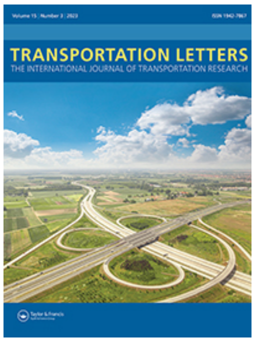

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Achieving robustness in the capacitated vehicle routing problem with stochastic demands

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ABSTRACT
Stochastic demands can impact the quality and feasibility of a solution. Robust solutions then become paramount. One way to achieve robustness in the Capacitated Vehicle Routing Problem with Stochastic Demands (CVRPSD) is to add a measure of the second-stage (recourse) distance to the objective function of the deterministic problem. We adopt variance as a measure of the recourse distance and propose a Mean-Variance (MV) model. To solve the model, a Hybrid Sampling-based solution approach is developed. Numerical experiments are conducted on benchmark instances and a selective waste collection system in Brazil. We compare our model with others from literature which also use a measure of the second-stage distance to attain robustness. The numerical results show that our model generates the most robust solutions. The comparison provides detailed features of each model and their advantages and disadvantages, helping decision-makers decide which model to utilize based on their different needs and priorities.

KEYWORDS
Vehicle routing problem; stochastic demand; robustness; mean-variance model; stochastic programming; selective waste collection; kerbside recycling collection

Introduction

Every decision-maker has to cope with uncertainty in daily logistics operations. The presence of uncertainty distinguishes today's decision-making problems from traditional deterministic mathematical optimization problems. One way to formalize uncertain information is modeling the uncertain input as a stochastic variable. Failing to recognize and deal with uncertainty may result in poor decisions (Calvet et al. 2019). Solutions designed without considering the stochasticity are very sensitive to perturbations on the uncertain input and can therefore become infeasible, suboptimal, or even both. This situation motivates great efforts on achieving solutions that are less affected by the various uncertainties encountered in a decision-making problem, namely robust solutions.

One decision-making problem facing a diversity of uncertainties is the Vehicle Routing Problem (VRP). The VRP is one of the most studied optimization problems in transportation and logistics and has many variants, e.g. VRP with Time Windows (VRPTW) (Schulze and Fahle 1999), Multi-Depot VRP (Azadeh and Farrokhi-Asl 2019), VRP with Pickup and Delivery (VRPPD) (Soleimani, Chaharlang, and Ghaderi 2018), and the Dial-a-Ride Problem (DARP) (Souza et al. 2021). The VRP can be defined as the problem of designing a route plan with minimum distance to attend a set of customers. The classical VRP is a decision-making problem under certainty, where all the problem inputs are available and fixed at the planning stage. Unfortunately, in the real world, uncertainties exist because of the time gap that separates the stages when route plans are designed and when they are executed (Toth and Vigo 2014). Uncertainties may come from different sources, among which customer demand is the most investigated. With consideration of stochastic demand and vehicle capacity constraints, the classical VRP becomes the Capacitated VRP with Stochastic Demand (CVRPSD). To solve the CVRPSD, usually a priori route plan is designed at the first stage (planning stage) considering the stochastic knowledge of demands, but their random nature could cause the a priori route plan to become infeasible. This means that when the real demand is revealed at the second stage (execution stage), the capacity constraint may fail to hold (Bertsimas and Sim 2004). This situation is referred to as a route failure and results in extra travel distance (second-stage distance). For instance, the additional distance covered by a vehicle to perform a detour to the depot for unloading since its capacity is depleted (Bernardo, Du, and Pannek 2021). The final distance traveled by a vehicle consists of first- and second-stage distances. A real-life example of the CVRPSD is waste collection systems. Although several papers have addressed this problem as the Capacitated VRP (CVRP), where demand is fixed and given explicitly (e.g. Teixeira, Antunes, and Sousa 2004; Hemmelmayr et al. 2013; Molina, Eguia, and Racero 2019; Tirkolaee et al. 2019), the volume of waste at any collection bin can be rarely a priori known with certainty in reality. Therefore, it may happen that after a vehicle completes part of its route, the quantity of collected waste reaches the maximum vehicle capacity. In this case, the vehicle has to return to the treatment location and unload before it resumes the following collection tasks according to the route plan. Such activity causes an additional traveled distance (Nambiar and Idicula 2013). To avoid route failure, and consequently additional traveled distance, techniques that ensure robustness have been proposed to solve the CVRPSD. While the definition of robustness is highly dependent on the decision-maker involved (Sörensen and Sevaux 2009), a robust solution is usually considered as a solution that resists perturbations in the stochastic inputs as much as possible (Solano-Charris, Prins, and Santos 2015). In this work, a route plan is considered robust when it is relatively insensitive to demand fluctuation, which means that minimum additional distance is incurred during the execution stage.

To guarantee the feasibility of a solution against fluctuation of the stochastic inputs, sub-optimal solutions are usually accepted as compromise (Sungur, Ordóñez, and Dessouky 2008), which indicates that robustness has a cost associated with it. This cost is called *Price of Robustness*, and it was introduced by Bertsimas and Sim (2004). This *Price* is the cost paid for being safe against changes in

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the stochastic inputs. Solutions at high *Price* are conservative in the sense that one has to sacrifice optimality to ensure robustness. A number of studies on the CVRPSD aimed to find not overly conservative solutions. Sörensen and Sevaux (2009); Bernardo and Pannek (2018) proposed mathematical models that are able to modify the level of robustness of route plans. Both models followed the idea of adding a measure of the second-stage distance to the objective function of the CVRP to achieve robustness. This idea was also adopted in the Stochastic Programming with Recourse (SPR) model, which has been applied to solve the CVRPSD (Salavati-Khoshghalb et al. 2019). Nevertheless, these models considered different measures of the second-stage distance. SPR models employed the expected value of the second-stage distance; Sörensen and Sevaux (2009) model applied the standard deviation; and Bernardo and Pannek (2018) model included a measure similar to the mean absolute deviation.

This paper aims to develop a mathematical model for the CVRPSD to achieve robustness by incorporating a measure of the route-failure distance to the objective function of the CVRP model. The objective function of the proposed model is based on the Mean-Variance (MV) function introduced by Mulvey, Vanderbei, and Zenio (1995). A solution hardly remains both robust and optimal for all realization of the uncertainties, and therefore, the objective function in the proposed model combines both aspects, optimality and robustness, into a scalar one by incorporating a weight. To solve the MV model, we propose a Hybrid Sampling-based (HyS) solution approach. To validate the proposed mathematical model, we compare the MV solutions with the solutions obtained with four models from the literature on benchmark instances. The comparison is made in terms of not only the objective function value at the planning stage and after the execution stage but also the incurred additional distance at the execution stage. The latter shows the level of robustness of the solutions. The four models are deterministic (Integer Linear Programming – ILP), SPR and the models proposed in Sörensen and Sevaux (2009); Bernardo and Pannek (2018). By adopting the deterministic model to solve the CVRPSD, we assume that the demands are precisely known and equal to the nominal values. By following this approach, which we call deterministic approach, we disregard all scenarios different than the one (nominal scenario) used to calculate the optimal solution. For these scenarios, the solution calculated using the nominal values might be suboptimal or even infeasible. Therefore, the deterministic approach serves in this study as the baseline approach for not considering the impact of uncertainties on the quality and feasibility of the solution. That is, by using the deterministic approach to solve the CVRPSD one aims just at optimality for the nominal values. On the other hand, SPR Sörensen and Sevaux (2009),; Bernardo and Pannek (2018) models aim at robustness by including different measures of the second-stage distance in the CVRP objective function. The proposed solution approach is also applied to a real-life selective waste collection problem in Brazil as a case study.

The contributions of this study are fourfold. (1) We propose a MV model for the CVRPSD, which is able to calculate solutions without incurring any second-stage distance (as shown in the computational results). To solve the proposed MV model, (2) we developed a multi-stage HyS solution approach that is a simple and practical way to achieve robustness in the CVRPSD. HyS uses a set of scenarios to create a single instance of the stochastic problem, and then solves this instance via three well-established heuristics with capability to handle large-scale CVRPSD problems. (3) We show that by solving a single instance, one can find a solution that incurs little additional distance during the execution stage, i.e. a robust solution, and that although the variable demand is not present in the CVRPSD's objective function, the demand values used to calculate a priori solution impact the level of robustness of the solution. (4) We provide a comprehensive comparison with multiple models from literature on both benchmark instances and real-life case study. The comparison clearly shows the features of each model and their advantages and disadvantages, supplying valuable information to help decision-makers decide which model to utilize based on their different needs and priorities.

The structure of the paper is as follows. In the subsequent section, we present the relevant literature. The literature review first includes a review of the concept of robustness and its *Price*, and then a review of solution methods to achieve robustness in the CVRPSD. In Section 3, the formulation of the proposed mathematical optimization model is shown together with the models from the literature used for comparison. The proposed solution approach is presented in Section 4. In Section 5, we describe the performance measures used in the comparison and present the computational experiments on benchmark instances and the application of the solution approach to a real-life selective waste collection system in Brazil. Finally, the work is concluded in Section 6.

Literature review

In this section, we first discuss the concept of robustness and the cost decision-makers must pay when choosing robust solutions over nominal ones, the so-called *Price of Robustness*. After that, we introduce the deterministic approach, and then review the literature on solution approaches to achieve different types of robustness in the CVRPSD.

Definitions of robustness and its price

The definition of robustness can differ greatly in the literature. Yet, what most decision-makers refer to as a robust solution is a solution that behaves well in (slightly) different conditions, meaning that it is immune to small changes in the conditions it was designed for (Barrico and Antunes 2006).

Mulvey, Vanderbei, and Zenio (1995) differentiated two terms: solution robustness and model robustness. Solution robustness corresponds to optimality, whereas model robustness to feasibility. The term solution robustness appears when a solution remains optimal for all the scenarios of the input data. On the other hand, a solution is said to be model robust when it remains feasible for all the scenarios of the input data. Bernardo and Pannek (2018) acknowledged robustness similarly to Mulvey, Vanderbei, and Zenio (1995), but instead of solution robust and model robust they used the terms optimality and robustness, respectively. Another term for model robustness is the strict robustness introduced by Soyster (1973). A strictly robust solution is one that is feasible for any realization of the uncertain inputs. To address a robustness level between solution and model robustness, Liebchen et al. (2009) introduced the term recoverable robustness. A recoverable robust solution is a solution that is 'almost' feasible for any realization of the uncertain inputs and therefore needs few recoveries (second-stage cost) when the real inputs are revealed. Similarly, Sörensen and Sevaux (2009) referred to a robust/flexible solution as the one with low additional second-stage cost. Instead of recoverable robustness, Ben-Tal et al. (2004); Dhamdhere et al. (2005); Thiele, Terry, and Epelman (2010) adopted the terms adaptive robustness, demand-robustness, and two-stage robustness, respectively.

In this study, we use the term optimality to refer to solution robustness and strict robustness to refer to model robustness, and the term robustness by itself refers to the capacity of a route plan to deal with variations on the stochastic demand. We assume that a solution (route plan) is strictly robust when it can endure perturbation of demands without second-stage distance, i.e. a route plan that is completely insensitive to changes in the demands defined in a possible demand range. Whereas a recoverable robust route plan is one that requires little additional second-stage distance as the real demands are revealed.

Imposing protection by creating solutions that are (recoverable or strictly) robust leads to a price, called *Price of Robustness* (Bertsimas and Sim 2004). This price refers to the price one needs to pay to allow for certain deviations within the stochastic variables, and it is usually defined as the difference between the cost of a robust solution and the cost of the nominal solution (deterministic approach). The *Price of Robustness* is a consequence of restricting the set of feasible solutions to the (in general smaller) set of robust solutions (Marotta et al. 2017). Therefore, to protect a solution against fluctuations on the stochastic inputs, we need to accept a suboptimal solution (Sungur, Ordóñez, and Dessouky 2008). Risk-tolerant decision-makers accept less protection and acquire a reduced *Price*. In contrast, risk-averse decision-makers seek higher protection, but are subjected to a higher *Price*. Robust solutions can be too conservative when we must give up too much of optimality for the nominal problem to ensure robustness (Bertsimas and Sim 2004). A solution will hardly remain both robust and optimal for all realization of the uncertainty (Mulvey, Vanderbei, and Zenio 1995). Thus, there exists a trade-off between optimality and robustness.

Optimality and robustness in the CVRPSD

Based on the definitions of robustness presented in the previous section, we classify the approaches to solve the CVRPSD into three groups: approaches that aim at optimality, approaches that aim at recoverable robustness, and approaches that aim at strict robustness (see Figure 1). Examples of these approaches are explained as follows.

A simple modeling approach that aims at optimality is the deterministic approach. In this approach, the CVRPSD is formulated as the CVRP. Accordingly, one chosen instance of the stochastic demands is supplied to an ILP model. The chosen instance represents the most likely estimate of the realization of the demands in the future. In other words, the value of the demands in the chosen instance are equal to their expected (nominal) values. This is mathematically expressed as follows.

The CVRPSD can be represented on a fully connected undirected graph $G = (N, A)$, where $N = (0, 1, 2, 3 \dots n)$ is the node set (node 0 is the depot) and $A = \{(i,j) | i,j \in N, i \neq j\}$ is the arc set. There are, thus, $|A| = n(n+1)/2$ arcs in G. d_i denotes the demand

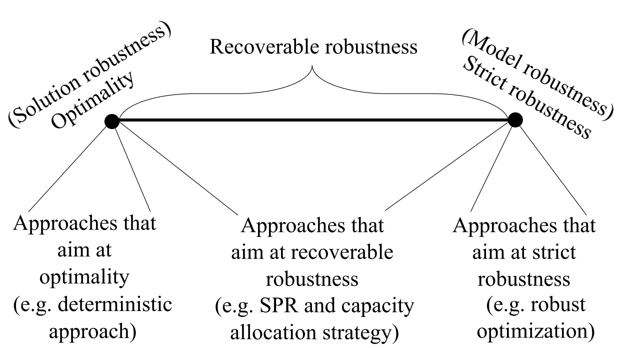

Figure 1. Levels of robustness and respective approaches.

that has to be collected from (delivered at) customer $i \in N$ ($d_0 = 0$). Suppose that customer demands are known as random variables, $d_i : \Omega_i \to \mathbb{R}_0^+ \quad i \in N$ with sampling spaces Ω_i, and its exact values are only revealed at runtime. The vehicle fleet $K = \{1, 2, \dots, k\}$ is homogeneous, hence there exist k vehicles with identical capacity C. The distance between customer i and j is represented by c_{ij}. This coefficient is calculated by using the Euclidean distance. The decision variable x_{ij} indicates whether a vehicle proceeds from customer i to customer j. A solution y to the problem is called route plan and consists of k feasible routes, that is, number of routes equals number of vehicles used in the solution. The total distance of a solution y is described by $J(y)$. A feasible route u_k is executed by vehicle k that departures from the depot, attends a subset of nodes whose total demand does not surpass C, and then returns to the depot. If $T \subseteq N$ is an arbitrary subset of nodes, then $m(T)$ denotes the minimum number of vehicles required to serve T. $m(T)$ can be calculated by solving a Bin Packing Problem (BPP) with item set T and bin's capacity C. For T, let $\delta(T) = \{(i,j) : i \in T, j \notin T \text{ or } i \notin T, j \in T\}$ and $\delta(i) := \delta(\{i\})$ be a singleton set $T = \{i\}$ 2014. The ILP model, proposed by Laporte, Nobert, and Desrochers (1985), is presented as follows. In this paper, we call this model the Deterministic Model (DM).

Definition 1 (DM)

$$\min_y J_0(y) := \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} [0], \tag{1}$$

$$\text{s.t.} \sum_{(i,j) \in \delta(i)} x_{ij} = 2 \qquad i \in N \backslash \{0\}, [0], \tag{2}$$

$$\sum_{(i,j) \in \delta(0)} x_{ij} = 2K, [0], \tag{3}$$

$$\sum_{(i,j) \in \delta(S)} x_{ij} \geq 2m(T) \qquad T \subseteq N \backslash \{0\}, T \neq \emptyset, [0], \tag{4}$$

$$x_{ij} \in \{0, 1\} i, j \notin \delta(0), [0], \tag{5}$$

$$x_{ij} \in \{0, 1, 2\} i, j \in \delta(0). \tag{6}$$

In this model, objective function (1) is minimizing the total traveled distance. Constraint (2) ensures that each node is visited by one incoming and one outgoing vehicle. Constraint (3) guarantees that k routes are created. Connectivity and vehicle capacity requirements are imposed by Constraint (4) by forcing a sufficient number of arcs to enter each subset of nodes. Constraints (5) and (6) determine the decision variables. In this model, if $i, j > 0$, then x_{ij} can only take the value 0 or 1. However, if $i = 0$, x_{ij} can be equal to 2 when a vehicle visits a single client. As the BPP is a NP-hard problem, $m(T)$ could be approximated from below by any lower bound, such as $\sum_{i \in N} E[d_i \backslash C]$

The deterministic approach described above does not take into account the impact of uncertainties on the quality and feasibility of the solutions. Hence, the solutions designed with this approach are likely to be sensitive to perturbations of demands, becoming infeasible, suboptimal, or even both, at the execution stage. The goal of this approach is to achieve optimality (solution robustness) as it designs a route plan that is optimal for the nominal values of the uncertain demands. In contrast, there are approaches that aim at (recoverable or strict) robustness in the CVRPSD. The most adopted approaches to achieve robustness in the stochastic VRP are Stochastic Programming (SP) and Robust Optimization (RO) (Ordóñez 2014). Another approach for attaining robustness in the CVRPSD encountered in the literature is called capacity allocation strategy. It is important to highlight that despite the ability of these approaches of producing robust route plans yet not all studies that apply them mention the term robustness. In the following, we review those approaches and present papers that have acknowledged the concept of robustness while applying them.

SP aims at achieving recoverable robustness. There exist two general modeling techniques to formulate the CVRPSD using a two-stage SP approach, Chance-Constrained Programming (CCP) and SPR. The difference between these techniques lies in the goal of the first stage (Pillac et al. 2013). In the CCP, the goal is to find a priori route plan of lowest cost while ensuring an upper bound on the probability of a route failure, regardless of the second stage expected cost, whereas SPR aims at minimizing the a priori route plan cost plus the expected second-stage cost (the total expected cost) (Bernardo and Pannek 2018). Although these two modeling approaches can achieve recoverable robustness in the CVRPSD, to the best of our knowledge, there is no paper that adopts them for such purpose. SP models were proposed by Sörensen and Sevaux (2009); Bernardo and Pannek (2018) to attain recoverable robustness in the CVRPSD. Sörensen and Sevaux (2009) replaced the second term of the SPR model's objective function by the objective value standard deviation in a set of demand scenarios. On the other hand, Bernardo and Pannek (2018) replaced the same term by a measure similar to the objective function mean absolute deviation in a demand scenarios set. Another approach that aims at recoverable robustness is the capacity allocation strategy. This strategy is a natural way to cope with uncertain demands in which vehicle capacity is saved to handle situations when the real demand is greater than the nominal demand value (Sungur, Ordóñez, and Dessouky 2008). In the capacity allocation strategy, only a percentage of the vehicle capacity is considered at the planning stage. The remaining capacity works as a buffer to deal with demand fluctuations that might happen in the execution stage. When the buffer is not enough, then a route failure happens and second-stage distance is incurred. This strategy was adopted in the CVRPSD by Juan et al. (2011).

RO aims at achieving strict robustness. This method assumes that the uncertain demands belong to a deterministic set (Lee, Lee, and Park 2012) and that all uncertain realizations are equiprobable (Eufinger and Buchheim 2018). RO has been adopted to attain strict robustness in the CVRPSD by Sungur, Ordóñez, and Dessouky (2008); Gounaris, Wiesemann, and Floudas (2013), and Gounaris et al. (2016). RO avoids the second-stage distance as it looks for a route plan that is feasible for any realization of the uncertainty set. That is, RO calculates a route plan that performs well even in the worst case of the deterministic set. However, it is not always easy or even tractable to analytically determine the worst-case performance of a given solution (Sörensen and Sevaux 2009). RO solutions are usually conservative (Maggioni, Potra, and Bertocchi 2015). The solution that has the best worst-case performance (RO) will generally be more conservative than the one that has the best average-case performance (SP) (Sörensen and Sevaux 2009), resulting in a higher *Price of Robustness*.

In this study, we do not follow RO's idea of seeking to determine a route plan that remains feasible for all possible demand realizations – and be likely to pay a high *Price*. We want to allow decision-makers to choose the solution's level of robustness, and consequently the *Price* they need to pay for that safety. Juan et al. (2011) argued that finding solutions of different robustness levels could be achieved through the capacity allocation strategy by changing the percentage of the vehicle capacity available at the planning stage. Nevertheless, we do not want to limit/restrict the resources available for vehicle routing. Thus, we propose a mathematical model that can trade-off optimality and robustness. In the MV proposed model, optimality is represented by the mean of the first-stage distance, and robustness is expressed as the variance of the second-stage distance. The SPR model and the models proposed in Sörensen and Sevaux (2009) and Bernardo and Pannek (2018) also represent robustness as a second-stage distance measure, but they adopted different measures. In this way, a comparison between the performance of the proposed model against the performance of SPR, Sörensen and Sevaux's and Bernardo and Pannek's models is relevant. This comparison is made with respect to first-stage and second-stage distances and *Price of Robustness*. We also consider the performance of the deterministic approach in the comparison.

To solve the proposed MV model, and make a comprehensive comparison analysis between the models mentioned above, we introduce the HyS solution approach. HyS is a simple and practical approach to achieve robustness in the CVRPSD. It uses a set of demand scenarios to create a robust CVRP instance, and then solves this instance via Clarke and Wright (C&W) savings (Clarke and Wright 1964), 2-Opt Local Search (2-Opt LS) (Lin 1965), and Simulated Annealing (SA) (Kirkpatrick, Gelatt, and Vecchi 1983). By calculating and then solving a robust CVRP instance, one does not ignore the concept of robustness. The demands in this instance are likely to be higher than their expected demand values. Therefore, fewer customers will be attended in a single route, designing a shorter route. Accordingly, route failures will tend to not occur, and second-stage (recourse) distance will be lower. Since a robust solution is one that incurs little additional distance during the execution stage, by solving this CVRP instance HyS creates a robust route plan. In this study, we also want to demonstrate that although demands do not appear in the objective function of the CVRPSD, the demand values adopted for finding a priori route plan influence how robust this route plan can be. As the solution approach reduces the complex CVRPSD to a CVRP instance, HyS can then make use of these fast and extensively tested heuristics. This adds confidence on the quality of the solution and enables the solving of CVRPSD instances of any size. Another advantage of the proposed solution approach is that it can consider any probability distribution with a known mean and different distribution for each demand.

Model formulation

In this section, we first describe three models from the literature, which adopted a measure of the second-stage distance to achieve robustness. These models, namely SPR, Sörensen and Sevaux's (SD model), and Bernardo and Pannek's (Rob model), are used in this study for comparison. After that, we present the mathematical formulation of the proposed MV model.

Multiple models with robustness

The CVRPSD is usually formulated as a SPR model with an objective function that minimizes the final distance. The final distance consists of the a priori route plan distance and the expected second-stage distance. In SPR, some capacity constraints are relaxed and included in the objective function, assuming that violations induced by random demand during the execution of the a priori route plan can be fixed by recourse (corrective) actions (Solano-Charris, Prins, and Santos 2015). A typical SPR model for the CVRPSD is introduced as follows (Birge and Louveaux 2011).

Definition 2 (SPR)

$$\min_{y} J(y) := J_0(y) + E[Q(y)], \tag{7}$$

s.t. (2), (3), (4), and (6)

where $J_0(y)$ is the objective function value of the a priori route plan (solution designed with the expected demand values) and $Q(y)$ is the distance traveled during the second stage. The latter depends on the recourse policy defined by the decision-maker. A recourse policy outlines the actions to be taken for correcting the solution after a route failure (Oyola, Arntzen, and Woodruff 2018). There are mainly two corrective policies in the CVRPSD, preventive restocking and detour-to-depot. Detour-to-depot is the most used recourse policy in the literature (Laporte, Louveaux, and van Hamme 2002; Erera, Morales, and Savelsbergh 2010; Juan et al. 2011), and requires the vehicle to return to the depot to deplete when its capacity is exceeded. We can evaluate objective function (7) only a posteriori, that is, upon completion when all real demands are revealed, or via stochastic analysis. To partially overcome this, Monte Carlo sampling of the stochastic parameters can be used. A set of samples, defined as

$$P = \{p_\alpha = (d_1^\alpha, d_2^\alpha, d_3^\alpha \ldots, d_n^\alpha) | \alpha = 1, \ldots, z\}, \tag{8}$$

in which a sample determines the demand for each customer $i \in N$, and a set of scenarios as

$$S = \{s_\alpha = (p_\alpha, N, A) | \alpha = 1, \ldots, z\}, \tag{9}$$

are used to compute the second-stage distance. As all the scenarios have the same set of nodes N and arcs A and identical coefficients c_{ij}, each scenario s_α can be considered as a deterministic instance of the CVRPSD. The recourse model with detour-to-depot is obtained by introducing an additional binary recourse variable $r_{k,i}^\alpha$. This variable indicates whether the capacity of vehicle k is depleted, while the vehicle executes its route u_k (in the a priori route plan), at customer $i \in u_k$ considering s_α. When the capacity of vehicle k is depleted at customer i considering s_α, then

$$r_{k,i}^\alpha = \begin{cases} 1, & ifcapacityconstraintholds \\ 0, & else. \end{cases} \tag{10}$$

The expected second-stage distance could be thus approximated by

$$E[Q(y)] = \frac{1}{z}\left(\sum_{\alpha=1}^{z}\sum_{k\in K}\sum_{i\in u_k} r_{k,i}^\alpha (2c_{i0})\right). \tag{11}$$

Consequently, the objective function (7) considering detour-to-depot as the corrective action becomes

$$\min_{y} (y) := J_0(y) + \frac{1}{z}\left(\sum_{\alpha=1}^{z}\sum_{k\in K}\sum_{i\in u_k} r_{k,i}^\alpha (2c_{i0})\right). \tag{12}$$

Since SPR model's goal is to minimize not only the distance of the a priori solution but also the expected second-stage distance due to route failures, solutions calculated by the SPR model are recoverable robust. In a SPR model, the expected second-stage distance is added to the objective function of the classical CVRP model $J_0(y)$ to achieve recoverable robustness. The expected second-stage distance term was replaced by a measure similar to the mean absolute deviation in the Rob model proposed by Bernardo and Pannek (2018). The objective function of the Rob model is as follows.

$$\min_{y} J_{Rob}(y) := J_0(y) + \omega\left(\frac{1}{z}\right)\left(\sum_{\alpha=1}^{z}(Q(y, s_\alpha) - Q(y))\right), \tag{13}$$

where $Q(y, s_\alpha)$ is the second-stage distance of solution y in scenario s_α, and ω – which takes a value $[0, \infty)$ – reflects the risk preference of the decision-maker. That is, decision-makers that have risk aversion could increase ω so that the designed solutions are more robust against demand fluctuation.

The model proposed by Sörensen and Sevaux (2009) also incorporated the risk preference of the decision-maker through ω, but the measure of the second-stage distance which is multiplied by the decision-maker's weight is different. Sörensen and Sevaux (2009) added the final distance standard deviation to the CVRP's objective function to attain recoverable robustness. Hence, the objective function (7) was replaced by

$$\min_{y} J_\sigma(y) := J_0(y) + \omega\sqrt{\left(\frac{1}{z-1}\right)\sum_{\alpha=1}^{z}\left(J_f(y, s_\alpha) - J_0(y)\right)^2}, \tag{14}$$

where $J_f(y, s_\alpha)$ is the objective function value of solution y in scenario s_α. This objective function value is the final traveled distance considering the demands defined by p_α, the sets N and A, and detour-to-depot as the corrective policy. Considering Equation (10), then

$$J_f(y, s_\alpha) = J_0(y) + \sum_{k\in K}\sum_{i\in u_k} r_{k,i}^\alpha (2c_{i0}). \tag{15}$$

We call the model proposed by Sörensen and Sevaux (2009) SD model. It can be seen that the difference among the objective functions of SPR (12), Rob (13), and SD (14) models is associated with the term that measures robustness.

Mean-variance formulation

According to Mulvey, Vanderbei, and Zenio (1995) and Sörensen and Sevaux (2009), any measure of variability can represent robustness. In this paper, the variance of the final distance is adopted as a measure of robustness in the CVRPSD. The complete formulation of the MV model is presented as follows.

We include p_0 and s_0 in the set of samples (8) and the set of scenarios (9), respectively. Thus, the sample set (8) becomes

$$P = \{p_\alpha = (d_1^\alpha, d_2^\alpha, d_3^\alpha \ldots, d_n^\alpha) | \alpha = 0, \ldots, z\}, \tag{16}$$

and the set of scenarios (9) becomes

$$S = \{s_\alpha = (p_\alpha, N, A) | \alpha = 0, \ldots, z\}. \tag{17}$$

In sample p_0, demand is equal to its expected value, $d_i^0 = E[d_i]$, and s_0 is called the nominal scenario. The vector of final distances of route plan y associated with the set of scenarios is represented by

$$M = \{J_f(y, s_0), J_f(y, s_1), \ldots, J_f(y, s_z)\}. \tag{18}$$

The measure of robustness is the sample estimator of the variance of the vector of final distances,

$$\sigma^2(y) = \frac{1}{z-1}\sum_{\alpha=1}^{z}\left(J_f(y, s_\alpha) - \hat{M}\right)^2, \tag{19}$$

and the measure of optimality is the mean sample estimator of this vector,

$$\hat{M} = \frac{1}{z}\sum_{\alpha=1}^{z} J_f(y, s_\alpha). \tag{20}$$

By combining these two equations with a weight ω, we get

$$J_{MV}(y) = \hat{M} + \omega\sigma^2. \tag{21}$$

Given that $\hat{M} = J_f(y, s_0) = J_0(y)$, the objective function of the proposed MV model for the CVRPSD becomes

$$J_{MV}(y) = J_0(y) + \omega\sigma^2, \tag{22}$$

and Eq. (19) is changed to

$$\sigma^2(y) = \frac{1}{z-1}\sum_{\alpha=1}^{z}\left(J_f(y, s_\alpha) - J_0(y)\right)^2. \tag{23}$$

To sum up, the complete MV model for the CVRPSD is shown as follows.

Definition 3 (MV)

$$\min_y J_{MV}(y) := J_0(y) + \omega\left(\frac{1}{z-1}\right)\sum_{\alpha=1}^{z}\left(J_f(y, s_\alpha) - J_0(y)\right)^2 \tag{24}$$

$$s.t., (2), (3), (4), (5) \text{ and } (6).$$

where ω represents the parameter of choice for decision-makers to adjust the weight of each goal in the objective function. Specially, when $\omega = 0$, the MV model becomes the DM. In other words, when $\omega = 0$ we are solving the problem for when all demands are assumed to be equal to their expected values. The solution obtained from the DM is very sensitive to demand change, whereas the route plan obtained with greater ω is much less sensitive (Mulvey, Vanderbei, and Zenio 1995; Bernardo and Pannek 2018).

Hybrid sampling-based solution approach

In this section, a Hybrid Sampling-based (HyS) solution approach is developed, which consists of four stages and three heuristics, C&W savings, 2-Opt LS, and SA. A complete flow of the solution approach is shown in Figure 2. Similar to the Sample Average Approximation (SAA) method introduced by Verweij et al. (2003), the HyS approach adopts a scenario set based on the distribution of the stochastic inputs to create a single deterministic optimization problem, which is then solved by deterministic optimization techniques. The difference between the proposed solution approach and SAA method is that the latter uses a set of scenarios to obtain different candidate solutions to the stochastic problem, whereas HyS uses a scenario set to design a single robust scenario, and then a robust solution. We introduce HyS because one of the study's aims was to demonstrate that the values adopted for the demands in the first stage – when a priori route plan is designed – can influence how robust the resulting route plan is, though the demand variable is not present in the CVRPSD's objective function. The HyS solution approach adopts the set of scenarios S, presented in Eq. (17), to design the so-called MV scenario represented by

$$I^{MV} = \{d_1^{MV}, d_2^{MV}, d_3^{MV} \ldots, d_n^{MV}\}. \tag{25}$$

In the MV scenario, each demand is computed as follows

$$d_i^{MV} = d_i^0 + \omega\left(\frac{1}{z-1}\right)\sum_{\alpha=1}^{z}\left(d_i^\alpha - d_i^0\right)^2 \qquad i \in N. \tag{26}$$

Eq. (26) is developed based on the objective function (24). For comparison purposes, HyS also uses the set of scenarios S to design one robust scenario for each of the three models from literature that achieve robustness, namely, Rob, SPR, and SD (stage 3). For that, HyS uses three demand equations (one for each model). Table 1 presents both the robust scenario and demand equation adopted for the three models. For instance, the demand equation used to create the robust scenario I^{Rob}, i.e. Eq. (13), is derived from the objective function (13). In total, four robust scenarios are created ($I^{MV}, I^r, I^{Rob}, I^\sigma$). After that, HyS solves each of these robust scenarios (stage 4), computing one route plan for each scenario ($y^{MV}, y^r, y^{Rob}, y^\sigma$). It is important to highlight that all four models (MV, Rob, SPR, and SD) follow the idea of adding a second-stage distance measure to the CVRP's objective function to achieve robustness. The difference between these models is that each adopts a different measure. That is why in this study we compare the solutions obtained with these demand equations. We also compare the robust solutions with the route plan calculated with the deterministic approach. In the deterministic approach, which is the baseline approach for aiming only at optimality, all demands are assumed to be equal to their expected values.

Since each robust scenario ($I^{MV}, I^r, I^{Rob}, I^\sigma$) is a deterministic instance of the problem, well-established heuristics can be applied to searching optimal solutions. First, we use C&W savings to calculate an initial route plan. The C&W algorithm is one of the most used constructive heuristics for VRPs. The reason is its simplicity and the ease with which it can be adapted to handle VRP variations (Stewart and Golden 1983). C&W is based on the concept of savings. The algorithm begins from a solution formed by a set of n round routes ($0 - i - 0$). At each iteration, two routes with i and j as the last customers are merged if a cost saving ($S_{ij} = c_{0i} + c_{0j} - c_{ij}$) can be obtained and capacity constraints are respected. The savings of all possible mergers are stored in decreasing order. C&W then merges the routes that provide the largest saving. The procedure continues until no further merges are possible. Then, the C&W solution is improved by the 2-Opt LS approach. Constructive heuristics, such as C&W, are usually followed by an improvement heuristic. An improvement heuristic, such as the 2-Opt LS approach, examines a neighborhood of a feasible solution in order to find an improving solution. The 2-Opt LS approach used in this work is the 2-Opt with intra-route improvements. It means that the method performs swaps between all combinations of customers at two positions within all routes in the C&W solution in order to find a route plan with improved

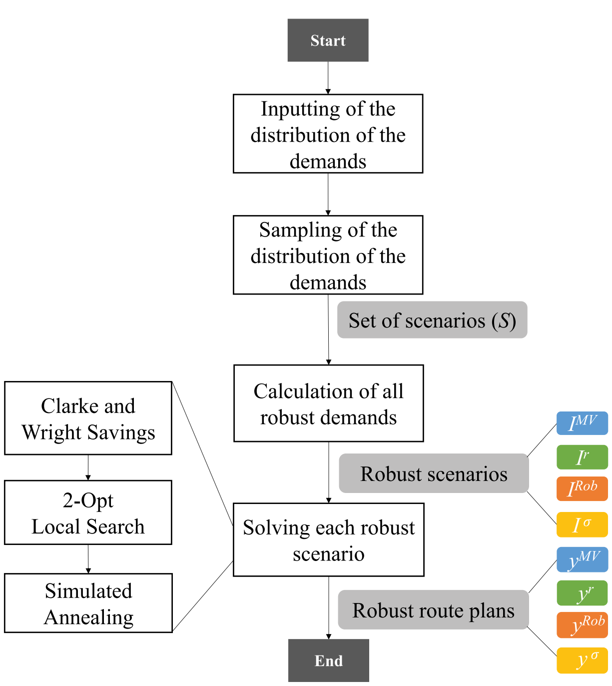

Figure 2. Flowchart of the proposed HyS solution approach.

Table 1. Robust scenario and demand equation for SPR, Rob, and SD models.

Model	Robust scenario	Demand equation ($i \in N$)	
SPR	$I^r = \{d_1^r, d_2^r, d_3^r \ldots, d_n^r\}$ (27)	$d_i^r = d_i^0 + \frac{1}{z}\sum_{a=1}^{z} \theta_i^m,$ (28)	where $\theta_i^m = \begin{cases} d_i^a - d_i^0, & if1mu1mud_i^a > d_i^0 \\ 0, & else \end{cases}$ (29)
Rob	$I^{Rob} = \{d_1^{Rob}, d_2^{Rob}, d_3^{Rob} \ldots, d_n^{Rob}\}$ (30)	$d_i^{Rob} = d_i^0 + \omega\left(\frac{1}{z}\right)\sum_{a=1}^{z}(d_i^a - d_i^0)$ (31)	
SD	$I^{\sigma} = \{d_1^{\sigma}, d_2^{\sigma}, d_3^{\sigma} \ldots, d_n^{\sigma}\}$ (32)	$d_i^{\sigma} = d_i^0 + \omega\sqrt{\left(\frac{1}{z-1}\right)\sum_{a=1}^{z}(d_i^a - d_i^0)^2}$ (33)	

objective function value. The solution generated by the 2-Opt LS approach is further improved by the SA. SA starts with an initial feasible solution. At each iteration, a neighbor solution is generated, and its objective function value is compared with that of the current solution. Improved solutions are always accepted, while non-improved solutions are accepted with a probability $p = e^{-\frac{\Delta C}{T}}$ to escape a local optimum. This probability is a function of the current temperature (T) and objective function value difference between the neighbor and current solutions (ΔC). The temperature, and thus p, decreases at each iteration. T value varies from a large to a small value (close to zero). A cooling schedule specifies the initial temperature, temperature decrement between successive iteration, and the number of iterations for each temperature. For the SA cooling schedule, we adopt the most used one in the literature, which was proposed in Osman (1993). For solving DM, we use these three heuristics. Examples of application of C&W, 2-Opt LS, and SA in the CVRPSD can be found in Dror and Trudeau (1986), Mendoza et al. (2010), and Goodson, Ohlmann, and Thomas (2012), respectively.

Numerical experiments

In this section, we first introduce the performance measures employed in the comparison analysis. After that, we present and discuss the computational results on instances from a benchmark dataset. Finally, we apply the proposed solution approach to a case study of the CVRPSD, which is a selective waste collection system in Brazil, and then analyze the results.

Performance measures

To compare the proposed MV model with other models in the literature, five performance measures are selected from literature studies: planned distance, final distance, planned number of routes, final number of routes, and *Price of Robustness*. Planned distance and planned number of routes are characteristics of the a priori route plan (route plan available at the planning stage). *Price of Robustness* is defined as the additional cost incurred if a robust formulation is applied instead of employing the deterministic approach. Hence, if y is a minimizer for the deterministic model and $\tilde{y}$ is a minimizer for any model that seeks (recoverable or strict) robustness, then the *Price of Robustness* for a minimization problem is given by

$$Price := J_0(\tilde{y}) - J_0(y). \quad (34)$$

Monte Carlo simulation and the distribution of the demands are employed to estimate the final number of routes and final distance. As the problem is dynamic and the solution approach is static, we only know the final distance after the real demands are revealed. However, the final distance can be estimated based on simulated demand. By simulating the demands, we can infer how many times a route failure happens, and consequently how often recourse actions are required. A failure happens when the vehicle capacity is exceeded by the demand along the route. In this work, we assume that if a failure occurs, a detour-to-depot is performed. Therefore, if a route failure takes place, the recourse distance is the sum of the distances traveled on the detours to the depot and the final distance is the sum of the planned distance plus recourse distance. Similarly to the final distance, the final number of routes is only known when all vehicles completing serving the customers in their routes. Moreover, we assume that an extra route arises when a detour-to-depot is implemented. The final number of routes is thus equal to the amount of planned routes plus extra number of routes.

Results on benchmark instances

In this section, we conduct a series of numerical experiments using benchmark instances. These instances are designed based on the 14 instances created by Christofides, Mingozzi, and Toth (1979). Their CVRP instances can be classified both in instances without (instances CMT01-05 and CMT11-12) and with maximum tour length constraint (instances CMT06-10 and CMT13-14). As we do not consider maximum tour length restriction in our problem, we removed this constraint from instances CMT06-10 and CMT13-14. Similar to the many other works on the CVRPSD, e.g. Tillman (1969), Laporte, Louveaux, and van Hamme (2002), and Mendoza and Villegas (2013), we assume that stochastic demands follow independent Poisson distributions. We also assume that demand expected values are equal to the corresponding values in the deterministic dataset. The solution approach was coded in Qt/C++. All experiments are implemented on a desktop with an Intel Dual Core i5 processor and 16GB RAM. Although some of the robust models presented in this study can reflect the risk preference of the decision-maker via the weight ω, namely, MV, SD, and Rob, we did not aim to analyze the impact of ω on the performance of their solutions. Therefore, we adopted $\omega = 1$ for all of them.

For choosing the number of discrete scenarios used in the solution approach (z), see Eq. (17), we adopted $z = 10, 20, 40, 80, 100,$ and 200, and solved each of the 14 instances 20 times for each z, and then computed the average objective function value. The Pearson Correlation test was used to check the relationship between z and the average objective function value in the 20 runs. Correlation was tested at a 95% confidence level with a degree of freedom = 4. The results from the statistical tests provided enough evidence to fail to reject the null hypothesis of 'No statistically significant relationship exists between z and the average objective function value in 20 the runs'. We can then say that at a 95% confidence level the correlation between these two variables equals zero. Accordingly, for illustration purpose, $z = 40$ is used in the numerical experiments.

To validate the efficiency of the proposed solution approach, we compare the performance of HyS with that of the solution approach introduced by Tarantilis and Kiranoudis (2002) on the instances originally designed without maximum tour length constraint (instances CMT01-05 and CMT11-12) in terms of objective function value and computational time. Their solution approach is a population-based heuristic called BoneRoute. We also present the optimal value of the objective function for these instances in the literature. The code and the solutions are available to interested readers upon request.

Tables 2 and 3 present an overview of the computational results on the 14 instances. Table 2 shows the planned, final, and recourse distances calculated for each model, and Table 3 compares the five models based on planned, final, and extra number of routes. In Table 2, the column n represents the number of customers in the corresponding instance, whereas column Opt displays the optimal value of the CVRP objective function. Columns Best and Minutes below BoneRoute show the best distance and the computational time for BoneRoute. Similarly, the columns Best and Minutes below DM display the best distance and run time of the deterministic approach, and the column % shows the deviation from the optimal solution. The column Avg displays the average objective function value in 20 runs. Finally, in Table 3, the (planned, final, and extra) number of routes represents the number of routes in the best solution found in 20 runs.

By comparing the DM best objective function values with the optimal ones, it can be seen that the proposed solution approach HyS is able to find optimal route plans in low computational times. HyS found five optimal solutions and the deviation from the optimal value was no greater than 0.31%. Although BoneRoute outperformed HyS in regards to objective function value in two instances, HyS was faster in finding optimal solutions than BoneRoute. Comparing DM with the robust solutions, in most instances DM solutions exhibit not only the worst final distance among the five models, but also the worst recourse distance. Most of the route plans of the Rob model present comparable planned distances to DM route plans. Similarly, these Rob solutions also have resembling final and recourse distances to the DM solutions. In all instances, SPR route plans are inferior to Rob solutions regarding planned distance. Nonetheless, plans generated from the SPR model show better final and recourse distances than Rob route plans. In turn, solutions fashioned via the SD model show worse planned distances but improved final and recourse distances compared to SPR solutions. Compared to the SD solutions, the MV route plan shows longer planned and final distance but zero recourse distance. It is worth mentioning that in all instances, MV route plan presents identical planned and final distances, therefore, all MV solutions are featured with no recourse distance.

Table 3 shows that in all instances but one, the planned number of routes of DM solutions is exactly equal to the number of routes of optimal solutions. This is also the case for Rob solutions in most of the instances. Therefore, the majority of DM and Rob route plans show a similar final number of routes. DM and Rob route plans are the ones with the largest number of extra routes among all solutions. In all instances, the planned number of routes of SPR solutions is lower than SD and MV solutions. This pattern is also seen in the final and extra number of routes in most of the instances. MV route plan shows equal planned and final number of routes. As a result, all MV route plans do not cause extra routes.

It can be seen that solutions generated from the MV model do not show any route failure when the real demand is revealed, which indicates that MV route plan incurs neither recourse distance nor extra routes based on simulated demand. Thus, such a route plan is the most robust one among all solutions. However, the *Price of Robustness* of MV solution is the highest among all solutions. Figure 3 compares this *Price* for the solutions. The *Price* payed by a decision-maker while implementing MV route plan instead of DM solution reaches up to 81% of the planned distance of the DM solution. As expected, the higher the level of robustness is, the higher its *Price* will be. Therefore, in a decreasing order of the *Price of Robustness*, the MV route plan is followed by the solutions calculated via SD, SPR, and Rob models. DM solutions show the least robust, since they present the worst recourse distances. It is important to highlight that although MV solutions are the most robust among all solutions, SD route plan performs better in the sense that they show the lowest final distance.

In addition, by comparing SD and SPR route plans, more customers are attended in single route in SPR route plans than in SD plans, and therefore, the average number of customers per route of a SPR route plan is higher than that of a SD route plan, as shown in Figure 4. In this way, the average travel distance per route of a SPR route plan is greater than that of the SD plan, as shown in Figure 5. However, SPR route plans tend to fail more often and, therefore, present a higher recourse distance. In the end, the final distance of SPR solutions turns out to be higher than that of SD solutions. On the contrary, as solutions become more robust, reduced number of customers are served in individual route in MV plans. In this case, route failure is less likely to happen and, thus, recourse distances of MV solutions are lower. As a compromise, more routes are required in MV route plan, and consequently the planned number of routes is higher. This situation clearly indicates the trade-off between optimality (planned distance) and robustness (recourse distance).

Case study – a selective waste collection problem in Brazil

In order to test the proposed model under real circumstances, we demonstrate the application of the MV model to a selective waste collection problem in Teresina, Brazil. In 2010, the Brazilian government created a national law to regulate the solid waste management in cities, under which each Brazilian city must have a selective waste collection system. Teresina, which is a city with population over 0.8 million, implemented a selective waste collection system in 2013. This system is managed by a municipal company, called Coordenação Especial de Limpeza Pública – CELIMP, together with the cooperative of waste pickers. The municipal company is responsible for the operational cost involved in collection and transportation, while the cooperative administers the recycling facility. In this facility, the collected materials are separated, assessed, stored, and commercialized. The collection is performed by two trucks of 10 tons serving a set of 38 locations, such as hotels, hospitals, condominium, and voluntary delivery points. Voluntary delivery points are containers of 2.5 tons with 4 compartments (plastic, glass, paper, and metal) in which anyone can drop off recyclable materials. It is estimated that the collection points produce a day approximately 19 tons of recyclables.

Problem settings

The selective waste collection problem is considered as a CVRPSD. In this way, the recycling facility is set as the depot ($d_i = 0$) and the 38 locations are treated as customers ($n = 39$). Each customer's demand is assumed to follow normal probability distribution. We thus estimate the mean and the standard deviation of the amount of materials generated by each customer based on the data provided by CELIMP. The geographic coordinates of each customer have been collected based on GPS data. To construct the road network (A) and calculate the distance matrix, Geographic Information System (GIS) – QGIS and database – PostGIS are adopted. All remaining cost items are defined as fixed costs. Homogeneous trucks are used with identical capacity of 10 tons. Figure 6 displays the locations of customers and depot.

Table 2. Planned, final, and recourse distances of each model.

			Planned distance																							
			BoneRoute		DM				Rob		SPR		SD		MV		Final distance					Recourse distance				
Data file	n	Opt	Best$^{(a)}$	Minutes[1]	Best$^{(a2)}$	%	Avg[3]	Minutes[1]	Best[2]	Avg[3]	Best[2]	Avg[3]	Best[2]	Avg[3]	Best[2]	Avg[3]	DM	Rob	SPR	SD	MV	DM	Rob	SPR	SD	MV
CMT01	51	**524.61**	**524.61**	0.11	**524.61**	0	558.02	0.1	536.78	575.34	547.36	589.09	550.97	625.06	761.00	795.00	1074.36	1141.47	755.58	695.56	761.00	549.75	604.69	208.22	144.59	0
CMT02	76	**835.26**	**835.26**	4.56	**835.26**	0	892.13	1.45	932.04	949.28	941.12	945.89	1003.34	1016.60	1415.28	1427.28	1813.85	1763.53	1561.22	1316.51	1415.28	978.59	831.49	620.09	313.16	0
CMT03	101	**826.14**	**826.14**	7.66	**826.14**	0	896.45	4.34	829.44	900.33	930.94	932.97	964.18	984.55	1208.00	1273.00	1629.27	1460.93	1446.97	1051.51	1208.00	803.13	631.48	516.02	87.31	0
CMT04	151	**1028.42**	1030.88	9.13	1032.52	0.16	1143.16	5.56	1034.18	1187.34	1050.84	1222.45	1185.23	1295.51	1635.37	1735.37	2495.06	2745.46	1878.28	1357.69	1635.37	1462.53	1711.28	827.44	172.46	0
CMT05	200	**1291.29**	1314.11	16.97	1318.18	0.31	1496.95	10.45	1322.27	1509.76	1422.11	1589.72	1689.34	1669.19	2259.68	2315.68	3545.74	3125.44	2702.67	2105.54	2259.68	2227.56	1803.17	1280.57	416.21	0
CMT06	51	–	–	–	585.43	–	559.40	0.05	650.42	651.83	823.62	834.42	855.42	897.96	955.42	1085.42	1220.55	1351.76	1223.86	1091.09	1155.42	635.12	701.34	400.23	235.67	0
CMT07	76	–	–	–	989.68	–	912.65	0.65	1041.90	1042.03	1142.75	1153.02	1220.48	1345.78	1370.48	1462.48	1805.16	1844.24	1673.31	1588.27	1370.48	815.48	802.34	530.56	367.89	0
CMT08	101	–	–	–	895.94	–	869.56	3.78	913.45	974.05	1043.10	1063.55	1068.87	1078.54	1375.87	1445.87	1886.54	1876.90	1489.66	1370.99	1375.87	990.60	1003.45	446.56	312.12	0
CMT09	151	–	–	–	1170.40	–	1200.12	2.31	1177.66	1177.82	1272.41	1272.73	1328.89	1329.45	1831.89	1988.89	2738.50	2628.89	2107.30	1785.65	1531.89	1568.10	1451.23	834.89	456.76	0
CMT10	200	–	–	–	1425.72	–	1468.90	10.67	1603.56	1614.32	1690.88	1691.84	1724.90	1801.12	2024.90	2153.90	3882.44	3971.45	3045.11	2032.70	1824.90	2456.71	2367.89	1354.23	307.80	0
CMT11	121	**1042.12**	**1042.12**	0.21	**1042.12**	0	1105.31	0.18	1123.03	1247.92	1155.86	1578.54	1215.47	1476.93	1893.00	2033.00	2340.32	2468.70	1973.73	1562.25	1893.00	1296.23	1345.67	817.87	346.78	0
CMT12	101	**819.56**	**819.56**	0.1	**819.56**	0	823.47	0.06	920.45	922.74	1063.37	1163.67	1125.12	1324.89	1415.12	1455.12	3260.24	3341.09	2321.01	1247.52	1415.12	2440.68	2420.64	1257.63	122.40	0
CMT13	121	–	–	–	1552.81	–	1573.89	9.56	1561.66	1562.57	1694.68	1715.30	1443.26	1834.45	2177.26	2197.26	3920.20	3774.11	2959.89	2300.02	1977.26	2367.39	2212.45	1265.21	856.76	0
CMT14	101	–	–	–	886.37	–	871.78	0.05	978.56	978.58	1006.44	1012.60	1035.09	1056.89	1204.09	1219.09	3231.79	3546.46	2971.89	2239.41	2224.09	2365.42	2567.90	1965.45	1204.32	0

(a)Optimal solutions are in boldface.
(1)Time for reaching the best value for the first time.
(2)Best objective function value in 20 runs.
(3)Average objective function value in 20 runs.

Table 3. Planned, final, and extra numbers of routes of each model.

		Planned number of routes					Final number of routes					Extra number of routes				
Data file	Opt	DM	Rob	SPR	SD	MV	DM	Rob	SPR	SD	MV	DM	Rob	SPR	SD	MV
CMT01	5	5	6	6	7	11	8	10	8	8	11	3	4	2	1	0
CMT02	10	11	12	12	14	21	15	15	14	15	21	4	3	2	1	0
CMT03	8	8	8	9	10	16	13	12	13	11	16	5	4	4	1	0
CMT04	12	12	12	13	15	20	18	19	16	16	20	6	7	3	1	0
CMT05	17	17	17	19	22	35	24	23	23	24	35	7	6	4	2	0
CMT06	6	6	7	10	10	13	10	12	13	12	13	4	5	3	2	0
CMT07	11	11	12	13	14	16	18	19	16	16	16	7	7	3	2	0
CMT 08	9	9	9	11	11	17	15	15	14	13	17	6	6	3	2	0
CMT 09	14	14	14	16	17	22	19	19	19	19	22	5	5	3	2	0
CMT 10	18	18	19	19	20	28	26	26	23	22	28	8	7	4	2	0
CMT11	7	7	7	7	8	12	13	13	10	10	12	6	6	3	2	0
CMT 12	10	10	10	12	13	18	15	15	14	14	18	5	5	2	1	0
CMT 13	11	11	11	13	15	23	15	14	15	17	23	4	3	2	2	0
CMT 14	11	11	12	13	13	19	19	21	19	17	19	8	9	6	4	0

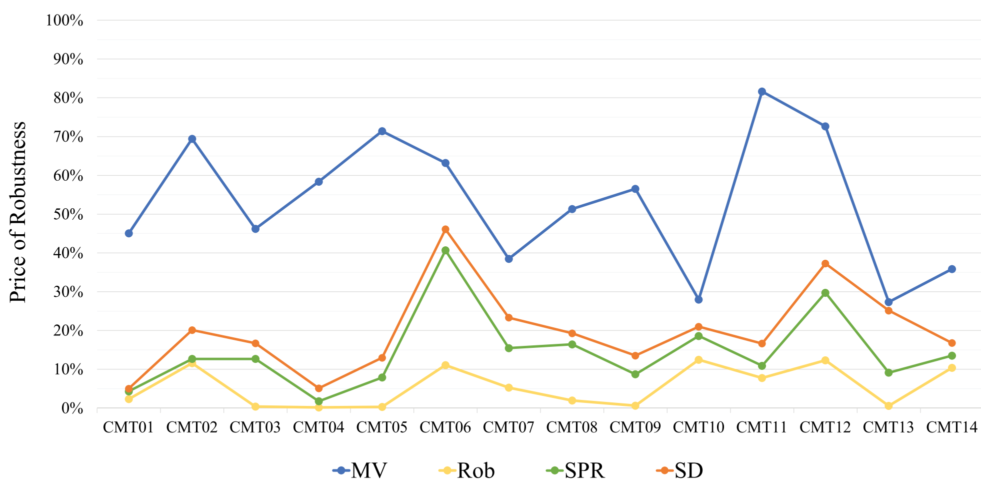

Figure 3. Comparison on the *Price of Robustness.*

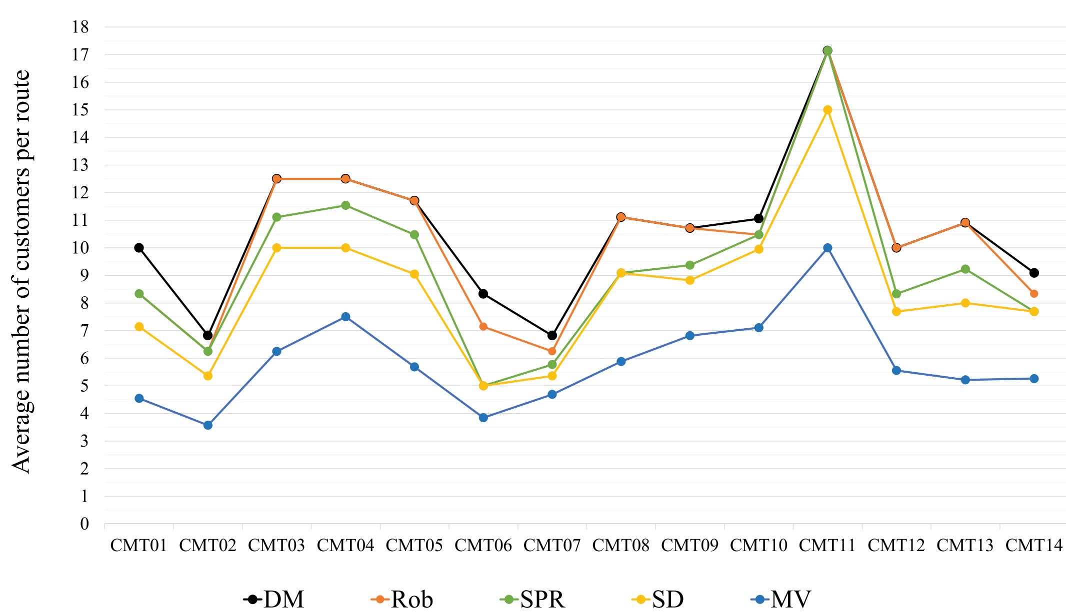

Figure 4. Comparison on the average number of customers per route.

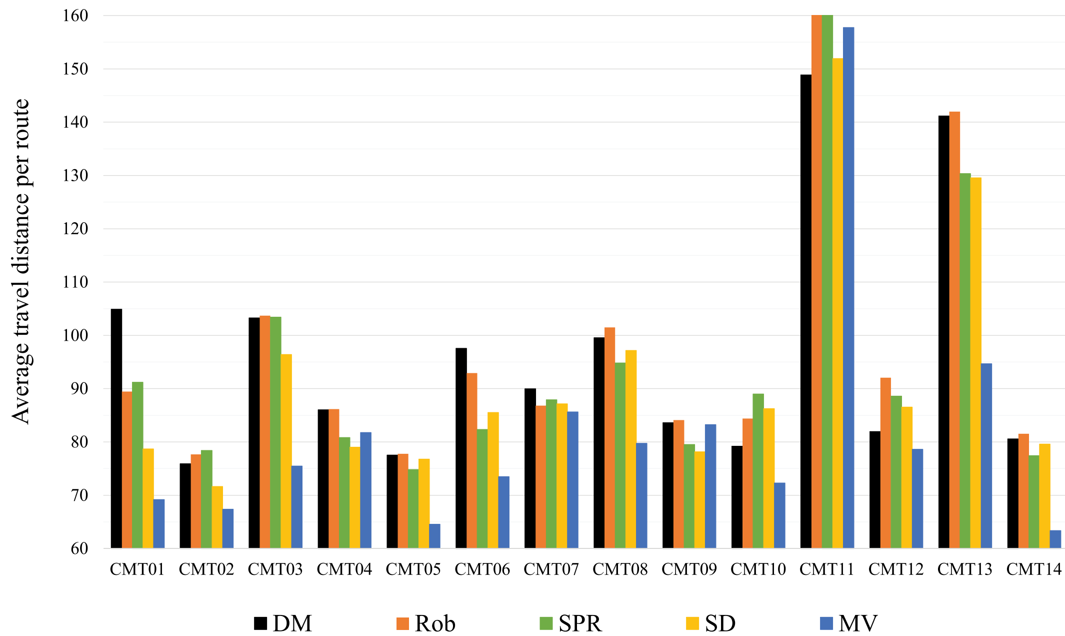

Figure 5. Comparison on the average travel distance per route.

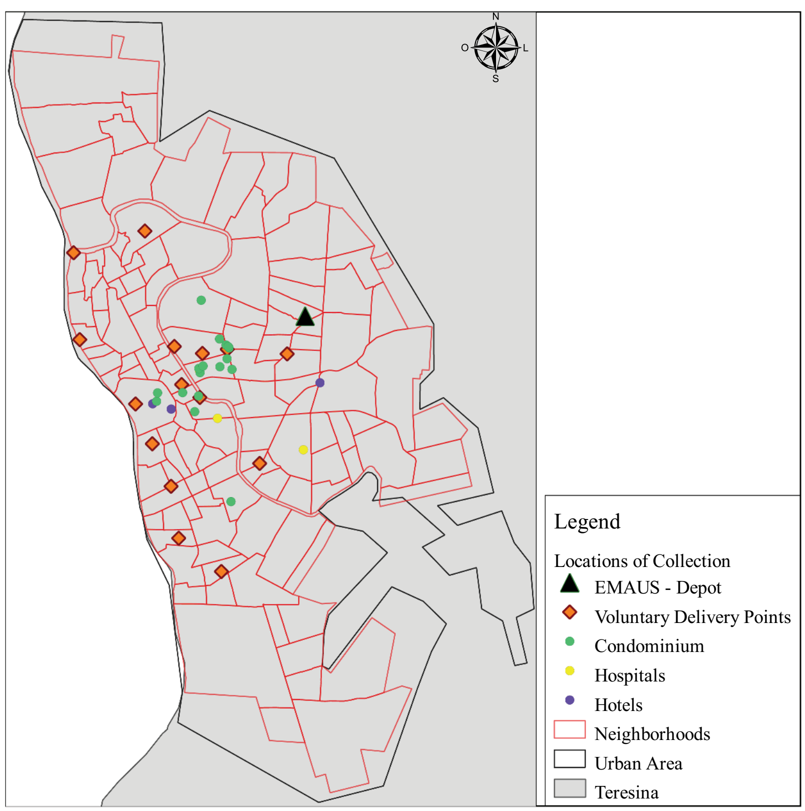

Figure 6. Spatial distribution of 38 customers and 1 depot over the road network in Teresina.

Numerical results and analysis

In this section, the selective waste collection problem is solved via the proposed HyS solution approach. For comparison purposes, we adopt the same performance measures described in Section 5.1. To simplify the analysis for illustration purpose, the travel distance by each truck is considered as the only variable cost in operation.

Table 4 compares solutions obtained from the aforementioned five models based on the performance measures. It can be observed that the results in the case study are similar to the results on benchmark instances. The DM solution performs best regarding the planned distance, and the MV solution outperforms the other solutions on the final distance. When the real amount of recyclable materials from every customer is simulated, all other solutions,

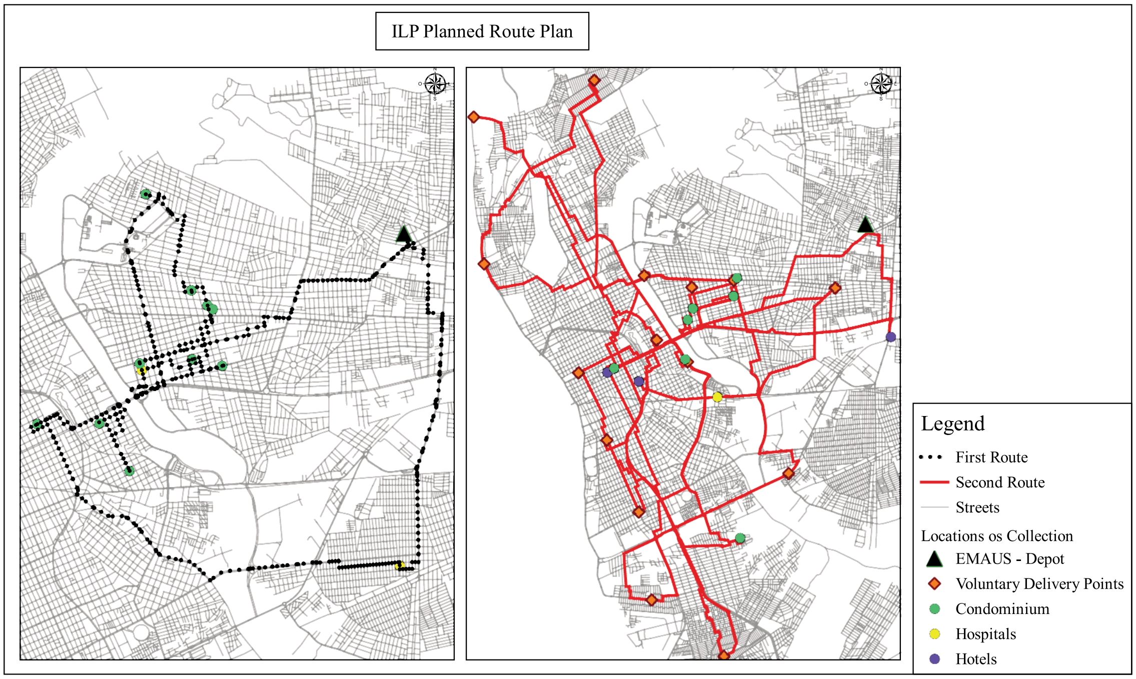

Figure 7. Planned DM route plan.

Table 4. Comparison results of the five models.

Model	Planned distance	Final distance	Recourse distance	Planned number of routes	Final number of routes	Extra number of routes
DM	138,156.59	149,130.05	10,973.46	2	3	1
Rob	138,458.02	148,820.60	10,362.58	2	3	1
SPR	139,658.65	148,102.31	8,443.66	2	3	1
SD	139,852.78	141,105.24	1,252.46	2	3	1
MV	141,710.60	141,710.60	0	3	3	0

except MV route plan, suffer a route failure. A priori (planned) DM route plan, for instance, is displayed in Figures 7, and 8 shows the final DM route plan. The DM, SPR, Rob, and SD solutions present one extra route, as shown in Table 4, while the planned and final route plan in the MV solution is equal (see Table 4 and Figure 9). The numerical results indicate that the MV solution does not fail, and the corresponding cost to guarantee such robustness is not high in this case study. The *Price of Robustness* of the MV solution is only 3% of the planned distance in the DM solution. The final distance is the distance run by trucks to collect recyclable materials from each customer, which is reported to CELIMP at the end of each working day. In this sense, the SD route plan is the best solution

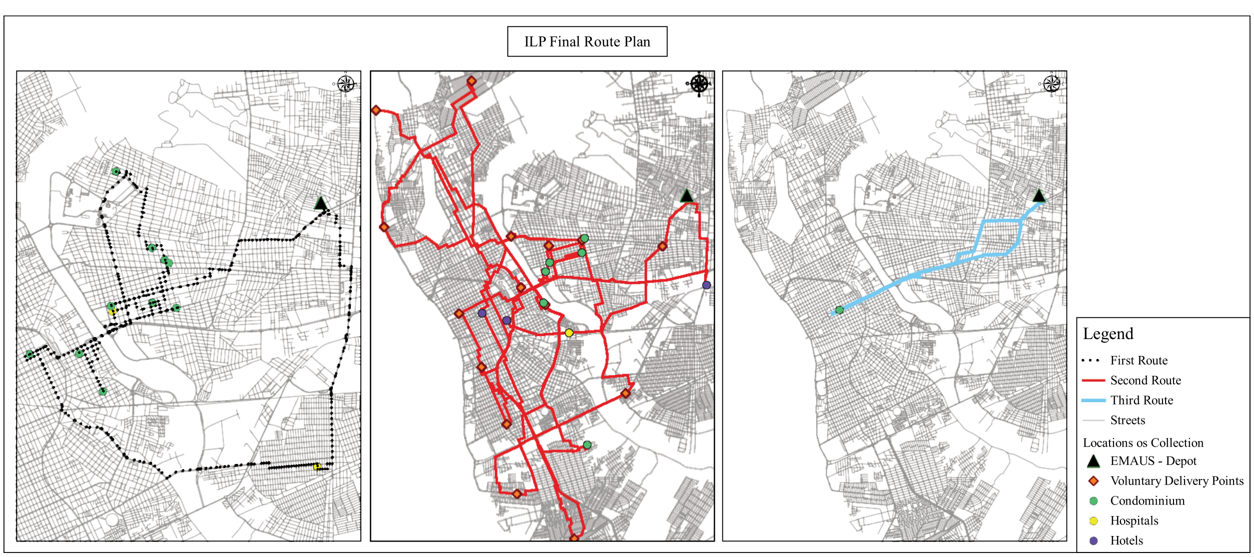

Figure 8. Final DM route plan (with route failure).

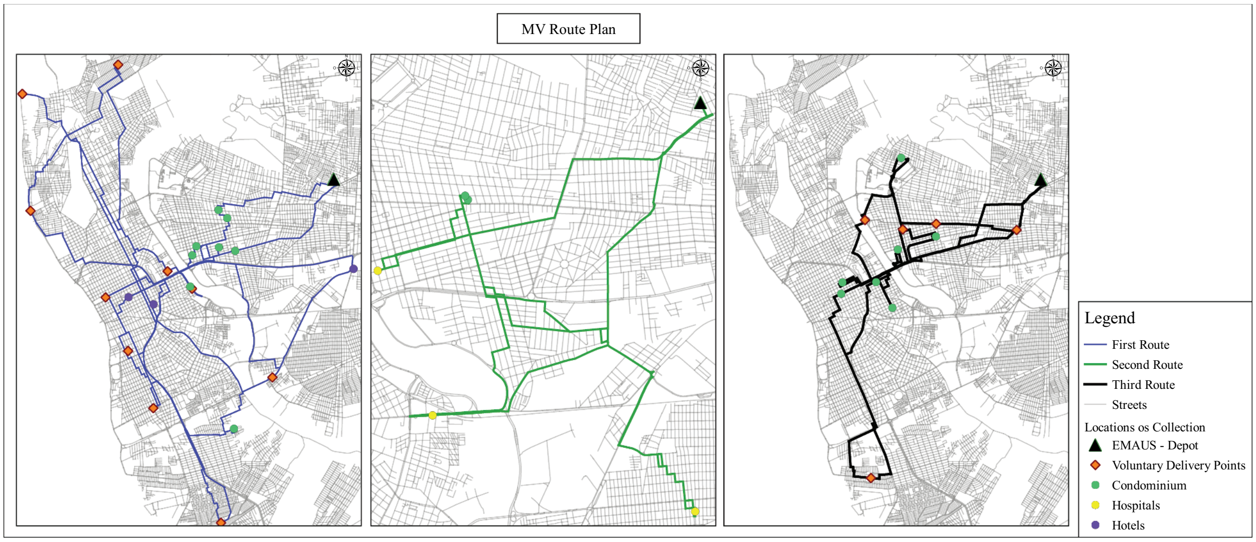

Figure 9. Planned and final MV route plan.

since it shows the lowest final distance in Table 4. Nevertheless, drivers do not have to perform any detour to the depot when the MV solution is implemented by CELIMP for performing the selective waste collection.

Conclusion

Uncertainties affect a wide range of decisions decision-makers have to make. Uncertainty in demands complicate the task of route planning for fleet managers. The stochastic CVRP is more complicated to handle than its deterministic counterpart. The CVRPSD raises concepts that the deterministic problem does not, such as robustness. A (recoverable or strictly) robust route plan, which offers protection against fluctuations on the stochastic input, has a cost associated with it, called *Price of Robustness*. To protect a solution, fleet managers need to accept a suboptimal solution. Accordingly, they must balance optimality and robustness.

This paper proposed a MV model for the CVRPSD, where the objective function combined optimality and robustness. Optimality was represented by the minimization of the mean of first-stage distance, while robustness was expressed by the minimization of the variability in the expected value (second-stage distance). To solve the MV model, and other three models from the literature that achieve robustness, a multi-stage HyS solution approach was developed. We adopted the same heuristics employed by HyS to solve DM (deterministic approach). Both benchmark instances and a real-life case study of a selective waste collection problem in Brazil were used to test the solution approach. Numerical results showed that the MV model was the only one not suffering from route failure compared to the other four models, and hence, no recourse action or extra routes were needed in the MV solutions. Nevertheless, considering the real distance traveled by trucks to attend all customers (final distance), the model introduced by Sörensen and Sevaux (2009) performed the best. This was mainly because of the *Price of Robustness* incurred by the MV solutions, as a compromise to its robustness. The comparison results clearly showed features of each model and their advantages and disadvantages, which provided valuable information to help fleet managers decide which model to utilize based on their different needs and priorities. We believe that our proposed solution approach is a simple and practical way for fleet managers to find optimal route plans. HyS relies on simple demand equations and well-established heuristics. By applying our proposed solution approach in vehicle route planning, fleet managers can not only obtain solutions of optimal distance but also solutions with a high level of robustness (solutions that do not need to be changed when the real demands are revealed).

Although the numerical results showed the good performance of the MV model and its potential as an effective operational planning tool for the CVRPSD, this work has certain limitations to be overcome. For example, we assumed that the probability distributions of customer demands were known, and we considered the distance traveled by each truck as the only variable cost in operation. As future research, we plan to implement a data-driven approach to calibrate the demand distribution and compare our proposed approach with a dynamic solution method. Moreover, we would like to discretize the parameter ω and parametrize the MV objective function. In this way, we will allow decision-makers to choose different levels of robustness using a single model, and then make the solution approach more flexible to satisfy the level of risk of decision-makers.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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