Flexible Route Optimization for Demand-Responsive Public Transit Service

Ailing Huang[1]; Ziqi Dou[2]; Liuzi Qi[3]; and Lewen Wang[4]

Abstract: Traditional customized buses travel on fixed routes, which cannot satisfy passengers' flexibility and convenience requirements. This paper studies a demand-responsive transit (DRT) service that can continuously adjust the path based on passengers' dynamic demand. The path optimization model is established with more realistic constraints to create a bus travel plan within a specified area, and the model not only considers the preferred time windows of passengers but also maximizes the benefits of the system. Based on simulated annealing, a dynamic genetic algorithm is designed to generate the static initial travel path, and the dynamic travel path is continuously updated to satisfy the real-time demand. To evaluate the proposed model and algorithm, a case study in a typical residential community of Beijing, China, is conducted based on transit smart card records. According to the case study results, the convenience, travel time, and economic and environmental benefits of the DRT service are assessed via comparison with traditional buses and private cars. The analysis results demonstrate the feasibility and significance of the method, and it can be used by transit planners to design a superior DRT service. **DOI: 10.1061/JTEPBS.0000448.** *© 2020 American Society of Civil Engineers.*

Author keywords: Customized bus; Demand-responsive transit; Route optimization; Real-time demand; Dynamic genetic algorithm; Benefit assessment.

Introduction

With the rapid development of urbanization, motor vehicle ownership increases year by year, and the car has become the main mode of transportation in many cities. As a result, many problems related to the overuse of private cars have emerged, for example, road traffic congestion, the mismatch between parking supply and demand, and air and noise pollution.

The public transit (PT) system is regarded as the most effective solution for addressing these problems. However, the available public transit service is not sufficiently comfortable or flexible for use by many travelers. In addition, the travel time of riding a bus is typically much longer than that of riding in a private car. Therefore, less interest is paid to the use of the public transit service. To attract additional passengers to the public transit service, innovative transit forms that can provide a faster, more comfortable, personalized, high-quality service are urgently needed. Scholars have proposed such a transit mode: a customized bus (CB), or demand-responsive transit (DRT), service (Mageean and Nelson 2003).

A CB or DRT bus system is a new hybrid transit system that integrates traditional fixed-route, fixed-schedule PT systems and demand-interactive collective transit systems such as carpooling, car-sharing, and subscription buses (TCRP 1999; Shaheen 2001), which can provide advanced, flexible, and personalized transit services for passengers, especially for commuters (Liu et al. 2016). With this mode, passengers can reserve the service through computers or handheld devices such as smartphones. Based on information about reserved services, bus operators will define suitable stops (Iqbal and Ghani 2014) and arrange operational routes to satisfy passengers' travel demands. DRT systems that are based on telematics, which use ZigBee as the communication medium, have been introduced and operated in rural areas (Mageean and Nelson 2003; Iqbal et al. 2011; Velaga et al. 2012). Regarding the development of a DRT service, scholars have identified seven potential market niches for DRT (Davison et al. 2014; Ryley et al. 2014; Cheng et al. 2019): (1) rural and general public, (2) suburban and rural shopping, (3) general public, (4) airport access for passengers and employees, (5) rail station access for passengers, (6) workplaces that are outside the urban core for employees, and (7) hospitals and other destinations for special needs.

Various types of customized buses have appeared in China. Considering Beijing as an example, there have been customized buses in the forms of business shuttle buses and direct shuttle buses, among others. However, these buses' operation modes are similar to that of a conventional bus, namely the operational route is fixed and cannot satisfy passenger requests in real time; hence, they are equivalent to small chartered services. Therefore, this paper proposes a new customized shuttle bus system that can provide DRT services with flexible routing. Connecting with transfer junctions or main traffic hubs, this system can satisfy the real-time trip demand within a specified area. It is especially adaptable for addressing the so-called last mile problem of access to subway stations for commuters.

Compared with a traditional public transit service, the DRT service with flexible routing that is proposed in this paper does not have fixed routes or stops, except various key stops such as subway

[1]Associate Professor, Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Beijing Jiaotong Univ., Beijing 100044, China (corresponding author). ORCID: https://orcid.org/0000-0002-9948-6463. Email: alhuang@bjtu.edu.cn

[2]Bachelor, Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Beijing Jiaotong Univ., Beijing 100044, China. Email: 16251059@bjtu.edu.cn

[3]Master' Candidate, Collaborative Innovation Center for Transport Studies, Dalian Maritime Univ., Liaoning 116026, China. Email: 16251071@bjtu.edu.cn

[4]Master' Candidate, Key Laboratory of Transport Industry of Big Data Application Technologies for Comprehensive Transport, Beijing Jiaotong Univ., Beijing 100044, China. Email: 16251074@bjtu.edu.cn

Note. This manuscript was submitted on September 25, 2019; approved on June 15, 2020; published online on September 17, 2020. Discussion period open until February 17, 2021; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Transportation Engineering, Part A: Systems***, © ASCE, ISSN 2473-2907.

stations, transfer stops, and main traffic hubs. In addition, when using this service, passengers are allowed to make reservations. When passengers want to take a trip, they will convey trip information, such as the origin point, destination, and earliest and latest departure and arrival times, to the travel dispatch center (TDC) through a handheld device or the internet. Then, the TDC will design the routing and stops, schedule the timetable, and dispatch buses to realize a door-to-door service. Therefore, with the service, the passenger's walking distance and travel time can be reduced, thereby increasing passenger satisfaction and the service level.

In addition, because the routing need not be fixed, the service can respond to the real-time demand while the bus is operating. If trip requests are being submitted to the TDC after the bus scheduling plan has been determined, or even while the bus is running on the predesigned route, the TDC can accept the requests and replan the route to satisfy the requirements of passengers, but under the conditions that vehicle carrying capacity is available and other passengers' travel experiences, including their preferred latest alighting times, are not affected. Because the algorithm of the system must satisfy the constraint of the preferred time window to regenerate the route, the alighting times of the passengers who want to use the system will be met within their acceptable ranges, and their decisions regarding their use of this system will not be affected.

The origin and destination points of these real-time requests should be within the service area, and they need not be alternative DRT stops but could be other bus stops. Furthermore, this service not only designs the route by optimizing the benefits of the system while satisfying passengers' time window requirements but also satisfies the passengers' real-time demands by fully utilizing the bus capacity while avoiding impacts on on-board passengers; this benefit can significantly improve the vehicle utilization and reduce the waste of carrying resources. This flexible operation renders the service especially suitable for application in large residential communities, urban peripheral areas, or areas where residents have large trip demands to access to railway stations in peak hours. Overall, the system is an optimization and extension of the service that is provided by traditional DRT.

To develop a successful DRT system, key steps are to design a suitable bus path optimization model that satisfies various constraints and obtain the solutions accurately and efficiently. The path optimization problem, which is a subproblem of network design (Ceder 2007), has attracted substantial interest to this field in recent years. Researchers proposed a comparative model of combined fixed-route conventional and flexible-route subscription bus systems and optimized both (Chang and Schonfeld 1991). An innovative layout of a local shuttle bus system was proposed by Xie et al. (2010), and it was based on the multiple-route, maximal-covering/shortest-path (MSMCSP) model and was suitable for local shuttle bus route optimization in Hepingli District, Dongcheng District, Beijing. Based on the previously proposed model, integrated models were developed and intended for conceptual comparisons of services rather than detailed planning and operations, for optimizing the flexible bus service (Kim and Schonfeld 2011).

Although the concept of flexible transit services has been proposed for over 40 years, these innovative systems have been provided only by a small percentage of transit agencies (Potts et al. 2010). One of the challenges of providing flexible transit services is that the uncertain travel demand in low-demand areas is difficult to forecast and cater to (Velaga et al. 2012). Qiu et al. (2014) dealt with this problem by proposing a dynamic station strategy to improve the performance of flex-route transit in operating environments with uncertain travel demand. A path-based multisubstance network flow model was proposed for optimizing the path and resource allocation (Jin et al. 2015). Additionally, researchers established road demand models for shuttle buses and logit models for passenger flow allocation (Zhu et al. 2017). Subsequently, an optimization model that was based on a multicommodity network flow was developed (Tong et al. 2017).

Various optimization objectives have been considered in the path optimization model for addressing DRT or similar problems, such as the dial-a-ride or feeder service, for example, with more focus on minimizing the operating cost (Cordeau 2006; Cordeau and Laporte 2007; Amirgholy and Gonzales 2016; Tabassum et al. 2017; Drakoulis et al. 2018), along with the environmental costs and traffic congestion costs (Ma et al. 2017), or minimizing the walking time (Yu et al. 2015) or bus travel time (An et al. 2019) of each passenger while maximizing the service quality (Chandra and Quadrifoglio 2013) and optimizing multiple objectives (Chevrier et al. 2012). Time window restrictions also began to be considered in the modeling (Chevrier et al. 2012; Zhou et al. 2017; Tong et al. 2017; Guo et al. 2019). Amirgholy and Gonzales (2016) revealed that the distribution of requested times for DRT service, as well as the operating capacity of the system, are the main parameters that can influence the efficiency of DRT system; dynamic pricing strategies were presented that can be implemented to manage demand and reduce the total cost of the DRT system.

Although more research has been conducted on designing the DRT service, a passenger-oriented and applicable DRT system needs to be developed further, such as focusing on maximizing the benefit of the DRT system (Huang et al. 2020) from the perspectives of both users and operators, considering a variety of passenger demands within more practical restrictions, such as the passengers' preferred trip times, the balance of cost and benefit, the setting of the route length according to the trip behavior, and suitable vehicle capacity for DRT operation, which should also be clearly defined in the modeling framework.

Regarding solution methods, scholars have found that heuristic algorithms can provide better results for path optimization problems in various scenarios on small road networks (Jerby and Ceder 2006). Among the heuristic algorithms, the genetic algorithm (GA) is widely applied in solving transit network design problems. Additionally, the algorithms of simulated annealing (SA), ant-colony optimization, greedy search, and tabu search (TS), among others, are gradually being investigated for this problem. The TS method yields better results for a large-scale experiments regarding vehicle routing problems (Nguyen et al. 2013; Guo et al. 2019). However, for the path optimization problem of shuttle bus service, the results obtained by GA are more reliable and efficient (Xiong et al. 2015; Masmoudi et al. 2017; Guo et al. 2018).

In addition, other algorithms, such as Lagrangian-based algorithms, the branch and bound (B&B) method, and the branch-and-cut (B&C) algorithm, have been presented. Tong et al. (2017) developed a solution algorithm that is based on the Lagrangian decomposition for the primal problem and a space-time prism-based method for reducing the solution search space, and they demonstrated the effectiveness of algorithm and the sensitivity under various practical CB operating conditions. To flexibly transport passengers and obtain higher returns for a CB service, a B&B algorithm that is based on the shortest path of the selected station was proposed and found to be reliable (Dikas and Minis 2014; Ma et al. 2017; Cao and Wang 2017). Padberg and Rinaldi (1991) presented a B&C algorithm for solving the large-scale traveling problem effectively. However, when Guo et al. (2018) compared B&C with GA algorithms on the CB route design solution problem, they found that GA is more efficient with lower complexity. The available algorithms only address the demands that are known in advance in the design of the CB route, and demand interactivity is not considered sufficiently.

Recently, more interests are attracted to catering to dynamic or random demand in the DRT service. The common ways to address this problem include simulation-based approaches and analytical models (Jung and Jayakrishnan 2014; Ronald et al. 2015), or the two solution strategies (Berbeglia et al. 2010): (1) solving a static problem each time based on the real-time information; and (2) solving the static problem only once initially and updating the current solution with heuristic methods, such as insertion heuristics (Van Engelen et al. 2018). In addition, a two-phase optimization strategy was proposed. Kong et al. (2018) proposed a two-stage method that consists of travel demand forecasting and dynamic route planning and can optimize the last mile problem, and they found that the B&B method can effectively solve the route selection model. Huang et al. (2020) proposed a two-phase optimization model to separate the trip request processing (dynamic phase) and vehicle routing (static phase) problems.

The maturity of the DRT service system requires continuous evaluation. Scholars suggested that a satisfactory DRT service revitalizes public transit services (Mulley and Nelson 2009) and the practical experience (Brake et al. 2004; Nelson et al. 2010), and key policy issues that are related to the implementation of future DRT services have been identified (Mulley and Nelson 2009). According to Davison et al. (2014), DRT is important for providing access and geographic coverage. After comparing the performance metrics of private cars, public transportation, and CB between two cities, namely, Auckland, New Zealand, and Paris, France, Liu et al. (2016) found that DRT is more effective than private cars and traditional public transit services for commuter travel. By studying the various indicators of CB in Beijing, main factors that influence people's choice of customized shuttle bus have been suggested (Cao and Wang 2016). However, the lack of assessments of detailed DRT projects in various application levels or areas requires additional and deeper research because a comprehensive assessment of the overall benefits will help convince planners to implement DRT service.

In light of the review, although increasingly, many contributions have been made to the improvement of DRT service in demand analysis, technology development, model specification, and algorithm design, several questions remain to be addressed further. In terms of path optimization, most studies regarded the DRT routing model as a pick-up and delivery problem and focused on static path design according to the demand that was specified in advance, without considering the real-time demand; hence, they cannot realize the full advantages of DRT service, namely, flexibility and demand interactivity. Accordingly, algorithm design for the dynamic path optimization problem still needs to be explored to improve the efficiency of the DRT service.

In addition, the specified needs of passengers cannot be closely catered to, for example, without considering that passengers want to be served within their preferred time windows, which are sometimes specified after the DRT bus has been scheduled and has departed. Moreover, the more practical constraints, such as the route length and vehicle capacity for DRT operation, must be clearly defined in the modeling framework to well satisfy the realistic requirements of passengers. DRT service can be provided not only by a public agency but also by private companies. Because DRT is a new hybrid transit system that has a high likelihood of no need to be supported by the government and of even making a profit, the benefits of the whole system, not just the cost, should be seriously considered by whoever operates it. Hence, the fare pricing should be correlated with the revenue and demand. However, these issues were rarely considered in previous studies.

To address the research limitations that are discussed in the preceding paragraphs, this paper proposes a DRT service with flexible routing by establishing a path optimization model with a time window constraint and designing a GA algorithm that is based on SA and includes both static and dynamic path solving processes to satisfy the real-time trip demand for the realization of multivehicle flexible routing optimization. The model is used to maximize the revenue of the DRT system in consideration of the spatial-temporal demands of passengers, along with the operating cost.

Moreover, aiming at satisfying the commuting mobility requirements of commuters from the Tiantongyuan residential community to the Dongzhimen subway station in Beijing, based on bus and rail transit smart card records, the authors conduct a case study to evaluate the model, and they conduct a multiperspective detailed analysis of the experimental results to assess the benefits in terms of convenience, travel time, economics, and the environment of DRT service via comparison with traditional buses (TBs) and private cars (PCs). The pricing of the DRT fare is also discussed to probe the relationship between demand and the fare of TBs.

The remainder of this paper is organized as follows. In the second section, a DRT optimization model with flexible routing within a service scope is proposed. The third section designs a solution algorithm under the condition that DRT buses must respond to the dynamic demand. In the fourth section, considering the Beijing Tiantongyuan Community as the research area, based on trip demand data obtained by processing transit smart card records, this paper selects candidate stops and applies the proposed model and algorithm to design DRT service. The results of the case study are presented and compared from multiple perspectives in the fifth section. The conclusions of this study and the outlook are discussed in the final section.

Flexible Route Optimization Model for Demand-Responsive Transit

In this paper, the authors define the flexible route optimization problem as follows: within a specified service area, there are alternative stops and a fixed fleet-size of buses with flexible routes for serving the residents in the area. Passengers input their arrival time windows at boarding and drop-off stops into the DRT system, including their preferred earliest and latest arrival times at the stop. Then, the bus will pick up passengers from their designated boarding stops and take them to their designated drop-off stops within their requested time windows. Because the route has been preplanned based on passenger requests, the arrival times of the bus at the stops will be specified to the passengers in advance by the TDC, which will help them determine when to begin walking to the stop.

For the expected earliest arrival times at the drop-off stops that are inputted by passengers, the DRT system requires that the value must be greater than or equal to the shortest delivery time between each passenger's boarding and drop-off stops. A passenger can reserve the service at any time, not only before the bus departs but also while the bus is running. Hence, the DRT system collects the requests and generates the static routing scheme before the bus departs. While the bus is running along the predetermined route, if there is demand from passengers, the DRT system can replan the routing of this bus to satisfy the real-time demand under the condition that the rerouting has no impact on the demands of other passengers that have been considered in the predetermined scheme.

However, the satisfaction of all potential origins or destinations (Os/Ds) of passengers simultaneously by a bus would be costly and inefficient for the system. To avoid this scenario, various conditions are imposed on the system. First, the DRT bus is confined to service within a specified area, such as the residential community, urban peripheral area, or areas where residents have large trip demands for access to railway stations at peak hours. Second, the Os and Ds of real-time requests must be within the service area. Third, factors that would increase the operational cost should be seriously considered in the model constraints to prevent the system from being costly, such as the setting of the length of a DRT route and the penalty time cost. Then, based on these restrictions of Os and Ds and these constraints, the system will be allowed to generate multiple routes to satisfy all demand within this area. For some extremely special real-time demands that are costly to the system, the fare will need to be specially priced.

Problem Description and Hypothesis

In the selected service area, the alternative DRT stops are predetermined among the TB stops according to various principles; hence, their locations are known in advance. The load capacity and maximum traveling distance of the vehicle are fixed, and the number of passengers who board and alight from the bus are also known. Then, based on the passengers' travel demand information, the most reasonable route is arranged. On that route, the objective function is optimized in the case in which each stop is accessed, and the scheme is obtained simultaneously. The basic assumptions are as follows:

1. There is only one vehicle center for dispatching buses to provide DRT service. This paper selects the TDC as the starting point of routes, and vehicles must return to the TDC after completing their services.
2. All delivery vehicles are of the same type. Each bus has the same maximum load capacity, and the passenger load must not exceed the maximum capacity during services. In addition, all passengers are equally important.
3. The location coordinates of passengers (including new passengers who are making dynamic service requests) and of the vehicle dispatching center are known.
4. Each passenger is served by only one bus.
5. Each passenger has their own service time window.
6. Each vehicle travels at a uniform speed during service, and factors that cannot be modified, such as weather conditions and road congestion, are not considered in this paper.
7. Dynamic requests for transit services are real-time requests.
8. Once a dynamic request has been received, the TDC can obtain basic information.
9. The time window in the problem is a soft time window, which allows the vehicle to serve outside the time window at a specified penalty cost.
10. The passenger demand at each stop is less than the capacity of the vehicle.

Model Objective Function

The objective of the model aims to maximize the satisfaction of the passengers' requirements while making bus operation more profitable. The costs consist of the fixed cost, the operating cost of the vehicle, and the time cost. The fixed cost refers to the cost that does not change with the running of the vehicle. The fixed cost typically includes vehicle depreciation, vehicle insurance, repair and maintenance costs, the vehicle annual inspection fee, and the driver's salary. The operating cost is the cost that is due to the running of the vehicle, which mainly includes the fuel, oil and tire costs. Considering the higher level of service required by passengers who are willing to take the DRT service, this paper sets a penalty time cost. The penalty time cost refers to the penalty to the system when the vehicle is late or arrives earlier at a stop, which can result in DRT requesters or on-board passengers waiting too long or getting off late at this stop; such a result will lower the level of DRT service and decrease the satisfaction of users.

The model is designed to optimize the operating route of the vehicle on the basis of satisfying passenger reservation requirements, thereby improving the service level of the bus while increasing the revenue of the DRT system. The objective function is presented as Eq. (1)

$$\max Z = f_{\text{drt}}\sum_{k}\sum_{i} nu_i^k - \left[\left(c_1\sum_{k=1}^{K}\sum_{j\in N} x_{oj}^k + c_2\sum_{k=1}^{K}\sum_{i\in F}\sum_{j\in F} x_{ij}^k \cdot d_{ij}/v\right) + m_1\sum_{i\in F} w_i^k \cdot nn_i^k + m_2\sum_{i\in F} l_i^k \cdot (nu_i^k + nd_i^k)\right] \tag{1}$$

This objective function consists of five terms, which are all expressed in terms of monetary cost. The first term on the right of Eq. (1) refers to the total fare revenue of the service, and the second and third terms are the fixed cost and the operating cost of vehicle use, respectively. The final two parts correspond to the penalty cost of the system. The former represent the penalty that is incurred when the vehicle arrives too early; the penalty time cost will be calculated in terms of the number of on-board passengers that must wait for extra time at this stop. The latter term represents the penalty that is incurred when the vehicle is late; in this scenario, the boarding passengers must wait to be served and the alighting passengers are late to get off at this stop. Here, the on-board passengers may need to spend extra time on the bus, but their final alighting time at the destination stop should be within their preferred time windows; otherwise, a penalty will be incurred by the system. Thus, the late penalties at the intermediate stops for on-board passengers are not considered.

Constraints

Because the main feature of DRT is to provide a higher level of transit service, it is better to provide a "one seat one passenger" service (Liu and Ceder 2015) than to allow too many standees to ensure an uncrowded and comfortable on-board environment. Therefore, the sectional load of each vehicle should not exceed its maximum vehicle capacity Q, which should be set at a suitable level [Eq. (2)]. For smooth operation in communities in which the road lanes are relatively narrow and to provide a flexible and high-frequency service, minibuses or medium-sized vehicles are suitable as DRT buses; hence, the recommended capacity is $20 \le Q \le 40$ passengers/vehicle

$$0 \le \sum_{i}(nu_i^k - nd_i^k) \cdot y_i^k \le Q, \quad i \in N_o \tag{2}$$

For each DRT bus, the service vehicle should depart and eventually return to the TDC. Due to the flexibility feature, the DRT bus need not stop at any stop at which no passenger is boarding or alighting except the starting stop. Eqs. (3) and (4) ensure that the trip starts from the TDC and ends at the TDC

$$\sum_{i\in N} x_{oi}^k = 1, \quad \forall\ k \in K \tag{3}$$

$$\sum_{j\in N} x_{jo'}^k = 1, \quad \forall\ k \in K \tag{4}$$

When vehicles are running, the path of each vehicle should be guaranteed not to be repeated, namely for each bus, there must be

J. Transp. Eng., Part A: Systems, 2020, 146(12): 04020132

only one path to arrive at a demand stop and one path to leave the demand stop. It is also necessary to ensure that each stop is accessed only once by one vehicle

$$\sum_{i \in N_o} x_{ij}^k = 1, \quad \forall\ k \in K, \quad \forall\ j \in N, \quad \text{and} \quad i \neq j \tag{5}$$

$$\sum_{m \in N_{o'}} x_{jm}^k = 1, \quad \forall\ k \in K, \quad \forall\ j \in N, \quad \text{and} \quad j \neq m \tag{6}$$

The time when the bus arrives at each stop must be within the passengers' request time windows

$$a_i \leq r_i^k + w_i^k \leq b_i, \quad \forall\ k \in K,\ i \in N \tag{7}$$

$$x_{ij}^k (r_i^k + s_i^k + t_{ij}^k - r_j^k - w_j^k) \leq 0, \quad \forall\ k \in K,\ (i,j) \in A \tag{8}$$

The waiting and lateness times at each demand stop for a vehicle are

$$w_i^k = \max((a_i - r_i^k), 0), \quad \forall\ k \in K,\ i \in N \tag{9}$$

$$l_i^k = \max((r_i^k - b_i), 0), \quad \forall\ k \in K,\ i \in N \tag{10}$$

The service time at each demand stop for a vehicle is

$$s_i^k = p \cdot \max(nu_i^k, nd_i^k), \quad \forall\ k \in K,\ i \in N \tag{11}$$

In Eq. (12), the first subequation expresses that while a vehicle is moving, each required stop $i \in N$ is only stopped at by one vehicle k to ensure that the passengers at stop i can be served by one vehicle on this trip. The second subequation represents the total number of vehicles that are departing from or returning to the TDC

$$\sum_{k=1}^{K} y_i^k = \begin{cases} 1, & i \in N \\ k_0, & i \in \{o,\ o'\} \end{cases} \tag{12}$$

Constraint Eq. (13) ensures that for the vehicle k to visit a required stop j, it must begin exactly from one predecessor stop i that may be either a required stop $i \in N$ or the TDC. Constraint Eq. (14) ensures that once vehicle k has visited a required stop i, it must depart for a successor stop j that may be either another required stop $j \in N$ or the TDC

$$\sum_{i \in N_o} x_{ij}^k = y_j^k, \quad \forall\ k \in K,\ \forall\ j \in N \tag{13}$$

$$\sum_{j \in N_{o'}} x_{ij}^k = y_i^k, \quad \forall\ k \in K,\ \forall\ i \in N \tag{14}$$

After completing the pick-up and drop-off tasks at stop j, the vehicle will leave that stop

$$\sum_{i \in N_o} x_{ij}^k = \sum_{m \in N_{o'}} x_{jm}^k, \quad \forall\ k \in K,\ j \in N \tag{15}$$

The length of the DRT route should satisfy the minimal and maximal length requirements, which are related to passenger's travel distance and the vehicle operation capability

$$l_{\min} \leq \sum_{i \in F} \sum_{j \in F} x_{ij}^k \cdot d_{ij} \leq l_{\max}, \quad \text{and} \quad i \neq j \tag{16}$$

In actual operation, if the length of the route is set to be too long, a larger vehicle fleet will be required to ensure the DRT service frequency, which will increase the investment and operational cost

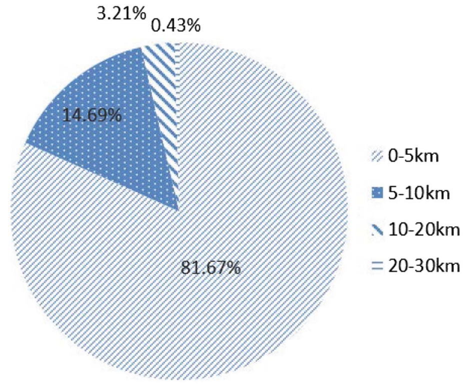

Fig. 1. Percentage of the different travel distances by bus in Beijing.

to the bus operators. Additionally, a long path may decrease the reliability and the punctuality rate due to road traffic conditions, which will lower the level of DRT service. However, if the route length is too short, the vehicle utilization rate will decrease, and the service will not be able to well satisfy the long-distance trip demand. By analyzing the travel patterns of Beijing bus users from smart card data, the authors find that approximately 96.36% of passengers who take the bus travel not more than 10 km in a single trip (Fig. 1). Therefore, except the route's section that links to the terminal, the main part of a route that covers the most spots within a trip demand area should not be longer than $l_{\max} = 10$ km. Wang (2006) suggested that the minimal length of the PT line should be 5 km under the operational requirements in China; hence, considering the features of flexibility and timely response, this paper recommends that the CB route have a minimal length of $l_{\min} = 3$ km.

The passenger arrival process obeys a Poisson distribution. The value of λ differs between peak hours and low-demand times

$$p_{nu_i^k} = \frac{\lambda^{nu_i^k}}{nu_i^k!} e^{-\lambda}, \quad nu_i^k = 0, 1, 2, \ldots \tag{17}$$

Dynamic Genetic Algorithm Based on Simulated Annealing

Problem-Solving Process

The complete process of solving the DRT bus problem with a flexible route can be described as static vehicle routing optimization that is continuously conducted throughout the planning and dispatching cycle at a suitable time and under the current known conditions. This process can be divided into the following three phases:

1. Initial path construction phase. This phase begins before the vehicle service, and the TDC obtains the requests of all passengers, each of which includes the boarding and alighting passenger numbers, the stops and their coordinates, and the time window requirements. According to the trip requests, the TDC constructs an initial path optimization scheme that maximizes the revenue of the bus operators while satisfying various constraints. The scheme will provide service along the designed route until a new real-time demand is submitted; then, the scheme must be updated.
2. Real-time dynamic request collection phase. The passengers' dynamic demand is also being collected and updated cumulatively after the static path scheme has been generated. Hence, the real-time demand starts being counted from the moment

J. Transp. Eng., Part A: Systems, 2020, 146(12): 04020132

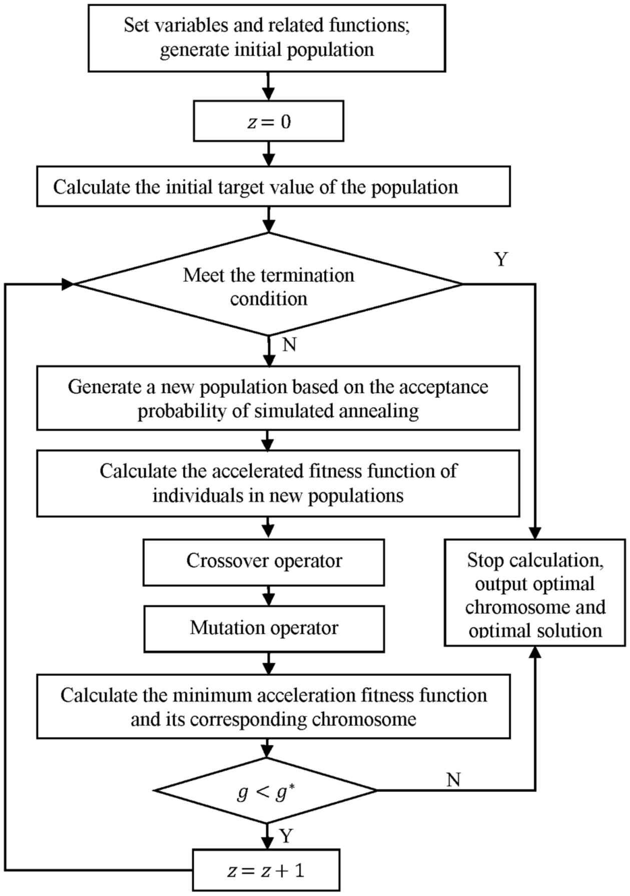

Fig. 2. Algorithm flowchart.

when the first dynamic demand is generated from the TDC and stops being accumulated into this trip at the moment at which or before the path starts to be updated by the system.

3. Path update phase. After the initial path begins to be executed, the TDC continues to accept new real-time requests and adjusts the initial path dynamically while optimizing the objective function.

The strategy of solving the dynamic demand problem that is considered in this paper is to divide the time (after the start of the service) into several time periods with interval T. During each time period, real-time requests are accepted by the TDC. When the end of a time period is reached, namely when the initial time of the next time segment is reached, the real-time demand that is received in this time period is inserted uniformly and integrated into the initial path to be addressed.

Algorithm Steps

The model constructed in the preceding section is a vehicle routing problem with soft time window constraints that considers the bus operating costs and the passenger time costs. Therefore, this study uses heuristic algorithms to solve the flexible route optimization problem for DRT. Moreover, because the GA suffers from premature convergence and weak local search ability, the SA algorithm is embedded into the GA, which reduces the difficulty in selecting the GA parameters and expands the search field.

The GA that is based on SA can avoid local minimum conditions during operation; however, if the problem has multiple extrema, it may not be guaranteed that the resulting approximate globally optimal solution is the optimal solution that has been obtained throughout the search process. Therefore, to improve the quality of the solution, the authors add a memory device into the algorithm so that it can remember the optimal result that has been obtained in the search process. A flowchart of the algorithm is presented in Fig. 2.

The steps of the process are as follows:

- Step 1: set the variables and the related functions. The maximum population size is denoted as NP. Set the temperature variable t_z. Given initial temperature control parameters $t_z = t_0$, generate the initial population $P(z)$, where $z = 0$. Compute the initial target value of the population $g(\gamma)$. Identify the chromosome γ that minimizes the fitness function $g_\gamma(t_z)$ and the value $g(\gamma)$ of this function. Set the memory variable, where $\gamma^* = \gamma$ and $g^* = g$; $\gamma \in P(z)$; and $g_\gamma(t_z)$ is the target value of individual γ at a temperature of t_z.
- Step 2: after the computation in Step 1, judge whether the termination condition is satisfied. If the termination condition is satisfied, this chromosome is the optimal chromosome, and the objective function value is the current optimal solution. Then, output the optimal chromosome γ^* and optimal solution g^*. If the termination condition is not satisfied, then an individual must be randomly selected in the neighborhood of each chromosome in the population $P(z)$. According to the acceptance probability of simulated annealing, acceptance or rejection β is determined, and, finally, a new population $P_{\text{new1}}(z)$ is generated. In this problem, the termination condition refers to the maximum number of iterations NI, namely if the current evolutionary number of the algorithm exceeds NI, the algorithm is terminated. If it does not exceed NI, the termination condition is not satisfied, and the algorithm continues. The value of NI depends on the scale of the problem and the time it takes; it is typically between 100 and 500. In this paper, the value of NI is set as 200. The probability of acceptance is

$$p = \min\left\{1, e^{-\frac{g_\beta(t_z) - g_\gamma(t_z)}{t_z}}\right\} \tag{18}$$

- Step 3: for the new generation of the population, the fitness function of individuals in $P_{\text{new1}}(z)$ is computed. Then, the population $P_{\text{new2}}(z)$ is formed from $P_{\text{new1}}(z)$ according to the probability distribution.
- Step 4: according to GA, $P_{\text{cross}}(z)$ is obtained after crossing $P_{\text{new1}}(z)$ with crossover probability P_c, and a new population $P_{\text{mut}}(z)$ is formed by mutating with mutation probability P_m.
- Step 5: let $P(z+1) = P_{\text{mut}}(z)$. Calculate the value of the function $g_\gamma(t_z)$, and identify the chromosome γ that minimizes the function $g_\gamma(t_z)$, along with the corresponding function value $g(\gamma)$. If $g < g^*$, then let $\gamma^* = \gamma$, $g^* = g$, and $t_{z+1} = \text{alph} \cdot t_z$, $z = z + 1$, and return to Step 2. The iteration step size for each cooling is nt.

Model Validation

To evaluate the feasibility of the proposed model and algorithm, this section considers the commuting requirements from Tiantongyuan community to Dongzhimen metro transfer station in Beijing during the early peak hours as an example and applies proposed method to conduct a simulation and result analysis.

Study Area

Tiantongyuan residential community, which covers an area of approximately 480,000 m^2, is densely populated with more than

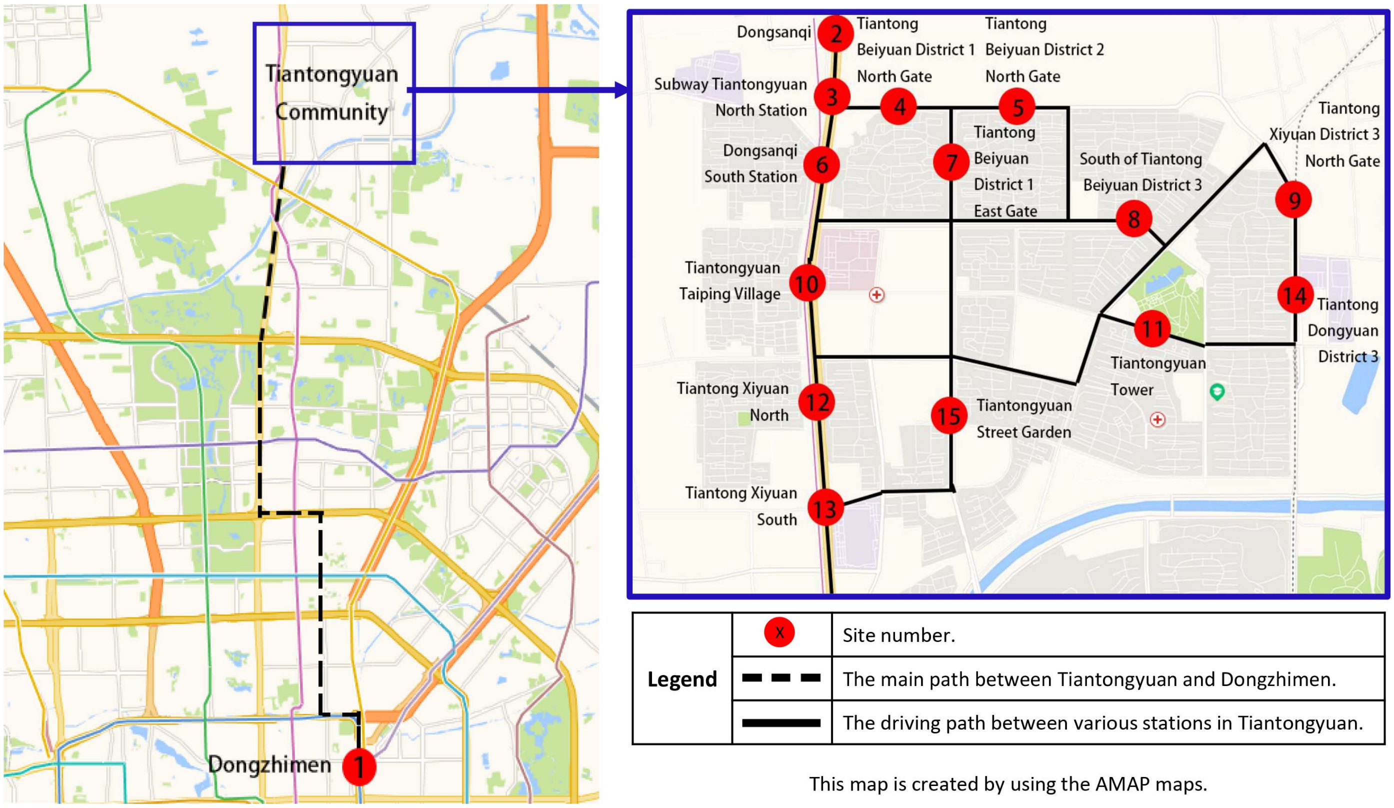

Fig. 3. Tiantongyuan-Dongzhimen road network map. (AMAP@2020 Autonavi—GS(2019)6379, Class A of surveying and mapping qualification certificate No. 11002004, Beijing ICP 070711.)

300,000 permanent residents and more than 600,000 transient residents. There is a subway line, namely, Line 5, and multiple bus lines that directly connect the community to the urban center. For the commuters of Tiantongyuan, the Dongzhimen metro station, which is a key traffic hub and an intersection among the subway's Line 2, Line 13, and the Airport Line, is regarded as one of the most important stations for transfer and access to workplaces. Fig. 3 shows the route network of Tiantongyuan-Dongzhimen, and the site number is the stop label, as presented in Table 1.

According to statistics that on rail transit smart card records, during the early peak hours (7:00–9:00 a.m.) on a workday, more than 53,000 passengers board at the rail stops of Subway Tiantongyuan North, Tiantongyuan, and Subway Tiantongyuan South of Line 5, which serve the residents of this community who commute to the urban center. This huge subway trip demand results in a heavy passenger load, a low quality of rail transit service, and a substantial loss of time due to passengers being unable to board the train in time at peak hours as a result of the train's limited capacity.

By analyzing the trip chains of Tiantongyuan residents using rail and bus transit smart card data, it was found that few residents took the bus to Dongzhimen because it typically required multiple bus transfers. The number of people who take the subway to Dongzhimen or need to transfer at Dongzhimen to other stops is approximately 1,000 in Tiantongyuan; hence, there is a large potential demand by commuters of Tiantongyuan to connect to Dongzhimen. Considering the advantages of the DRT system, such as higher service quality and shorter travel time without transfers, it is reasonable and necessary to provide DRT service in this area.

Table 1. Total number of the boarding passengers at major stops in Tiantongyuan community

Label	Stop	Number of boarding passengers
1	Dongzhimen (origin stop)	—
2	Dongsanqi	1,200
3	Subway Tiantongyuan North Station	1,569
4	Tiantong Beiyuan District 1 North Gate	146
5	Tiantong Beiyuan District 2 North Gate	165
6	Dongsanqi South Station	2,084
7	Tiantong Beiyuan District 1 East Gate	112
8	South of Tiantong Beiyuan District 3	139
9	Tiantong Xiyuan District 3 North Gate	567
10	Tiantongyuan Taiping Village	738
11	Tiantongyuan Tower	257
12	Tiantong Xiyuan North	189
13	Tiantong Xiyuan South	892
14	Tiantong Dongyuan District 3	74
15	Tiantongyuan Street Garden	50

Alternative Stop Selection Based on the Passenger Flow

To define the alternative stops for DRT service in the Tiantongyuan community based on the passenger flow, the authors collected bus smart card data for public transit in Beijing on a weekday (Thursday February 16, 2017) and obtained 366,427 records. Because the entry–exit charging system has been applied in the Beijing PT system since December 2014, detailed information regarding the buses that pass through each stop and the numbers of people boarding and alighting at each stop can be collected. By sorting and analyzing the data, valuable information is screened out as the basic data. First, six bus lines that pass through the Tiantongyuan community and 32 bus stop sites within this area

Table 2. Sample of alternative stop coordinates

Stop label	Latitude	Longitude	X	Y
2	40° 5′ 22.4154″ N	116° 24′ 25.9914″ E	4442330.194	705306.8564
3	40° 4′ 53.7954″ N	116° 24′ 29.2674″ E	4441449.408	705408.3852
4	40° 4′ 41.0874″ N	116° 24′ 38.2674″ E	4441063.161	705632.2776
5	40° 4′ 40.9794″ N	116° 25′ 2.496″ E	4441075.413	705206.5114
6	40° 4′ 31.3314″ N	116° 24′ 26.424″ E	4440754.6	705359.7504
12	40° 3′ 51.228″ N	116° 24′ 25.1634″ E	4449516.641	705363.366
13	40° 3′ 30.3834″ N	116° 24′ 26.244″ E	4448874.326	705406.3625

Table 3. Sample of the distance matrix between the candidate stops (meters)

Stop label	2	3	4	5	6	7
2	0	859.0	1,334.7	1,906.0	1,390.4	2,051.7
3	859.0	0	649.0	1,869.3	702.5	1,372.7
4	1,334.7	649.0	0	576.4	552.8	726.4
5	1,906.0	1,869.3	576.4	0	1,128.9	540.8
6	1,390.4	702.5	552.8	1,128.8	0	1,280.9
7	2,051.7	1,372.7	726.4	540.8	1,280.9	0

are identified. Then, based on the smart card swiping records of these lines, the numbers of boarding and alighting passengers at stops within the community are estimated. After accumulating the demand of each stop, 12 key stops with more than 100 boardings are defined as the alternative stops for DRT service (which are listed in Table 1 and specified in the upper right of Fig. 2).

Moreover, in addition to the residential buildings, there are a primary school and middle school near the Tiantong Dongyuan District 3 stop, and a flower market and a furniture shopping center near the Tiantongyuan Street Garden stop; hence, there are more potential DRT users. To cover a larger service area and to attract more potential DRT users, these two stops are also considered and labeled as Stops 14 and 15.

Shortest Distance between Alternative Stops

To plan the route more exactly, the distances between the candidate stops must be calculated. First, these 15 alternative stops' latitude and longitude coordinates are obtained from the map website, and they are converted into planar coordinates. Table 2 lists a sample of the alternative stops' coordinates. Finally, according to the real road network, the lengths of the shortest routes between the candidate stops are calculated, and samples are presented in Table 3.

Experimental Results

To evaluate the method, considering the real conditions of the community in Beijing, the parameters of the model are set as specified in Table 4. MATLAB version 2018b was used to program and execute the algorithm. MATLAB is a productive software package that can conduct various types of mathematical operations, such as expressing matrices and arrays, plotting graphics, and programming algorithms. When the proposed algorithm is executed on a computer with an Intel i5-6200u processor to compute a similar amount of data to that for the case in this paper, it takes no more than 13 s to obtain the final results. If there is a real-time request to be added, an additional time of approximately 2 s is required for obtaining the dynamic optimization results; hence, the algorithm demonstrates satisfactory performance and is especially suitable for making decisions regarding users who make dynamic requests.

Because not all traditional bus passengers will decide to take the DRT bus, this paper extracted the DRT trip demand randomly in terms of the activity degree of a stop, namely the boarding and alighting numbers among the 15 candidate stops (Table 5). Three boarding requests are made at Stop 15 when the bus is starting to depart from the TDC. Here, the departure TDC and return TDC are

Table 4. Parameter settings for the flexible route optimization model

Index	Parameter	Remarks
Vehicle type, $Q_{\max}$	40 passengers/vehicle	Considering the large demand in this community, a medium-sized bus is more suitable
Operating speed, v	25 km/h	Average CB bus operating speed in Beijing
Fixed cost of each bus, c_1	2.28 yuan/vehicle · h	Refers to the vehicle depreciation cost. The purchase cost of a medium bus is 0.2 million yuan, and the average service life is 10 years.
Operating cost of each bus, c_2	30 yuan/vehicle · h	Refers to the fuel consumption cost. The fuel consumption per 100 km for a medium bus is 20 L, and the oil price is 6 yuan/L.
Waiting penalty cost coefficient, m_1	1,000,000 yuan/person · h	The vehicle is not allowed to arrive earlier in this case
Late penalty cost coefficient, m_2	35.28 yuan/person · h	According to the standard for the average salary that was released by the Beijing Municipal Bureau of Statistics in 2017
One-way flat fare, f_{drt}	5 yuan/person	Set to the same value as TB, which is the average of mileage fare
Line length, $l_{\min}$	3 km	Defined in the section "Flexible Route Optimization Model for Demand-Responsive Transit"
Line length, $l_{\max}$	10 km	Determined by the average travel distance of commuters in Beijing
Population size, NP	70	Reference range: 20–100
Maximum number of iterations, NI	200	Reference range: 100–500
Crossover probability, P_c	0.9	Reference range: 0.4–0.99
Mutation probability, P_m	0.1	Reference range: 0.0001–0.1
Initial annealing temperature, t_0	15	Determined by two factors: the probability of finding a globally optimal solution and the computing time
Annealing attenuation coefficient, alph	0.9	Reference range: 0.9–0.99
Iteration step size for each cooling, nt	20	Suitable for finding the optimal solution

J. Transp. Eng., Part A: Systems, 2020, 146(12): 04020132

Table 5. Boarding and alighting demand and time window for DRT service at candidate stops

Stop label	Demand: Boarding number (passengers)	Demand: Alighting number (passengers)	Time window: Earliest arrival time (a.m.)	Time window: Latest arrival time (a.m.)
1	0	4	8:00	08:30
2	1	2	8:30	09:00
3	4	2	9:00	09:30
4	2	1	8:40	09:00
5	1	4	8:40	09:00
6	2	2	9:00	09:20
7	4	2	8:30	09:00
8	4	5	8:30	09:00
9	3	3	8:40	09:00
10	2	4	9:00	09:20
11	5	5	8:30	09:00
12	4	1	8:40	09:00
13	5	4	8:50	09:20
14	2	3	8:40	09:00
15	3 (real-time demand)	0	8:10	08:40
Total	42	42	—	—

the same stop, namely, Stop 1 (Dongzhimen). Additionally, the time windows that are requested by the boarding and alighting passengers at various stops are presented in Table 5.

Based on the proposed model and algorithm, the optimization results are presented as follows. All the routes are designed to start from Stop 1 and return to Stop 1. The digital labels in this section are presented in Table 1.

Static Optimization Result

Before the bus operates, the TDC will schedule the vehicle path in terms of all demand that is requested by the time of bus departure. In this case, before 8:00 a.m., three original routes are generated to satisfy the trip demand (except three real-time boarding requests at Stop 15), as presented in Table 5, via the static algorithm, which are as follows: (1) Vehicle 1 path: 1-8-12-1, (2) Vehicle 2 path: 1-9-11-7-5-13-6-1, and (3) Vehicle 3 path: 1-2-14-4-3-10-1. Fig. 4 shows the path maps in the plane coordinate system, which include the travel paths of Vehicles 1–3.

Dynamic Demand Processing Result

When the real-time demand from Stop 15 (Table 5) is put forward after the static scheduling scheme has been generated, the route is optimized further based on the static result that is described previously. Fig. 5 presents the general travel routes before and after dynamic optimization. The path schemes of Vehicles 2 and 3 are not affected, and Vehicle 1 changes its route midway because dynamic Stop 15 is inserted, and the vehicle still completes all transit services. Then, the path of Vehicle 1 is adjusted to 1-15-8-12-1.

Table 6 summarizes the arrival times of the DRT bus at various stops, along with the route lengths and travel times for various paths, which satisfy the constraints of the model well. Additionally, the response time that is needed for generating the schemes shows that the algorithm can respond to the demand quickly. Because the time window is well satisfied, the DRT system even generates a profit with the same pricing as TB. However, if some time windows of the demand cannot be satisfied, penalty costs will be incurred; hence, the fare should be repriced to reduce the system losses.

Result Assessment

To assess the feasibility of the optimization results, the authors use the with-and-without method to compare the benefits of DRT service with those of TB or PC. Because the DRT service is a special form of CB and, at present, there is no other form of CB running in the route direction of the case study, no information about additional CBs is available. Thus, this paper does not compare DRT service with other CBs. Because the DRT service is mainly designed according to commuter requirements and it strives to increase the service level while reducing the transfer times, travel time, and travel distance, a comparative analysis will be conducted in terms of convenience, travel time, and economic and environmental benefits.

Convenience

With the exception of the DRT service, there is no direct bus route that passes through the Tiantongyuan community and links it to Dongzhimeng subway station. When the DRT service is provided, it can not only link these two major locations directly but also visit as many requested stops as possible within the service area. In addition, due to the flexibility of the DRT service, the vehicle can use bus lanes and can adjust the driving path according to the traffic conditions, thereby ensuring that its operating speed is comparable to that of a taxi and is significantly higher than that of a TB. Table 7 compares the TB and DRT services in terms of convenience.

Travel Time

This paper presents the bus running times that are required for reaching Dongzhimen from various stops in the community. The travel times of the DRT bus are obtained based on the simulated results. The travel times by TB are calculated by the Baidu map app in terms of the shortest time path on the bus route network with transfers. Table 8 compares the TB and DRT services in terms of travel time.

According to Table 8, compared with TB, the DRT bus shortens the average travel time by 36.48%. For some stops, it can shorten the travel time by up to approximately 40%–50%. Hence, from the perspective of travel time, the commuting time is substantially improved.

Economic Benefits

The economic benefits are studied from the perspectives of commuters and bus operators.

Time and Travel Costs

For commuters, two aspects of the benefit realized by using DRT service, namely the time cost and the travel cost, are analyzed (Table 9). Here, the time cost is estimated according to the average salary level in Beijing in 2017.

Compared with a TB, a DRT bus has a shorter travel time by 8.9–49.8 min according to the results in Table 8, namely the time cost is reduced by 5.23–29.28 yuan. Comparing with PC, although the travel time via a DRT bus is longer by approximately 7 min, the time savings that is realized by traveling via PC is canceled out by the time that must be spent finding a parking space.

Regarding the cost of travel, in practice, the fare for taking a DRT bus is typically higher than that for a TB, but the quality of DRT service is higher because users can directly reach their destinations without needing to transfer. The on-board experience is more comfortable because each passenger has a seat. Moreover, compared with PC, DRT service has no parking costs. Thus, the use of DRT service is cost-effective for commuters.

J. Transp. Eng., Part A: Systems, 2020, 146(12): 04020132

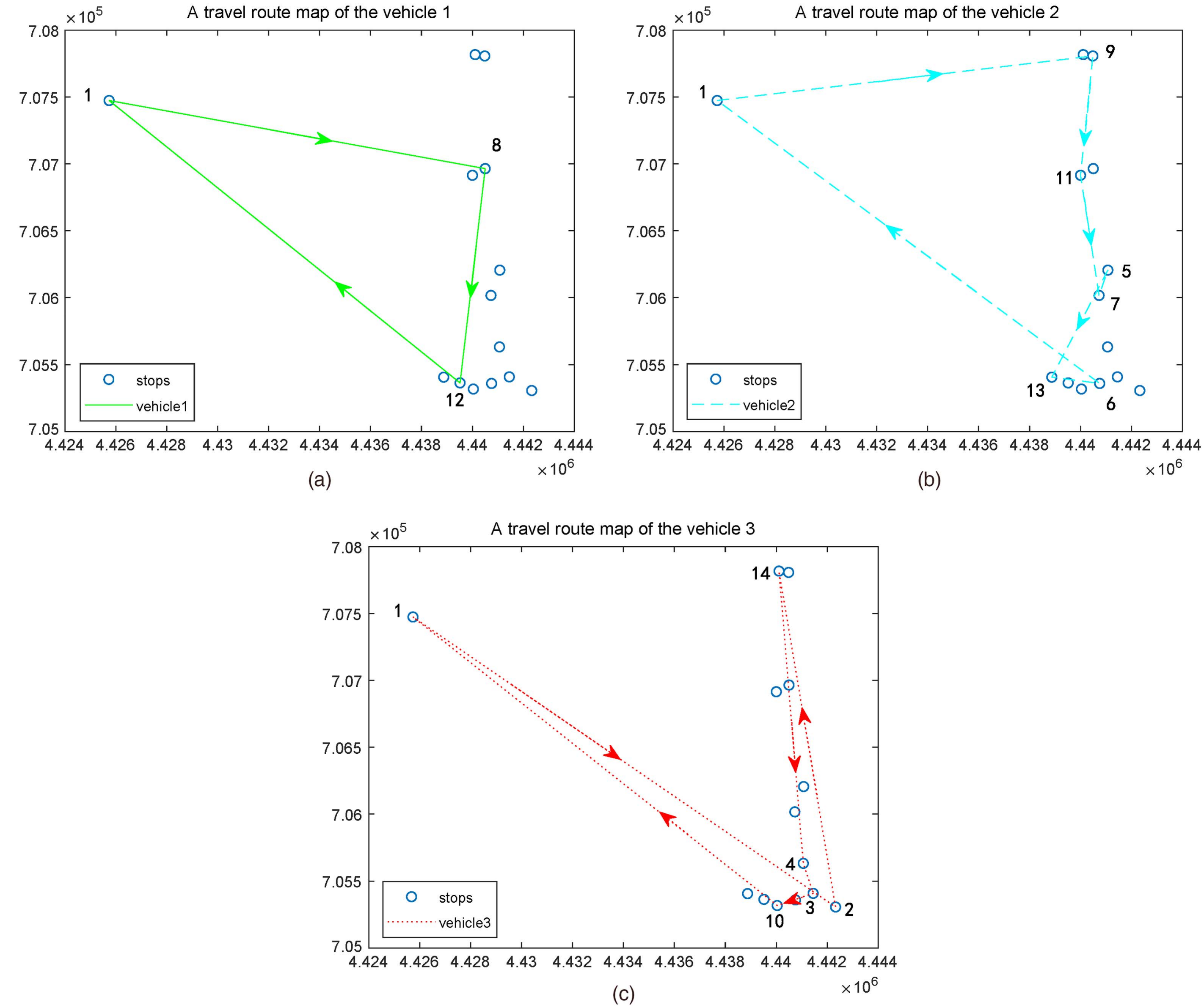

Fig. 4. Travel route maps: (a) Vehicle 1; (b) Vehicle 2; and (c) Vehicle 3.

Vehicle Use Cost

This paper focuses only on the fixed and operating costs of using a bus, which will be analyzed by comparing DRT with TB. Because most of the fixed costs are constant for both DRT and TB, the fixed cost in this paper refers only to the vehicle depreciation cost. The operating cost is the fuel consumption cost. In terms of the fixed and operating costs presented in Table 4, the total vehicle use cost per hour for a DRT bus is calculated as approximately 32.28 yuan. For a TB, the vehicle is typically large-sized, and its purchase cost is 1.9 million yuan. Additionally, the average service life is 10 years. Hence, the fixed cost of a TB is 21 yuan/h on average. Its fuel consumption per 100 km is 30 L, and the average number of operating kilometers per hour is 15 km in Beijing (due to the larger number of stops than a DRT bus); therefore, the operating cost is 27 yuan/h. Coupled with the fixed cost, the total vehicle cost per hour is approximately 48 yuan.

Fig. 6 presents the comparison results, which demonstrate that although the operating cost of a DRT bus is slightly higher than that of a TB due to its lower fuel economy, the total cost per hour is still lower; hence, the provision of DRT bus service is also cost-effective for operators.

Fare Revenue

To study the revenue, the authors conduct a fare sensitivity analysis to examine the relationship among the fares of TBs and DRT buses and the demand.

The actual trip demand is uncertain, which can result in different route schemes. Hence, the average travel time from the stops in the community to Dongzhimen is used to estimate the average vehicle cost. According to the average travel time (Table 8) and the total vehicle cost per hour (Fig. 6), the average costs of driving TB and DRT buses are calculated as 57.68 and 24.53 yuan/trip, respectively. Given that the capacity of a TB is 60 passengers, the revenue for a bus is

$$n_p \times f_c - \left\lceil \frac{n_p}{60} \right\rceil \times 57.68 \tag{19}$$

where $\lceil\rceil$ = integer ceiling operation.

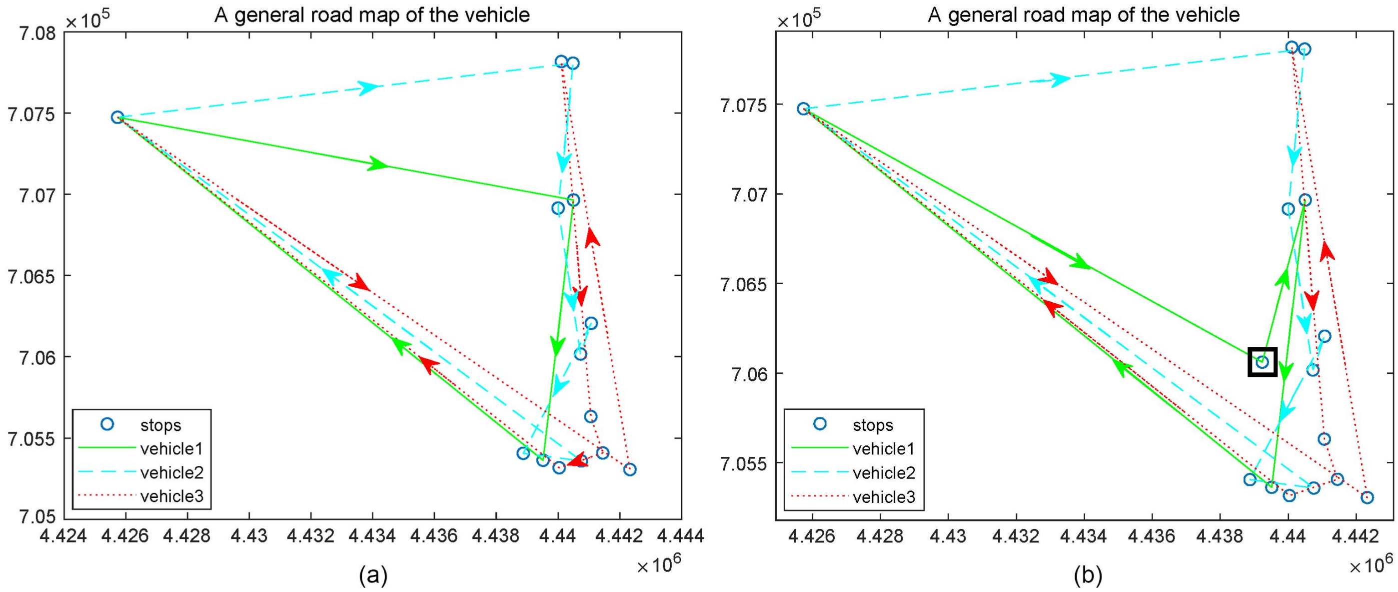

Fig. 5. Comparison of the general travel routes maps: (a) static optimization result; and (b) dynamic optimization result.

Table 6. Path optimization results by the static and dynamic algorithm

Route no.	Stop label and arriving time									Route length (km)	Travel time (min)	Revenue (yuan)	Response time (s)
	Static optimization scheme											48.02	13
Route 1	Stop label	1	8	12	1			—	—	3.2225	7.73		
	Arriving time (a.m.)	8:00	8:39	8:47	9:22	—	—	—	—				
Route 2	Stop label	1	9	11	7	5	13	6	1	8.9462	21.47		
	Arriving time (a.m.)	8:00	8:41	8:44	8:49	8:51	8:58	9:02	9:40				
Route 3	Stop label	1	2	14	4	3	10	1	—	9.3123	22.35		
	Arriving time (a.m.)	8:00	8:41	8:51	8:58	9:00	9:03	9:39	—				
	Dynamic optimization scheme											47.67	15
Route 1	Stop label	1	15	8	12	1			—	5.4143	12.99		
	Arriving time (a.m.)	8:00	8:18	8:39	8:47	9:22	—	—	—				
Routes 2 and 3 are the same as the static optimization results													

Note: Route length and travel time refer to the path length and travel time within the community. The distance and travel time between Dongzhimen (Stop 1) and the community are not included because there is no stop in this section.

Table 7. Comparison of the convenience between TB and DRT services

Type of transit service	Transfer time from the community to Dongzhimen (times)	Walking distance	Waiting time
TB	≥2	Need to walk to the stop, and the walking distance is relatively long	Because only approximately 18% of bus lines provide real-time vehicle arrival information in Beijing, passengers may not be able to obtain the vehicle arrival time and their time could be wasted
DRT	≥1	The service arranges the stop as close as possible to the door of the requesting passenger; then, the walking distance is shortened	Passengers can acquire real-time vehicle arrival information, thereby substantially reducing the waiting time

Similarly, the revenue for a DRT bus is

$$n_p \times f_{\mathrm{drt}} - \left\lceil \frac{n_p}{40} \right\rceil \times 24.53 \tag{20}$$

If the DRT service is more profitable than the TB service, it should follow that

$$n_p \times f_{\mathrm{drt}} - \left\lceil \frac{n_p}{40} \right\rceil \times 24.53 \geq n_p \times f_c - \left\lceil \frac{n_p}{60} \right\rceil \times 57.68 \tag{21}$$

Therefore, the price of DRT service must satisfy the following condition:

$$f_{\mathrm{drt}} \geq f_c - \left(\left\lceil \frac{n_p}{60} \right\rceil \times 57.68 - \left\lceil \frac{n_p}{40} \right\rceil \times 24.53 \right) \Big/ n_p \tag{22}$$

According to the investigation, the fare f_c by taking a TB from Tiantongyuan to Dongzhimen is approximately 5 yuan/person on average. Assuming that various proportions of the total demand, which is set as 1,000, will take the DRT service, the fare f^*_{drt} of

Table 8. Comparison of the travel time from community stops to Dongzhimen between TB and DRT services

Stop label	Travel time to stop 1 (min) TB	Travel time to stop 1 (min) DRT bus	Saving time (min)	Savings (%)
2	67	58.1	8.9	13.28
3	65	39.2	25.8	39.69
4	72	40.7	31.3	43.47
5	79	49.3	29.7	37.59
6	63	37.4	25.6	40.63
7	76	50.6	25.4	33.42
8	75	42.5	32.5	43.33
9	77	58.9	18.1	23.51
10	62	35.8	26.2	42.26
11	79	55.3	23.7	30.00
12	61	34.8	26.2	42.95
13	63	42.0	21.0	33.33
14	98	48.2	49.8	50.82
Average	72.1	45.6	26.5	36.48

DRT buses for maintaining the same profitability level as TB service is estimated in Table 10. Although f^*_{drt} fluctuates and is not linear with respect to the demand due to the variety of vehicle capacity utilizations, the DRT bus fare is always lower than the TB fare. In addition, the service can make a profit with a breakeven point of f^*_{drt} compared with TB. Therefore, because the DRT system can provide a higher level of service to passengers, its pricing should be set at a higher but affordable level to attract more users while increasing the profit margin.

Environmental Friendliness

Greenhouse gas CO_2, which is the main gas emission from gasoline combustion, is estimated to represent the environmental impact. Because the travel time of a DRT bus is 45.6 min on average, its operating distance can be calculated as 19.0 km per trip based on the speed. Similarly, a TB runs 18.0 km per trip. According to the statistical data that are discussed previously, the oil consumptions by a DRT bus and a TB can be derived as 0.2 and 0.3 L/km, respectively. Because burning 1 L of diesel fuel emits 1.58×10^6 L of CO_2, a DRT bus emits 2.53×10^6 fewer liters of CO_2 per trip than a TB.

In addition, because the DRT service utilizes medium-sized buses, which emit less exhausted gas than large-sized buses, and a DRT bus makes fewer stops and idle operations when running, which further reduces its emissions, the DRT service produces less exhaust emissions than TB service. In summary, DRT service is more environmentally friendly than TB service. The comparison results are presented in Table 11.

Conclusions and Future Work

Aiming at overcoming the disadvantages of traditional PT services and satisfying the diverse travel requirements of passengers, especially the flexibility and comfort requirements, this paper presented a DRT service with flexible routing for solving the last mile problem. This flexible routing is realized by establishing a path optimization model with a time window constraint and maximizing the DRT system benefit. This model is solved via dynamic GA based on SA. Considering the connection of the Tiantongyuan community to Dongzhimen traffic hub in Beijing during the early peak hour as an example, the paper evaluated the proposed model and algorithm and conducts a comprehensive assessment of the case study results. Table 12 summarizes the key results of the analysis.

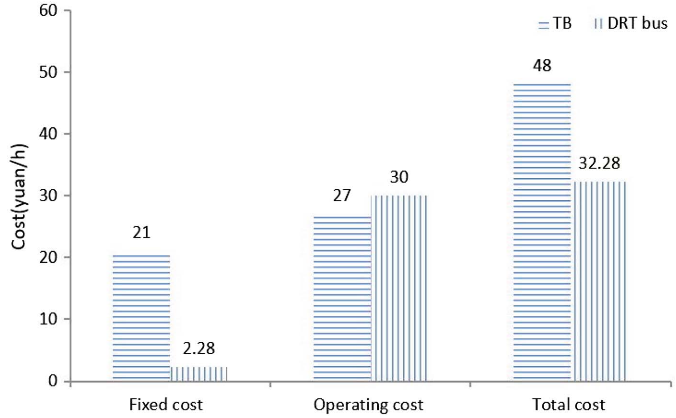

Fig. 6. Comparison of the fixed and operating cost between TB and DRT bus.

For commuters, compared with the TB service, the DRT service with flexible routing can reduce the travel time consumption, decrease the time cost, and improve the service level. Compared with PC service, the increased operating time of DRT service can be offset because a PC user must find a parking space and the higher parking cost is also avoided; thus, DRT service decreases the commuting time and the travel cost. For bus operators, when considering the fixed and operating costs of a bus, the DRT service can reduce vehicle use costs compared with TB service due to the reasonable selection of the vehicle type. The fare sensitivity analysis shows that DRT service can generate a profit margin even if its fare is priced lower than that of TB services. In addition, from the perspective of environmental friendliness, a DRT service can decrease exhaust emissions and conform to the concept of green and sustainable development.

In summary, it is concluded that DRT service with flexible routing is a reasonable, future-oriented, and environmentally friendly PT system. The results also demonstrated that the proposed model and algorithm can consider the benefits to both passengers and bus operators and can well satisfy the passengers' dynamic and flexible demands. Hence, DRT service with flexible routing is applicable in practice.

Although contributions have been made regarding the modeling and solution of the DRT path optimization problem with flexible routing in this paper, additional studies are required on, for

Table 9. Comparison of the economic benefit from the perspective of commuters who use the DRT service

Compared with	DRT bus: Time spent	DRT bus: Time costs	DRT bus: Travel cost
TB	Reduced by 8.9–49.8 min	Reduced by 5.23–29.28 yuan	Decreased due to the improved service quality
PC	Increased by about 7 min	Increased by 4.13 yuan	Reduced greatly because of no parking fees

Table 10. Fare sensitivity analysis of DRT bus on the TB fare and demand

Amount total demand (%)	n_p (passengers)	DRT bus (vehicles)	f_c (yuan/person)	f^*_{drt} (yuan/person)	DRT revenue (yuan)
80	800	20	5	4.60	3,192.48
60	600	15		4.65	2,423.2
50	500	13		4.60	1,980.88
30	300	8		4.69	1,211.6
20	200	5		4.46	769.28
10	100	3		4.58	384.64

Table 11. Comparison of CO_2 emission between TB and DRT bus

Type of transit service	Average travel time (min/trip)	Speed (km/h)	Operating distance (km/trip)	Fuel consumption (L/km)	CO_2 emissions (10^6 L)	Exhaust emissions
TB	72.1	15	18.0	0.3	8.53	More
DRT bus	45.6	25	19.0	0.2	6.00	Less

Table 12. Summary of the analysis of the case study results

Mode of transport	Convenience	Travel time	Passenger cost	Operator cost	Environment impact
DRT bus	Need to walk to the stop, transfer-free, and the ride experience is better	8.9–49.8 min less than that of TB; no need to find a parking space	Lower	Hourly cost is 15.72 yuan/h less than that of TB	Environmentally friendly, reduces CO_2 emissions by 2.53×10^6 L/trip compared with TB
TB	Need to walk a longer distance to the stop, more transfers, and the ride experience is poor	More time-consuming	Lowest	Higher	Environmentally friendly
PC	No need to walk, no transfer, and no crowding	Less travel time, but additional time to find a parking space	Higher	—	Not environmentally friendly

example, (1) mining the DRT demand more accurately from multiple sources of the traffic data, such as taxi trajectory, online car-haling services, and even mobile phone data, in addition to transit smart card data, and (2) defining the candidate stops to well satisfy the trip demands by establishing an improved model.

In addition, this paper only evaluates the performance of the proposed method in a typical community in which the road network is not highly complicated. However, with the increasing interest in DRT service, more real-time demands will need to be satisfied simultaneously, and road networks of a larger scale, including more stops and roads, should be considered, which will present substantial challenges in terms of the efficiency and effectiveness of the algorithm. These aspects must to be addressed urgently. Last, additional studies to further investigate passenger's variable requirements, to quantitatively evaluate the quality of DRT service and consider various application scenarios should be conducted, such as studies on DRT service for a large-scale airport hub or a high-speed rail station, to improve the modeling performance.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available from the corresponding author by request (algorithm code, boarding and alighting data in the studied area, stop coordinates, and distance matrix).

Acknowledgments

This study is funded by the National Key R&D Program of China (Grant No. 2018YFB1601200) and the Foundation for Innovative Research Groups of the National Nature Science Foundation of China (Grant No. 71621001). The authors are grateful to the anonymous reviewers for providing helpful suggestions for the study.

Notation

The following symbols are used in this paper:

A = set of network streets (graph links), $A = \{(i,j) : i, j \in N; i \neq j\}$;
$[a_i, b_i]$ = preferred time window at stop i that is requested by the passenger;
alph = annealing attenuation coefficient;
c_1 = fixed cost of vehicle use;
c_2 = operating cost of vehicles per unit time;
d_{ij} = travel distance of vehicles from stop i to stop j;
$F = N \cup \{o'\} \cup \{o\}$ = collection of all stops;
f_c = one-way flat fare per passenger by traditional bus;
f_{drt} = one-way flat fare per passenger by DRT bus;
$g_\gamma(t_z)$ = fitness function;
i, j, m = stops i, j, and m that have the demand, where i is the predecessor stop of j and m is the successor stop of j;

K = set of all vehicles in the TDC, $K=\{1,2,\ldots,k_0\}$;
l_i^k = lateness time of vehicle k at stop i;
m_1 = waiting penalty coefficient per passenger per hour if the vehicle arrives at stop i earlier than a_i;
m_2 = late penalty coefficient per passenger per hour if the vehicle arrives at stop i later than b_i;
N = set of all stops except the TDC, $N=\{1,2,\ldots,n\}$;
$N_o = N\cup\{o\}$;
$N_{o'} = N\cup\{o'\}$;
n_p = number of sectional passengers during the peak hours;
nd_i^k = number of passengers who are alighting from vehicle k at stop i;
nn_i^k = number of passengers on vehicle k at stop i;
nu_i^k = number of passengers who are boarding vehicle k at stop i;
$\{o,o'\}$ = TDCs, where o is the departure TDC and o' is the return TDC;
p = service time that is required by each passenger;
Q = vehicle capacity limit;
r_i^k = time when vehicle k arrives at stop i;
s = basic fare;
s_i^k = time interval between vehicle k arriving at stop i and leaving for the next stop, which includes w_i^k and other service times;
t = price per passenger for 1 km;
t_{ij} = travel time from stop i to stop j, $t_{ij}=d_{ij}/v$;
v = vehicle average operating speed;
w_i^k = extra waiting time of vehicle k at stop i due to arriving earlier than a_i;
x_{ij}^k = a decision variable with value 0 or 1, where if vehicle k goes from stop i to stop j, the value is 1, and otherwise, the value is 0;
y_i^k = a decision variable with value 0 or 1, where if required stop i is visited by vehicle k, the value is 1, and otherwise, the value is 0;
z = number of iterations; and
λ = number of passengers who are arriving per unit time.

References

Amirgholy, M., and E. J. Gonzales. 2016. "Demand responsive transit systems with time-dependent demand: User equilibrium, system optimum, and management strategy." *Transp. Res. Part B: Methodol.* 92 (Part B): 234–252. https://doi.org/10.1016/j.trb.2015.11.006.

An, J., R. Song, M. Bi, and S. Xue. 2019. "Optimization flexible route design for high-speed railway station feeder bus." *J. Transp. Syst. Eng. Inf. Technol.* 19 (5): 150–155. https://doi.org/10.16097/j.cnki.1009-6744.2019.05.021.

Berbeglia, G., J. F. Cordeau, and G. Laporte. 2010. "Dynamic pickup and delivery problems." *Eur. J. Oper. Res.* 202 (1): 8–15. https://doi.org/10.1016/j.ejor.2009.04.024.

Brake, J., J. D. Nelson, and S. Wright. 2004. "Demand responsive transport: Towards the emergence of a new market segment." *J. Transp. Geogr.* 12 (4): 323–337. https://doi.org/10.1016/j.jtrangeo.2004.08.011.

Cao, Y., and J. Wang. 2016. "The key contributing factors of customized shuttle bus in rush hour: A case study in Harbin city." *Procedia Eng.* 137: 478–486. https://doi.org/10.1016/j.proeng.2016.01.283.

Cao, Y., and J. Wang. 2017. "An optimization method of passenger assignment for customized bus." *Math. Prob. Eng.* 2017: 1–9. https://doi.org/10.1155/2017/7914753.

Ceder, A. 2007. *Public transit planning and operation: Theory, modelling and practice*. London: Elsevier.

Chandra, S., and L. Quadrifoglio. 2013. "A model for estimating the optimal cycle length of demand responsive feeder transit services." *Transp. Res. Part B: Methodol.* 51 (May): 1–16. https://doi.org/10.1016/j.trb.2013.01.008.

Chang, S. K., and P. M. Schonfeld. 1991. "Optimization models for comparing conventional and subscription bus shuttle services." *Transp. Sci.* 25 (4): 281–298. https://doi.org/10.1287/trsc.25.4.281.

Cheng, Y., A. Huang, G. Qi, and B. Zhang. 2019. "Mining customized bus demand spots based on smart card data: A case study of the Beijing public transit system." *IEEE Access* 7: 181626–181647. https://doi.org/10.1109/ACCESS.2019.2959907.

Chevrier, R., A. Liefooghe, L. Jourdan, and C. Dhaenens. 2012. "Solving a dial-a-ride problem with a hybrid evolutionary multi-objective approach: Application to demand responsive transport." *Appl. Soft Comput.* 12 (4): 1247–1258. https://doi.org/10.1016/j.asoc.2011.12.014.

Cordeau, J. F. 2006. "A branch-and-cut algorithm for the dial-a-ride problem." *Oper. Res.* 54 (3): 573–586. https://doi.org/10.1287/opre.1060.0283.

Cordeau, J. F., and G. Laporte. 2007. "The dial-a-ride problem: Models and algorithms." *Ann. Oper. Res.* 153 (1): 29–46. https://doi.org/10.1007/s10479-007-0170-8.

Davison, L., M. Enoch, T. Ryley, M. Quddus, and C. Wang. 2014. "A survey of demand responsive transport in Great Britain." *Transp. Policy.* 31 (Jan): 47–54. https://doi.org/10.1016/j.tranpol.2013.11.004.

Dikas, G., and I. Minis. 2014. "Scheduled paratransit transport systems." *Transp. Res. Part B: Methodol.* 67 (67): 18–34. https://doi.org/10.1016/j.trb.2014.05.001.

Drakoulis, R., F. Bellotti, I. Bakas, R. Berta, P. K. Paranthaman, G. R. Dange, P. Lytrivis, K. Pagle, A. De Gloria, and A. Amditis. 2018. "A gamified flexible transportation service for on-demand public transport." *IEEE Trans. Intell. Transp.* 19 (3): 921–933. https://doi.org/10.1109/TITS.2018.2791643.

Guo, R., W. Guan, and W. Zhang. 2018. "Route design problem of customized buses: Mixed integer programming model and case study" *J. Transp. Eng., Part A: Systems* 144 (11): 04018069. https://doi.org/10.1061/JTEPBS.0000185.

Guo, R., W. Guan, W. Zhang, F. Meng, and Z. Zhang. 2019. "Customized bus routing problem with time window restrictions: Model and case study." *Transportmetrica A* 15 (2): 1804–1824. https://doi.org/10.1080/23249935.2019.1644566.

Huang, D., Y. Gu, S. Wang, Z. Liu, and W. Zhang. 2020. "A two-phase optimization model for the demand-responsive customized bus network design." *Transp. Res. Part C: Emerging Technol.* 111 (Feb): 1–21. https://doi.org/10.1016/j.trc.2019.12.004.

Iqbal, R., and M. U. Ghani. 2014. "Intelligent bus stops in the flexible bus systems." *J. Eng. Sci. Technol. Rev.* 7 (4): 59–65. https://doi.org/10.25103/jestr.074.10.

Iqbal, R., K. Yukimatsu, and T. Ichikawa. 2011. "The flexible bus systems using zigbee as a communication medium." In *Proc., 4th IFIP Int. Conf. in New Technologies, Mobility and Security*, 1–5. New York: IEEE.

Jerby, S., and A. Ceder. 2006. "Optimal routing design for shuttle bus service." *Transport. Res. Rec.* 1971 (1): 14–22. https://doi.org/10.1177/0361198106197100102.

Jin, J. G., K. M. Teo, and A. R. Odoni. 2015. "Optimizing bus bridging services in response to disruptions of urban transit rail networks." *Transp. Sci.* 50 (3): 790–804. https://doi.org/10.1287/trsc.2014.0577.

Jung, J., and R. Jayakrishnan. 2014. "Simulation framework for modeling large-scale flexible transit systems." *Transp. Res. Rec.* 2466 (1): 31–41. https://doi.org/10.3141/2466-04.

Kim, M., and P. Schonfeld. 2011. "Conventional, flexible, and variable-type bus services." *J. Transp. Eng.* 138 (3): 263–273. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000326.

Kong, X., M. Li, T. Tang, K. Tian, L. Moreira-Matias, and F. Xia. 2018. "Shared subway shuttle bus route planning based on transport data analytics." *IEEE Trans. Autom. Sci. Eng.* 15 (4): 1507–1520. https://doi.org/10.1109/TASE.2018.2865494.

Liu, T., and A. Ceder. 2015. "Analysis of a new public-transport-service concept: Customized bus in China." *Transp. Policy* 39 (Apr): 63–76. https://doi.org/10.1016/j.tranpol.2015.02.004.

Liu, T., A. Ceder, and R. Bologna. 2016. "Commuting by customized bus: A comparative analysis with private car and conventional public transport in two cities." *Public J. Transp.* 19 (2): 55–74. https://doi.org/10.5038/2375-0901.19.2.4.

Ma, J., Y. Yang, W. Guan, F. Wang, T. Liu, W. Tu, and C. Song. 2017. "Large-scale demand driven design of a customized bus network: A methodological framework and Beijing case study." *J. Adv. Transp.* 2017 (3): 1–14. https://doi.org/10.1155/2017/3865701.

Mageean, J., and J. D. Nelson. 2003. "The evaluation of demand responsive transport services in Europe." *J. Transp. Geogr.* 11 (4): 255–270. https://doi.org/10.1016/S0966-6923(03)00026-7.

Masmoudi, M. A., K. Braekers, M. Masmoudi, and A. Dammak. 2017. "A hybrid genetic algorithm for the heterogeneous dial-a-ride problem." *Comput. Oper. Res.* 81 (May): 1–13. https://doi.org/10.1016/j.cor.2016.12.008.

Mulley, C., and J. D. Nelson. 2009. "Flexible transport services: A new market opportunity for public transport." *Res. Transp. Econ.* 25 (1): 39–45. https://doi.org/10.1016/j.retrec.2009.08.008.

Nelson, J. D., S. Wright, B. Masson, G. Ambrosino, and A. Naniopoulos. 2010. "Recent developments in flexible transport services." *Res. Transp. Econ.* 29 (1): 243–248. https://doi.org/10.1016/j.retrec.2010.07.030.

Nguyen, P. K., T. G. Crainic, and M. Toulouse. 2013. "A tabu search for time-dependent multi-zone multi-trip vehicle routing problem with time windows." *Eur. J. Oper. Res.* 231 (1): 43–56. https://doi.org/10.1016/j.ejor.2013.05.026.

Padberg, M., and G. Rinaldi. 1991. "A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems." *SIAM Rev.* 33 (1): 60–100. https://doi.org/10.1137/1033004.

Potts, J. F., M. A. Marshall, E. C. Crockett, and J. Washington. 2010. *A guide for planning and operating flexible public transportation services*. TCRP Rep. No. 140. Washington, DC: Transportation Research Board, National Research Council.

Qiu, F., W. Li, and J. Zhang. 2014. "A dynamic station strategy to improve the performance of flex-route transit services." *Transp. Res. Part C: Emerging Technol.* 48 (Nov): 229–240. https://doi.org/10.1016/j.trc.2014.09.003.

Ronald, N., R. Thompson, and S. Winter. 2015. "Simulating demand-responsive transportation: A review of agent-based approaches." *Transp. Rev.* 35 (4): 404–421. https://doi.org/10.1080/01441647.2015.1017749.

Ryley, T. J., P. A. Stanley, M. P. Enoch, A. M. Zanni, and M. A. Quddus. 2014. "Investigating the contribution of demand responsive transport to a sustainable local public transport system." *Res. Transp. Econom.* 48 (Dec): 364–372. https://doi.org/10.1016/j.retrec.2014.09.064.

Shaheen, S. 2001. "Commuter-based carsharing: Market niche potential." *Transport. Res. Rec.* 1760 (1): 178–183. https://doi.org/10.3141/1760-23.

Tabassum, S., S. Tanaka, F. Nakamura, and A. Ryo. 2017. "Feeder network design for mass transit system in developing countries (case study of Lahore, Pakistan)." *Transp. Res. Procedia* 25: 3129–3146. https://doi.org/10.1016/j.trpro.2017.05.343.

TCRP (Transit Cooperative Research Program). 1999. *Guidelines for enhancing suburban mobility using public transportation*. TCRP Rep. No. 55. Washington, DC: Transportation Research Board, National Research Council.

Tong, L. C., L. Zhou, J. Liu, and X. Zhou. 2017. "Customized bus service design for jointly optimizing passenger-to-vehicle assignment and vehicle routing." *Transp. Res. Part C: Emerging Technol.* 85 (Dec): 451–475. https://doi.org/10.1016/j.trc.2017.09.022.

Van Engelen, M., O. Cats, H. Post, and K. Aardal. 2018. "Enhancing flexible transport services with demand-anticipatory insertion heuristics." *Transp. Res. Part E: Logistics Transp. Rev.* 110 (Feb): 110–121. https://doi.org/10.1016/j.tre.2017.12.015.

Velaga, N. R., N. D. Rotstein, N. Oren, J. D. Nelson, T. J. Norman, and S. Wright. 2012. "Development of an integrated flexible transport systems platform for rural areas using argumentation theory." *Res. Transp. Bus. Manage.* 3 (Aug): 62–70. https://doi.org/10.1016/j.rtbm.2012.05.001.

Wang, W. 2006. *Planning methodology and management technology of urban public transportation system*. Beijing: Science Press.

Xie, C., H. Gong, and F. Wang. 2010. "A solution for the last mile problem of the Beijing rapid transit network: Local shuttle bus system." In *Proc., 18th IEEE Int. Conf. on Geoinformatics*, 1–6. New York: IEEE.

Xiong, J., Z. He, W. Guan, and B. Ran. 2015. "Optimal timetable development for community shuttle network with metro stations." *Transp. Res. Part C: Emerging Technol.* 60 (Nov): 540–565. https://doi.org/10.1016/j.trc.2015.10.007.

Yu, Y., R. B. Machemehl, and C. Xie. 2015. "Demand-responsive transit circulator service network design." *Transp. Res. Part E: Logistics Transp. Rev.* 76 (Apr): 160–175. https://doi.org/10.1016/j.tre.2015.02.009.

Zhou, C., W. Guan, J. Xiong, and S. Jiang. 2017. "A study on dynamic dispatching strategy of customized bus." In *Proc., 3rd IEEE Int. Conf. in Control Science and Systems Engineering*, 751–755. New York: IEEE. https://doi.org/10.1109/CCSSE.2017.8088034.

Zhu, Z., X. Guo, J. Zeng, and S. Zhang. 2017. "Route design model of feeder bus service for urban rail transit stations." *Math. Prob. Eng.* 2017: 1090457. https://doi.org/10.1155/2017/1090457.

J. Transp. Eng., Part A: Systems, 2020, 146(12): 04020132