirements were viewed as a demand point. By contrast, the passengers at the same point who have varying time window requirements were viewed as different demand points (virtual demand points). The passengers' demands at each demand point were smaller than the bearing capacity of vehicles.
- d) Once the passenger's travel demand is responded, it cannot be cancelled, and the passenger is not allowed to get off in the middle of the journey. The service time of each demand point is a fixed value, and the passenger can get on the feeder bus within the specified service time.
- e) The RFT system mainly provides feeder services to passengers between the demand point and transit station. The passengers are classified into two types. The first type consists of passengers from the different demand points to the transit station. The other type comprises passengers from the transit station to the different demand points. The running route of vehicles for the second type is the inverse process of the first one. Hence, this study opts for the first type of passengers as research objects according to [44].

# B. Joint Optimization Model Based on Quantitative Batch Decision Points

Existing studies of real-time demands are mainly based on the insertion method. The remaining routes of the current running bus trip are mainly optimized by heuristic algorithms, including neighborhood searching [65], [66] and tabu search [67]. Although the real-time demand processing strategy based on the insertion method can achieve a good scheduling scheme, it is only a local optimization strategy, and it mainly focuses on the optimization of the remaining routes of the current running bus trip, thereby failing to assure the global optimization of the system. Frequent adjustment and optimization calculations of routes may increase the working loads and dispatching costs of public transportation enterprises.

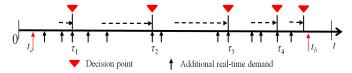


Fig. 2. Quantitative batch processing diagram.

On this basis, a global optimization strategy based on the batch processing of real-time demands was proposed in this study (Fig. 2). First, real-time demand applications were placed in an order according to the time axis. Batch processing was implemented when the newly increased real-time demands reach the preset threshold  $\theta$ . The point in time when real-time demands reaches the threshold  $\theta$  is defined as a decision point for running route optimization. The time moments that correspond to different decision points are recorded as  $\tau$ ,  $\tau+1,...$ , Therefore, the real-time demand processing can be expressed by a series of submodels (sub  $f(\tau)$ ).

The concept of "key point," which was proposed by Chen et al. [48], was introduced for the convenience of describing the dynamics of real-time demands. In this study, the key point was defined as each node that the vehicle passed through in the road network, including not only the demand point but also the other nodes in the road network. At time  $\tau$ , some bus trips that implement the initial running route have left the transit station. However, positions and state information of the bus trips that have departed can be obtained. All passengers' demands in the system can be divided into four types according to the position of vehicles, namely, finished demands in the plan, demands in service or of vehicles heading to the service point, unfinished demands in the plan, and newly increased real-time demands. No attentions are needed to finished demands in the plan, and they can be directly deleted in the residual demand tasks. The demands points in service or of vehicles heading to the service point are called key points of submodels because their service plan cannot be altered, that is, vehicle resources have been occupied. Therefore, the joint optimization based on the dispatching submodel at  $\tau$  focuses on the unfinished demands and the newly increased real-time demands. The optimization problem occurs when vehicles departs from the key points or transit stations, and it re-optimizes the remaining routes of all running bus trips and the running route and scheduling of all subsequent bus trips (the number of subsequent bus trips might increase due to the increased real-time demands) while meeting the constraints of time window and demand quantity of these demand points. Accordingly, it can realize the optimal total costs of the RFT system.

1) Total Cost of the Systems: The total cost of the RFT system  $f(\tau)$  includes the running cost of vehicles and traveling time costs of passengers:

$$f(\tau) = y(\tau) + w_1 f_1(\tau) + w_2 f_2(\tau)$$
 (1)

where  $y(\tau)$  is the cost of the RFT system, which has been generated at  $\tau$  before the current bus trip arrives at the key point; and  $w_1$  and  $w_2$  are the weight coefficients of the running cost of vehicles and traveling time cost of passengers at  $\tau$  to meet the traveling demands of passengers, respectively. Specifically, the running cost of vehicles at  $\tau$  to meet the

traveling demands of passengers can be expressed as follows:

$$f_{1}(\tau) = G \sum_{j \in DE} \sum_{r \in R_{0}(\tau)} \sum_{m=1}^{M} x_{ojr}^{m} + \alpha \sum_{i \in N} \sum_{r \in R_{co}(\tau)} \max[(e_{i} - t_{i}^{r}), 0] y_{ir} + \beta \sum_{i \in DE} \sum_{j \in DE} \sum_{r \in R_{co}(\tau)} \sum_{m=1}^{M} T_{ijr}^{m} x_{ijr}^{m}$$
(2)

The right of (2) includes the starting cost of vehicles, waiting cost of vehicles upon early arrival, and traveling time cost of vehicles. In (2), G is the starting cost of each bus trip, DE is the set of nodes in the road network and demand points, o is the transit station, D is the set of demand point, and M is the number of segments of time-dependent function.  $x_{iir}^m \in \{0, 1\}$  is a decision variable. If the bus trip r starts from the node i to the node j in the time interval m, then  $x_{ijr}^m = 1$ ; otherwise,  $x_{ijr}^m = 0$ .  $\alpha$  is the penalty cost of vehicles in unit time for early arrival.  $[e_i, l_i]$  is the time window of passengers at the demand point i.  $t_i^r$  is the time for the bus trip r to arrive at the demand point i.  $y_{ir} \in \{0, 1\}$  is a decision variable. If the demand point i is served by bus trip r, then  $y_{ir} = 1$ ; otherwise,  $y_{ir} = 0$ .  $\beta$  is the vehicle cost per unit traveling time.  $T_{ijr}^m$  is the traveling time for bus trip r from nodes *i* to *j* within the time interval m,  $T_{ijr}^m \neq T_{jir}^m$ .  $R_o(\tau)$  denotes the set of subsequent bus trips at  $\tau$ .  $R_c(\tau)$  is the set of current running bus trips at  $\tau$ .  $R_{co}(\tau)$  refers to the set of current running bus trips and subsequent bus trips at  $\tau$ .

The traveling time cost of passengers at  $\tau$  can be expressed as follows:

$$f_{2}(\tau) = \gamma \sum_{i \in N} \sum_{r \in R_{co}(\tau)} q_{i} \max[\left(t_{i}^{r} - l_{i}\right), 0] y_{ir}$$

$$+ \delta \sum_{i \in N} \sum_{r \in R_{co}(\tau)} \left\{t_{s} + \max[\left(e_{i} - t_{i}^{r}\right), 0] y_{ir} B_{i}\right\}$$
(3)

The right of (3) includes the waiting time cost of passengers and those of passengers on the bus at demand points. In (3),  $\gamma$  is the penalty costs of passengers in unit time for early arrival, and N is the set of rest demand points, including newly added demand, at  $\tau$ .  $q_i$  is the number of passengers getting on the bus at demand point i, and  $\delta$  is the unit waiting time cost of passengers on the bus at the demand point.  $t_s$  is the service time of the feeder bus at demand point (it is a fixed value, and it is 0 at nodes in the road network).  $B_i$  is the number of passengers in vehicles when the bus trip r reaches the demand point i.

<sup>2</sup> 2) Joint Optimization Model of Running Route and Scheduling:

$$\min f(\tau) \tag{4}$$

$$T_{ijr}^{m} = \sum_{(k,l)\in\mathbf{\Psi}_{klr}^{m}} \sum_{t=m}^{M} \tau_{klr}^{t} \theta_{klr}^{t}, \mathbf{\Psi}_{klr}^{m} \in \mathbf{\psi}_{klr}^{m}$$
 (5)

$$t_i^r = t_j^r + T_{jir}^m + t_s + \max\left[\left(e_j - t_j^r\right), 0\right], \ \forall i \in N$$
 (6)

Subject to 
$$\sum_{r \in R_{co}(\tau)} y_{ir} = 1, \quad \forall i \in D$$
 (7)

$$\sum_{i \in N} \sum_{r \in R_{co}(\tau)} y_{ir} = |D| \tag{8}$$

$$N_r \cap N_{\bar{r}} = \emptyset, \quad \forall r \neq \bar{r}$$
 (9)

$$\sum_{i \in N} q_i y_{ir} \le Q_e - Q_r(\tau), \quad \forall r \in R_{co}(\tau) \quad (10)$$

$$B_i + q_i y_i^r \le Q_e - Q_r(\tau), \quad \forall i \in N$$
 (11)

$$\sum_{i \in DE} x_{ojr}^m = \sum_{i \in DE} x_{ior}^m \le 1, \quad \forall r, m$$
 (12)

$$\sum_{j \in DE} \sum_{r} \sum_{m=1}^{M} x_{ojr}^{m} = \sum_{i \in DE} \sum_{r} \sum_{m=1}^{M} x_{ior}^{m}$$
 (13)

$$\sum_{i \in DE} \sum_{j \in DE} T_{ijr}^{m} x_{ijr}^{m} + \sum_{i \in N} \left\{ \max \left[ \left( e_{i} - t_{i}^{r} \right), 0 \right] \right\}$$

$$+t_s$$
}  $y_{ir} \le T_{\max} - T_r(\tau)$ ,  $\forall r \in R_{c0}(\tau)$ ,  $\forall m$  (14)

where  $\psi_{klr}^m$  is the set of accessible routes for the bus trip r from nodes i to j within the time interval m,  $N_r$  is the set of demand points served by bus trip r,  $Q_e$  is vehicle capacity,  $Q_r(\tau)$  is the cumulative load of bus trip r at  $\tau$ ,  $T_{\text{max}}$  is the maximum one-way running time of vehicles, and  $T_r(\tau)$  is the cumulative running time of bus trip r at  $\tau$ .

Equation (4) is an objective function of the minimum total cost. Equations (5)-(6) are the calculation formulas of traveling time in the effective road section. Equations (7)-(9) mean that only one bus trip is available at each demand point, and the demand points for different bus trips vary. Equations (10)-(11) is the capacity constraint of vehicles at  $\tau$ . Equation (12) is the transit station constraint in the return trip. Specifically, all vehicles start from the transit station and return to the transit station through demand points and nodes in the road network. Equation (13) is the balance constraint of the number of bus trips, which requires that the sum of number of bus trips from the transit station is equal to that of bus trips returning to the transit station. Equation (14) is the running time constraints of vehicles at  $\tau$ .

During the entire dispatching cycle, a series of submodels  $\operatorname{sub} f(\tau)$  can be constructed on the basis of the quantitative batch decision points in response to the continuous submissions of real-time demand information of passengers to describe the joint optimization for RFT under mixed demands. The model in the initial moment is the joint optimization model with only reservation demand, and the transit station is the key point, while all reservation demands form the set of unfinished demand points. In this case, the solution is the initial running route and scheduling plan under reservation demand.

# C. Solving of Models

1) Solving the Traveling Time and Shortest Route Under the Time-Dependent Road Network: In the time-dependent road network, if the feeder bus starts from the starting point at the specified departure time and travels at the speed corresponding to the departure time, then the feeder bus will not be able to pass through the road section within time segment corresponding to departure speed when the moment arriving at a node is greater than the maximum moment of the time interval

corresponding to the travel speed. The speed step function proposed by Ichoua *et al.* [68] was applied to describe the time dependence of traveling speed. The dispatching cycle was divided into M time intervals according to changes of speeds, which were recorded as  $\pi_m = [c_m, \xi_m](m = 1, 2, ..., M)$ , for the convenience of calculation and without loss of generality. This study hypothesizes that the traveling speed in each time interval is constant.

The calculation steps of the road section traveling time when the departure time is given are as follows:

Step 1: Input departure time and node numbering of the road network,  $m \leftarrow 1$ ,  $d \leftarrow d_{ij}$ ,  $t \leftarrow t_i$ , and  $c_{ij}(t_i) \leftarrow 0$ ;

Step 2: Judge the time interval of the departure time. If  $t_i \le \xi_m$ , then go to Step 3; otherwise, m = m+1 until the time interval of the departure time is determined, and then go to Step 3;

Step 3: If  $(t + d/v_{ij}(\pi_m)) > \xi_m$ , then go to Step 4; otherwise,  $c_{ij}(t_i) \leftarrow c_{ij}(t_i) + d/v_{ij}(\pi_m)$ , and  $t_j \leftarrow t_i + c_{ij}(t_i)$ . Output  $c_{ij}(t_i)$  and  $t_j$ ;

Step 4: If  $c_{ij}(t_i) \leftarrow c_{ij}(t_i) + (c_{m+1} - t), d \leftarrow d - v_{ij}(\pi_m) \times (c_{m+1} - t), t \leftarrow c_{m+1}$ , and  $m \leftarrow m+1$ , then return to Step 3.

where  $t_i$  is the departure time of node i,  $d_{ij}$  is the length of the road section from nodes i to j,  $c_{ij}(t_i)$  is the traveling time from nodes i to j at  $t_i$ , m is the serial number of the time intervals, and  $v_{ij}(\pi_m)$  refers to the traveling speed corresponding to all directed road sections in the time interval m.

If T is used to express the minimum traveling time of bus trip r from nodes i to j, then the improved shortest route model for vehicles in the time-dependent road network can be expressed as follows:

$$T = \min T_{ijr}^{m} = \sum_{(k,l)\in\Psi_{klr}^{m}} \sum_{t=m}^{M} \tau_{klr}^{t} \theta_{klr}^{t}$$
 (15)

Subject to 
$$\sum_{(k,l)\in\Psi_{lt}^{m}}\sum_{t=m}^{M}\theta_{klr}^{t}\leq 1$$
 (16)

where  $\Psi^m_{klr}$  is a feasible route for bus trip r from nodes i to j within the time interval m, (k,l) is the directed sections in the feasible route  $\Psi^m_{klr}$ , and  $\theta^t_{klr}$  is a decision variable. If the bus trip r starts from nodes i to j within t intervals, then  $\theta^t_{klr} = 1$ ; otherwise,  $\theta^t_{klr} = 0$ .  $\tau^t_{klr}$  is the traveling time of bus trip r in the road section (k,l), and  $\tau^t_{klr} \neq \tau^t_{lkr}$  exists.

Equation (16) demonstrates that the directed road sections in the road network can only be used once at most for each bus trip.

2) Genetic Algorithm (GA): The two-stage strategy of generating initial running route and real-time dynamic adjustment is adopted in the joint optimization model. In the dynamic optimization stage of real-time demand, the new passenger real-time demand process is transformed into several instantaneous static sub-processes by introducing the concept of key points. The mixed integer non-linear programming model, which is constructed by (7)-(16), represents a more complicated vehicle routing problem (VRP) that is usually solved by heuristic algorithm [69]–[73], such as Genetic

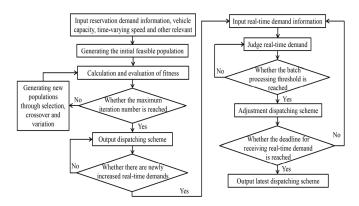


Fig. 3. Process of the adaptive GA.

Route layer	0	1	2	3	4	0	5	6	7	8	0	9	10	0
Scheduling layer	1	1	2	3	5	2	3	3	4	5	3	5	6	0

Fig. 4. Two-layer real number encoding diagram of chromosome.

Algorithm (GA), Ant Colony Algorithm (ACA), and Neighborhood Search Algorithm (NSA). In detail, GA is widely and commonly used in DVRP during studies. At first, Hanshar and Ombuki-Berman [74] used genetic algorithm to solve the DVRP model. After that, GA was used improved to solve the DVRP model conducted by Nakamura *et al.* [75], Ghannadpour *et al.* [50], Abdallah *et al.* [76]. As a result, these studies have shown that the GA has strong robustness and strong global convergence, which is particularly suitable for large-scale and multi-constraint optimization problems. The use of GA can efficiently solve the mixed-integer programming problem with a guaranteed accuracy of the solution [44], [77], [78].

In this section, an adaptive GA is designed for solving the joint optimization model by redefining an encoding scheme, a heuristic algorithm for generating initial population, and genetic operation based on the problem features. The detailed process of our proposed algorithm is shown in Fig 3.

a) Encoding scheme: A real number encoding system that involves two layers of "route layer + scheduling layer" was designed using chromosome coding to solve double decision-making on running route and scheduling in the joint optimization (Fig. 4). The route layer gene is directly represented by serial number of the node. The length of the chromosome is N + M + 1, where N is the number of nodes; M is the number of bus trips. The transit station is used to end the current running route and start a new one. In the scheduling layer, gene is the departure time of vehicles at nodes corresponding to the route layer.

The above-mentioned chromosome means that the passengers' demands can be met by three bus trips only, including bus trip I (departure at time 1): 0-1-2-3-4-0, bus trip II (departure at time 2): 0-5-6-7-8-0, and bus trip III (departure at time 3): 0-9-10-0.

b) Nearest-neighbor initialization of population: When the initial population is generated, the generation of a feasible solution itself is a NP-hard problem. Since the solving quality and efficiency of the heuristic algorithms depend on the quality of the initial solution to a large extent, this study designed an improved nearest-neighbor algorithm on the basis of the minimum cost (NNC algorithm) to generate the initial feasible population according to the initialization method of Solomon [79]. The process of NNC algorithm is introduced as follows:

Step 1: Given a fixed departure time, one bus trip departs from the transit station.

Step 2: The unfinished demand point nearest to the last service demand point in "distance" is chosen. If the demand point meets the constraint condition, it is inserted to the current running route.

Step 3: Repeat Step 2. If the demand point reaches the capacity limit or maximum running time limit of the current bus trip, then increase one bus trip until all demand points are finished. End of the calculation.

In this algorithm, "distance" (insertion cost) between two demand points  $(f_{ij})$  is defined as follows:

$$f_{ij} = \alpha_1 t_{ij}^r + \alpha_2 E_{ij}^r + \alpha_3 L_{ij}^r + \alpha_4 W_{ij}^r, \tag{17}$$

where  $t_{ij}^r$  is the traveling time of bus trip r,  $E_{ij}^r = \left| e_j - t_j^r \right|$  is the difference between the arrival and the starting times,  $L_{ij}^r = \left| l_j - t_j^r \right|$  denotes the difference between the arrival and the ending times,  $W_{ij}^r = t_{ij}^r + \max(0, (e_j - t_{ij}^r - t_s - l_i))$  is the sum of the traveling time and the minimum possible waiting time, and  $\alpha_i$  is a random parameter, and  $\sum \alpha_i = 1$ .

c) Individual evaluation and selection: The reciprocal of the cost objective function and penalty of infeasible routes was chosen as the fitness function for genetic operations. Finally, the chromosome with the maximum fitness was chosen as the optimal solution. The fitness function (F) is expressed as follows:

$$F = \frac{1}{f + M \times P_{10}} \tag{18}$$

where f is the objective function value, M is the number of infeasible routes, and  $P_w$  is the penalty weight of infeasible routes.

d) Crossover and mutation operations: During the selection operation, individuals with the maximum fitness in the population were directly replicated to the next generation. Subsequently, individuals were chosen according to the roulette wheel selection method. The adaptive GA proposed by Srinvivas [80] was applied to adjust the crossover and mutation stages because the double chromosome structure coding was applied. The route layer used sequential crossover and reversal mutation method, while the scheduling layer utilized the two-point crossover and k-mutation method. The adaptive crossover probability ( $P_c$ ) and mutation probability ( $P_m$ ) are expressed as follows:

$$P_{c} = \begin{cases} k_{1} - \frac{(k_{1} - k_{2})(F' - \bar{F})}{F_{\text{max}} - \bar{F}} & F' \geq \bar{F} \\ k_{1} & F' \leq \bar{F} \end{cases}$$
(19)

$$P_{m} = \begin{cases} k_{3} - \frac{(k_{3} - k_{4})(F - \bar{F})}{F_{\text{max}} - \bar{F}} & F \geq \bar{F} \\ k_{3} & F \leq \bar{F} \end{cases}$$
(20)

where  $k_i$  (i = 1, 2, 3, 4) is a constant within (0,1), F' is the maximum fitness of the two crossover individuals,  $\bar{F}$  is the

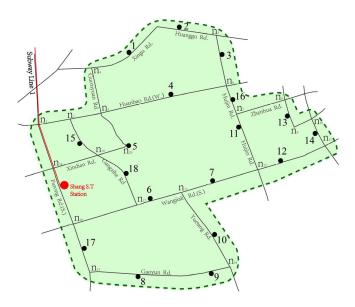


Fig. 5. Distribution of service scope and demand points.

TABLE I COORDINATE DISTRIBUTION OF DEMAND POINTS

Point	Longitude	Latitude	Point	Longitude	Latitude
0	112.998829	28.068004	10	113.018358	28.061095
1	113.007291	28.082976	11	113.021951	28.074738
2	113.013794	28.086864	12	113.026694	28.070833
3	113.01904	28.084363	13	113.027873	28.076308
4	113.012932	28.078498	14	113.031214	28.074332
5	113.007398	28.072283	15	113.001537	28.073379
6	113.010596	28.065717	16	113.020599	28.0780960
7	113.017979	28.068088	17	113.002076	28.059705
8	113.007434	28.056569	18	113.007286	28.068821
9	113.017998	28.056919			

mean fitness in each generation, and  $F_{\text{max}}$  is the maximum fitness of the population.

## III. CASE STUDY

#### A. Setting of the Basic Parameters

This study takes the surrounding area of Shang Shuangtang Station of Changsha Metro Line 1 in Changsha City as the service scope (dotted line in Fig. 5). Since the passenger flow in the service scope was mainly generated by commuting, it does not have the passenger flow conditions for providing traditional FRT service. Based on the analysis of passenger demand in the morning peak, the dispatching cycle of RFT system was set to 7:00-8:00. The passengers' demand points were 1-18. The longitudes and latitudes of all demand points were collected by the coordinate picker of the Baidu Map (Table I). The road network covered 20 nodes, which were  $n_1$ ,  $n_2, \dots, n_{20}$ . The traveling time of the road section is shown in Table II (only the study area). Within the service scope, the system had 85 passengers with reservation demands, and the distribution of reservation demand is shown in Table III. The system maintains 10 vehicles, the vehicle capacity  $Q_e =$ 15 people. The starting cost of vehicles was 20 yuan/trip. The time period for receiving real-time demand information was

TABLE II
TIME-VARYING SPEED (ONLY PART)

Time interval	Road section	Speed (km/h)	Time interval	Road section	Speed (km/h)
7:00-7:03	$(0, n_{18})$	33.7	7:03-7:06	$(n_1, n_2)$	29.5
7:00-7:03	$(0, n_{16})$	32.5	7:03-7:06	$(n_2, n_3)$	22.7
7:00-7:03	$(n_1, n_{18})$	31.8	7:03-7:06	$(n_2, n_{19})$	21.0
7:00-7:03	$(n_1, n_2)$	27.4	7:03-7:06	$(n_{16}, n_{17})$	32.4
7:00-7:03	$(n_2, n_3)$	23.1	7:06-7:09	$(0, n_{18})$	30.6
7:00-7:03	$(n_2, n_{19})$	25.3	7:06-7:09	$(0, n_{16})$	28.6
7:00-7:03	$(n_{16}, n_{17})$	35.2	7:06-7:09	$(n_1, n_{18})$	27.3
7:03-7:06	$(0, n_{18})$	31.2	7:06-7:09	$(n_1, n_2)$	22.9
7:03-7:06	$(0, n_{16})$	30.4	7:06-7:09	$(n_2, n_3)$	18.5
7:03-7:06	$(n_1, n_{18})$	28.5	7:06-7:09	$(n_2, n_{19})$	20.2

TABLE III
RESERVATION DEMAND INFORMATION

Point	Time	Number of	Point	Time	Number of
	window	passengers		window	passengers
1	[7:23,7:28]	4	7'	[7:42,7:47]	2
2	[7:16,7:21]	4	8	[7:05,7:10]	4
2'	[7:40,7:45]	3	8'	[7:22,7:27]	3
3	[7:30,7:35]	4	9	[7:08,7:13]	2
3'	[7:36,7:41]	3	9'	[7:27,7:32]	4
3"	[7:47,7:52]	4	10	[7:12,7:17]	2
4	[7:27,7:32]	2	10'	[7:24,7:29]	3
4'	[7:36,7:41]	4	11	[7:16,7:21]	2
4"	[7:50,7:55]	3	11'	[7:25,7:30]	2
5	[7:09,7:14]	3	12	[7:29,7:34]	2
5'	[7:50,7:55]	3	12'	[7:40,7:45]	2
6	[7:07,7:12]	3	13	[7:23,7:28]	3
6'	[7:18,7:23]	3	14	[7:32,7:37]	3
6"	[7:42,7:47]	2	14'	[7:38,7:43]	2
7	[7:36,7:41]	4			

set to 7:10-7:50. In addition,  $w_1 = 0.4$ ,  $w_2 = 0.6$ ,  $t_s = 1$  min,  $T_{\text{max}} = 40$  min, and  $T_{\text{min}} = 10$  min, and the quantitative batch threshold was  $\theta = 3$ . In the constructed model, the  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  values are economic costs per min, which are 0.5, 1.0, 0.3, and 0.2 yuan, respectively.

#### B. Optimization Results

This study implemented programming and solving according to above-mentioned passengers' demands and parameter setting on the basis of MATLAB 2018a by using a computer with 8-GB memory and Core i7 3.20-GHz CPU. The population size was set to 100, and the maximum iteration times were 200. During the crossover and mutation operations,  $k_1 = 0.8$ ,  $k_2 = 0.5$ ,  $k_3 = 0.1$ , and  $k_4 = 0.001$ . This experiment sets the departure interval at 5 min because the departure interval of the buses in the urban area of Changsha City is generally 5 min. Several vehicles can be sent at the same point in time, or departure at two intervals is acceptable.

1) Situations With Reservation Demand Only: Under the reservation demand in Table IV, the departure time, running routes, and running efficiency of the RFT system are obtained by applying the proposed model and algorithm (Table V).

TABLE IV
REAL-TIME DEMAND INFORMATION

Submission time	Point	Time window	Number of passengers
7:11	15	[7:33,7:38]	1
7:13	17	[7:16,7:21]	1
7:16	11	[7:19,7:24]	1
7:17	15	[7:23,7:28]	2
7:18	10	[7:28,7:33]	2
7:19	16	[7:25,7:30]	1
7:20	7	[7:24,7:29]	2
7:21	1	[7:32,7:37]	2
7:21	6	[7:41,7:46]	3
7:22	15	[7:24,7:29]	1
7:23	5	[7:24,7:29]	2
7:25	16	[7:33,7:38]	1
7:26	13	[7:36,7:41]	2
7:30	18	[7:32,7:37]	3
7:32	9	[7:43,7:48]	2
7:33	12	[7:41,7:46]	2
7:35	4	[7:44,7:49]	1
7:36	2	[7:43,7:48]	2
7:37	16	[7:40,7:45]	2
7:49	3	[7:42,7:47]	2

The convergence process of the optimal results is shown in Fig. 6(a). The algorithm converged at approximately 120 generations of iterations. Finally, the calculated total cost of the RFT system was 463.2 yuan.

2) Situations After the Newly Increased Real-Time Demands: Under the reservation demand in Table III, the newly increased real-time demands of the RFT system are shown in Table IV. When the batch processing threshold is  $\theta = 3$ , the proposed model and algorithm were applied, thereby obtaining the departure time, running route, and running efficiency of the RFT system after the newly increased real-time demands were obtained (Table VI). The converging process of the optimal results is shown in Fig. 6(b)-5(c). During the processing of the first batch of real-time demands, the algorithm converged at approximately 140 generations of iterations. As processes continue, the times of convergence iteration of the algorithm continuously decrease because the total number of unfinished demand points is declining. Finally, the calculated total cost of the RFT system was 534.7 yuan.

# C. Comparative Analysis Under Different Real-Time Demand Processing Strategies

The reference value of batches for the batch processing strategy was first set to  $\theta=3$  under the premise of fixed other conditions to analyze influences of different batch processing strategies (when  $\theta=1$ , the technique is immediate processing strategy) on the running efficiency of the RFT system. This study hypothesized that the proposed model and algorithm were applied when  $\theta=1$  and  $\theta=5$ , respectively. In this way, the running efficiencies of the time-dependent road network under different real-time demand processing strategies are shown in Table VII.

Table VII shows that when the immediate processing strategy is applied, the number of violation passenger time windows is increased by 6, which is attributed to the increased route adjustment frequency. The number of bus trips is increased by 1, and the average seat utilization is decreased by 4.2% with the increase in passengers with responsive real-time demands. The total cost of the RFT system is increased by 11.4 yuan. When the quantitative batch processing  $\theta=5$  is applied, the number of bus trips remains the same. However, the average seat utilization declines by 8.1% because the system decreases the number of passengers with responsive real-time demands. The number of violation passenger time windows is increased by 11 because the newly increased real-time demands are not processed timely. The total cost of the RFT system is lowered by 13.5 yuan.

# D. Effects of the Parameter Values on the Running Efficiency of the RFT System

1) Effects of Traveling Speed on Running Efficiency: This study hypothesized under other fixed conditions that vehicles travel at a constant speed to analyze the influences of time-dependence of road network on results. The results of investigation and data analysis indicated that the average traveling speed of medium buses in the urban area of Changsha City is about 20 km/h. On this basis, the traveling speed was increased and decreased by 15% to analyze the influences of the average traveling speed of urban roads on the practical operation of the RFT system. The calculated results are listed in Table VIII.

Table VIII demonstrates that the time-varying and constant traveling speeds lead to different average seat utilization and total costs of the RFT system even though the number of bus trips is the same. The average seat utilization of 9 bus trips under time-varying speed is 4.5% lower than that under a constant traveling speed (the number of responses to passengers' real-time demands is decreased), whereas the total cost of the RFT system is 32.6 yuan higher (the number of passengers violating the time window at demand points is increased). This finding reflects that the time-varying characteristics of vehicles influence the running efficiency of the RFT system to a certain extent, thereby agreeing with the practical situation. When the simulated traveling speed is 15% higher than the average one, the number of bus trips is decreased by 1, and the average seat utilization is increased by 7.5%. Accordingly, the objective function value is lowered by 43.9 yuan. When the simulated traveling speed is 15% lower than the average one, the number of bus trips is increased by 1, and the average seat utilization is decreased by 9.4%. Accordingly, the objective function value is increased by 54.8 yuan. If the traveling speed continues to decrease, then the system cannot meet a feasible solution to the constraint.

2) Effects of Vehicle Capacity and the Maximum One-Way Traveling Time on Running Efficiency: The influences of vehicle capacity on the running efficiency of the RFT system were analyzed by setting the reference value as 15 passengers/vehicle with a fluctuation range of 20% and fixing other conditions. Later, the influences of the maximum one-way traveling time on the running efficiency of the RFT system

249.4

85

70.8%

Summation

Departure Bus Travel Number of Seat End Running route time trip time (min) passengers served utilization time 7:00 24.9 7:25 1  $0-n_{16}-n_{15}-8-9-n_{14}-10-n_{13}-6-n_{17}-n_{16}-0$ 11 73.3% 7:05 60.0%  $0-n_{18}-n_{19}-5-n_3-n_4-2-n_5-n_6-4-n_3-n_2-n_1-n_{18}-0$ 35.2 9 7:41 7:10 3 30.8 10  $0 - n_{16} - n_{17} - 6 - n_{13} - n_{12} - 11 - n_7 - n_8 - 13 - n_9 - n_{10} - n_{11} - 12 - n_{12} - n_{13} - n_{17} - n_{16} - 0$ 66.7% 7:41 80.0% 7:10  $0-n_{18}-n_{1}-n_{2}-n_{3}-n_{4}-1-n_{5}-3-n_{6}-4-n_{3}-n_{2}-n_{1}-n_{18}-0$ 39.7 12 7:50 7:15 5 30.4 53.3%  $0 - n_{16} - n_{17} - n_{13} - 10 - n_{14} - n_{13} - n_{12} - 11 - n_7 - n_8 - n_9 - n_{10} - 14 - n_{11} - n_{12} - n_{13} - n_{17} - n_{16} - 0$ 8 7:46 7:25 35.2 15 100% 8:01  $0-n_{16}-n_{15}-8-9-n_{14}-n_{13}-7-n_{12}-n_{7}-n_{6}-3-n_{5}-n_{6}-n_{3}-n_{2}-n_{1}-n_{18}-0$ 26.9 7:25  $0\hbox{-} n_{16}\hbox{-} n_{17}\hbox{-} n_{13}\hbox{-} n_{12}\hbox{-} n_7\hbox{-} n_6\hbox{-} 3\hbox{-} n_5\hbox{-} 2\hbox{-} n_4\hbox{-} n_3\hbox{-} 4\hbox{-} n_6\hbox{-} n_3\hbox{-} 5\hbox{-} n_{19}\hbox{-} n_{18}\hbox{-} 0$ 12 80.0% 7:52 7:30 53.3%  $0 - n_{18} - n_{19} - n_{20} - n_3 - n_6 - n_7 - n_8 - n_9 - n_{10} - 14 - n_{11} - 12 - n_{12} - 7 - n_{13} - 6 - n_{17} - n_{16} - 0$ 26.3 8 7:57

 $\label{thm:table V} \text{Departure Time and Running Routes of Vehicles Under Reservation Demand}$ 

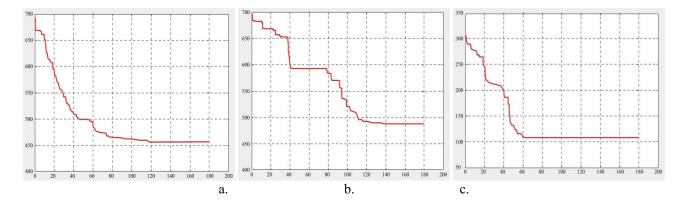


Fig. 6. Iterative curve. (a) Condition with reservation demand only. (b) The first time the newly increased real-time demands are processed. (c) The last time the newly increased real-time demands are processed.

TABLE VI
DEPARTURE TIME AND RUNNING ROUTES AFTER THE NEWLY INCREASED REAL-TIME DEMANDS

Departure time	Bus trip	Running route	Traveling time (min)	Number of passengers served	Seat utilization	End time
7:00	1	$0-n_{16}-n_{15}-8-9-n_{14}-10-n_{13}-6-n_{17}-n_{13}-n_{12}-11-n_{7}-n_{6}-n_{3}-n_{2}-15-n_{19}-n_{18}-0$	32.2	14	93.3%	7:33
7:05	2	$0-n_{18}-n_{19}-5-n_3-n_4-2-n_5-n_6-n_{17}-11-n_{12}-7-n_{13}-n_{14}-9-n_{15}-n_{16}-0$	36.7	15	100.0%	7:42
7:10	3	$0 - n_{16} - n_{17} - 6 - n_{13} - n_{12} - 11 - n_{7} - 16 - n_{6} - n_{7} - n_{8} - 13 - n_{9} - n_{10} - n_{11} - n_{12} - 7 - n_{13} - n_{17} - n_{16} - 0$	35.4	12	80.0%	7:46
7:15	4	$0-n_{16}-17-n_{15}-8-n_{14}-10-n_{13}-n_{12}-12-n_{11}-14-n_{10}-n_{9}-n_{8}-n_{7}-n_{6}-4-n_{3}-n_{2}-n_{1}-0$	38.9	13	86.7%	7:54
7:20	5	$0 - n_{16} - n_{17} - 18 - n_{19} - n_{20} - n_3 - n_6 - n_7 - n_8 - 13 - n_9 - n_{10} - n_{11} - 12 - n_{12} - n_{13} - 6 - n_{17} - n_{16} - 0$	33.6	10	66.7%	7:54
7:20	6	$0-n_{18}-n_{19}-15-n_2-n_3-n_4-1-n_5-3-n_6-16-n_7-n_6-4-n_3-n_2-n_1-0$	31.5	14	93.3%	7:52
7:25	7	$0-n_{18}-n_{19}-5-n_3-4-n_6-3-n_5-n_6-16-n_7-n_{12}-n_{13}-6-n_{17}-n_{16}-0$	29.6	11	73.3%	7:55
7:30	8	$0 - n_{18} - n_{19} - 18 - n_{17} - n_{13} - 7 - n_{12} - 12 - n_{11} - 14 - n_{10} - n_{11} - n_{12} - n_{13} - n_{17} - n_{16} - 0$	34.1	9	60.0%	8:05
7:35	9	$0-n_{18}-n_1-n_2-n_3-n_4-2-n_5-3-n_6-4-n_3-5-n_{19}-n_{18}-0$	26.9	13	86.7%	8:02
Summati	on	-	298.9	111	82.2%	-

TABLE VII

EFFECTS OF DIFFERENT REAL-TIME DEMAND PROCESSING STRATEGIES UNDER THE TIME-DEPENDENT ROAD NETWORK ON THE RUNNING EFFICIENCY

Processing strategy	Number of bus trips	Average seat utilization	Number of violation passenger time windows	Number of route adjustments	Total cost (yuan)
Immediate processing ( $\theta = 1$ )	10	78.0%	13	20	546.1
Batch processing ( $\theta = 3$ )	9	82.2%	7	7	534.7
Batch processing ( $\theta = 5$ )	9	74.1%	18	4	521.2

were discussed by setting the reference value as 40 min with a fluctuation range of 15%. The results are listed in Tables IX–X.

Table IX shows that when the vehicle capacity is increased to 18, the number of bus trips is unchanged. However, the average seat utilization is decreased by 10.6%, and the total cost of the RFT system is lowered by 17.8 yuan. When the vehicle capacity is decreased to 12, the number of bus trips is increased by 1 because the number of passengers with

reservation demand is unchanged. The average seat utilization is increased by 5.3%, and the total cost of the RFT system is increased by 16.5 yuan.

Table X shows that when the maximum one-way traveling time of vehicles is prolonged to 46 min, the number of bus trips is decreased by 1, the average seat utilization is increased by 4.5%, and the total cost of the RFT system is lowered by 19.3 yuan. When the maximum one-way traveling time of vehicles is shortened to 34 min, the number of bus trips is

TABLE VIII
EFFECTS OF TRAVELING SPEED ON RUNNING EFFICIENCY

Traveling speed	Number of bus trips	Average seat utilization	Total cost (yuan)
Time-varying speed	9	82.2%	534.7
Constant speed 1 (20km/h)	9	86.7%	502.1
Constant speed 2 (17km/h)	10	77.3%	556.9
Constant speed 3 (23km/h)	8	94.2%	458.2

TABLE IX
EFFECTS OF CARRYING CAPACITY OF VEHICLES ON
THE RUNNING EFFICIENCY

Vehicle capacity	Number of bus trips	Average seat utilization	Total cost (yuan)
12	10	87.5%	551.2
15	9	82.2%	534.7
18	9	71.6%	516.9

TABLE X
EFFECTS OF ONE-WAY TRAVELING TIME ON RUNNING EFFICIENCY

Maximum one-way traveling time (min)	Number of bus trips	Average seat utilization	Total cost (yuan)
34	10	73.3%	547.6
40	9	82.2%	534.7
46	8	86.7%	515.4

increased by 1, the average seat utilization is decreased by 8.9%, and the total cost of the RFT system is increased by 12.9 yuan.

#### IV. CONCLUSION

This study constructs a joint optimization model of running route and scheduling for multiple RFT vehicles under time-dependent road network. The batch processing strategy of real-time demands is proposed and the proposed model is created based on designing an adaptive genetic algorithm. Finally, the scientific of the proposed model and the validity of the algorithm are verified according to the case studies and the following conclusions can be obtained based on our studies:

- a) The proposed quantitative batch processing strategy for the newly increased real-time demands at different moments can save the calculation loads and route adjustment frequency significantly compared with the immediate processing strategy. This strategy is convenient for the management of the dispatching center. However, the batch processing strategy can decrease the number of passengers with real-time demands because it does not process the real-time demands of passengers. Nevertheless, the batch processing strategy can further decrease the responses of passengers with real-time demands with the increase in threshold value. If the threshold value of the batch processing strategy continues to increase, then the RFT system may be unable to respond to the real-time demands of passengers.
- b) The time-varying speed of the practical road network increases the number of violation passenger time windows compared with that under constant traveling speed, thereby decreasing the average seat utilization and adding to the total cost of the RFT system which reflect this concept relatively conforming to the practical situations.
- c) Given the same demands, the vehicle capacity and the maximum one-way traveling time limit can influence the total cost of the RFT system to a certain extent.

In addition, further studies will focus on the running route and scheduling of multiple vehicles of various modes which can simultaneously pick-up and delivery modes.

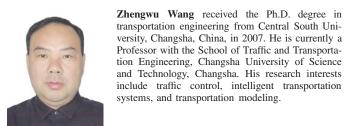
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