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Bus Timetable Design to Ensure Smooth Transfers in Areas with Low-Frequency Public Transportation Services

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Abstract. This study investigates the timetable design problem in areas with low-frequency public transportation services. In Japan, rural areas face sparse populations and rapid growth in the percentage of elderly people. In these areas, many bus lines offer fewer than 10 services per day. In addition to low-frequency services, it is also inconvenient to transfer to another bus or train service. Thus, there is a strong need to design a timetable that ensures smooth transfers among buses and trains. We tackle this problem by adopting existing bus lines and train timetables as much as possible to avoid drastic changes such as an increase in the number of services. Based on this approach, we present a mathematical optimization model to generate a revised bus timetable that shortens waiting time for transfers compared with the current timetable. We apply our model to a part of the Tohoku District in Japan and demonstrate its usefulness in the real world.

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Keywords: timetable design • mathematical optimization • bus and railway network • event–activity network

1. Introduction

Japan has faced severe social phenomena including an aging population and the uneven distribution of its residents. The negative effects of these phenomena are especially serious in rural areas. From the viewpoint of transportation services, a sparse population and a decrease in activity make traffic demand too low for the system to be self-supporting.

In rural areas, many people rely on private cars and rarely use public transportation because the service frequency of trains and buses is low. The more aged our society, the lower the number of drivers of private cars carrying elderly people. Thus, there is an urgent need to promote the use of public transportation.

Table 1 presents an example of a bus timetable in a rural area of Japan, showing only three or four buses per day for each destination. Because the public transportation network in the area is rudimentary, people often have to change buses and/or trains multiple times to reach their destination. Unfortunately, bus and train services are scheduled within each bus/railway company, and their timetables are not designed for smooth transfers among different companies. As a result, passengers often wait for an unexpectedly long time. To promote the use of public transportation, it is thus important to design a bus timetable that shortens waiting time compared with the current timetable.

In this study, we focus on areas with low-frequency public transportation services and present a mathematical optimization model to generate bus timetables that ensure smooth transfers among buses and trains. Most Japanese timetables are designed to permit only one-way transfers. For example, if the timetables are designed for smooth transfers from one (bus) to the other (train), a passenger traveling in the opposite direction will miss the bus and have to wait for the next one. In cases where the bus line has low-frequency services, the next bus will arrive after a few hours. Thus, low-frequency services combined with one-way transfers lead to substantially inconvenient transfers.

Our aim is to design bus timetables without drastic changes such as an increase in the number of services. To improve timetables under low-frequency services, we keep existing transfers available and add opposite transfers as much as possible. This leads to the *mutual connections* of buses and trains, which enable passengers to transfer in both directions. Because bus lines with different routes are intricately connected in rural areas of Japan, it is important to realize mutual connections not only between a bus and a train but also between buses. For this purpose, we need to deal with individual transfers in terms of time and space, because bus lines offer an extremely small number of services. This is made possible by using an event–activity network, as explained in Section 4.1.

Mutual connections can be achieved at the cost of making buses stand at bus stops for some extra time. Thus, there is a trade-off between the gain for transfer

Table 1. A Timetable at the Setamai-Eki-Mae Bus Stop in the Tohoku District in Japan

To Morioka	To Nakai	To Sumita High School
6:46 a.m.	8:18 a.m.	7:27 a.m.
7:46 a.m.	1:08 p.m.	11:17 a.m.
12:46 p.m.	4:28 p.m.	4:17 p.m.
4:46 p.m.	6:43 p.m.	

passengers and loss for on-board passengers. We formulate this problem as a mixed integer programming problem (MIP), where we find the departure and arrival times of each bus and when and where we should have mutual connections.

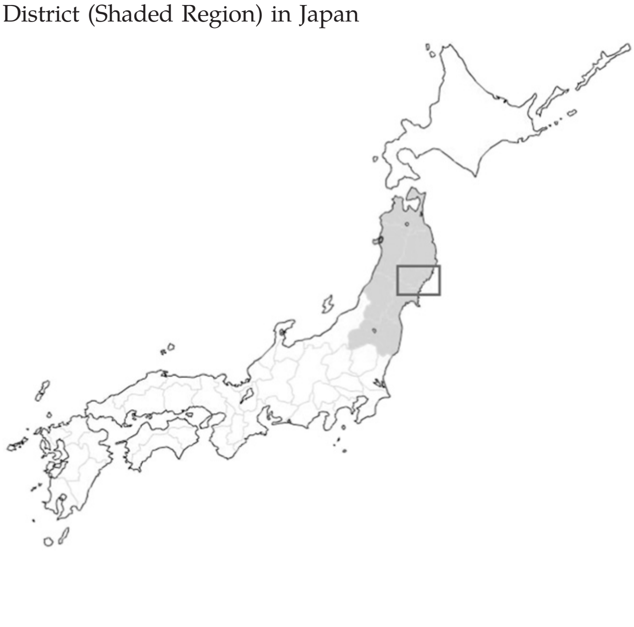
We apply our model to the area on the Pacific Ocean side of the Tohoku District in Japan (see Figure 1) and show that the obtained timetable succeeds in realizing mutual connections with little negative effect on standing time.

The organization of the rest of this paper is as follows. Section 2 describes related work. In Section 3, we state the significance of this research for the revitalization of rural areas in Japan. In Section 4, we recapitulate an event–activity network and provide a mathematical optimization model to generate a bus timetable with mutual connections among buses and trains. In Section 5, we explain how to narrow down candidates for mutual connections based on the network structure. In Section 6, we apply our model to the area on the Pacific Ocean side of the Tohoku District in Japan. We further explain that the model is also useful for the analysis of bus routes in Section 7. Finally, Section 8 concludes this paper.

2. Related Work

The area of railway optimization has advanced, especially in Europe (Wilson and Nuzzolo 2004, 2009; Schöbel 2006; Geraets et al. 2007; Schmidt 2014), and it has been shown that mathematical optimization techniques are useful for line planning, timetabling, rolling stock scheduling, and crew scheduling.

Figure 1. The Target Area (Rectangle) in the Tohoku District (Shaded Region) in Japan



The train scheduling problem has been studied extensively (Cordeau, Toth, and Vigo 1998). The design of timetables that ensure smooth transfers is an important task. A variety of models have been proposed to minimize total passenger waiting time (Ceder 1991, Nachtigall 1996, Nachtigall and Voget 1996, Odijk 1996, Vansteenwegen and Van Oudheusden 2007), maximize the number of passengers on direct connections (Bussieck, Kreuzerb, and Zimmermann 1997), and so on.

Mutual connections are a key concept in timetable design, especially for buses. This is because buses are less attractive than cars when the waiting time for transfers is long. The problem of maximizing the number of simultaneous bus arrivals is called the synchronization problem, and optimization techniques are used to solve it (Ceder, Golany, and Tal 2001; Eranki 2004; Ceder et al. 2013).

A concept similar to mutual connections can be found in the delay management problem. In this problem, we decide whether a connecting bus should wait for a delayed train or depart on time. If the connection is not maintained, a passenger cannot board the scheduled bus. On the contrary, if the bus waits for the train, the sum of delays over all vehicles becomes large. The delay management problem is an important subject that has been studied intensively in recent years (Schöbel 2006, 2007; Heilporn, De Giovanni, and Labbé 2008; Schachtebeck and Schöbel 2010; Dollevoet et al. 2012, 2015). Another similar concept is the meeting constraint in D'Ariano, Pacciarelli, and Pranzo (2007), which enables two trains to be together at a station to exchange passengers or goods.

The delay management problem has the same difficulty as ours because in this problem there is a trade-off between the gain for passengers who can board the scheduled bus and loss for passengers who arrive late at the destination. The delay management problem is formulated as an MIP by using an event–activity network (Schöbel 2007, Schachtebeck and Schöbel 2010). In this study, we also formulate our problem as an MIP by using an event–activity network.

A major difference between previous work and this study is in service frequency. As shown in Table 1, our target areas have low-frequency services. In these areas, missing a train or a bus strongly affects the travel time from the origin to the destination. Hence, we have to determine when and where we should realize mutual connections with careful consideration of the bus and railway network structure. In many previous works, including D'Ariano, Pacciarelli, and Pranzo (2007), important interchange stations are specified in advance. In Wong et al. (2008), these stations are determined by solving an MIP. In our approach, we narrow down candidates for mutual connections before formulating an MIP based on the network structure. As shown in the numerical example, this leads to results applicable in practice and short computational time.

3. Setting of Our Problem Toward Revitalization of Rural Areas

Let us explain the significance of this research for the revitalization of rural areas in Japan. A population decrease in rural areas is seen as a serious social problem. Whereas the transportation system in Tokyo is highly developed, bus companies in rural areas face a variety of difficulties, as explained below.

Take the case of the bus lines in Iwate Prefecture of the Tohoku District as an example. Figure 2 shows the average number of passengers on a bus, which is published in the reports[1] on the official website of Iwate Prefecture. The number of bus lines with at most four passengers on average on a bus increases from 2011 to 2016, whereas the number of total bus lines decreases from 77 to 68. About 60 passengers can board a bus altogether. Thus, the number of passengers in Figure 2 is extremely low, and these bus lines are obviously unprofitable.

Figure 3 shows detailed data[2] on Kitakami City of Iwate Prefecture, where each point corresponds to a bus line. In most of the 16 bus lines, a bus has fewer than five passengers on average, regardless of the number of bus services. The bus line where a bus has more than 25 passengers on average is bound for the high school, and many high school students may use it.

Figure 2. The Average Number of Passengers on a Bus in Iwate Prefecture

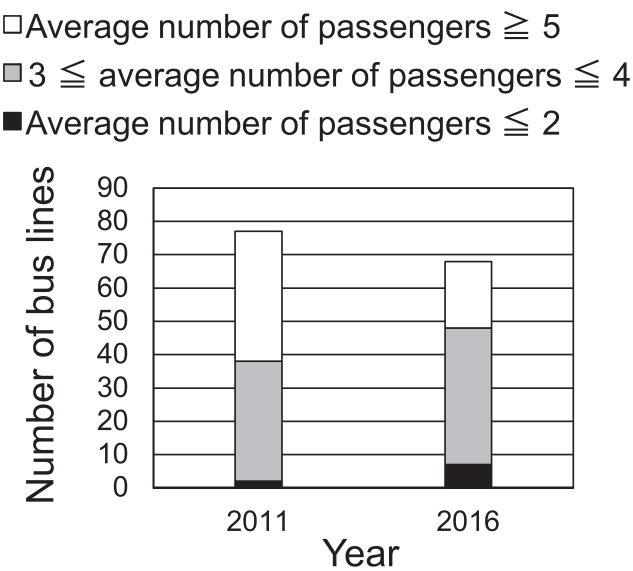
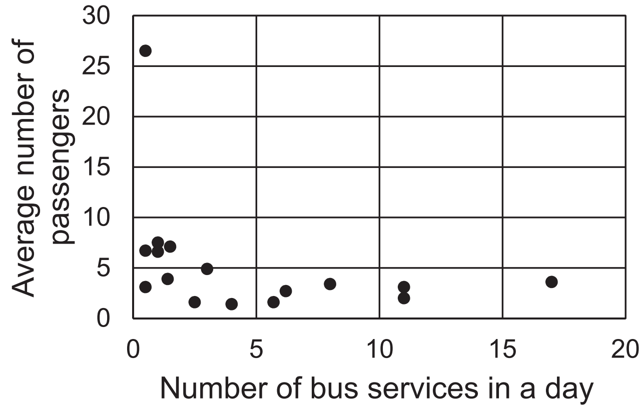


Figure 3. The Number of Services and the Average Number of Passengers on a Bus in Kitakami City



In rural areas of Japan, bus companies often withdraw from unprofitable bus lines, and hence the level of public transportation services is declining rapidly. This leads to a further decrease in passengers, which results in a larger number of unprofitable bus lines. Thus, bus companies in such areas get caught in a negative spiral.

To revitalize such rural areas, it is important to raise the level of public transportation services. We anticipate that the number of passengers will increase and bus companies will acquire new passengers by enhancing the quality of public transportation services. As shown in Figures 2 and 3, the current number of passengers is extremely low. Moreover, the bus services are inconvenient, and hence the current origin–destination (OD) data may not show potential traffic demand. Therefore, we should avoid depending only on the current OD data. Our ultimate aim is to design public transportation systems that enable passengers to travel comfortably from any origin to any destination in order to promote the use of buses.

In rural areas, bus companies cannot afford to expand bus lines or increase bus services. The most realistic approach without incurring costs is to change bus timetables. We expect that we can improve bus timetables effectively by exploiting optimization techniques. To show the power of optimization is the first step in breaking the negative spiral in rural areas, and further may lead to a big change in the trend of bus timetable design in Japan. Unlike bus lines, railway lines run over several areas. Because a change in the train schedule affects not only our target area but also other extensive areas, it would be better to keep the train schedule unchanged. In this study, we assume that train timetables are fixed. This setting is realistic and suitable for the situation in rural areas of Japan. In Section 4, we provide a mathematical optimization model to generate a bus timetable that provides comfortable travel from any bus stop (or station) to any bus stop (or station) without depending on the current OD data.

4. Bus Timetable Design

4.1. Event–Activity Network

In this subsection, we explain an event–activity network, which represents both the timetables of vehicles and the behavior of passengers. Event–activity networks are widely used in timetable design (Serafini and Ukovich 1989, Schöbel 2006, Schmidt 2014). A similar tool used to analyze Japanese railways is called a time–space network (Taguchi 2005).

Given a timetable Π, we construct an event–activity network as follows. Let V be the set of stations and bus stops and F be the set of vehicles (trains or buses). We denote the time needed for the transfer by L_{change} and define

$$\mathcal{P} = \{(v,u) \mid v,u \in V, v \neq u, \text{a passenger can walk from } v \text{ to } u \text{ in } L_{\text{change}} \text{ minutes}\}.$$

This means that bus stops v and u satisfying $(v,u) \in \mathcal{P}$ are located nearby, and passengers can transfer between v and u.

Let us define

$$\mathcal{E}_{\text{arr}} = \{(g,v,\text{arr}) \mid \text{vehicle } g \in F \text{ arrives at station } v \in V\},$$
$$\mathcal{E}_{\text{dep}} = \{(g,v,\text{dep}) \mid \text{vehicle } g \in F \text{ departs from station } v \in V\},$$

where $\mathcal{E}_{\text{arr}}$ and $\mathcal{E}_{\text{dep}}$ represent arrival events and departure events. Each $i \in \mathcal{E}_{\text{arr}} \cup \mathcal{E}_{\text{dep}}$ has the arrival or departure time in the timetable Π, which is denoted by Π_i.

In this study, we deal with connections between buses as well as connections between a train and a bus. Consider $(g,v,\text{arr}) \in \mathcal{E}_{\text{arr}}$ and $(h,u,\text{dep}) \in \mathcal{E}_{\text{dep}}$ with $(v,u) \in \mathcal{P}$. If $\Pi_{(h,u,\text{dep})} - \Pi_{(g,v,\text{arr})} \geq L_{\text{change}}$ holds, then (g,v,h,u) is called a *connection*. We denote the set of connections by $\mathcal{U} \subseteq F \times V \times F \times V$.

Next, consider buses g and h that belong to different bus lines but share bus stop v. Although we should deal with a connection (g,v,h,v) in a similar way to connections in $\mathcal{U}$, there are too many pairs of g and h to discuss them individually. In Section 5.1, we select important connections among pairs of buses sharing bus stops. The set of selected connections (g,v,h,v) is denoted by $\mathcal{U}_{\text{bus}}$.

We define

$$\mathcal{A}_{\text{drive}} = \{((g,v,\text{dep}),(g,u,\text{arr})) \in \mathcal{E}_{\text{dep}} \times \mathcal{E}_{\text{arr}} \mid \text{vehicle } g \text{ goes directly from } v \text{ to } u\},$$
$$\mathcal{A}_{\text{wait}} = \{((g,v,\text{arr}),(g,v,\text{dep})) \in \mathcal{E}_{\text{arr}} \times \mathcal{E}_{\text{dep}}\},$$
$$\mathcal{A}_{\text{change}} = \{((g,v,\text{arr}),(h,u,\text{dep})) \in \mathcal{E}_{\text{arr}} \times \mathcal{E}_{\text{dep}} \mid (g,v,h,u) \in \mathcal{U} \cup \mathcal{U}_{\text{bus}}\},$$

where $\mathcal{A}_{\text{drive}}$, $\mathcal{A}_{\text{wait}}$, and $\mathcal{A}_{\text{change}}$ represent driving activities, waiting activities, and changing activities, respectively. In addition, we define

$$\mathcal{A}_{\text{next}} = \{((g,v,\text{dep}),(g',v,\text{dep})) \in \mathcal{E}_{\text{dep}} \times \mathcal{E}_{\text{dep}} \mid g' \textit{is} \text{ the next vehicle to } g\}.$$

In the event–activity network, the set of vertices and set of arcs are given by

$$\mathcal{E} = \mathcal{E}_{\text{arr}} \cup \mathcal{E}_{\text{dep}}, \quad \mathcal{A} = \mathcal{A}_{\text{drive}} \cup \mathcal{A}_{\text{wait}} \cup \mathcal{A}_{\text{change}} \cup \mathcal{A}_{\text{next}}.$$

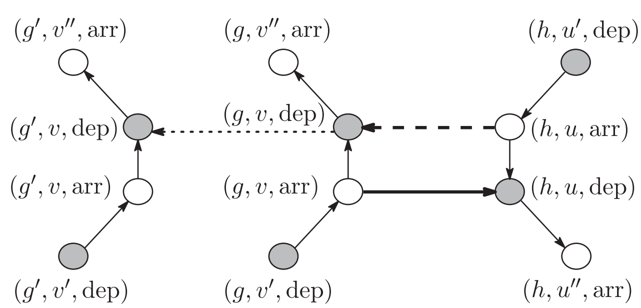
Figure 4 shows the arcs in the event–activity network.

Let us have a closer look at $\mathcal{A}_{\text{change}}$ and $\mathcal{A}_{\text{next}}$. Assume that bus stop v is shared by buses g, g', and g'' that belong to *different* bus lines, and that $(g,v,g',v),(g,v,g'',v),(g',v,g'',v) \notin \mathcal{U}_{\text{bus}}$. The event–activity network is given in Figure 5, where the arcs of $\mathcal{A}_{\text{change}}$ do not exist. Consider a passenger who changes from bus g to g''. In the event–activity network, the passenger moves along $((g,v,\text{arr}),(g,v,\text{dep})) \in \mathcal{A}_{\text{wait}}$, $((g,v,\text{dep}),(g',v,\text{dep})) \in \mathcal{A}_{\text{next}}$, and $((g',v,\text{dep}),(g'',v,\text{dep})) \in \mathcal{A}_{\text{next}}$. Thus, $\mathcal{A}_{\text{next}}$ is useful when we express changing activities without the arcs of $\mathcal{A}_{\text{change}}$. The combination of $\mathcal{A}_{\text{next}}$ and $\mathcal{A}_{\text{change}}$ requires far fewer arcs to represent changing activities than the use of only $\mathcal{A}_{\text{change}}$.

Table 2 summarizes the four kinds of pairs of $g,h \in F$. Our aim is to increase mutual connections, as defined in Section 4.2. Connections in $\mathcal{U} \cup \mathcal{U}_{\text{bus}}$, which are expressed by the arcs of $\mathcal{A}_{\text{change}}$ in the event–activity network, are candidates for mutual connections. By contrast, a connection between g and h with $(g,v,h,v) \notin \mathcal{U}_{\text{bus}}$ is not a candidate for mutual connections, and changing activities can be expressed by $\mathcal{A}_{\text{next}}$.

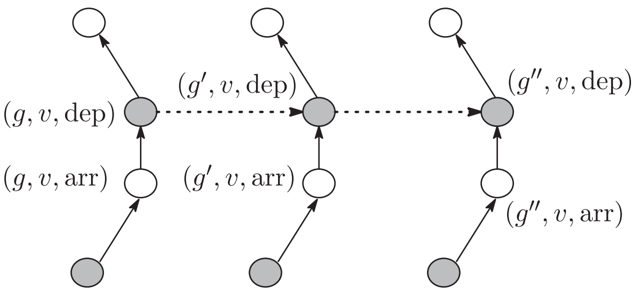
Figure 6 depicts the bus and railway network (left) and the corresponding event–activity network (right) of the target area in Figure 1.

Figure 4. An Event–Activity Network



Notes. The solid lines denote the arcs of $\mathcal{A}_{\text{drive}}$ and $\mathcal{A}_{\text{wait}}$, the dotted line corresponds to an arc of $\mathcal{A}_{\text{next}}$, the bold line represents an arc of $\mathcal{A}_{\text{change}}$, and the dashed line indicates that passengers cannot change from h into g. White and gray vertices represent arrival (arr) and departure (dep) events, respectively.

Figure 5. An Example of Changing Activities with $\mathscr{A}_{\text{next}}$



Note. The dotted line corresponds to an arc of $\mathscr{A}_{\text{next}}$. Arr, arrival; dep, departure.

4.2. Mutual Connections

We first explain the typical situation in Japan. Consider the event–activity network given in Figure 4. Let us assume that a passenger arrives at bus stop v by bus g. He (or she) will walk from v to train station u and board train h by $\Pi_{(h,u,\text{dep})} - \Pi_{(g,v,\text{arr})} \geq L_{\text{change}}$.

In Japan, standing time at a bus stop is usually at most only one minute, which leads to $\Pi_{(h,u,\text{arr})} > \Pi_{(g,v,\text{dep})}$ in the majority of cases. In this case, no one can change from train h to bus g. Many Japanese timetables are designed to permit only one-way transfers; however, one-way transfers are inconvenient when the next bus does not arrive for a few hours.

To design a timetable that ensures smooth transfers, we would like to allow passengers to change from train h to bus g if they can change from bus g to train h. We call such a pair of g and h a mutual connection, which this study aims to increase.

We call (h, u, g, v) a *missed connection*[3] if

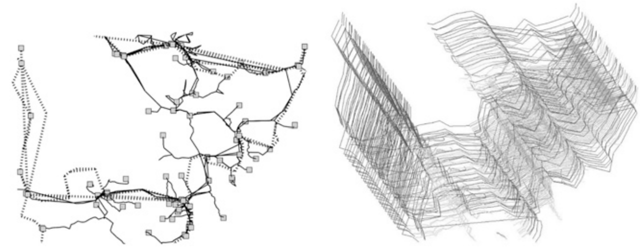
$$((g, v, \text{arr}), (h, u, \text{dep})) \in \mathscr{A}_{\text{change}}, \Pi_{(g,v,\text{dep})} - \Pi_{(h,u,\text{arr})} < L_{\text{change}}. \tag{1}$$

We remark that (h, u, g, v) becomes a missed connection only if the reverse connection (g, v, h, u) exists. Let $\mathscr{U}^- \subseteq F \times V \times F \times V$ be the set of selected missed connections. Section 5.2 explains the way in which to find $\mathscr{U}^-$. For a missed connection (h, u, g, v), the arc

Table 2. The Four Kinds of Pairs of g and h, Where ✓ Represents Candidates for Mutual Connections

	Mutual connections	Remarks
$(g, v, h, u) \in \mathscr{U}$ with $v \neq u$	✓	v and u are located nearby each other
$(g, v, h, u) \notin \mathscr{U}$ with $v \neq u$		
$(g, v, h, v) \in \mathscr{U}_{\text{bus}}$	✓	g and h share v and the connection is regarded as important
$(g, v, h, v) \notin \mathscr{U}_{\text{bus}}$		

Figure 6. (Left) The Bus and Railway Network and (Right) Event–Activity Network of the Target Area Shown in Figure 1



Notes. In the left network, the solid and dotted lines represent bus lines and railway lines, respectively. The gray boxes indicate terminal bus stops and train stations. Arr, arrival; dep, departure.

$a = ((h, u, \text{arr}), (g, v, \text{dep}))$ is called *activated* if $\tilde{\Pi}_{(g,v,\text{dep})} - \tilde{\Pi}_{(h,u,\text{arr})} \geq L_{\text{change}}$ in a revised timetable $\tilde{\Pi}$. A pair of g and h becomes a mutual connection by activating a. We also deal with the situation that both g and h are buses.

In Section 4.3, we determine which connections in $\mathscr{U}^-$ should be activated by solving an MIP. If $|\mathscr{U}^-|$ is large, the range of the choice of arcs is wide, and bus stops with adopted mutual connections can be scattered over a large area. Because this is an unfavorable result for bus companies, we need to determine $\mathscr{U}^-$ carefully before solving an MIP, as explained in Section 5.2.

Reconsider the situation in Figure 4. To realize a mutual connection between g and h, the revised timetable $\tilde{\Pi}$ needs to satisfy

$$\begin{aligned} \tilde{\Pi}_{(h,u,\text{dep})} - \tilde{\Pi}_{(g,v,\text{arr})} &\geq L_{\text{change}}, \\ \tilde{\Pi}_{(g,v,\text{dep})} - \tilde{\Pi}_{(h,u,\text{arr})} &\geq L_{\text{change}}. \end{aligned} \tag{2}$$

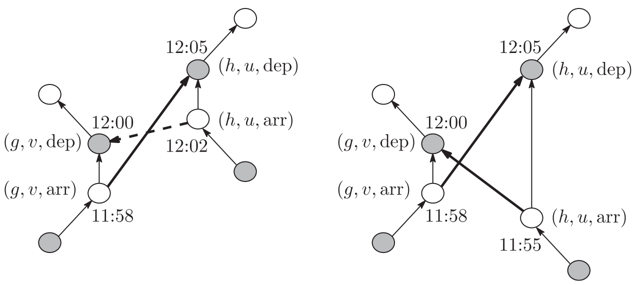
For example, if the following three conditions are satisfied, condition (2) holds:

$$\begin{aligned} \tilde{\Pi}_{(g,v,\text{dep})} &\geq \tilde{\Pi}_{(h,u,\text{dep})} + L_{\text{change}}, \\ \tilde{\Pi}_{(h,u,\text{dep})} &\geq \tilde{\Pi}_{(h,u,\text{arr})}, \\ \tilde{\Pi}_{(h,u,\text{arr})} &\geq \tilde{\Pi}_{(g,v,\text{arr})} + L_{\text{change}}. \end{aligned}$$

However, this leads to an increase in standing time $\tilde{\Pi}_{(g,v,\text{dep})} - \tilde{\Pi}_{(g,v,\text{arr})}$ at v. Thus, there is a trade-off between the benefit of mutual connections and cost of standing time. We demonstrate that we can improve bus timetables with little negative influence on standing time by introducing mutual connections effectively.

Example 1. Consider buses g and h in Figure 7. In the left network, (h, u, g, v) is a missed connection, and passengers cannot change from h to g. By activating the arc $a = ((h, u, \text{arr}), (g, v, \text{dep}))$, we realize a mutual connection in the right network. The standing time of bus h increases from 3 minutes to 10 minutes in this example.

Figure 7. (Left) A Missed Connection and (Right) a Mutual Connection



4.3. Mathematical Optimization Model

Let Π be the current timetable and $(\mathscr{E}, \mathscr{A})$ be the corresponding event–activity network. We now define

$$\mathscr{A}^{-}_{\text{change}} = \{((h,u,\text{arr}),(g,v,\text{dep})) \in \mathscr{E}_{\text{arr}} \times \mathscr{E}_{\text{dep}} \mid (h,u,g,v) \in \mathscr{U}^{-}\}.$$

For the event–activity network $(\mathscr{E}, \mathscr{A} \cup \mathscr{A}^{-}_{\text{change}})$, we use the following variables:

- $\tau(i)$ is the arrival or departure time of $i \in \mathscr{E}$;
- $y(a)$ is a 0–1 variable for $a \in \mathscr{A}^{-}_{\text{change}}$ such that $y(a) = 1$ if a is activated and $y(a) = 0$ otherwise.

We first describe the constraints for the vertices of $\mathscr{E}$. The vertex set $\mathscr{E}$ is partitioned into the vertex set $\mathscr{E}_{\text{rail}}$ for train stations and the vertex set $\mathscr{E}_{\text{bus}}$ for bus stops. Because we do not change the train timetables, we have $\tau(i) = \Pi_i$ for any $i \in \mathscr{E}_{\text{rail}}$. For the vertices of $\mathscr{E}_{\text{bus}}$, we assume $0 \le \tau(i) \le 25 \times 60$minutes for any $i \in \mathscr{E}_{\text{bus}}$. This means that buses can run from midnight on the present day to 1:00 on the next day.

The constraints for the arcs of $\mathscr{A}$ are given by

$$\begin{aligned} \tau(j) - \tau(i) &= \Pi_j - \Pi_i \qquad \forall (i,j) \in \mathscr{A}_{\text{drive}}, \\ 0 \le \tau(j) - \tau(i) &\le L_{\text{wait}} \qquad \forall (i,j) \in \mathscr{A}_{\text{wait}}, \\ \tau(j) - \tau(i) &\ge 0 \qquad \forall (i,j) \in \mathscr{A}_{\text{change}} \cup \mathscr{A}_{\text{next}}. \end{aligned}$$

The first two equations for trains are always satisfied because the train timetable is fixed. The first equation says that the travel time between bus stops is the same as in the current timetable. The second equation allows the situation that standing time at bus stops is at most a given number L_{wait}. The third equation states that waiting time for transfers may be shorter or longer.

Next, we introduce the big M–type constraints for the arcs of $\mathscr{A}^{-}_{\text{change}}$:

$$\tau(j) - \tau(i) + M(1 - y(a)) \ge L_{\text{change}} \qquad \forall a = (i,j) \in \mathscr{A}^{-}_{\text{change}}.$$

This constraint implies that if $y(a) = 1$, we have $\tau(j) - \tau(i) \ge L_{\text{change}}$, and hence a is activated.

Finally, we provide an objective function. Increasing mutual connections has the following positive and negative effects on passengers:

- Passengers who use mutual connections, *transfer passengers*, have shorter travel times.
- Passengers on direct connections, *direct passengers*, have longer travel times because the increase in mutual connections can lengthen standing time at bus stops.

Our aim is to benefit transfer passengers with little negative effect on direct passengers.

We denote the set of buses by B. For a bus $b \in B$, let $f_b = (b, v, \text{dep})$ and $l_b = (b, u, \text{arr})$ be the vertices in $\mathscr{E}$ corresponding to the first bus stop and the last bus stop, respectively. Then, the travel time is equal to $\tau(l_b) - \tau(f_b)$. We now define an objective function:

$$\max \sum_{a \in \mathscr{A}^{-}_{\text{change}}} w_a y(a) - \sum_{b \in B} w_b\big(\tau(l_b) - \tau(f_b)\big),$$

where the variables are $y(a)$, $\tau(l_b)$, and $\tau(f_b)$, and w_a, w_b are the weights explained in Section 4.4. The first term represents the sum of gains for transfer passengers. In the second term, $\tau(l_b) - \tau(f_b)$ corresponds to the change in the sum of standing time at bus stops, because the travel time between bus stops is fixed. Thus, the objective function represents the gain for transfer passengers minus the loss for direct passengers.

The MIP formulation for our problem is as follows:

$$\max \sum_{a \in \mathscr{A}^{-}_{\text{change}}} w_a y(a) - \sum_{b \in B} w_b\big(\tau(l_b) - \tau(f_b)\big) \tag{3}$$

$$\text{s.t.} \quad \tau(j) - \tau(i) = \Pi_j - \Pi_i \qquad \forall (i,j) \in \mathscr{A}_{\text{drive}}, \tag{4}$$

$$0 \le \tau(j) - \tau(i) \le L_{\text{wait}} \qquad \forall (i,j) \in \mathscr{A}_{\text{wait}}, \tag{5}$$

$$\tau(j) - \tau(i) \ge 0 \qquad \forall (i,j) \in \mathscr{A}_{\text{change}} \cup \mathscr{A}_{\text{next}}, \tag{6}$$

$$\tau(j) - \tau(i) + M(1 - y(a)) \ge L_{\text{change}} \quad \forall a = (i,j) \in \mathscr{A}^{-}_{\text{change}}, \tag{7}$$

$$\tau(i) = \Pi_i \qquad \forall i \in \mathscr{E}_{\text{rail}}, \tag{8}$$

$$0 \le \tau(i) \le 25 \times 60(\text{minutes}) \quad \forall i \in \mathscr{E}_{\text{bus}}, \tag{9}$$

$$y(a) \in \{0, 1\} \qquad \forall a \in \mathscr{A}^{-}_{\text{change}}. \tag{10}$$

The number of real variables $\tau(i)$ is equal to $|\mathscr{E}|$, whereas that of integer variables $y(a)$ is equal to $|\mathscr{A}^{-}_{\text{change}}|$. The number of constraints is equal to $|\mathscr{E}| + |\mathscr{A} \cup \mathscr{A}^{-}_{\text{change}}|$.

A similar MIP formulation is given in Schöbel (2007) and Schachtebeck and Schöbel (2010) for the delay management problem. Our formulation corresponds to a special case with $\mathcal{A}_{\text{head}} = \emptyset$ in Schachtebeck and Schöbel (2010), where the objective function is defined by combining the delay over all vehicles and the number of connections such that changing activities are not maintained.

4.4. Weights in the Objective Function

We define the weights w_a and w_b based on the network structure. We now introduce the *traffic index* to evaluate the importance of a pair of bus lines. Consider line 1 and line 2 that run through bus stops $v_1, \ldots, v_k, w_1, \ldots, w_c, v_{k+1}, \ldots, v_n$ and $u_1, \ldots, u_l, w_1, \ldots, w_c, u_{l+1}, \ldots, u_m$, as shown in Figure 8(a). The traffic index between them is defined as $k(m-l) + l(n-k)$. This is equal to the number of OD pairs that use both lines 1 and 2 under the assumption that one unit of traffic is assigned to each pair of bus stops. The traffic index in Figure 8(a) is $18(= 4 \times 3 + 3 \times 2)$, that in (b) is 0, and that in (c) is $6(= 2 \times 2 + 1 \times 2)$. If two lines have no common bus stops, the traffic index is defined as zero.

The definitions of w_a and w_b are given by

$$w_a = \gamma_a(\Pi_{j^+} - \Pi_i) \quad \text{for } a = (i,j) \in \mathcal{A}^-_{\text{change}}, \tag{11}$$

$$w_b = \sqrt{\frac{n_b(n_b-1)}{2}} \quad \text{for } b \in B. \tag{12}$$

We first explain $\Pi_{j^+} - \Pi_i$ in (11). Consider $(i,j) \in \mathcal{A}^-_{\text{change}}$ and $(i,j^+) \in \mathcal{A}_{\text{change}}$, where the vehicles in j and j^+ belong to the same bus/train line. In the current timetable Π, waiting time for transfers is equal to $\Pi_{j^+} - \Pi_i$. If (i,j) is activated in the revised timetable, waiting time will become relatively small compared with $\Pi_{j^+} - \Pi_i$. Hence, we regard the revised waiting time as zero and assume that activating (i,j) reduces waiting time by $\Pi_{j^+} - \Pi_i$. This is because in areas with low-frequency services, an example of $\Pi_{j^+} - \Pi_i$ is about four hours and the revised waiting time is only a few minutes.

The importance γ_a of a in (11) is estimated from the traffic index. For $a = (i,j)$ such that the vehicles of i and j are buses, let c_a denote the traffic index between the corresponding bus lines. We then define $\gamma_a = \sqrt{c_a}$, where we adopt the square root so that we do not make little of a with a small traffic index in our model. If a vehicle of i or j is a train, we define $\gamma_a = \max_{a'} \sqrt{c_{a'}}$, where the maximization is taken over all the arcs a' between buses. This is because a connection between a train and a bus is more important than one between buses.

In (12), n_b denotes the number of stops of bus b. The value $n_b(n_b-1)/2$ represents the number of pairs of departure and arrival bus stops, and the square root is adopted for a reason similar to that in the case of transfer passengers.

Passengers tend to perceive waiting time to be longer than it actually is. Although the objective function in this study does not distinguish the values of waiting time and driving time, the values of time have been discussed in studies of public transport (Wardman 2004). For example, Vansteenwegen and Van Oudheusden (2006, 2007) and Xuan and Yang (2015) adopt objective functions in which waiting time and driving time have different weights.

5. Selection of Candidates for Mutual Connections

5.1. Bus Stops with Important Connections

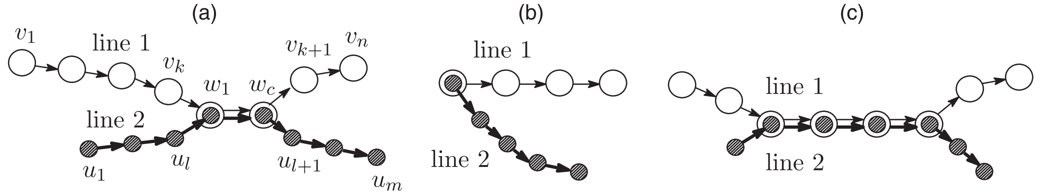
Table 1 shows that three bus lines share the Setamai-Eki-Mae bus stop v. Consider bus g bound for Sumita High School with the arrival/departure time 7:27 a.m. and bus h bound for Morioka at 7:46 a.m. As explained in Section 4.1, if a connection (g, v, h, v) belongs to $\mathcal{U}_{\text{bus}}$, we have a corresponding arc in $\mathcal{A}_{\text{change}}$. On the contrary, we have an arc in $\mathcal{A}_{\text{next}}$ if $(g, v, h, v) \notin \mathcal{U}_{\text{bus}}$.

Because most bus stops are shared by several bus lines, there are many possible connections between buses. Among them, we extract the important connections to determine the set $\mathcal{U}_{\text{bus}}$. In the following, we evaluate the importance of a pair of bus lines by using the traffic index and finding the set V_{bus} of bus stops shared by an important pair of bus lines. Then, $\mathcal{U}_{\text{bus}}$ is defined by

$$\mathcal{U}_{\text{bus}} = \{(g, v, h, v) \mid \Pi_{(h,v,\text{dep})} - \Pi_{(g,v,\text{arr})} \geq L_{\text{change}}, v \in V_{\text{bus}}\}.$$

Let α be a threshold for selecting pairs of bus lines. If a pair of bus lines has a traffic index larger than α, their common bus stop v is expected to have important connections, and hence we should add v to V_{bus}. To make the network structure around v flexible, we further add to V_{bus} bus stops that bind bus lines through v.

Figure 8. Examples of a Pair of Bus Lines with Common Bus Stops



The set V_{bus} is determined as follows. We first divide bus lines into groups, where i and j belong to different groups for each pair $\{i, j\}$ with a traffic index larger than α. Then, we define V_{bus} as the set of bus stops shared by a pair of lines such that one and the other of a pair belong to different groups. We provide an example of grouping and V_{bus}.

Example 2. Consider bus lines 1, 2, and 3 and bus stops u, v, and w in Figure 9. Assume that the traffic index between line 1 and line 2 is larger than α and that the traffic indices between the other pairs are not. Because line 1 and line 2 are in different groups, at least one pair of {line 1, line 3} and {line 2, line 3} belongs to different groups. This fact indicates $u \in V_{\mathrm{bus}}$ and v and/or $w \in V_{\mathrm{bus}}$. Bus stop u is selected because u has the important connection between line 1 and line 2. On the contrary, v and w are selected because v and w help the network structure around u become flexible. By grouping bus lines, we can find them simultaneously.

The problem of grouping bus lines is reduced to the vertex coloring problem. We construct an undirected graph $G_{\mathrm{ln}} = (V_{\mathrm{ln}}, E_{\mathrm{ln}})$ that represents the relationship between bus lines. The vertex set V_{ln} is the set of bus lines, and the edge set E_{ln} is defined by

$$E_{\mathrm{ln}} = \{(i, j) \mid (\text{traffic index between } i \text{ and } j) > \alpha\}.$$

If a pair $\{i, j\}$ of bus lines has a traffic index larger than α, the end vertices of $(i, j) \in E_{\mathrm{ln}}$ have to be assigned different colors. Thus, the problem reduces to vertex coloring, for which we use the algorithm of Welsh and Powell (1967). Figure 10 shows three examples of G_{ln}, where the second case is used in Section 6.

5.2. Evaluation of Missed Connections

Among the missed connections determined by (1), we select the critical connections to determine $\mathcal{U}^-$. Let $\mathcal{G}$ be the bus and railway network, as shown on the left-hand side of Figure 6. The vertices in $\mathcal{G}$ consist of those in V (the set of stations and bus stops) and copies of v with $(g, v, h, v) \in \mathcal{U}_{\mathrm{bus}}$, whereas the arcs are spatial arcs corresponding to $\mathcal{A}_{\mathrm{drive}} \cup \mathcal{A}_{\mathrm{change}}$. Each

Figure 9. Bus Lines 1, 2, and 3 in Example 2

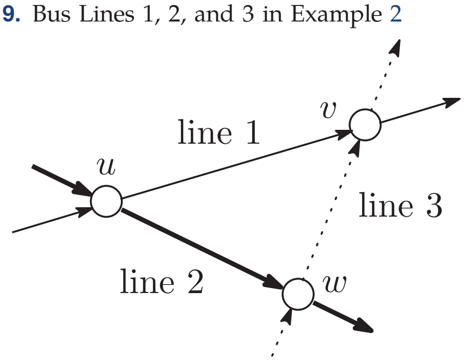
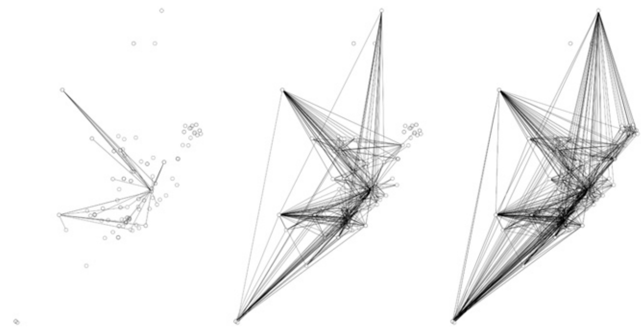


Figure 10. Comparison of $G_{\mathrm{ln}} = (V_{\mathrm{ln}}, E_{\mathrm{ln}})$ for (Left) $\alpha = 208$, (Middle) $\alpha = 84$, and (Right) $\alpha = 25$, Where Each Vertex is Located at the First Bus Stop of the Corresponding Bus Line



arc length is determined by the average travel time or average transfer time. We find connections whose shortcut greatly improves the average path length in $\mathcal{G}$.

Let $L(\mathcal{G})$ denote the average path length of $\mathcal{G}$ defined by

$$L(\mathcal{G}) = \frac{1}{|S|(|S| - 1)} \sum_{u,v \in S, u \neq v} l(u, v),$$

where S is the set of terminals of the bus and railway lines shown on the left-hand side of Figure 6, and $l(u, v)$ denotes the shortest path length from u to v. Here, we adopt S instead of the vertex set of $\mathcal{G}$ to reduce the computational time.

We first set $H \leftarrow \emptyset$. For each arc a corresponding to the arcs in $\mathcal{A}_{\mathrm{change}}$, we denote by $\mathcal{G}_a$ the network obtained by setting the length of a to zero. Let a^* be the arc minimizing $L(\mathcal{G}_a)$. Because a^* is expected to be critical for improving the timetable, we add a^* to H. We then set $\mathcal{G} \leftarrow \mathcal{G}_{a^*}$ and repeat this procedure until $|H|$ is equal to a given number β. The set $\mathcal{U}^-$ is defined as the set of all missed connections corresponding to H and those between a train and a bus.

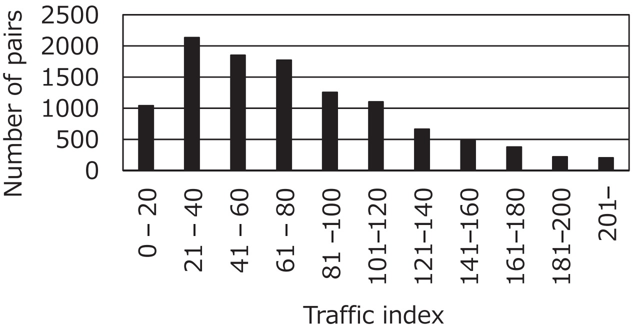
Let us discuss α in Section 5.1 and β. The smaller a threshold α, the larger is the number of vertices in $\mathcal{G}$. Thus, the value of α directly affects the computational time for determining $\mathcal{U}^-$. In Section 6, we compare four cases with different values of α and β.

6. Application to the Target Area in the Tohoku District of Japan

6.1. Data on the Target Area

In this section, we apply our mathematical optimization model to the area on the Pacific Ocean side of the Tohoku District in Japan to demonstrate the usefulness of the model. Figure 1 shows our target area, which includes Kamaishi City, Ofunato City, Rikuzentakata City, Kesennuma City, and Ichinoseki City. This area was damaged by the 2011 Great East Japan Earthquake and subsequent tsunami. As part of

Figure 11. The Distribution of the Traffic Index



recovery efforts, it requires bus timetables that ensure smooth transfers among buses and trains. In the target area, we have 803 bus stops, 258 bus lines, and 88 train stations (see also Figure 6).

6.2. Numerical Results

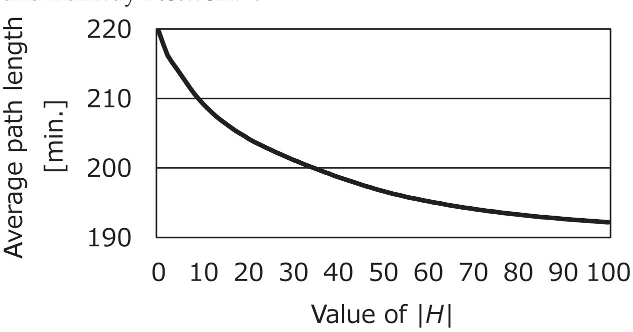
There exist 11,101 pairs of bus lines that have common bus stops. To determine $\mathcal{U}_{\text{bus}}$ and $\mathcal{U}^-$, we compute the traffic index of each pair. Figure 11 shows the distribution of the value of the traffic index.

We adopt $\alpha = 84$ as a threshold for selecting pairs of bus lines in Section 5.1. Then, the number of pairs with a traffic index larger than α is 3,977 (35.8%). Figure 12 shows the change in the average path length in Section 5.2. This change slows with increasing $|H|$. By setting $\beta = 50$, we thus obtain $\mathcal{U}^-$ on the left-hand side of Figure 13. The bus and railway network $\mathcal{G}$ has 2,282 vertices and 9,746 arcs. Each iteration to find a^* takes an average of 305 seconds in Windows 7 on an Intel Core i7 central processing unit at 3.4 GHz with 8 GB of RAM.

Table 3 summarizes the numbers of vertices and arcs of the event–activity network $(\mathcal{E}, \mathcal{A} \cup \mathcal{A}^-_{\text{change}})$. We solve the MIP given in Section 4.3, where L_{change} is set to 3 minutes and L_{wait} is 10 minutes. Table 4 presents the results. We use the integer programming solver ILOG CPLEX 12.6.3 (IBM 2016) on Windows 7.

The right-hand side of Figure 13 depicts the train stations and bus stops that have at least one mutual connection. Most train stations have some mutual

Figure 12. Changes in the Average Path Length of the Bus and Railway Network $\mathcal{G}$



connections. Let us observe the mutual connections between buses. A mutual connection requires sufficient space for buses to stand simultaneously. On the right-hand side of Figure 13, a gray box just inside a circle indicates that the bus stop is located in front of the corresponding station. These bus stops generally have sufficient space. However, this may not be the case for bus stops located apart from stations. In this example, only four bus stops are selected to improve the timetable. Because the number of selected bus stops is small and they are located separately, it is anticipated that we can create some space for mutual connections around these bus stops. In practical applications, it would be better to incorporate such a space constraint into the formulation.

Finally, we discuss the size of the event–activity network and computational results. Let us compare the following four cases, where (i) is the case explained previously:

(i) Set $\alpha = 84$ and $\beta = 50$.
(ii) Set $\alpha = 25$ and $\beta = 50$.
(iii) Set $\alpha = 84$.
(iv) Set $\alpha = 25$.

In (iii) and (iv), we skip the procedure in Section 5.2 and define $\mathcal{U}^-$ as the set of all missed connections corresponding to $\mathcal{U} \cup \mathcal{U}_{\text{bus}}$.

Table 5 summarizes the computational results for the four cases. The set $\mathcal{A}^-_{\text{change}}$ in (iii) and (iv) contains many unimportant connections. The values of the first and second terms in (3) for (iii) and (iv) are larger than the value for (i). Whereas the increase in the first term indicates an improvement in connections, that in the second one means that the sum of standing time at bus stops becomes larger. Both values of the two terms for (ii) are better than those for (i). For $\alpha = 25$ and $\beta = 50$ in (ii), however, each iteration to find a^* in Section 5.2 takes an average of 1,287 seconds, which is much longer than the 305 seconds for $\alpha = 84$ and $\beta = 50$ in (i).

Thus, the phase in Section 5.1 excludes connections with a small traffic index, and the phase in Section 5.2 finds frequently selected missed connections. A larger value of α leads to a shorter computational time in the latter phase. Introducing β is thus useful so that $\mathcal{A}^-_{\text{change}}$ does not become too large, which results in the short computational time of the MIP.

6.3. An Example of a Revised Timetable

Table 6 shows the current bus timetable at the Konashi-Eki-Mae bus stop, a revised bus timetable obtained from the optimal solution, and the train timetable at Konashi train station. We see that the standing time of two buses in the revised timetable is slightly longer than that in the current one.

We check that the revised timetable improves connections for the underlined bus/train services. In the revised timetable, we can change in both

Figure 13. Train Stations and Bus Stops (Left) in $\mathcal{U}^-$ and (Right) Those Where at Least One Mutual Connection Is Realized

Note. The dots represent train stations/bus stops, circles correspond to connections between a train and a bus, and gray boxes correspond to connections between buses.

directions between the bus at 4:57–5:04 p.m. and the train at 5:01 p.m., which may not be possible in the current one. The same is true for the connection between the bus at 7:39–7:46 p.m. and the train at 7:43–7:44 p.m. Thus, we have succeeded in achieving mutual connections by making minor alterations to the arrival and departure times of buses.

We next compare travel times by applying Dijkstra's algorithm to the event–activity network, in which each arc length is determined by the timetables. Imagine that we want to arrive at one of the hospitals before 9:00 a.m. We analyze which timetable allows us to start later. For the 27 bus stops located in front of hospitals, we compute the latest departure time at each bus stop such that we can arrive at a hospital before 9:00 a.m. Table 7 shows that we can start after 8:30 a.m. at 98 bus stops in the current timetable. In the revised one, this number of bus stops increases by one. Similarly, the number of bus stops increases in each time slot, especially 7:00–8:00 a.m., because most of these long trips contain some transfers. As a result, the revised timetable allows a later departure time than the current one at about 100 bus stops. This is a noteworthy result because the objective function of the MIP does not contain a direct term to reduce the travel time for trips to specific destinations, including hospitals.

Figure 14 compares the travel time from each terminal of the bus and railway lines, which are shown on the left-hand side of Figure 6, to each bus stop under the assumption that they start at 7:00 a.m. The revised timetable improves the travel time if $\Delta t > 0$ and does not if $\Delta t < 0$. If $\Delta t = 0$, the two timetables have the same travel time. The revised timetable becomes worse for 33,485 OD pairs, but about 90% of those pairs belong to the case with $-20 \le \Delta t < 0$. On the contrary, it improves for 32,787 OD pairs, and about 17% of those pairs belong to the case with $\Delta t \ge 20$. Furthermore, about 62% pairs with $\Delta t \ge 20$ belong to the case with $\Delta t \ge 40$. Thus, the revised timetable benefits OD pairs with a long distance at the expense of OD pairs with a short distance.

Finally, we check that in the revised timetable, standing time at a bus stop is similar to the corresponding standing time in the current timetable. Standing time is at most $L_{\text{wait}} = 10$ *minutes* from constraint (5). Table 8 shows that standing time remains the same at 16,639 spots. Thus, there is little negative effect on standing time.

7. Further Applications

The proposed MIP model is useful for not only timetable design but also other applications. We find critical intervals that preclude mutual connections in Section 7.1 and bus routes to be reviewed in Section 7.2.

Table 3. Size of the Event–Activity Network $(\mathcal{E}, \mathcal{A} \cup \mathcal{A}^-_{\text{change}})$

$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{A}_{\text{change}}\rvert$	$\lvert\mathcal{A}^-_{\text{change}}\rvert$
59,558	111,338	24,187	4,156

Table 4. Information on the Optimal Solution

Computational time (seconds)	4.35
Value of the first term in (3)	1,231,859
Value of the second term in (3)	2,071,672
# of activated arcs in $\mathcal{A}^-_{\text{change}}$	360

Table 5. Comparison of the Computational Results for Cases (i)–(iv)

	(i)	(ii)	(iii)	(iv)
Computational time (seconds)	4.35	4.26	61.82	119.26
Value of the first term in (3)	1,231,859	1,378,372	1,665,465	2,059,977
Value of the second term in (3)	2,071,672	2,070,489	2,099,235	2,102,222
# of arcs in $\mathcal{A}^{-}_{\text{change}}$	4,156	4,879	22,149	36,506

Table 6. The (Left) Current Bus Timetable, (Middle) Revised One, and (Right) Train Timetable at Konashi

Current timetable	Revised timetable			Train timetable	
ARR/DEP	ARR	DEP	Standing time (in minutes)	ARR	DEP
				⋮	⋮
7:21 a.m.	7:19 a.m.	7:19 a.m.	0	3:40 p.m.	3:41 p.m.
5:01 p.m.	<u>4:57 p.m.</u>	<u>5:04 p.m.</u>	7	<u>5:01 p.m.</u>	<u>5:01 p.m.</u>
5:35 p.m.	5:31 p.m.	5:31 p.m.	0	6:36 p.m.	6:37 p.m.
7:35 p.m.	<u>7:39 p.m.</u>	<u>7:46 p.m.</u>	7	<u>7:43 p.m.</u>	<u>7:44 p.m.</u>

Note. ARR, Arrival; DEP, departure.

7.1. Sensitivity Analysis

Many candidates for mutual connections are not selected in our model. One reason why they remain one-way transfers is the fixed travel time between successive bus stops. If we could shorten this, mutual connections might increase.

Let $\mathcal{A}^{*}_{\text{change}}$ denote the set of activated arcs in $\mathcal{A}^{-}_{\text{change}}$ obtained by solving the MIP. We consider the following linear programming problem with the variables λ_a for $a \in \mathcal{A}^{-}_{\text{change}} \setminus \mathcal{A}^{*}_{\text{change}}$, r_a for $a \in \mathcal{A}_{\text{drive}}$, and $\tau(i)$ for $i \in \mathcal{E}$:

$$\begin{aligned} \min \quad & \sum_{a \in \mathcal{A}^{-}_{\text{change}} \setminus \mathcal{A}^{*}_{\text{change}}} \lambda_a \\ \text{s.t.} \quad & r_a = \tau(j) - \tau(i) \quad \forall a = (i,j) \in \mathcal{A}_{\text{drive}}, \\ & \lambda_a \geq -\big(\tau(j) - \tau(i)\big) \quad \forall a = (i,j) \in \mathcal{A}^{-}_{\text{change}} \setminus \mathcal{A}^{*}_{\text{change}}, \\ & r_a \geq \Pi_j - \Pi_i \quad \forall a = (i,j) \in \mathcal{A}_{\text{drive}}, \\ & \tau(j) - \tau(i) \geq 0 \quad \forall (i,j) \in \mathcal{A}^{*}_{\text{change}}, \\ & (5), (6), (8), (9). \end{aligned}$$

In this linear programming problem, r_a denotes the travel time for $a \in \mathcal{A}_{\text{drive}}$, and λ_a implies that $a \in \mathcal{A}^{-}_{\text{change}} \setminus \mathcal{A}^{*}_{\text{change}}$ becomes activated if the bus arrives $\lambda_a + L_{\text{change}}$ minutes earlier.

Let us consider the third constraint $r_a \geq \Pi_j - \Pi_i$ for $a = (i,j) \in \mathcal{A}_{\text{drive}}$. We compute the shadow price of this constraint, which is denoted by p_a. From the definition of the shadow price, the objective value improves by p_a if $\Pi_j - \Pi_i$ decreases by one.

Decreasing $\Pi_{(g,u,\text{arr})} - \Pi_{(g,v,\text{dep})}$ for $a = ((g,v,\text{dep}),(g,u,\text{arr})) \in \mathcal{A}_{\text{drive}}$ affects the constraints for all $a' \in \mathcal{A}_{\text{drive}}$ corresponding to travel from v to u regardless of time. Hence, for arc (v,u) in the bus and railway network, we find that "price" $q_{(v,u)}$ is defined by

$$q_{(v,u)} = \sum_{a \in \mathcal{A}_{\text{drive}}:\ \text{travel from } v \text{ to } u} p_a.$$

The larger $q_{(v,u)}$, the more potential to increase mutual connections the interval (v,u) has. Thus, we can find critical intervals that preclude mutual connections by using $q_{(v,u)}$.

7.2. Bus Route Modification Based on Standing Time

In Section 4.3, we assume that buses stay at bus stops at most L_{wait}. Consider an MIP obtained by replacing (5) with

Table 7. Comparison of the Number of Bus Stops at Which the Latest Departure Time Belongs to Each Time Slot

	8:30–9:00 a.m.	8:00–8:30 a.m.	7:30–8:00 a.m.	7:00–7:30 a.m.	Total
Current	98	114	196	244	652
Revised	99	120	234	303	756

Figure 14. The Number of OD Pairs Such That the Travel Time Decreases by Δt Minutes in the Revised Timetable

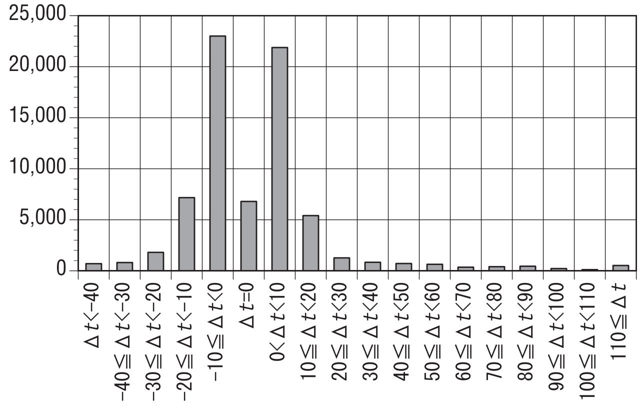


Table 8. Comparison of the Standing Time of Each Bus at Each Bus Stop

	Difference in standing time (minutes)										
	0	1	2	3	4	5	6	7	8	9	10
# of arcs in $\mathscr{A}_{\text{wait}}$	16,639	59	115	78	34	34	30	22	21	17	62

$$\tau(j) - \tau(i) \geq 0 \qquad \forall (i, j) \in \mathscr{A}_{\text{wait}}.$$

If $\tau(j) - \tau(i)$ is more than a few hours in the optimal solution, it may be better to divide the bus route through this bus stop v into two routes: a route with destination v and a route with origin v.

We solve this MIP for the case in Section 6. Table 9 summarizes the results of standing time, which remains zero at 16,458 spots but exceeds two hours at three spots. This finding indicates that we need to review the corresponding bus routes.

8. Conclusion

We focused on areas with low-frequency public transportation services and presented a mathematical optimization model to generate a bus timetable that ensures smooth transfers among buses and trains. In the application to the area on the Pacific Ocean side of the Tohoku District in Japan, we demonstrated that the obtained timetable improves the travel time for OD pairs with a long distance at the expense of OD pairs with a short distance.

Most Japanese timetables are designed to permit only one-way transfers. However, low-frequency services combined with one-way transfers lead to a substantially inconvenient transfer. To realize smooth transfers, we introduced mutual connections, which enable passengers to change in both directions. Our results indicate that mutual connections are essential to smooth transfers in areas with low traffic demand and low-frequency public transportation services.

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Endnotes

[1] See https://www.pref.iwate.jp/kendozukuri/koutsuu/koukyou/1005382.html, accessed June 17, 2019 (in Japanese).

[2] See https://www.city.kitakami.iwate.jp/life/kurashi_tetsuduki/douro/kokyokotsu/10198.html, accessed June 17, 2019 (in Japanese).

[3] The term *missed connection* means a nonmutual connection and is unrelated to delays in this study, although it is also used in the delay management problem.

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Table 9. Distribution of Standing Time

	Standing time (minutes)							
	0	1–30	31–60	61–90	91–120	121–150	151–180	Over 180
# of arcs in $\mathscr{A}_{\text{wait}}$	16,458	621	19	7	3	2	1	0

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