Research on the Critical Demand for Different Types of Public Transit Feeder Systems

Jinpei Li and Sida Luo(✉)

School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China
sdluo@bjtu.edu.cn

Abstract. In low demand areas, it has been proven that demand responsive transit (DRT) is more cost-effective compared to fixed route transit (FRT). The present study further examines the demand threshold of four public transit systems: DRT, FRT, fixed-route but non-fixed-point transit and dynamic stop transit, which are designated to feed urban rail transit. It constructs user cost models for these four transit systems, where we aim to find the system with the "best" user experience. The demand at which the user costs of two transit systems are equal is referred to as the critical demand. After deriving the analytical expression for the critical demand, a sensitivity analysis of the analytical solution is conducted. The results indicate that DRT is optimal for low demand density scenarios. If we allow more flexibility to transit services compared with FRT, fixed-route but non-fixed-point transit proves superior for higher demand. Additionally, the critical demands between DRT and FRT, as well as that between DRT and fixed-route but non-fixed-point transit, are influenced by the bus speed and passenger walking speed, where the impact has an upper bound; at the same time, the fleet size also affects the critical demands, where the impact has a lower bound.

Keywords: Public Feeder Systems · User cost · Critical demand

1 Introduction

Scholars have begun to explore how to merge traditional transit systems with taxi services for a more economical and flexible transit to satisfy people's demand for efficient and convenient mobility. Gupta et al. proposed DRT can provide door-to-door services, but it is not suitable for high-density areas [1]. Guo et al. and Aalipour and Khani proposed a transit model by combining FRT with other transit models [2, 3]. Abdelwahed et al. and Zhang et al. proposed a dynamic public transit system that combined passenger convenience with the sustainable development of transit [4, 5]. However, research on how to choose the appropriate transit system based on demand remains relatively scarce. Luca Quadrifoglio and Xiugang Li were among the first to study the critical demand for FRT and DRT in single and double-bus scenarios using a continuous approximation method, and derived analytical solution expressions [6]. Feng Qiu et al. building on their work, further studied the critical demand for fixed route and fixed-point but non-fixed-route transit [7]. Yue Zheng et al. established a user cost function, deriving the critical demand for the service transit point between route deviation strategy and point deviation strategy [8].

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In this paper, we establish models for four transit feeder systems: DRT, FRT, fixed-route but non-fixed-point transit, and dynamic stop transit. We will compare system performances under expected demand levels, identify the more critical threshold densities, derive analytical solution expressions, and conduct sensitivity analysis.

2 System Description

2.1 Service Areas and Trip Demand

The symbols used in this article are follow [8]. In this research, the focus is placed on public transit feeder system within a service area delineated by a rectangular shape. The width of this rectangle is W, and the length is L, divided by two transfer stations located at the terminals. Taking demand responsive transit as an example, as shown in Fig. 1. The transit system takes passengers from the transfer station to their destination or from the starting point to the transfer station. Passengers can transfer to the rail transit at the transfer station. Within the service area, there is only one route providing service. There are three types of passengers in the service area, with proportions γ_1, γ_2, and γ_3 respectively, defined as follows ($\gamma_1 + \gamma_2 + \gamma_3 = 1$): Type I: pick up and drop off both at transfer stations. Type II: pick up at transfer stations, and drop off not at transfer stations. Type III: pick up not at transfer stations, and drop off at transfer stations.

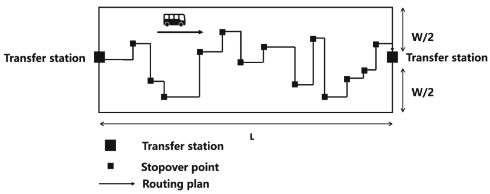


Fig. 1. Demand Responsive Transit Schematic Diagram.

2.2 Symbol Explanation and Measure of Performance

In the study, it is assumed that passengers rely on transit. The service vehicles have the following characteristics: considering the average speed of stopping due to traffic and pedestrian interference, V_b; the dwell time for sign stops or requested stops, ${T_d}^r$; the dwell time at transfer stations, ${T_d}^f$; the walking speed of passengers, V_{wk}; the demand rate in the service area, θ; and the fleet size, M. In the analysis, we define the performance measure of the transit system as the user cost function. The user cost function is constructed as a weighted sum of three indicators: expected walking time K, expected waiting time A, and expected riding time R, with weights of ω_K, ω_A, and ω_R, respectively. A lower value of F clearly indicates a better transit system.

$$F = \omega_K K + \omega_A A + \omega_R R \tag{1}$$

3 User Cost Model

3.1 Demand Response Transit

DRT is shown in Fig. 1, where vehicles travel from one transfer station to another and then return. The vehicles adopt a "no backtracking constraint strategy", and passengers are served in the order of horizontal coordinates as guided by [6]. The calculation of related costs is consistent with Sect. 4.3.1 of [8], and F is as follows:

$$F = \frac{6L + W + 6V_b T_d^f}{6MV_b - 2W\theta(\gamma_2 + \gamma_3) - 6\theta V_b T_d^r(\gamma_2 + \gamma_3)} \times \left[\omega_A \left(\begin{array}{l} \gamma_1 + \gamma_2 + \dfrac{\gamma_3 W(\gamma_2 + \gamma_3)\theta}{12V_b} \\ + \dfrac{\gamma_3 T_d^r(\gamma_2 + \gamma_3)\theta}{4} \end{array} \right) + \omega_R M \frac{1 + \gamma_1}{2} \right] - \frac{\omega_A\left(\gamma_3 W + 3\gamma_3 T_d^r V_b\right)}{12V_b} \tag{2}$$

3.2 Fixed Route Transit

The FRT operation strategy offers continuous service, with vehicles traveling along a fixed route from one transfer station to another, passing through n fixed stations before returning, as shown in Fig. 2.

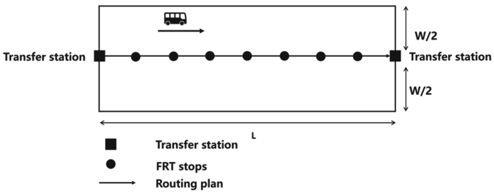


Fig. 2. Fixed Route Transit Schematic Diagram.

Assuming that customers walk to the nearest station according to the Manhattan distance, only Type II and III passengers need to walk, so expected walking time K is:

$$K = \frac{(\gamma_2 + \gamma_3)}{4V_{wk}} \left(\frac{L}{n + 1} + W \right) \tag{3}$$

The single-trip time T_r includes not only the driving time but also the dwell time:

$$T_r = \frac{L}{V_b} + nT_d^r + T_d^f \tag{4}$$

The headway of system is $2T_r/M$, so the expected waiting time for all passengers is Tr/M. The expected riding time of Type I passengers is T_r, while the expected riding time of Types II and III passengers is $T_r/2$. Therefore: the user cost F is:

$$F = \frac{\omega_K(\gamma_2 + \gamma_3)}{4V_{wk}} \left(\frac{L}{n + 1} + W \right) + \left[\frac{\omega_A}{M} + \frac{\omega_R(1 + \gamma_1)}{2} \right] \left(\frac{L}{V_b} + nT_d^r + T_d^f \right) \tag{5}$$

3.3 Fixed-Route but Non-Fixed-Point Transit

The fixed-route but non-fixed-point transit as shown in Fig. 3, vehicles travel along a route from one transfer station to another and stop when there is passenger demand.

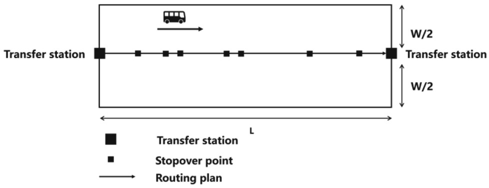


Fig. 3. Fixed-route but Non-fixed-point Transit Schematic Diagram.

The expected walking time K is:

$$K = \frac{(\gamma_2 + \gamma_3)W}{4V_{\mathrm{wk}}} \tag{6}$$

The calculation method of single-trip time T_r is consistent with that of Sect. 3.1.

$$T_r = \frac{ML + MV_b T_d^f}{MV_b - \theta V_b T_d^r(\gamma_2 + \gamma_3)} \tag{7}$$

The calculation method for the expected waiting time and riding time of passengers is consistent with Sect. 3.2, and the user cost F is

$$F = \frac{\omega_K(\gamma_2 + \gamma_3)W}{4V_{\mathrm{wk}}} + \left[\frac{\omega_A}{M} + \frac{\omega_R(1+\gamma_1)}{2}\right]\frac{ML + MV_b T_d^f}{MV_b - \theta V_b T_d^r(\gamma_2 + \gamma_3)} \tag{8}$$

3.4 Dynamic Stop Transit

Dynamic stop transit as shown in Fig. 4. Passengers within βW enjoy "door-to-door" service, and other passengers need to walk to the service area.

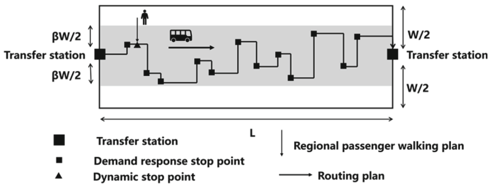


Fig. 4. Dynamic Stop Transit Schematic Diagram.

Not within βW range, passengers need to walk, so the average walking time K is

$$K = \frac{\beta(\gamma_2 + \gamma_3)W}{4V_{wk}} \tag{9}$$

The calculation method for the expected waiting time and riding time of passengers is consistent with Sect. 3.3, and the user cost F is

$$F = \frac{\beta(\gamma_2+\gamma_3)W\omega_K}{4V_{wk}} - \frac{\omega_A\left(\gamma_3 W\beta^2 + 3\gamma_3 T_d^r V_b\right)}{12V_b} + \frac{6L + W + 6V_b T_d^f}{\begin{pmatrix} 6MV_b - 2W\theta(\gamma_2+\gamma_3) \\ -6\theta V_b T_d^r(\gamma_2+\gamma_3) \end{pmatrix}} \times\left[\omega_A\left(\gamma_1+\gamma_2+\frac{\gamma_3 W\beta^3(\gamma_2+\gamma_3)\theta}{12V_b}+\frac{\gamma_3 T_d^r(\gamma_2+\gamma_3)\theta}{4}\right)+\omega_R M\frac{1+\gamma_1}{2}\right] \tag{10}$$

4 System Performance

4.1 Parameter Values

Similar to previous studies, ωA = 1 and ωR = 1 are follow [9] and set ωK = 2 is follows [10]. The remaining parameters are as follows (Table 1):

Table 1. Parameter values.

Parameters	Values	Parameters	Values
L	5 km	*Tdr*	12 s
W	1.6 km	*Tdf*	15 s
M	2	*γ1/γ2/γ3*	0.2/0.4/0.4
Vb	40 km/h	*n*	5
Vwk	5 km/h	*β*	0.5

4.2 System Performance Comparison

After substituting the value of parameter 4.1, we can obtain the user costs associated with the demand for four types of transit systems. The user costs of the four transit systems vary with demand density as shown in Fig. 5. It can be observed that under conditions of low demand, DRT is more advantageous to users, as it can provide more personalized services, reducing users' waiting and travel time. However, when demand reaches a certain level, the fixed route transit becomes more advantageous, as it can provide efficient and lower-cost services.

In this scenario, if we allow more flexibility to transit services compared with FRT, fixed-route but non-fixed-point transit would be a better choice. This is because the fixed-route but non-fixed-point combines the flexibility of Demand Responsive Transit and the efficiency of fixed route transit, able to provide more convenient direct services while meeting higher demands.

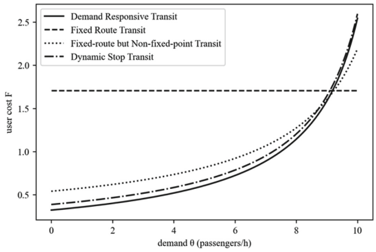


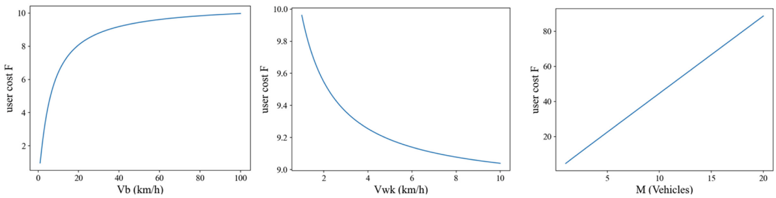
Fig. 5. System Performance Chart.

5 Critical Demand Analysis

5.1 Demand Responsive Transit and Fixed Route Transit

The critical demand is defined as the demand level where the user costs of two transit systems equal each other. We equate the F of DRT and FRT, and the analytical solution for the critical demand as follows.

$$\theta = \frac{\begin{array}{l}\frac{3MV_b(\gamma_2+\gamma_3)}{2V_{wk}}\left(\frac{L}{n+1}+W\right)\omega_K + 3n(1+\gamma_1)MV_bT_d^r\omega_R \\ +\left[6\gamma_3 L + \left(\left(\frac{M}{2}+1\right)\gamma_3 - 1\right)W + \left(6n+\frac{3M\gamma_3}{2}\right)V_bT_d^r + 6\gamma_3V_bT_d^f\right]\omega_A\end{array}}{\begin{array}{l}\frac{(\gamma_2+\gamma_3)^2}{2V_{wk}}\left(\frac{L}{n+1}+W\right)\left(W+3V_bT_d^r\right)\omega_K \\ +\left\{\begin{array}{l}\frac{1}{M}\left(\frac{L}{V_b}+nT_d^r+T_d^f\right)\left(2W+6V_bT_d^r\right)(\gamma_2+\gamma_3) \\ +\left(\frac{W}{12V_b}+\frac{T_d^r}{4}\right)\left(6L+3W+6V_bT_d^r+6V_bT_d^f\right)\gamma_3(\gamma_2+\gamma_3)\end{array}\right\}\omega_A \\ +\left(1-\gamma_1^2\right)\left(\frac{L}{V_b}+nT_d^r+T_d^f\right)\left(W+3V_bT_d^r\right)\omega_R\end{array}} \tag{11}$$



(a) Effect on V_b (b) Effect on V_{wk} (c) Effect on M

Fig. 6. Critical Demand Analysis.

This study substitutes the parameter values defined in Sect. 4.1 (except for the research variables) into Formula (11) for sensitivity analysis.

The Relationship Between Bus Speed and Critical Demand
The relationship between bus speed and critical demand is shown in Fig. 6(a). The critical demand increases with the increase of bus speed, indicating that the higher the bus speed, the more favorable it is for DRT. When the bus speed (V_b) approaches 0, the critical demand tends to 0, indicating that the system is close to FRT. Conversely, when V_b approaches infinity, the critical demand tends towards 10.56 passengers per hour, indicating that DRT has certain service limitations.

The Relationship Between Passenger Walking Speed and Critical Density
The relationship between passenger walking speed and critical demand is shown in Fig. 6(b). The critical demand decreases with the increase of passenger walking speed, indicating that the higher the passenger walking speed, the more favorable it is for FRT. When V_{wk} approaches infinity, the critical demand tends towards 8.87 passengers per hour, indicating that the impact of passenger walking speed on the critical demand has certain limitations; it cannot infinitely expand the advantages of FRT.

The Relationship Between Fleet Size and Critical Density
The relationship between fleet and critical demand is shown in Fig. 6(c). The critical density increases with the increase of fleet size, indicating that a larger fleet size is more beneficial for demand responsive transit. Moreover, the impact of fleet size on critical density remains significant.

5.2 Demand Responsive Transit and Fixed-Route but Non-Fixed-Point Transit

Due to the complexity of the analytical solution for the critical density between DRT and fixed-route but not-fixed-point transit, it is not provided here. Consistent with Sect. 5.1, the effects of bus speed, passenger walking speed, and fleet size on the critical demand density were analyzed, leading to conclusions similar to those in Sect. 5.1. For DRT and fixed-route but non-fixed-point transit, their critical demand densities are influenced by these three variables. The faster the bus speed, the higher the critical demand density, which is more favorable for DRT, and this influence is limited; the faster the passenger walking speed, the lower the critical demand density, which is more favorable for fixed-route but non-fixed-point transit, and the impact has an upper bound; the larger the fleet size, the higher the critical demand density, which is more favorable for DRT, and the impact has a lower bound.

6 Conclusion

The study reveals that DRT are the best choice when passenger demand is at a lower level. However, as the scale of demand increases to a certain threshold, FRT becomes more advantageous. If we allow more flexibility to transit services compared with FRT, fixed-route but non-fixed-point transit has more advantages than DRT in reducing user costs.

This study further explores the demand switching points from DRT to FRT, as well as from DRT to fixed-route but non-fixed-point transit, defining these demand switching points as critical demand. By applying continuous approximation methods, we have constructed a decision problem analysis model framework aimed at supporting planners and operators in making informed choices. Additionally, we compared the competitive utility functions under different operational policies to determine the critical demand representing the conditions for service mode transitions. In the analysis of parameter influences, we find that bus speed and passenger walking speed significantly affect both types of critical demand densities, where the impact has an upper bound; similarly, fleet size also affects both, where the impact has a lower bound.

In conclusion, this paper strongly recommends that traffic planners and operators adopt the findings of this research to select the most fitting public transit systems for varying regional needs. With the increasing performance requirements of transit system networks, the proper design of transit services is particularly crucial.

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