Evaluation of on-demand line-based bus services

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Abstract

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1 Introduction

Modern societies need to solve the trade-off between individual mobility and the necessary reduction of CO_2 emissions. As a consequence several alternative technologies like electric-driven vehicles or cars with hydrogen propulsion as well as mobility services like sharing economy (car sharing, bike sharing, e-scooter) or ridepooling services are evaluated. Currently it is an open question which of them should be used in which context and how they are ideally combined with traditional line-based public transport.

In this paper, we evaluate the potential of a different solution which can be classified between a traditional line-based bus service and a door-to-door ridepooling service. The first has the advantage of significant pooling, as all customers need to adapt themselves to the time schedule and the bus route. A bus is used best if the bus schedule addresses the mobility demand well for as many people as possible. However, this is not the case in off-peak hours or rural areas. In these cases, an on-demand ridepooling service has advantages, as the service is offered only in case of demand. Thus, no empty or oversized

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buses drive. Moreover, a ridepooling service has the advantage of door-to-door transportation which increases the comfort and the access to the mobility service especially for people with reduced mobility.

Our setting is between a traditional line-based bus service and a ridepooling service. We consider a line-based bus service, but the bus does not drive the entire line as scheduled but only on-demand. This means that customers need to announce their interest in the service upfront via smartphone application or telephone call. In practice, this is done with a lead time of two hours. Then, the bus drives only if it is required. Thus, we know the demand upfront such that resources can be used well and we avoid empty or oversized buses driving. This setting is currently used in rural areas in the north of Germany. In rural areas, villages are rather small and concentrated around a centre. Hence, transportation requests mainly occur between villages or between a village and the next city, which means that even a line-based bus is close to a door-to-door transportation.

The aim of the paper is to evaluate the concept of an on-demand line-based bus service that serves the pre-defined bus routes only if demand occurs. With respect to this aim our contribution is as follows:

- We present a network flow formulation for an on-demand line-based bus service if bus capacities are sufficiently high.
- We extend the approach to include buses of restricted size and driver breaks.
- We analyse the computational complexity of each of the aforementioned variants and provide clear borders between hard and easy problem variants.
- We use the models to evaluate the potential of on-demand line-based bus services as a mobility service especially in rural areas. To do so, we use real-world data.

By the last point we also address an issue named in the survey paper by Vansteenwegen et al. (2022) to evaluate the potential of bus services and how and where they are used best.

The paper is constructed as follows: First, we classify our setting into the literature (Section 2). Then, we give a formal problem description (Section 3) and present our model formulations as well as the complexity analysis (Section 4). Afterwards, the models are used to evaluate the potential of an on-demand line-based bus service in a comprehensive computational study (Section 5). Finally, the paper closes with a conclusion in Section 6.

2 Literature review

Our setting is, as already mentioned, a mixture between a traditional line-based bus service and a ridepooling service. We only need to serve bus tours with a positive demand or even only the parts with a positive demand. Therefore, the literature review is separated into the two research areas public bus services and ridepooling. Moreover, we shortly refer to literature regarding the shift scheduling of drivers, as we include driver breaks in our analysis in Section 4.4.

2.1 Public bus services

Public bus services are a backbone of the public transport system. Especially in rural areas, a main focus of public bus services is on the transportation of pupils to and from schools which leads to two demand peaks directly before and after school times while there is only a moderate demand in between. In line with the practical demand, a significant amount of papers focuses on school bus routing. However, during these peak times the demand can be predicted very well, as it is known how many pupils need to be transported between a stop and a school (pupils often have subsidized tickets for exactly their demand). Since our setting focuses on the low demand off-peak and weekend hours, we shortly refer to the survey on the school bus routing problem by Park and Kim (2010) for further details on school bus routing. Nevertheless, note that also these papers often use mixed-integer programming formulations for their solution approaches (Bektaş and Elmastaş (2007), **?**, Bögl et al. (2015)). Beside line-based public transport there are also other applications for bus services like event tours, e.g. in the tourism sector (Brandinu and Trautmann, 2014).

Our approach to serve bus tours only (partly) if there is a positive known demand can be classified as a flexible bus service. The survey paper by Errico et al. (2013) classifies a bus service as semi-flexible if there is some buffer time in the bus tour which can be used to deviate from the fixed bus route and serve nearby on-demand requests. As one setting the authors describe marked stops on the bus tour where users can ask for service by waving their hand (so called flag requests). In fact, our setting is very similar. We also have marked stops, as all bus stops are on-demand marked stops. However, the customers cannot simply wave their hand when the bus passes the stop because the bus only serves the stop if there is a pre-known demand. In this point, we deviate by the fact that users need to announce their request upfront by a smartphone app or via telephone. As we know upfront whether there is a demand at a stop, we only need to visit stops with demand and, hence, can optimize the travelled bus routes visiting only these stops.

Another approach allowing vehicles to deviate from the fixed line-based path to serve customers within a surrounding service area is Mobility Allowance Shuttle Transit (MAST) investigated amongst others by Quadrifoglio et al. (2007). A different variation of flexibility is presented by Qiu et al. (2014). They integrated the pickup and delivery points of accepted curb-to-curb customers as temporary stops in their system which can then be used by other customers.

The system investigated in Pei et al. (2019) is semi-flexible in the sense that the buses, serving the same line in both directions with a U-turn at the end of the line, can shorten the line at the end if there is no demand. This means that the buses can perform their U-turn before the last stop and thereby shorten their tours at the end as well as the beginning of the next tour serving the line backwards. In the paper at hand, we somehow extend this setting to several lines such that buses can interrupt the service of a line if the bus is empty and switch to another line. This can be at the end of a line, i.e. the rest of the line is skipped or somewhere in between if another bus serves the rest of the previous line with positive demand. A recent review paper on on-demand bus services is provided by Vansteenwegen et al. (2022).

Kim and Schonfeld (2014) coordinate conventional bus services with flexible doorstep services allowing transits. The authors use probabilistic optimization models to reduce the customers transfer times. The problem of integrating ridepooling requests into the public transport network, i.e. serving customers with a ridepooling vehicle and public transport while allowing transfers between both, is also known as the integrated dial-a-ride problem (Häll et al., 2009).

In practical applications, bus schedules underlie further restrictions like multiple depots such that each bus needs to return to its own depot in the evening and different bus types (e.g. with different capacities). Gintner et al. (2005) presented a two-phase method to solve such multiple depots and vehicle types bus scheduling problems close to optimality even for instances with thousands of scheduled bus tours. In the paper at hand, we restrict ourselves to a single depot. However, our second and third setting also allow to include multiple depots by assigning the vehicles to them. By this, we could also ensure that the vehicle returns to its own depot. Moreover, depots' capacities can be included (Kliewer et al., 2006). Besides, we do not consider any recharging or refueling operations. For a survey on electric bus planning and scheduling see Perumal et al. (2022).

From the methodological point of view, as already mentioned, many papers use mathematical programming formulations to solve bus scheduling problems as we do in this paper. However, there are also papers combining mathematical programming with further methods like heuristics (Gintner et al., 2005) or constraint programming (De Silva, 2001). A survey paper on modelling approaches for vehicle scheduling models is provided by Bunte and Kliewer (2009).

2.2 Ridepooling

A ridepooling service is a door-to-door on-demand service where customers can request a ride between a pickup and a delivery location usually via a smartphone app. Thereby, the pickup and the delivery location are almost fully flexible and the request time is either immediately or within a given future time frame (Schulz and Pfeiffer, 2024). We defined semi-flexible bus lines as bus lines where the bus can leave the line in between to serve some on-demand customers nearby. A ridepooling service is in this sense a fully flexible bus service where no predefined time table or bus line is given (Vansteenwegen et al., 2022).

It seems to be apparent that a line-based bus service is used best if there is a high transportation demand from similar pickup to similar delivery locations while a ridepooling service is more attractive if there are very heterogeneous transportation requests of a reasonable density within a service area. Then, the ridepooling provider can pool them such that still all customers are served between their pickup and delivery locations but not necessarily directly. Prior studies show that significant pooling rates are possible while still maintaining acceptable detours (Pfeiffer and Schulz, 2022). In comparison to a ridepooling service, our line-based service leads already to some kind of clustered requests. Customers cannot ask for a request from any location to any other but only between the line-based stops, i.e. the customers themselves cluster their requests to the bus line. Therefore, it is likely to reach a reasonable pooling rate. Of course, this is also the case in any conventional line-based bus service. However, as only those tours with positive demand need to be served, fewer buses are required. How many buses are required with the on-demand line-based setting is one of the questions we want to answer in this paper.

The underlying tour scheduling problem of a ridepooling provider is the Dial-a-Ride Problem (DARP) which has been investigated for decades (Psaraftis, 1980). The DARP was originally introduced to serve people with reduced mobility where it is also used in practice. Borndörfer et al. (1999) developed a vehicle scheduling approach called Berlin's Telebus. Telebus is a bus service for handicapped people who are not able to use the public transport system. For the DARP different solution approaches were developed: the three-index formulation (Cordeau, 2006), the two-index formulation (Ropke et al., 2007), the restricted fragments based formulation (Rist and Forbes, 2021) or the event-based formulation (Gaul et al., 2022, 2025).

Ridepooling services have also been investigated in cities (Pfeiffer and Schulz, 2022) as well as in rural areas with interrelated trips (Johnsen and Meisel, 2022). A further related concept is the line-based DARP (Reiter et al., 2024). In the line-based DARP, requests can only occur along a traditional bus line but without any time schedule. Thus, requests can occur at any time and at any bus stop while the vehicles can skip stops or turn to the opposite direction before reaching the final stop. Our setting differs in the aspect that we still assume a time schedule to be given.

2.3 Shift scheduling

As our paper does not mainly focus on shift scheduling but only addresses the effect of driver breaks on the planning solution, we only shortly refer to the survey on tour scheduling by Alfares (2004) as well as to the shift scheduling paper for ridepooling services by Berthold et al. (2024). However, note that we use the simplifying assumptions that breaks are already scheduled. In fact, there is some degree of freedom in the break scheduling due to labour regulations (Boyer et al., 2018). Moreover, we assume that there is a predefined assignment of one driver to a bus whenever the bus is outside the depot. Thus, driver changes are only allowed when the bus visits the depot between tours. Perumal et al. (2019) allow driver changes also outside the depot.

3 Problem description

We investigate a line-based bus service. The bus service needs to serve several bus lines at different times according to a given time schedule. We call the underlying problem the *Bus Line Servicing Problem* (*BLSP*). We consider every combination of a bus line and a starting time as one line $l \in L = \{1, ..., n\}$. A line l consists of m_l stops $s^l_1, ..., s^l_{m_l}$. Every stop is associated with a time $\bar{t}_{s^l_j}$, $l \in L$, $j = 1, ..., m_l$, at which the bus stops at the stop. We assume that the time for boarding and deboarding is in comparison to the driving time negligible such that the bus stops and departs at the same time at each bus stop. This means that $\bar{t}_{s^l_{m_l}}$ indicates the end of bus tour l. Furthermore, travel times between any pair of stops are given by $t_{s^l_j, s^{l'}_{j'}} > 0$, $l, l' \in L$, $j = 1, ..., m_l$, $j' = 1, ..., m_{l'}$. Travel times between the single depot D and stops of tour l as well as between stops of tour l and the depot are given by $t_{D, s^l_j} > 0$ and $t_{s^l_j, D} > 0$, $l \in L$, $j = 1, ..., m_l$, respectively. We assume the triangle inequality to hold for all travel times. The bus company uses $|K|$ buses $k \in K = \{1, ..., |K|\}$ with capacity Q_k to serve the bus lines. The objective function is to minimize the number of required buses.

In our first setting, we assume that all buses have a sufficiently high, i.e. non-limiting, capacity. This setting is motivated by the practice where capacity can significantly be increased by standing room. Moreover, we assume the buses to be autonomous in this setting. This means that drivers do not need to be considered.

In the second setting, buses have a capacity limitation. The buses can be homogeneous, but we consider the general case with heterogeneous buses, i.e. $Q_k \neq Q_{k'}$ might hold for some $k, k' \in K$. In this setting, we also need to include customer demand $d_{s^l_j, s^l_{j'}}$ for every line $l \in L$ and stops $j, j' = 1, ..., m_l$ with $j' > j$. Note here that we need at most $|K| = n$ buses, one for every line if assume that a single bus can serve the demand of a line as it is done in practice.

In the third setting, we additionally include drivers. Thus, one driver is assigned to every bus and a time a_k at which the bus is available (the driver's shift starts), a time b_k at which the associated bus driver starts a break of a given length p, and a time c_k at which the driver's shift ends are given. Thus, bus k can only serve tours such that the bus starts not before a_k at the depot, spends a break between time b_k and $b_k + p$ and ends the last tour not later than c_k at the depot. Hence, the difference between settings two and three is that in setting three the availability of the bus is reduced to the time intervals $[a_k, b_k]$ and $[b_k + p, c_k]$. Thereby, we assume that driver shifts are given by the time intervals and that drivers are assigned to buses according to their capabilities if necessary.

The first setting can be interpreted as the setting with homogeneous autonomous buses such that no differentiation between the vehicles is necessary. The second setting can be interpreted as the setting with heterogeneous autonomous buses. The third setting can be interpreted as the current situation with driver-driven buses. Thus, the three settings allow us to evaluate the potential of on-demand line-based bus services in today's situation with drivers as well as in a future situation with autonomous buses. Moreover, the different capacities allow us to also answer the question of ideal bus capacities. Vansteenwegen et al. (2022) asked to answer this question in the investigation of bus services.

Our aim in this paper is to evaluate the effect if only those parts of a bus tour are served for which a positive demand is given in comparison to the service of all bus tours independent of the demand. Thus, we minimize the number of buses, and thereby drivers, required to serve the bus tours. We do this analysis for all three settings.

Of course, we also use some simplifying assumptions in this paper which still allow us to obtain realistic results in our analysis. These are:

- We consider one common depot for all buses.
- We consider driver shift schedules fulfilling the labour regulations to be given.
- We assume fixed driver breaks fulfilling labour regulations to be given and assume that there is only one scheduled break in a shift (there might be further free time between serving lines).
- We assume that there is a predefined assignment of one driver to each bus whenever the bus is on tour. Thus, we assume that there is always a driver who has the capabilities necessary to drive the bus. Otherwise, the bus can simply be excluded from the analysis. This also includes that no driver changes are considered outside the depot.
- We do not consider any refueling or recharging operations but assume the buses to be refueled/recharged when starting in the depot and that the tank/battery is sufficient to serve all scheduled tours until the depot is reached again.
- We do not differentiate between bus configurations beside the capacity.

The last point can be integrated by excluding that a certain bus is allowed to serve a certain tour if the configuration is not sufficient.

4 Model formulations

We start with the model for the first setting in Section 4.1. Thereby, we model the case where all tours need to be served (Section 4.1.1) as well as the case where only the tours with positive demand are served (Section 4.1.2). Before proceeding with the second setting in Section 4.3, we present a complexity analysis of the *BLSP* using the results of Section 4.1 (Section 4.2). Finally, the third setting is modelled in Section 4.4.

4.1 Network flow formulation for the first setting

Let us first consider the case where all tours need to be served.

4.1.1 Service of all tours

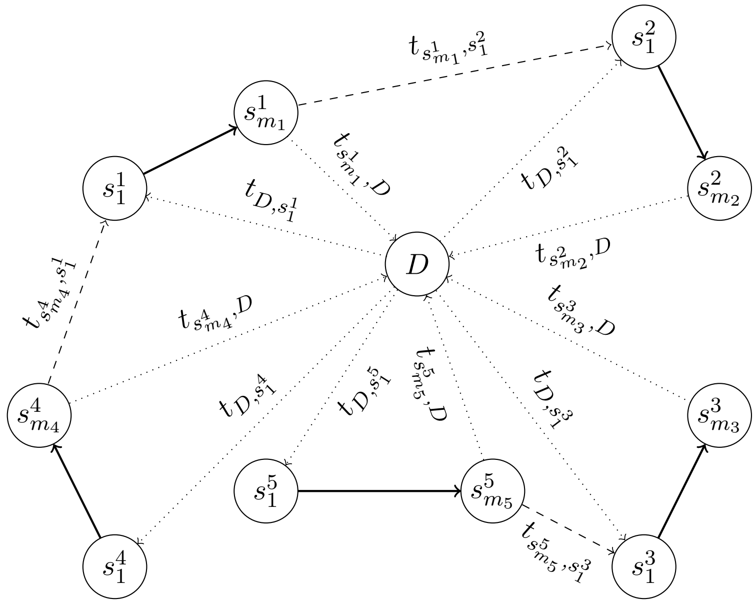


Figure 1: Graph for setting 1

The setting can be represented by a graph $G = (V, A)$ with

$$V = \{D, s^1_1, ..., s^n_1, s^1_{m_1}, ..., s^n_{m_n}\}$$

as in Figure 1. In the graph, a bus schedule with five lines is pictured. Moreover, the travel times t_{D,s^l_1} between the depot and the start of the line as well as between the end of the line and the depot $t_{s^l_{m_l},D}$ are given by dotted lines. These relations must always be possible, as otherwise a line could not be served (at least if triangle inequality holds). Moreover, dashed lines represent possible tour connections with travel times $t_{s^l_{m_l},s^{l'}_1}$. A connection of two tours is possible if $\bar{t}_{s^l_{m_l}} + t_{s^l_{m_l},s^{l'}_1} \leq \bar{t}_{s^{l'}_1}$ holds. Thus,

$$\begin{aligned} A =& \{(D, s^1_1), ..., (D, s^n_1), (s^1_{m_1}, D), ..., (s^n_{m_n}, D), (s^1_1, s^1_{m_1}), ..., (s^n_1, s^n_{m_n})\} \\ &\cup \{(s^l_{m_l}, s^{l'}_1) : \bar{t}_{s^l_{m_l}} + t_{s^l_{m_l},s^{l'}_1} \leq \bar{t}_{s^{l'}_1}\}. \end{aligned}$$

In the concrete setting, two tours are possible serving bus tours 4, 1 and 2 as well as 5 and 3, respectively, in this sequence. As all travel times are positive, $\bar{t}_{s^l_{m_l}} + t_{s^l_{m_l}, s^{l'}_1} \leq \bar{t}_{s^{l'}_1}$ implies that the graph is acyclic.

In the following, we present a network flow formulation for problem setting 1 using the variables x_{ij} which are 1 if a tour uses the arc $(i, j) \in A$ and 0 otherwise.

$$\min \sum_{j:(D,j)\in A} x_{Dj} \tag{1}$$

with the constraints

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = 0 \qquad \forall i \in V \tag{2}$$

$$x_{s^l_1, s^l_{m_l}} = 1 \qquad \forall l \in L \tag{3}$$

$$x_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{4}$$

The objective function (1) minimizes all outgoing flows of the depot, i.e. all starting bus tours. Constraints (2) are the flow conservation constraints which ensure that every node which is reached is left again. Constraints (3) ensure that all tours are served. As all nodes beside the depot node D have only one outgoing (s^l_1) or only one ingoing ($s^l_{m_l}$) arc, Constraints (2) and (3) also ensure that there is exactly one ingoing and outgoing flow for every node beside the depot node D. Constraints (4) are the non-negativity constraints for the flow variables.

The formulation can be interpreted as a minimum cost flow model with a sufficient number of buses, i.e. flow starting in the source (the depot), whereat a flow from the depot to the depot is possible and all arcs are cost-neutral beside those between the depot and the bus line starts which have costs of 1. Additionally, Constraints (3) ensure that every bus line is served (solid lines in Figure 1). We can replace the arc $(s^l_1, s^l_{m_l})$ by a sink $sink_l$ with inflow 1 and arc $(s^l_1, sink_l)$ as well as a source $source_l$ with outflow 1 and arc $(source_l, s^l_{m_l})$. This is equivalent to $x_{s^l_1, s^l_{m_l}} = 1$. Note that adding additional sources and sinks does not violate the network flow property, as we can add a supersource requiring the sum of all required flows and arcs between the supersource and the sources with the capacity of the source. We can proceed analogously with the sinks (Cormen et al., 2009). Thus, (1)–(4) is a minimum cost flow model formulation.

4.1.2 Service of tours with positive demand only

In this part of setting 1, we only serve those parts of a tour which have a positive demand. Therefore, V changes to

$$\bar{V} = \{D, s^1_1, ..., s^1_{m_1}, ..., s^n_1, ..., s^n_{m_n}\}$$

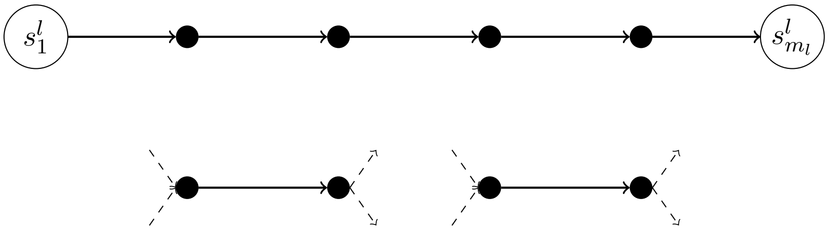
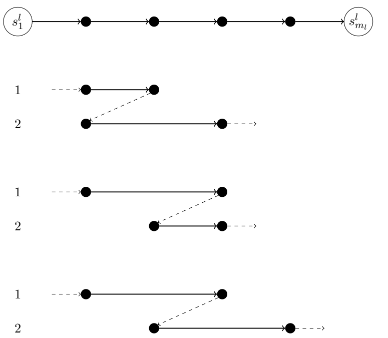


Figure 2: Graph for setting 1 (tours only served if demand positive)

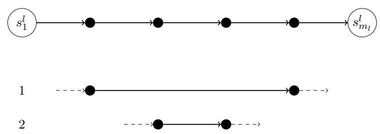
and A changes to

$$\bar{A} = \{(D, s_1^1), ..., (D, s_{m_n}^n), (s_1^1, D), ..., (s_{m_n}^n, D)\} \cup \{(s_i^l, s_j^{l'}) : \bar{t}_{s_i^l} + t_{s_i^l, s_j^{l'}} \leq \bar{t}_{s_j^{l'}}\}.$$

Figure 2 shows the situation. Instead, of serving the entire tour between s_1^l and $s_{m_l}^l$, the bus needs only to serve the parts between the second and third as well as between the fourth and fifth stop of the tour. However, it is not necessary that the same bus serves both of them and if the same bus does, it is allowed that another line is served in between. Generally, the approach in Section 4.1.1 can directly be applied by simply defining each customer trip as one tour. As each customer has one pickup and one delivery location without any (by this customer) required stop in between, $m_l = 2$ for all $l \in L$. Now several customer trips can share parts of one original bus line tour. If their



(a) partly overlapping customer trips



(b) one customer trip comprises the other

Figure 3: Possible overlapping situations

trips belong to the same bus line tour but do not overlap, this is no problem. However, if the trips overlap, we need to ensure that they can be served together by one bus. Figure 3 shows the two relevant cases. In Subfigure 3a, two customer trips overlap, but one of them joins and leaves the bus not later than the other. Then, we can simply allow to serve trip 2 after trip 1, i.e. add a corresponding arc to $\bar{A}$, although trip 1 is not finished before trip 2 begins. However, both trips share the same geographical path, so, they can be served together. The situation is more challenging if one trip fully comprises the other (Subfigure 3b). In this case, the first trip starts before and ends after the second trip. Thus, the bus would be free earlier if we add the second trip after the first.

For this reason, we use a different technique to model both cases. As buses are not capacity restricted, it is obvious that the solution cannot be worse if the bus serving trip 1 in Subfigure 3b also serves trip 2. Thus, we can simply combine both trips in the preprocessing. The same is true in the setting of Subfigure 3a as shown in Figure 4.

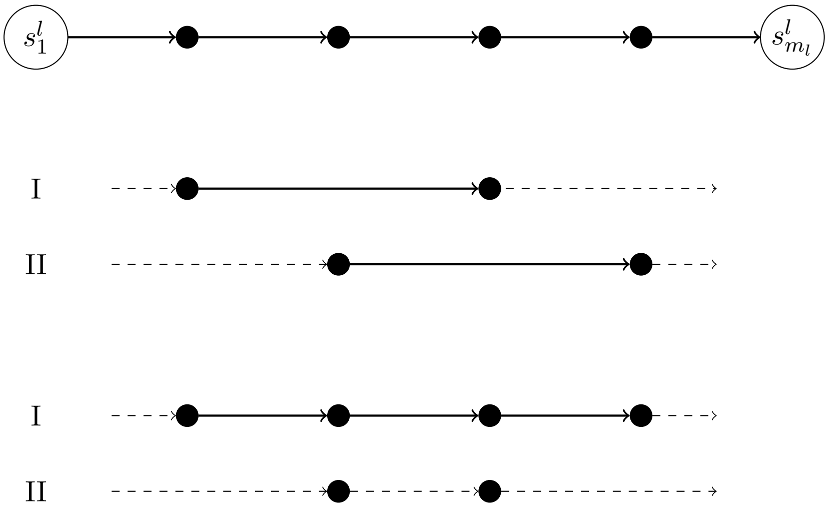


Figure 4: Preprocessing for overlappings

In the upper part of Figure 4, two buses I and II each serve one of the trips. In the lower part, bus I serves its trip until the overlapping part, serves then the overlapping part and proceeds with the trip of bus II while bus II serves its part until the overlapping part, then the overlapping part, and proceeds with the trip of bus I. We can swap the two buses in the overlapping part, as both of them drive on the same line while bus I enters the line before bus II and leaves it earlier. Thus, bus I can pickup all customers on the line served by bus II and bring them to their delivery stop when serving the rest of bus II's trip while bus I is empty when it leaves the line on its original trip such that bus II can take over here. Then, the setting in the lower part of Figure 4 results. As bus II has a feasible schedule even if serving parts of the pictured line between the two black dots, its tour is also feasible when skipping the two stops. Thus, we can combine the two pictured trips to one trip between the first and the last black dot in the preprocessing. If we do this for all overlapping trips, no overlapping trips exists any more, we can define each resulting non-overlapping trip as an individual bus line and use the approach in Section 4.1.1 to solve it.

4.2 Complexity analysis

In the second setting, buses have a capacity restriction. Before we discuss the resulting model formulation, we prove that capacity restrictions make the problem challenging.

Theorem 1.
BLSP is $\mathcal{NP}$-hard in the strong sense if bus capacities are limiting, even if buses are homogeneous, i.e. $Q_k = Q_{k'}$ for all $k, k' \in K$.

Proof:
We prove the theorem by a reduction from the *3-PARTITION* problem that is well-known to be $\mathcal{NP}$-hard in the strong sense (Garey and Johnson, 1979, SP15):

Given is a set I of $3q$ trips, a bound $B \in \mathbb{N}_+$, and a size $d_i \in \mathbb{Z}_+$ for each $i \in I$ such that $B/4 < d_i < B/2$ and $\sum_{i \in I} d_i = q \cdot B$. Can I be partitioned into q disjoint bus trips $I_1, ..., I_q$ such that, for $1 \leq j \leq q$, $\sum_{i \in I_j} d_i = B$. Note that each I_j must by construction contain exactly three elements from I.

Let an instance of the *3-PARTITION* Problem be given. We consider a single line and q buses of capacity B as given. We interpret I as the set of customer requests with demand d_i, $i \in I$, between two bus stops. Each of them is the only one requesting service from a stop, but all of them want to drop out at a common stop at the end of the line. Thus, the bus line consists of $|I| + 1$ stops, at the first $|I|$ a single customer requests starts their trip and at the last stop $|I| + 1$ all customers leave the buses. Thus, between stops $|I|$ and $|I| + 1$ all customers are in the buses and, therefore, all buses are filled up to the capacity limit. Hence, we need to find a partition of the $|I|$ customer requests into subsets $I_1, ..., I_q$, each of them served by one bus, such that $\sum_{i \in I_j} d_i = B$ holds for every $1 \leq j \leq q$, i.e. we need to solve the *3-PARTITION* instance. $\square$

There are two important remarks for Theorem 1:

Remark 2.
Theorem 1 holds also for heterogeneous buses, as BLSP with heterogeneous buses is a generalization of BLSP with homogeneous buses.

Remark 3.
It is decisive for Theorem 1 that several buses can serve parts of the same line in parallel. If a single bus needs to serve all customers, entering and leaving the bus, at a bus stop of a line, different buses can only serve one line if there is no demand between two consecutive bus stops (like in Figure 2). Thus, we can separate the bus line into different fragments such that the bus is empty when driving to the first stop of the fragment and empty when leaving the last but never in between. Then, we can easily calculate the bus capacity required to serve the fragment and interpret the entire fragment as a single customer request. If all buses are homogeneous, either the capacity is sufficient to serve all fragments and we are again in the first setting (Section 4.1) or the instance is infeasible.

Figure 5 summarizes the complexity results for *BLSP*.

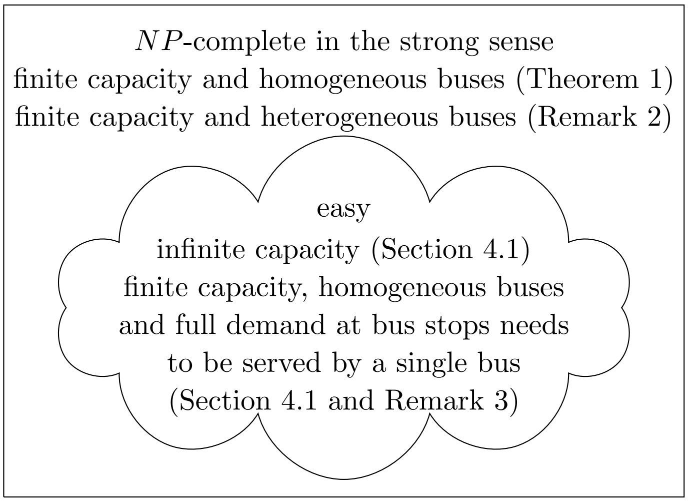


Figure 5: Summary of complexity results for *BLSP*.

4.3 Integer programming formulation for the second setting

To picture heterogeneous bus capacities, we include an index for the buses $k \in K$ (also in line with the complexity results) and need to redefine the arc set as follows:

$$
\begin{aligned}
\hat{A} =& \{(D, s_1^1), ..., (D, s_{m_n}^n), (s_1^1, D), ..., (s_{m_n}^n, D)\} \cup \{(s_i^l, s_j^{l'}) : \bar{t}_{s_i^l} + t_{s_i^l, s_j^{l'}} \leq \bar{t}_{s_j^{l'}}\} \\
& \cup \{(s_i^l, s_j^l) : i, j = 1, ..., m_l : i > j \wedge \exists k < j < i : d_{s_k^l, s_i^l} > 0 \wedge \exists k > j : d_{s_j^l, s_k^l} > 0\}.
\end{aligned}
$$

$\hat{A}$ allows, in contrast to $\bar{A}$, that the bus can go back to an earlier stop on the current line. We restrict the possibility to go back to an earlier stop in Constraints (9). Let x_{ijk} be 1 if tour k uses the arc $(i, j) \in \hat{A}$ between bus stops i and j and 0 otherwise. If the bus capacity is not sufficient to serve stops i or j, we set $x_{ijk} = 0$. The model formulation is as follows:

$$\min \sum_{k=1}^{K} \sum_{j:(D,j) \in \hat{A}} x_{Djk} \tag{5}$$

with the constraints

$$\sum_{j:(i,j) \in \hat{A}} x_{ijk} - \sum_{j:(j,i) \in \hat{A}} x_{jik} = 0 \qquad \forall i \in V, k \in K \tag{6}$$

$$\sum_{k=1}^{K} x_{ijk} = 1 \qquad \forall i, j \in V : (i, j) \in \hat{A} \wedge d_{ij} > 0 \tag{7}$$

$$\sum_{j:(D,j) \in \hat{A}} x_{Djk} \leq 1 \qquad \forall k \in K \tag{8}$$

$$x_{s_i^l s_j^{l'} k} \leq 1 - x_{s_h^l s_{h'}^l k}$$

$$\forall l, l' \in L, i = 1, ..., m_l, j = 1, ..., m_{l'} : (s_i^l, s_j^{l'}) \in \hat{A}, k \in K,$$
$$h, h' \in \{1, ..., m_l\} : h \leq i < h' \wedge d_{s_h^l s_{h'}^l} > 0 \wedge \bar{t}_{s_{h'}^l} + t_{s_{h'}^l s_j^{l'}} > \bar{t}_{s_j^{l'}} \quad (9)$$

$$\sum_{j,j'=1,...,m_l : s_j^l \leq s_i^l < s_{j'}^l} d_{s_j^l s_{j'}^l} \cdot x_{s_j^l s_{j'}^l k} \leq Q_k \quad \forall l \in L, i = 1, ..., m_l, k \in K \quad (10)$$

$$x_{ijk} \in \{0, 1\} \quad \forall (i, j) \in \hat{A}, k \in K \quad (11)$$

Constraints (6) are again the flow conservation constraints. Constraints (7) ensure that every request is served by a bus. Constraints (8) take care that every bus is used at most once. This is ensured by the fact that every bus leaves the depot at most once. The idea of Constraints (9) is that a bus can go back in time for overlapping trips like in Figure 3. However, then we need to ensure that only a trip on a different line can follow that can be reached after the last of the overlapping trips is completed. If this is not the case, the right side of Constraints (9) is 0, such that $x_{s_i^l s_j^{l'} k}$ is set to 0. Constraints (10) are the capacity constraints counting for every location s_i^l the number of customers served on that line who are currently in the bus. This number is not allowed to exceed the bus capacity Q_k. Constraints (11) are the binary constraints.

4.4 Adaptation for the third setting

The third setting is a very straightforward extension, as the information which bus serves which trip is already included in the model formulation (x_{ijk} variables). We simply need to set the corresponding variables x_{ijk} to 0 if the bus tour would violate working hours. This can directly be done in the preprocessing given the t_{D,s_i^l}, $t_{s_i^l,D}$, $\bar{t}_{s_i^l}$ parameters. Variables are fixed for every $k \in K$ as follows:

(i) $x_{Ds_i^l k} = 0$ if $\bar{t}_{s_i^l} - t_{Ds_i^l} < a_k$,

(ii) $x_{s_i^l Dk} = 0$ if $\bar{t}_{s_i^l} + t_{s_i^l D} > c_k$,

(iii) $x_{s_i^l s_j^l} = 0$ if $b_k < \bar{t}_{s_i^l} < b_k + p \vee b_k < \bar{t}_{s_j^l} < b_k + p \vee (\bar{t}_{s_i^l} < b_k \wedge b_k + p < \bar{t}_{s_j^{l'}})$,

(iv) $x_{s_i^l s_j^{l'}} = 0$ if $b_k < \bar{t}_{s_i^l} < b_k + p \vee b_k < \bar{t}_{s_j^{l'}} < b_k + p \vee (\bar{t}_{s_j^{l'}} - \bar{t}_{s_i^l} - t_{s_i^l s_j^{l'}} < p \wedge \bar{t}_{s_i^l} < b_k \wedge \bar{t}_{s_j^l} > b_k + p)$

(i) ensures that no trip can be served by bus k if the bus needs to depart at the depot before a_k. Due to (ii) no tour can be served by the bus if the trip ends after end of work. (iii) takes care that no tour can be served if the bus drives during the break. This is the case if the trip starts or ends within the break or starts before and ends after the break. Finally, (iv) pictures the case that a bus swaps between lines l and l'. The bus is not allowed to do so if the last stop on line l or the first on line l' is within the break or the last stop on line l is before the break, the first stop of line l' after the end of the break but the time in between is not sufficient to drive between both stops and still allows for the break. Note that if there is enough time for the break, we assume that the bus driver can either stop on the way or spends the break either at stop s_i^l or s_j^l. If it is not possible to spend the break there, further variables can be fixed to exclude stopping there. This also holds if the break should be spend at the depot.

5 Computational evaluation of on-demand line-based bus services

To assess the practical performance and operational implications of the proposed models for on-demand line-based bus services, we conducted a comprehensive set of computational experiments. The optimization models were implemented in Julia (Bezanson et al., 2017) using the JuMP modeling language(Lubin et al., 2023), and solutions were obtained using the HiGHS solver (Huangfu and Hall, 2018). All computations were performed on a MacBook Pro equipped with an Apple M1 Max processor and 32 GB of RAM.

The empirical basis for our study is real-world operational data from seven bus depots located in the German federal state of Mecklenburg-Vorpommern provided by the Verkehrsgesellschaft Ludwigslust-Parchim. The analysis centered on the scheduled operations for a representative day, August 22, 2024. For each depot, the dataset included the network of scheduled routes, stop locations, inter-stop travel times, and real passenger demands derived from booking information and GTFS data for Germany[1]. Driving times between stops on different routes were calculated based on the Euclidean distance, adjusted by a network detour ratio of 1.35 to account for actual road network paths, assuming an average speed of 70 km/h.

5.1 Operational scenarios and service scope

To systematically evaluate the impact of different operational constraints and service policies, we designed experiments based on four main operational scenarios and three variations in service scope. The *Operational Scenarions* vary the assumptions about vehicle capacity, driver constraints, and fleet size:

(1) *Unconstrained Baseline (OS1):* Assumes a theoretically unlimited fleet of buses, each with unlimited passenger capacity. This serves as an idealized lower bound.

(2) *Capacity Constraints Only (OS2):* Assumes a sufficient number of buses are available, but imposes specific, heterogeneous, capacities for each bus type. Driver shifts are not considered.

(3) *Capacity and Shift Constraints (OS3):* Incorporates both bus capacity limits and predefined driver shifts, including mandatory breaks. A sufficient number of buses is still assumed to be available.

[1] https://gtfs.de/en/

(4) *Capacity, Shift, and Fleet Constraints (OS4):* Represents the most realistic setting, imposing bus capacity limits, driver shifts/breaks, and a fixed, limited number of available buses for deployment.

The *Service Scope Scenarios* define which parts of the bus lines must be operated:

(A) *Serve All Scheduled Trips:* Requires all trips defined in the timetable to be operated, irrespective of actual passenger demand (mimicking traditional fixed-route service).

(B) *Serve Demanded Lines Fully:* Requires a complete scheduled line trip (from its start stop to its end stop) to be operated only if there is at least one passenger request associated with that specific trip.

(C) *Serve Only Demanded Segments:* Implements the core on-demand concept by requiring the service of only those specific segments of a line necessary to transport passengers who have booked trips. Entire lines or segments without demand are skipped.

Our computational study involved two primary analyses:

1. *Bus Fleet Minimization:* For each combination of the operational scenarios (OS1-OS4) and service scope scenarios (A-C), we solved the problem of minimizing the total number of buses needed to meet the specified service requirements. This allows us to quantify the potential fleet size reduction achievable through on-demand operations under different constraints and helps us to evaluate the computational time required for solving these problems.

2. *Demand Coverage Analysis:* Focusing on the most flexible service scope (C: Serve Only Demanded Segments) under the more realistic operational scenarios (OS3: Capacity and Shift Constraints) and (OS4: Capacity, Shift, and Fleet Constraints), we investigated the relationship between the required demand coverage and the necessary operational resources. Specifically, we determined the minimum number of buses required to satisfy progressively increasing minimum demand coverage levels, systematically evaluating this from 1 % to 100 % of total passenger demand in 1 % increments. This analysis provides a detailed understanding of the trade-off curve between the guaranteed percentage of demand served and the corresponding fleet size commitment.

5.2 Impact on required fleet size

This section details the findings from the *Bus Fleet Minimization* analysis, focusing on the primary question: How does implementing an on-demand service model affect the number of buses required compared to traditional operations? We calculated the minimum fleet size for all combinations of operational scenarios and service scopes outlined in 5.1. The results illustrate the potential savings under varying degrees of operational flexibility and constraints.

5.3 Trade-offs between Service Level and Fleet Size

While minimizing the total fleet size is a primary efficiency objective, operators must often balance resource allocation against service coverage guarantees for passengers or political considerations. This subsection directly addresses this trade-off by examining how the required number of buses changes as the mandated percentage of served passenger demand increases. This analysis focuses specifically on the 'Serve Only Demanded Segments' scope (C) and compares Operational Scenario 3 (OS3: sufficient buses with time-shifts) against Operational Scenario 4 (OS4: restricted buses with time-shifts). By calculating the minimum fleet required for coverage levels ranging from 1% to 100%, we map out the relationship between fleet size requirements and service guarantees, as depicted in Figure 6.

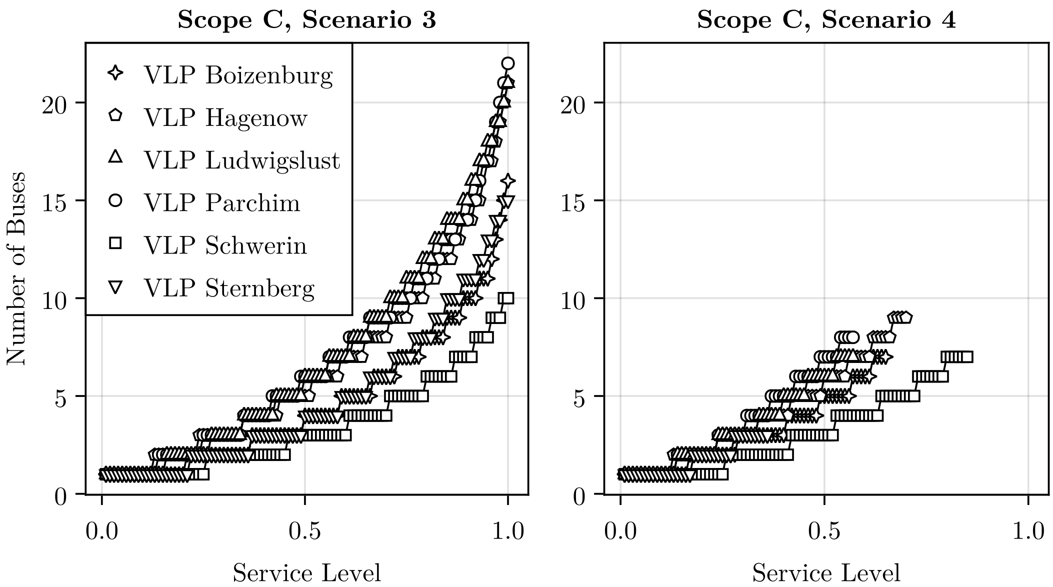


Figure 6: Comparison of fleet size versus service level for Operational Scenario 3 (sufficient buses with time-shifts) and Operational Scenario 4 (restricted buses with time-shifts). Both scenarios operate under the scope of serving only demanded segments (C).

Figure 6 illustrates the differing impacts of these two operational scenarios on the fleet size needed to achieve various service levels.

In OS3, where bus availability is assumed to be sufficient, the required fleet size grows monotonically with the target coverage level. While some depot regions can satisfy up to 20 % of demand with just a single bus, achieving 100 % demand coverage necessitates a minimum of 10 buses in most regions. Notably, the VLP Parchim depot region requires up to 22 buses to serve the entire demand under OS3.

In stark contrast, OS4 imposes the constraint of using only the buses initially available at the depot. This fleet restriction significantly caps the maximum achievable coverage level. For instance, in the VLP Sternberg depot region, the available fleet of 3 busses under OS4 can cover no more than 36 % of the passenger demand. However, in all other evaluated regions, the existing depot fleets under OS4 constraints were sufficient to cover at least 57 % of the total demand. In addition, the results indicate that the shift scheduling might have potential for improvements, as we could for example cover the demand of up to 49 % of the demands in VLP Sternberg with 3 busses with different shifts schedules. Furthermore, busses could be relocated between depots to ensure a more equal maximal coverage level.

6 Conclusion

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