The Integrated Dial-A-Ride-Problem: Fixed Route Selection

Sabreen Hassan, Islam Ali, and M. Nashat Fors
Industrial Engineering,
Alexandria University,
Alexandria, Egypt
eng-sabreen.hassan@alexu.edu.eg, islam.ali@alexu.edu.eg, nashat.fors@alexu.edu.eg

Abstract

The integrated dial-a-ride problem (IDARP) concerns of the optimal routes of a set of vehicles that serve the first and last miles of in combination journeys such that a fixed transport service may carry out the longest part of the journey. It implies some modeling and practical issues. Practical wise, it only integrates a single fixed line. It also does not accept any dial-a-ride problem (DARP) request. From the modeling perspective, the transfer node system enlarges quickly with each new request. It also requires more strengthening strategies to be adopted. The proposed integrated dial-a-ride problem fixed-route selection (IDARP-FRS) handles some of these issues. It extends the IDARP to include iterations over a two-line fixed service. An additional set of constraints is devised to easily switch the proposed model to the IDARP and even accommodate users from the dial-a-ride problem. The IDARP-FRS also introduces a set of newly constructed direction-based constraints, for controlling the service quality. Besides, the symmetry-breaking constraints are brought from the literature to the context of the IDARP-FRS to further strengthen the model. The computational tests are implemented on Gurobi and showed the significant role of the proposed constraints in reducing the problem complexity.

Keywords

Dial-a-ride problem, DARP, Public transportation, Bimodal transportation, and Vehicle-routing problem.

1. Introduction

According to the Worldometer's (2021) forecast, Egypt will be recording a considerable population growth during the next decades. Any upsurge in population causes an increase in mobility demand. Therefore, the transport infrastructure must be developed to not only meet the increasing demands but also to comply with social quality as claimed by Yatskiv et al. (2017). Shifting from private to public transport is useful in terms of transportation sustainability, public health, environmental conditions, and economic aspects (Elias and Shiftan 2012). Consequently, policymakers and planners aim to develop more accessible and time-saving public transport solutions (Saghapour et al. 2016). Public transport can be more effective by providing demand-responsive mobility of the door-to-door type (Yatskiv et al. 2017). Door-to-door services are typical applications of the well-known DARP (Cordeau and Laporte 2003). Häll et al. (2009) introduced a variant of this problem called IDARP, which is an extension of the DARP generated by Cordeau (2006). It is therefore an NP-hard problem. The IDARP is such an innovative bimodal transport option that combines demand-responsive and fixed-route services. It represents the bedrock for this paper. The IDARP only employs the expensive dial-a-ride service for the first and last miles of the journeys while making use of the existing stations of the cheaper service, fixed-route, for the longest part. Employing stations as transfer locations reduces the costs as noted by Masson et al. (2014).

The IDARP shows some modeling and practical issues. First, the network-based formulation makes the model size buildups increasingly and an exponential increase in the computational complexity is expected as the requests increase. Each request adds not only two nodes (pickup and drop-off), but it also adds the corresponding transfer nodes that equal to the number of fixed-route stations. This replication of physical stations brings a vital issue of how the transfer nodes are modeled. Second, the model lacks any consideration of the fixed-line direction. Third, in bimodal transport modeling, synchronization between the two services is generally vital but the model ignored it. One more issue is the consideration of one fixed line only. Posada et al. (2017) handled the synchronization and transfer nodes replication issues and implemented a different transfer structure.

1.1 Objectives

This work is inspired by the previously mentioned issues and proposes an upgraded model called the IDARP-FRS based on the IDARP. The contribution is four-fold (i) through a focused study, it is noted that the IDARP transfer nodes structure can be further strengthened and the number of arcs related to the replicated stations can be reduced by considering a direction for each user (ii) the IDARP is extended by considering iterations over two-fixed line service which reflects a more realistic aspect that determines which line is optimal for which user; in this way, the available fixed stations are more perfectly employed to escalate the utilization of the dial-a-ride vehicles (iii) the incorporation of symmetry- breaking constraints (iv) providing constraints for switching on/off the transfer nodes and converting the IDARP-FRS to IDARP or the regular DARP.

2. Literature Review

It is essential for this paper to review some earlier studies on the dial-a-ride problem with transfer (DARPT) in addition to the IDARP. It is also paramount to shed light on solution methods. The DARPT is different from the IDARP, but both apply transfer points during journeys. Achieving savings is the prime motivation for applying transfer locations as noted by Masson et al. (2012). In the DARPT switching between vehicles during the journey at particular points is allowed. Transfer points may be modeled as static or dynamic. For instance, Masson et al. (2011) introduced a Tabu search and minimized the total distance traveled making use of static transfer points. Later, Masson et al. (2012) developed a metaheuristic method, studied the feasibility of the request insertion approach, and modeled the transfer points as static locations. On the contrary, Deleplanque and Quilliot (2013) handled the transfer points dynamically; they located dependently once the request is inserted. Masson et al. (2014) completed their preliminary study and proved that the transfer point's application led to substantial savings.

Regarding the IDARP, it is said that Potter (1976) and Wilson et al. (1976) were the earliest papers that studied the idea of integrating two modes. Potter described the operational procedures employed in the computer-aided dispatching system. The integrated system combines 45 dial-a-ride vehicles that feed 36 express buses. Wilson et al. studied and modeled the problem of controlling integrated dial-a-ride and fixed-route services. They minimized the users' dissatisfaction in terms of wait time, ride time, total time, and pickup /delivery deviation. Similar studies then continued to emerge with a focus on real-life implementations. Liaw et al. (1996) formulated a bimodal dial-a-ride problem, integrating paratransit and fixed modes, and introduced a decision support system to establish efficient routes and schedules for both dynamic and static instances. The number of requests accommodated was increased by 10% and the number of paratransit vehicles decreased by 10% compared to the system when no buses are considered. Hickman and Blume (2001) proposed a two-stage heuristic for scheduling integrated itineraries to minimize the operating costs while handling the users` service level aspects as constraints. The results showed the underlying advantages of the integrated service in terms of cost and service quality. Aldaihani and Dessouky (2003) minimized the total traveled distance and total traveled time by developing a new heuristic that grounded on previous heuristics by enhancing the insertion approaches and adding a Tabu Phase. This approach was computationally efficient and enabled them to solve large-sized instances. More recently, Posada et al. (2017) extended the IDARP developed by Häll et al. (2009) including timetables for the fixed service, scheduling the arrival of vehicles at the transfer locations. They criticized how Häll et al. (2009) modeled the transfer nodes and developed a new approach inspired by Stålhane et al. (2014). Steiner and Irnich (2020) sought to decide on the bus lines or just segments for integration while a hybrid system connecting two modes was developed by Edwards et al. (2012), utilizing a demand-responsive transit to cover the first and last miles of the trip. From the solution method perspective, the complexity and reduced usefulness of solving NP-hard models to optimality let numerous scholars resort to heuristic algorithms (Anbuudayasankar et al. 2014). Furthermore, fewer efforts were seen on the exact methods (Braekers et al. 2016). The exact methods though are essential for testing the effectiveness of heuristic solutions (Nuha et al. 2018). The fact is that by constraining the problem tightly, bigger instances can be exactly solved (Cordeau 2006).

This review shows the effectiveness of integrating two modes of transport and accordingly more research is essential for the advancement of this approach, especially on the exact solution models. In response, this research extends the

IDARP to the IDARP-FRS and seeks to solve it to optimality using a commercial solver, namely Gurobi, through an interface implemented using Python programming language.

The rest of this paper is outlined as follows. Section 3 presents an arc-based, exact mathematical formulation for the IDARP-FRS and some enforcement strategies including newly employed ones, namely symmetry-breaking constraints, and direction constraints. In Section 4, an instance is tested, and computational results are discussed in Section 5. Conclusions are finally presented in Section 6.

3. Methods

The IDARP-FLS extends the IDARP proposed by Häll et al. (2009) by modeling two fixed lines. The model is based on a directed graph structure and formulated as a MILP that is static and deterministic. The fixed-route stations are incorporated as transfer locations. The newly incorporated idea, which was not considered in the IDARP, sets two directions for each fixed-line, inbound and outbound, and thus each user may travel to only one direction. The IDARP-FLS implemented this approach to control the service quality. The rest formulation aspects are similar to the IDARP. Table 1 shows all notation of the IDARP-FRS.

Table 1: Notation

	Notation	Definition
Parameters	n	no of requests
	Q	max no of passengers per vehicle
	T	maximum route duration time of a vehicle
	$g_{1,\, g2}$	no of stations for line 1,2
	g	total no of stations
	c_{ij}	the travel cost from node i to node j
	t_{ij}	the travel time from node i to node j
	q_r	no of persons/request
	d_i	service duration at node i
	e_i	the earliest time at which service may start at node i
	l_i	latest time at which service may start at node i
	e_k	the earliest time at which vehicle k can exit the depot
	l_k	latest time at which vehicle k can leave the depot
	L_i	maximum ride time of request i
	M	large positive number
	f_{ijr}	$= \begin{cases} 1, & \text{if node } i \text{ is the pick-up node of request } r \\ -1, & \text{if node } i \text{ is the drop-off node of request } r \\ 0, & \text{otherwise} \end{cases}$
	b_r	a direction parameter, each request takes a direction 1, -1, or 0
Sets	K	set of DAR vehicles
	R	set of requests
	P	set of pick-up nodes
	D	set of drop-off nodes
	C	set of all transfer nodes
	C_{rt}	set of all transfer nodes associated with request r
	C_{r1}	set of transfer nodes on line 1 associated with request r
	C_{r2}	set of transfer nodes on line 2 associated with request r
	N	set of all nodes, pick-up, drop-off, depot and transfer nodes
	A	set of all arcs (i,j) ∀ i in N and j in N
	F	set of fixed lines [1,2]

	Notation	Definition
Decision Variables	x_{kij}	a binary variable equal to *1* if arc *(i, j)* is used by vehicle *k, 0* otherwise
	y_{rij}	a binary variable equal to *1* if user *r* used arc *(i, j), 0* otherwise
	z_{1rij}	a binary variable equal to *1* if user *r* used arc *(i, j)* on line *1, 0* otherwise
	z_{2rij}	a binary variable equal to *1* if user *r* used arc *(i, j)* on line *2, 0* otherwise
	B_k	the time at which vehicle *k* exit the depot
	B_i	the time at which the service begins at node *i*
	D_i	the duration time (travel time) from depot to node *i*

3.1 Mathematical Formulation

The IDARP-FRS implies scheduling n requests that initiate a set of pickup nodes $P= \{1, 2,....,n\}$ and a set of corresponding drop-off nodes $D= \{ n + 1,......,2n\}$. In this sense, for each pickup node, there is a corresponding drop-off node $n + i$. The fleet of vehicles is homogeneous and uses a single depot. For the formulation purpose, node *0* represents the origin depot, and node *2n+1* is the destination depot. Another set of nodes is *C* that embraces all the transfer nodes for all requests. Each station can be a transfer location (drop-off/pickup) for each request. The number of transfer nodes equal to the total number of stops *g* times the number of requests *n*. Therefore, $C = \{2n + 2,...., 2n + 1+ ng\}$. All nodes can be combined in a global set called $N = \{P, D, C, 0, 2n+1\}$. For user *r*, C_{rt} represents all possible transfer nodes; C_{r1} and C_{r2}represent corresponding nodes at lines 1 and 2, respectively. The set of nodes *N* and their corresponding arcs *A* compose a directed graph network $G = (N, A)$. The travel time from *i* to *j* is t_{ij}, and the associated cost is c_{ij} for every arc $(i, j) \in A$. At each node *i*, service may begin at the earliest time e_i or at the latest time l_i and thus constructing a time window $[e_i,l_i]$. The set of dial-a-ride vehicles is K. The vehicles are homogeneous; *Q* and T are the vehicle capacity and max service duration in order. At each node $i \in N$ there is a service duration d_i (given that $d_0 = d_{2n+1}= 0$). The set of requests is denoted by *R,* and each request $r \in R$ has a corresponding load q_i and has a maximum ride time L_i. The set of fixed-route lines is F. At node $i \in N$; service starts at time B_i. Vehicle $k \in K$ leaves the depot 0 at time B_k. D_i is the duration time of a vehicle from leaving the depot until it reaches node *i*. At most, two transfer nodes in C_{r1} or C_{r2} are visited by each request r. The differences between the IDARP's formulation and the IDARP-FRS's are represented in the z_{1rij}, $z_{2rij,}$ the set *F* where $f \in F$ *and* C_{rt} that includes all transfer nodes for user *r.* Another difference is the newly employed direction idea where b is a set of directions, and each user may be inbound or outbound with respect to the fixed service. In the IDARP-FRS, any fixed line is modeled to have two directions denoted as 1 and -1.

The IDARP-FRS is formulated as follows:

$$min \sum_{i\in N} \sum_{j\in N} \sum_{k\in K} c_{ij}\, x_{kij} \qquad \textbf{(1)}$$

The total routing cost of the fleet is to be minimized by the objective function (1) which runs under a set of constraints.

$$\sum_{j\in N} \sum_{k\in K} x_{kij} = 1 \qquad \forall\, i \in P \cup D \qquad \textbf{(2)}$$

$$\sum_{j\in N} x_{k0j} = 1 \qquad \forall\, k \in K \qquad \textbf{(3)}$$

$$\sum_{j\in N} x_{kji} - \sum_{j\in N} x_{kij} = 0 \qquad \forall\, i \in P \cup D \cup C \qquad \textbf{(4)}$$

$$\sum_{i\in N} x_{i,2n+1} = 1 \qquad \forall\, k \in K \qquad \textbf{(5)}$$

Constraint (2) reveals that each pick-up and drop-off node is serviced strictly by only one vehicle, where the pick-up vehicle may differ from the drop-off vehicle. By constraints (3) and (5), each vehicle is enforced to begin its route from depot 0 and ends it at *2n+1*. Constraint (4) makes sure that each vehicle leaves the serviced node.

$$\sum_{j\in N} y_{rij} + \sum_{j\in C_{r1}} z_{1rij} - \sum_{j\in N} y_{rji} - \sum_{j\in C_{r1}} z_{1rji} = f_{ri} \qquad \forall\, r \in R, i \in C_{r1} \tag{6}$$

$$\sum_{j\in N} y_{rij} + \sum_{j\in C_{r2}} z_{2rij} - \sum_{j\in N} y_{rji} - \sum_{j\in C_{r2}} z_{2rji} = f_{ri} \qquad \forall\, r \in R, i \in C_{r2} \tag{7}$$

Constraints (6) and (7) belong to the IDARP-FRS and ensure that at any corresponding transfer node, the user may reach the node and get serviced either by the dial-a-ride or the fixed-route service; these constraints ensure the balance of the transfer nodes on both lines.

$$\sum_{j\in N} y_{rij} - \sum_{j\in N} y_{rji} = f_{ri} \qquad \forall\, r \in R, i \in N/C_{rt} \tag{8}$$

Constraint (8) depicts the balance of any node in N and not belongs to any transfer nodes of the target request. If a request reaches at any of those nodes, it also leaves.

$$\sum_{r\in R} q_r y_{rij} \le Q \sum_{k\in K} x_{kij} \qquad \forall\, i \in N, j \in N \tag{9}$$

$$B_j \ge B_k + t_{0j} - M\left(1 - x_{k0j}\right) \qquad \forall j \in N, k \in K \tag{10}$$

$$B_j \ge B_i + d_i + t_{ij} - M\left(1 - \sum_{k\in K} x_{kij}\right) \qquad \forall\, i \in N, j \in N \tag{11}$$

$$B_j \ge B_i + t_{ij} - M\left(1 - z_{1rij}\right) \qquad \forall\, r \in R, (i,j) \in (C_{r1} \times C_{r1}) \tag{12}$$

$$B_j \ge B_i + t_{ij} - M\left(1 - z_{2rij}\right) \qquad \forall\, r \in R, (i,j) \in (C_{r2} \times C_{r2}) \tag{13}$$

Constraint (9) shows the ride-sharing possibility by comparing the load requirements with the available capacity, ensuring that the vehicles' capacity is greater than the total number of users transferred from node i to node j. Constraints (10), (11), (12) and (13) satisfy the start time variable requirements and determine when the service at each node begins. Constraints (11) and (12) are for line 1 and line 2 respectively.

$$e_i \le B_i \le l_i \quad \forall\, i \in P \cup D \tag{14}$$

$$t_{i,n+i} \le B_{n+i} - (B_i + d_i) \le L_i \quad \forall\, i \in P \tag{15}$$

$$D_j \ge D_i + d_i + t_{ij} - M\left(1 - \sum_{k\in K} x_{kij}\right) \quad \forall\, i \in N, j \in N \tag{16}$$

$$D_i + d_i + t_{i,2n+1} \le T \quad \forall\, i \in N \tag{17}$$

The start time of the service at each pickup and drop-off node is bounded by constraint (14). The duration from pickup to a drop-off is ensured to be less than the maximum ride time specified for each request by constraint (15). Constraint (16) ensures that each vehicle only travels to a node that requires a service. By constraint (17) each vehicle`s duration time is set to be less than the maximum duration time. Both control the duration time of any vehicle to satisfy the maximum duration time.

$$e_k \le B_k \le l_k \quad \forall\, k \in K \tag{18}$$

$$x_{kij} \in \{0,1\} \quad \forall\, i \in N, j \in N, k \in K \tag{19}$$

$$y_{rij} \in \{0,1\} \quad \forall\, r \in R, i \in N, j \in N \tag{20}$$

$$z_{1rij} \in \{0,1\} \quad \forall\, r \in R, i \in N, j \in N \tag{21}$$

$$z_{2rij} \in \{0,1\} \quad \forall\, r \in R, i \in N, j \in N \tag{22}$$

Constraint (18) sets the time for each vehicle to depart from the depot. Constraints (19)-(22) represent the binary decision variables. Constraints (21) and (22) are for line 1 and line 2, respectively.

$$\sum_{j \in C_{r1}} \sum_{i \in C_{r1}} \mathrm{z}_{1\mathrm{rij}} = 0 \qquad \forall\, r \in R \tag{18}$$

$$\sum_{j \in C_{r2}} \sum_{i \in C_{r2}} \mathrm{z}_{2\mathrm{rij}} = 0 \qquad \forall\, r \in R \tag{19}$$

Constraints (23) can be switched off to solve the IDARP. If both (23) and (24) switched off the model solves the regular DARP. Both distinguish the IDARP-FRS from the IDARP.

$$\sum_{i \in C_{r1}} \sum_{j \in C_{r1}} z_{1rij} = 0 \qquad \forall\, r \in R, \qquad for\ i < j\ and\ br = 1, or\ i > j\ and\ b_r = -1 \tag{25}$$

$$\sum_{i \in C_{r2}} \sum_{j \in C_{r2}} z_{2rij} = 0 \qquad \forall\, r \in R\,, \qquad for\ i < j\ and\ br = 1, or\ i > j\ and\ b_r = -1 \tag{26}$$

Constraints (25)-(26) are the 'direction constraints' that newly developed and distinguish the IDARP-FRS. They present an approach for diminishing the transfer arcs.

3.2 Model Strengthening

For reducing the network size and curbing the run time, the next strategies are essential:

Arc elimination

The IDARP-FRS is defined over a directed graph *G = (N, A)*. Due to maximum ride time and time windows constraints, some arcs become infeasible and eliminating them reduces the number of binary variables. Some rules are dedicated to the original DARP introduced by Cordeau (2006) and some others are for the IDARP devised by Häll et al. (2009). The latter rules are related to at least one transfer node and are extended to match the purpose of the IDARP-FRS which includes more than one fixed-line. For more details, check Häll et al. (2009) and Cordeau (2006).

Variable substitution

Vehicles homogeneity and the implementation of a single depot allow removing some variables because the most important thing is to know the optimized routes not which vehicle is dedicated to which route. With this consideration, introducing the new variables v_{ij} is useful; they are binary variables equal to 1 if arc *(i, j)* is used by vehicle *k*, 0 otherwise. They are connected to x_{kij} as in constraint (27):

$$\mathrm{v}_{\mathrm{ij}} = \sum_{k \in K} x_{kij} \qquad \forall\, i \in N, j \in N \;\; (i,j) \neq (0,2n+1) \tag{27}$$

The implementation of *vij* entails some changes and substitution of the x_{kij} in the objective function and in constraints (2), (4), (5), (9), (11), and (16). However, in constraints (3) and (10), the variables x_{k0j} are essential to know when each vehicle starts, so they will not be replaced. Instead, constraint (19) can be replaced with:

$$v_{0j} = \sum_{k \in K} x_{k0j} \qquad \forall\, j \in P \cup D \cup C \tag{28}$$

$$v_{ij} \in \{0,1\} \qquad \forall\, i \in N, j \in N \tag{29}$$

$$x_{k0j} \in \{0,1\} \qquad \forall\, j \in N \tag{30}$$

Transfer node strengthening

Some constraints related to the transfer nodes are added.

$$\sum_{i \in C_{rt}} \sum_{j \in N} \mathrm{v_{ij}} <= 2 \qquad \forall\, r \in R \tag{31}$$

$$\sum_{j \in N} v_{ji} = \sum_{j \in C_{r1}} (z_{1ji} + z_{1ij}) \qquad \forall\, r \in R, i \in C_{r1} \tag{20}$$

$$\sum_{j \in N} v_{ji} = \sum_{j \in C_{r2}} (z_{2ji} + z_{2ij}) \qquad \forall\, r \in R, i \in C_{r2} \tag{21}$$

Constraint (31) does not allow any user to be transferred more than twice. Constraints (32)-(33) show that whenever a vehicle reaches a transfer node, an arrival at or a departure from this node by the fixed service should be started, for lines 1 and 2 respectively.

Symmetry-Breaking Constraints

These are newly adapted from the literature to the context of the IDARP-FRS. They eliminate symmetries and shrink the search space size.

$$B_k - B_{k+1} \;<=\; 0 \qquad for\ k\ \in K/|K| \tag{34}$$

$$\sum_{j \in N} t_{0j} x_{k0j} \;>=\; \sum_{j \in N} t_{0j} x_{(k+1)0j} \quad for\ k\ \in K/|K| \tag{22}$$

Constraint (34) ensures that the vehicles are used in sequence. Constraint (35) sets the travel time from the depot to node *j* for vehicle *k* is longer than for vehicle *k+1*.

4. Data Collection

The same 4-request instance introduced by Häll et al. (2009) is employed. For the IDARP-FRS, an additional line with three stations is added to the instance. In test 5, a new request is inserted. Fixed stations are replicated for each request, and the idea of applying a direction for each request for the fixed service is employed. Table 2 illustrates how the replicated nodes are numbered. Table 3 reveals the values of the parameters. Max ride time is assumed to be double the travel time between the pickup and drop-off nodes. The latest time (*li*) is calculated based on a time window of 900 sec. The earliest service time at pickup nodes is shown in Table 3. The earliest time at drop-off nodes = $e_i + d_i + t_{i,n+i}$.

Table 2: Replicated transfer nodes numbering

	Line 1			**Line 2**		
Request	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6
1	10	11	12	22	23	24
2	13	14	15	25	26	27
3	16	17	18	28	29	30
4	19	20	21	31	32	33

Gurobi 9 has been used to solve the model on a Windows 10 64-bit, Intel® Core™ i7 CPU @1.80 GHz 1.99 GHz and 7.89 GB usable memory laptop. The basic IDARP has been tested once and the IDARP-FRS has been tested 4 times (Table 4). The IDARP-T1 evaluates the IDARP for solving a 4-request, 12-transfer node instance after the implementation of the arc elimination, variable substitution, and transfer nodes strengthening strategies. The IDARP-FRS-T1 assesses the model’s ability to accommodate an extra fixed line with three stations and solving the same 4 requests but with 24 transfer nodes under the same augmentation strategies. IDARP-FRS-T2 tests the same 4 requests with the 24 transfer nodes and same strategies in addition to the symmetry-breaking constraints. IDARP-FRS-T3 additionally tests the role of direction-based constraints as a newly developed approach. The IDARP-FRS-T4 tests the insertion of a new request which adds 8 extra nodes to the model. The gap and computational time have been observed.

Table 3: Parameters

Parameter	**Value**	**Description**
n	5	no of requests
R	[1,2,3,4,5]	set of requests
br	[1, -1, -1,1]	set of request direction
K	[I,2]	set of vehicles
Q	20	maximum no of passengers
T	4000	max duration time (sec.)
g1, g2	*3, 3*	no of stations in line 1, line 2
q_r	[1,2,1,2,1]	no of passengers/ request
e_i	[50,350, 650, 950, 1200]	earliest time at pickups
l_i	[950, 1250, 1550, 1850, 2100]	latest time at drop-offs
Tw	900 sec	Time window at pick-ups and drop-offs
e_k	[0,0]	earliest service time
(xo, y0)	(4.5, 3)	depot coordinates
xp	[5,3.3,3.5,13]	XC pick-ups
yp	[4,10.5,8.8,4.5]	YC pick-ups
xd	[2,11,11.3,1]	XC drop-offs
yd	[11,1,3,9]	YC drop-offs
xf1	[2,5.2,12.8] * n	XC transfer nodes at line 1
yf1	[8.5,2.8,1.5] * n	YC transfer nodes at line 1
xf2	[5, 8.2, 15.8] * n	XC transfer nodes at line 2
yf2	[8.5,4,3.5] * n	YC transfer nodes at line 2

5. Results and Discussion

The results are summarized in Table 4 and visualized in Figure 1. The optimal routes are the same for the first four tests as they test the same number of requests; pick-up and drop-off locations and times; and depot. These results demonstrate the successful formulation of the IDARP-FRS. The reduced time in test 2 reveals the significance of symmetry-breaking constraints while the time reduction in test 4 underlines the substantial role of the newly developed idea of user's direction. The massive increase in the computational time (from test 1 to test 2; and from test 4 to test 5) complies with the literature (Cordeau 2006, Häll et al. 2009, Posada et al. 2017) and ensures that the problem is NP-hard, and its computational complexity exponentially grows when the problem size expands.

Table 4: Experimental test results

	IDARP-T1 4r-12tn	IDARP-FRS-T2 -4r-24tn	IDARP-FRS-T3 - 4r-24tn	IDARP-FRS-T4 -4r-24tn	IDARP-FRS-T5 -5r-30tn
Arc Elimination	yes	yes	yes	yes	yes
Variable substitution	yes	yes	yes	yes	yes
Transfer nodes strengthening	yes	yes	yes	yes	yes
Symmetry Constraints	no	no	yes	yes	yes
Direction Constraints	no	no	no	yes	yes
Obj. (min cost)	39	39	39	39	59
Gap %	**0**	0	0	0	45.9
Elapsed Time (sec)	**8**	380	130	35	2000
T1-T4 results: 1st vehicle route	0 → 1 → 3 → 2 → 5 → st1 → 8 → 9 St1-line1 used to drop-off r2 ,3 and pickup r4				
T1-T4 results: 2nd vehicle route	0 → st2 → 7 → 4 → 6 → st2 → 9 St2-line1 visited twice to pick up r 2,3 and later to drop-off r 4				

5.1 Numerical Results

Table 4 indicates the numerical outputs for 5 experimental tests. In test IDARP-T1-4r-12tn, the basic IDARP model of Hall et al. which includes 4 requests, and 12 transfer nodes is solved to the optimality in 8 seconds. In test IDARP-FRS-T2-4r-24tn, the IDARP-FRS which includes additional 12 transfer nodes are solved to optimality in 380 seconds. This shows how the run time increases dramatically with the insertion of additional transfer nodes. In test IDARP-FRS-T3- 4r-24tn, the run time is reduced to 130 seconds due to the implementation of the symmetry breaking constraints. This time further diminished to 35 seconds when the structure of transfer nodes is strengthened by the newly invented direction constraints. The considerable reduction in time reflects the effective role of the symmetry breaking constraints and the direction-based constraints. This encourages the search for more applicable constraints to make the model reasonably and tightly constrained (Cordeau 2006).

5.2 Graphical Results

For test 1 to test 4, the minimum-cost routes are illustrated in Figure 1. The first vehicle departs from the depot (0) and travels to pick request 1, 3, and 2 in order. It then drops off request 1 at node (5) and request 2 and 3 at station1 at line 1. At this station it waits for request 4 to arrive, then it travels to drop off request 4 at node (8) to finish its route by returning to the depot (9). The second vehicle starts its route from the depot to visit station 2 at line 1 to pick up request 2 and 3. It drops off request 3 at node (7) and continues its journey to pick up request 4 and drop of request 2 at node (6). It again revisits station 2 at line 1 to drop off request 4 to be transported by the fixed line service. It then returns to the depot.

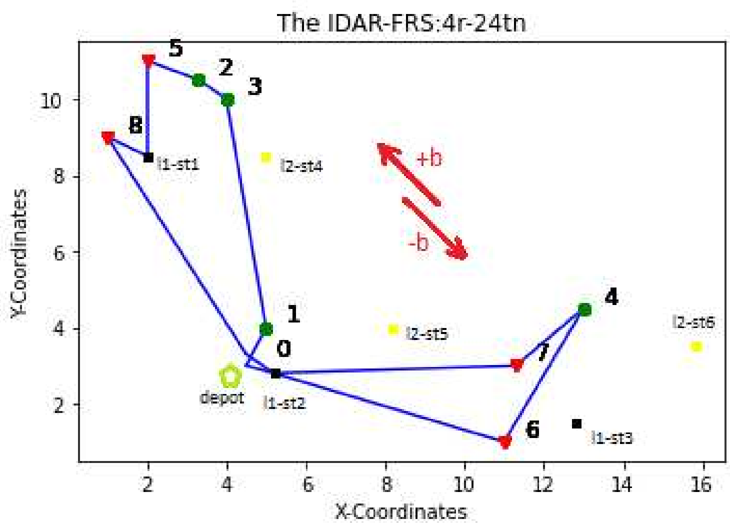

Figure 1: Illustration of the minimum-cost routes for T1-T4

5.3 Proposed Improvements

Test 5, IDARP-FRS-T5, comprises 5 requests and 30 replications of transfer locations. The elapsed time is 2000 seconds to realize a 45.9% gap. This result should be improved if the number of replicated transfer locations is reduced. Therefore, a new approach for modeling transfer locations differently and more effectively is essential.

5.4 Validation

The IDARP-FRS has extended the IDARP that was validated by Häll et al. (2009) to includes extra transfer stations distributed over two fixed lines. The same instance – 4 requests with the same pick-ups and drop-offs locations and times, and same depot coordinates– is tested by the basic IDAR and IDARP-FRS. The same minimum-cost routes are obtained by the two models when they are solved to the optimality. For the IDARP-FRS, the heuristics solution (before reaching the optimality) attempted successfully to iterate over the stations of the extra line which proves the capability of the model.

6. Conclusion

The IDARP represents an effective formulation for two-mode transport. The IDARP-FRS has effectively extended the IDARP to specify which fixed-line is perfect for which user in terms of operating costs. It also accommodates constraints to allow users to be transported by the IDARP or the regular dial-a-ride problem. A set of newly formed direction-based constraints are presented. The role of these constraints in suppressing the number of transfer nodes related arcs is proved. Moreover, the IDARP-FRS comprises the symmetry-breaking constraints which has a potential in strengthening the model. The IDARP-FRS is a rich problem that needs future research to be extended with the inclusion of numerous real-life scenarios. However, the utmost attention should be paid to advancing the transfer nodes' structure so that bigger instances can be solved within a reasonable time.

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Biographies

Sabreen Hassan is a Master of Science student in Industrial Engineering, Faculty of Engineering, Alexandria University. She earned Bachelor of Science (BSC) in Production Engineering from Alexandria University. Her research interests include optimization and public transport.

Islam Ali graduated from Alexandria with a BSc in Production Engineering in 2009. He obtained a MSc in Industrial Engineering from the same university in 2011 whilst researching container terminal operations. In 2017, he obtained his PhD degree from the School of Industrial Engineering at Purdue University, doing research on Humanitarian Logistics, after which he joined the teaching staff as an Assistant Professor at the Production Engineering Department, Alexandria University. Currently, his research focuses on the design, analysis, and application of efficient algorithms to solve different types of routing and scheduling problems in the fields of Supply Chain Management and Logistics.

M. Nashat Fors is Professor Emeritus of Industrial Engineering, Faculty of Engineering, Alexandria University, Egypt. He has written or co-authored numerous research papers and articles on Mathematical Programming & Simulation Techniques Applications, Supply Chain Management, Operations Management, Maintenance planning, Water Management, Scheduling & Distribution, Aggregate Planning, Facilities Layout. He has many joint projects, consultation, and training programs with industry and other universities in the area of Operations Planning & Scheduling, Maintenance Planning, and Water Management. Dr. Fors is advisor and co-advisor of more than 50 earned M.Sc. and 10 Ph.D. in the different areas of Industrial Engineering. He participated in designing, innovation, development, and advisor of educational programs, curricula at graduate and undergraduate levels at Alexandria University, Cairo University, Aim Shams University, E-Just University, and AAST&MT. He was involved in the switching plan from credit hour system to yearly system during his stay at King Abdulazziz University, SA. He is a team leader in Restructuring and Human Resource Development at Faculty of Engineering - Alexandria University CIQAP. He is member of Many Alexandria Universities Committees (AU Faculty Grievance, AU Council executive, AU Prizes and Awards, AU bylaws).