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Robust Periodic Vehicle Routing Problem with Time Windows under Uncertainty: An Efficient Algorithm

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Abstract

In a competitive environment, distributors engage in intense rivalry to meet the customers' demands and accordingly earn maximum profits. Due to changes in customers' demands, changes in a planned combination of customers to be visited in various days, traffic changes, etc., a sequence of visiting customers by competitors will be changed. So, planning for serving customers ahead of competitors will be uncertain. On the other hand, total transportation costs are also of high importance in distribution of goods. By keeping this issue in mind, we develop a bi-objective mathematical model to evaluate a Periodic Vehicle Routing Problem (PVRP) with time windows under uncertainty for companies competing to provide services to customers. The model is established using an improved scenario-based robust optimization approach. Given that the PVRP is an NP-hard problem, an Improved Differential Evolution (IDE) algorithm is used to identify efficient solutions for the model. The results on small-scale problems were compared with those obtained using the CPLEX solver. To evaluate the performance of the proposed IDE algorithm, a few sample tests on large-scale problems are conducted, and the results are compared with those derived using two other differential evolution algorithms. The findings show that the IDE algorithm exhibits suitable accuracy and performance in solving the presented model.

Keywords: *periodic vehicle routing problem, robust optimization, time window, differential evolution algorithm*

1. Introduction

Transportation is one of the major sectors in every country and regarded as one of the most important divisions that account for a proportion of the total cost of final products. The consequent rising transportation costs ultimately increase service time to customers and cause dissatisfaction. Operating costs can be reduced and productivity can be increased by using a Vehicle Routing Problem (VRP), which has many applications in optimizing transportation, distribution, and logistical costs (Kos and Karaoglan, 2016; Archetti *et al.*, 2016).

The VRP concept, which was first introduced by Dantzig and Ramser (1959), encompasses a series of routes that homogeneous vehicles use to deliver products and services from depots to customers who are spread out in different geographical locations; customer demands are fulfilled by only one distributor, and no competition exists (Wang *et al.*, 2015). A VRP with time Windows (VRPTW) is one of the most important kinds of VRPs that has found extensive application in distribution systems. A VRPTW finds use in different dimensions, such as just-in-time manufacturing; the transportation of dangerous materials; the addressing of the corrupted distribution and production of oil and petroleum; the distribution of groceries; postal delivery; and distribution of raw materials (Yan *et al.,* 2015). It is an extension of a VRP in which distributers are required to satisfy the needs of each customer in interval $[l_i, u_i]$, where u_i and l_i are defined as the latest and earliest times at which delivery to a customer should be achieved. A VRPTW is categorized into two classes, namely, the VRP with a hard time window and the VRP with a soft time window. In the former, deviation from time windows is prohibited under any condition, whereas in the latter, such deviation is acceptable but imposed a penalty.

With attention to the levels of customer dispersion and the changes in customer demand, researchers expanded the problem into a periodic VRP with time windows (PVRP), in which customer demand is fulfilled in a few days, and customers can

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choose several combinations of periodic days for servicing (Kohl and Madsen, 1997). The PVRP model presents various practical applications, such as the resolution of refuse collection problems, the distribution of spare parts (Hemmelmayr *et al.,* 2011). In a PVRP, each customer $n_i \in N$ indicates a set of ϕ_i schedules that clarify the combination of days for servicing. In a seven-day period, for example, if customer n_i specifies two servicing combinations {1, 4, 5} and {3, 6, 7} and ultimately chooses the first combination, a provider should serve the customer in the first, fourth, and fifth days. If the customer chooses the second combination, the provider should service the customer in the third, sixth, and seventh days (Coene *et al.*, 2010).

Some variants of the PVRP are discussed in the literature, with scholars proposing different related objective functions, such as minimizing the travel distance, driving time, or total transportation cost. A number of vehicles and service quality can also be incorporated as parts of an optimization function. Researchers use different constraints, which can be divided into constraints concerning the (1) design of customer visits (different frequencies, restrictions on certain days, etc.), (2) type of demand (constant or variable; discussed later in the paper), and (3) vehicles. Under these restrictions, a PVRP is mostly situated at the tactical and operational levels. Another variant is the PVRP with intermediate facilities, in which vehicles can unload (or reload) and thus renew capacity along a route (Coene *et al.*, 2010; Pourghaderi *et al.*, 2008).

As indicated in the literature review, a PRVP is resolved with the assumptions that a product distribution company has monopoly and that the volume of distributed products exerts no effects on the distributor's profit. The effects of response rate on customer demands are disregarded. In this scenario, fulfilling the customer demand sooner than a rival through competitive vehicle routing poses considerable effects on achieving liquidity. However, in reality, distribution companies compete to gain market share and optimize the logistics management process. Faced with this challenge, these companies are compelled to deal with constant changes in market opportunities and strive to serve customers faster than their competitors. For these reasons, Tavakkoli-Moghaddam *et al.* (2011) introduced a competitive VRP model that maximizes profits from liquidity while simultaneously minimizing transportation costs. The problem with this model is that in the real world, distribution companies do not adopt a certain daily visit pattern (Li *et al.*, 2015). The classical VRP assumes that the number of distributed items exerts no effects on a distributer's profit. As previously stated, the real world is characterized by competition in earning liquidity; disregarding this issue reduces the efficiency of the paths that VRP models generate for increasing liquidity. If customer service is provided later than competitors, market share will be reduced. Correspondingly, distribution companies endeavor to reduce the costs associated with the transportation of goods to acquire the largest market share (Norouzi *et al.*, 2012). To help distributors increase their incomes, Alinaghian *et al.* (2012) considered *a* new mathematical PVRP model, wherein companies operate in a competitive environment, but the service time of competitors are assumed certain.

Given the considerable effects of a competitor's service time on profit in actual competitive environments, distributors must revise the design of fleet routes, with particular attention paid to scenarios featuring their rivals' operations to optimize profit. Resolving this problem necessitates the consideration of certain parameters and variables to calculate the efficient time at which customers are visited and maximize the profit earned by distributors. A PVRPTW cannot derive the best solution to this issue. Meanwhile, researchers have proposed different approaches to dealing with uncertainty, such as robust optimization. Stochastic robust optimization (Mulvey *et al.*, 1995) is one of the most functional and flexible models for robust optimization problems. this approach enables the justification of an optimal solution and the guarantee that uncertain parameters are close to optimal in most possible scenarios (List *et al.,* 2003). Sungur *et al*. (2008) and Gounares *et al.* (2013) used a robust optimization model of a VRP with limited capacity and uncertain demand. The results showed that the model satisfactorily solves the aforementioned problem.

Lee *et al.* (2012) developed a robust VRP model that considers uncertainty in customer demand and vehicle transit time, with the aim of minimizing travel distance and with consideration for expected service time to customers. Considering uncertainty in VRP optimization significantly increases the complexity of problem optimization. Thus, a VRP is an NP-hard problem (Lenstra and Rinnooy, 1981). Recent years have seen the development of different meta-heuristic methods of solving VRPs; examples are Tabu search (Jia *et al*., 2013), simulated annealing (Fazel zarandi *et al.,* 2014), and particle swarm optimization (Goksal *et al.,* 2013). However, the inapplicability of these exact methods to large-scale problems prompted the use of Differential Evolution (DE) algorithms in evaluating large-scale solutions. DE algorithms are some of the best evolution algorithms for identifying the optimal solutions to various problems because of their simple configuration, ease of use, rapid convergence, and stability (Das and Suganthan, 2011).

Innovations implemented in this paper are briefly explained as follows: by considering authors' knowledge, lack of certainty existing in competitors' starting time of rendering service is taken into consideration in the proposed model for the first time. Also, instead of an estimated starting time of service, a set of scenarios for service start time are considered in modeling, and a robust model is provided as for encountering the stated uncertainty. In the proposed model, profit expectation-maximization and transportation costs minimization are also considered as bi-objectives. In continuation, a meta-heuristic method based on the DE algorithm is proposed for problem solving. Eventually, quality of the results obtained from the algorithm is dealt with.

The rest of the paper is organized as follows. Section 2 explains the robust optimization approach employed in this work. Section 3 presents the mathematical model of the PVRPTW under uncertainty. Section 4 discusses the IDE algorithm for solving the model and explains a method of continuous solution presentation. Section 5 describes the solution of small- and large-scale problems and the computational results. Section 6 concludes the paper.

2. Robust Optimization

As suggested by (Mulvey *et al.,* 1995), there are two important definitions in modeling robust optimization problems (i.e., robust response and robust model), which are individually referred to in continuation. A response to the optimization model is considered as robust when it remains close to the optimized value for all of defined scenarios. Again, the model is called robust when it is almost justified for all input data scenarios. In the robust optimization model, there are two design and control variables. In design variables, input data are fixed and certain with no changes, under any circumstances. However, in control variables, input data are uncertain. Consider the following linear programming model for better expression of aforementioned.

$$\mathrm{Min} R\phi + Q\Psi \tag{1}$$

s.t.

$$A\phi \geq G \tag{2}$$

$$F\phi + \pi\Psi \geq H \tag{3}$$

$$\phi, \Psi \geq 0 \tag{4}$$

where ϕ is a vector of design variables, and Ψ is considered as control variables' vector. Also, R, Q, A, F, and π are model parameters while G and H are considered as vector of values mentioned on the right side. To define a robust optimization problem, each of uncertain parameters is defined under one scenario, and probability of occurrence of uncertain parameters of s scenario including $\{Q_s, F_s, \pi_s \text{ and } H_s\}$ is defined by Ps symbol. A set of scenarios are defined by Ω, whose indices are s and s'. $\Omega = \{1, 2, \ldots, S\}$, where $\sum_{s=1}^{S} p_s = 1$ The control variable vector is denoted by Ψ. When a scenario occurs, Ψ_s can be defined for each scenario. The uncertain parameters in the proposed model are infeasible for certain scenarios. Therefore, in each scenario, δ_s demonstrates the infeasibility of the model. When the model is infeasible, δ_s is greater than 0; otherwise, its value is equal to 0. A robust optimization model is described as follows (Pan and Nagi, 2010):

$$\mathrm{Min}\, \sigma(\phi\Psi_1, \Psi_2, \ldots, \Psi_s) + \omega(\delta_1, \delta_2, \ldots, \delta_5) \tag{5}$$

s.t.

$$A\phi \geq G \tag{6}$$

$$F_s\phi + \pi_s\Psi + \delta_s \geq H_s \qquad \forall s \in \Omega \tag{7}$$

$$\phi \geq 0, \Psi_s \geq 0, \delta_s \geq 0 \qquad \forall s \in \Omega \tag{8}$$

In the first section of Eq. (5), control variables $\{\Psi_1, \Psi_2, \ldots, \Psi_s\}$ are defined for each $s \in \Omega$. The second term denotes model robustness weighted by ω, which contains solutions with penalties and does not meet demand in a scenario or contravenes other restrictions, such as those related to capacity. Variable δ_S is considered as the value of violation of restriction in sth scenario. Also, a weight is related to this variable. In fact, using ω, a balance may be created between robustness of answer and that of the model. The term $f(x, \Psi)$ denotes the cost and benefit functions, and for each scenario, $\xi_s = f(x, \Psi_s)$. Term ξ represents $f(x, \Psi)$, which is a benefit or cost function, and $\xi_s = f(x, \Psi_s)$ for scenario s. A variance represents the level of risk in a decision. In another statement, a slight modification to uncertain parameters can cause significant changes in the objective function. To provide a robust-solution nonlinear quadratic term, the following equation is considered (Mulvey *et al.*, 1995):

$$\sigma(\mathrm{o}) = \sum_{s\in\Omega} p_s\xi_s + \lambda\sum_{s\in\Omega} p_s\Big(\xi_s - \sum_{s'\in\Omega} p_{s'}\xi_{s'}\Big)^2 \tag{9}$$

To demonstrate solution robustness, λ is a positive constant value that indicates the weight placed on the variance of a solution. To modify the quadratic term to the absolute deviation, Yu and Li (2000) put forward the following equation:

$$\sigma(\mathrm{o}) = \sum_{s\in\Omega} p_s\xi_s + \lambda\sum_{s\in\Omega} p_s\Big|\xi_s - \sum_{s'\in\Omega} p_{s'}\xi_{s'}\Big| \tag{10}$$

This equation is a nonlinear function. Instead of minimizing the sum of absolute differences in Eq. (10), Leung and Chen (2009) presented two variables that are minimized according to basic and additional restrictions as follows:

$$\mathrm{Min}\sum_{s\in\Omega} p_s\xi_s + \lambda\sum_{s\in\Omega} p_s\Big[\Big(\xi_s - \sum_{s'\in\Omega} p_{s'}\xi_{s'}\Big) + 2\theta_s\Big] \tag{11}$$

s.t.

$$\xi_s - \sum_{s\in\Omega} p_s\xi_s + \theta_s \geq 0 \qquad \forall s \in \Omega \tag{12}$$

$$\theta_s \geq 0 \qquad \forall s \in \Omega \tag{13}$$

where θ_s is a positive variable used to linearize the absolute function. Because $\sum_{s\in\Omega} p_s\xi_s = 1$ is less than ξ_s, $\theta_s = 0$. If ξ_s is less than $\sum_{s\in\Omega} p_s\xi_s$, then $\theta_s = \sum_{s\in\Omega} p_s\xi_s - \xi_s$. Objective function $\alpha(\delta_1, \delta_2, \ldots, \delta_s)$ in the second section of Eq. (5) that is used to penalize the infeasibility of the model and compensate for control restriction errors in some scenarios. An error in control constraints means that infeasible solutions can be achieved in some scenarios. In accordance with this argument, the objective function is denoted by:

$$\mathrm{Min}\sum_{s\in\Omega} p_s\xi_s + \lambda\sum_{s\in\Omega} p_s\Big[\Big(\xi_s - \sum_{s'\in\Omega} p_{s'}\xi_{s'}\Big) + 2\theta_s\Big] + \omega\sum_{s\in\Omega} p_s\delta_s \tag{14}$$

3. Proposed Model Formulation

Before presenting the mathematical model, hypotheses are provided as follows:

- There are specified numbers of customers, whose demands are estimated for a specified period of time.
- Each customer has a certain numbers of distinct combinations. Each combination includes a number of days from within programming period. In case one of customer's com-

binations would be selected for serving, the customer has to be visited in all days of the selected combination.
- A number of homogeneous vehicles are available on various days of programming period, and all vehicles are used every day.
- The fleet is homogenous and the capacity of vehicles is fixed and identical.
- The maximum distance for travel of vehicles is fixed and identical.
- Starting time of service in comparison to that of competitor affects level of sales.
- Starting time of service of competitor is uncertain and follows different scenarios.

The assumptions that underlie a PVRP with a time window are as follows. On the basis of real-world observations, customer demand is divided into two categories. The first consists of an independent request that does not depend on the time of service initiation by competitors, and all customer demands should be satisfied. The second category involves time-dependent demand; that is, if a company's start-of-service time to customers is before the competitor initiates it's, the time-dependent demand of customer is satisfied and profit will increase. The objective function related to the minimization of the transportation cost and maximization of the expected value of fulfilling the time-dependent demands of customers are presented by:

$$\begin{aligned}\text{Max } Z = &\sum_{s\in\Omega} p_s\left[\sum_{i\in N}\sum_{d\in D}\eta_{ids}de_{tdi}+\gamma_{ids}\left(\frac{u_{ids}-t_{id}}{u_{ids}-l_{ids}}\right)de_{ini}\right]\\ &-\lambda\sum_{s\in\Omega}p_s\left[\begin{array}{l}\displaystyle\sum_{i\in N}\sum_{d\in D}\eta_{ids}de_{tdi}+\gamma_{ids}\left(\frac{u_{ids}-t_{id}}{u_{ids}-l_{ids}}\right)de_{ini}-\\ \displaystyle\sum_{s'\in\Omega}p_{s'}\left(\sum_{i\in N}\sum_{d\in D}\eta_{ids'}de_{tdi}+\gamma_{ids'}\left(\frac{u_{ids'}-t_{id}}{u_{ids'}-l_{ids'}}\right)de_{ini}\right)+2\theta_s\end{array}\right]\\ &-\omega\sum_{s\in\Omega}\sum_{k\in K}\sum_{d\in D}p_s\delta_{kds}-\sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\sum_{d\in D}Co_{ij}x_{ijk}^d\end{aligned} \tag{15}$$

s.t.

$$\sum_{v\in V_i}a_{iv}=1,\quad i\in N/\{0\} \tag{16}$$

$$am_{id}=\sum_{v\in V_i}ad_{ivd}.a_{iv},\quad i\in N/\{0\},d\in D \tag{17}$$

$$am_{0d}=1,\quad d\in D \tag{18}$$

$$\sum_{k\in K}\sum_{j\in N}x_{ijk}^d=am_{id},\quad d\in D,\ \ i\in N/\{0\} \tag{19}$$

$$\sum_{j\in N}x_{0jk}^d=1,\quad d\in D,k\in K \tag{20}$$

$$\sum_{i\in N}\sum_{d\in D}\sum_{k\in K}x_{iik}^d=0, \tag{21}$$

$$\sum_{i\in N}x_{ijk}^d-\sum_{i\in N}x_{jik}^d=0,\quad d\in D,\ \ j\in N,k\in K \tag{22}$$

$$\sum_{k\in K}x_{ijk}^d\le\frac{am_{id}+am_{jd}}{2},\quad d\in D,\ \ i,j\in N \tag{23}$$

$$\sum_{i\in N}\sum_{j\in N}st_i x_{ijk}^d+\sum_{i\in N}\sum_{j\in N}tr_{ij}x_{ijk}^d\le MT,\quad d\in D,k\in K \tag{24}$$

$$t_{jd}=\sum_{k\in K}\sum_{i\in N}(t_{id}+st_i+tr_{ij})x_{ijk}^d,\quad d\in D,j\in N \tag{25}$$

$$\sum_{i\in N}(de_i-de_{tdi}\varpi_{ids})\sum_{j\in N}x_{ijk}^d-\delta_{kds}\le Ca,\quad d\in D,k\in K,s\in\Omega \tag{26}$$

$$\eta_{ids}+\gamma_{ids}\le 1,\quad i\in N,d\in D,s\in\Omega \tag{27}$$

$$(l_{ids}-t_{id})+M(1-\eta_{ids})\ge 0,\quad i\in N,d\in D,s\in\Omega \tag{28}$$

$$(u_{ids}-t_{id})+M(1-\gamma_{ids})\ge 0,\quad i\in N,d\in D,s\in\Omega \tag{29}$$

$$(u_{ids}-t_{id})-M(1-\varpi_{ids})\le 0,\quad i\in N,d\in D,s\in\Omega \tag{30}$$

$$\eta_{ids}\le am_{id},\qquad i\in N,d\in D,s\in\Omega \tag{31}$$

$$\gamma_{ids}\le am_{id},\qquad i\in N,d\in D,s\in\Omega \tag{32}$$

$$\varpi_{ids}\le am_{id},\qquad i\in N,d\in D,s\in\Omega \tag{33}$$

$$\begin{aligned}&\sum_{i\in N}\sum_{d\in D}\eta_{ids}de_{di}+\gamma_{ids}\left(\frac{u_{ids}-t_{id}}{u_{ids}-l_{ids}}\right)de_{ini}-\\ &\sum_{s'\in\Omega}p_{s'}\left(\sum_{i\in N}\sum_{d\in D}\eta_{ids'}de_{di}+\gamma_{ids'}\left(\frac{u_{ids'}-t_{id}}{u_{ids'}-l_{ids'}}\right)de_{ini}\right)+\theta_s\ge 0,s\in\Omega\end{aligned} \tag{34}$$

$$\sum_{i,j\in Q}x_{ijk}^d\le|Q|-1,Q\subset N,2\le|Q|,d\in D,k\in K \tag{35}$$

$$\begin{aligned}&t_{jd},\theta_s\ge 0\\ &x_{ijk}^d,a_{iv},am_{id}\eta_{ids},\gamma_{ids},\varpi_{ids}\in\{0,1\}\end{aligned} \tag{36}$$

Equation (15) consists of objective function. The objective function consists of four terms. The first term is an average of the obtained profit in each scenario. The second term shows the changes of the objective function in each scenario and the third term penalizes for not following the capacity constraint in different scenarios, and fourth term is the transportation cost. Constraint (16) means every customer must be chosen in only one of the proposed combinations. Constraint (17) chooses the selected combination days for servicing the customers. Restriction (18) guarantees that depot is available on all days. Constraint (19) is about servicing customers in selected combination days. Constraints (20) guarantee that all the vehicles in all days must start moving from a depot. Constraint (21) removes the ring. Constraint (22) states that if vehicle *k* in day *d* enters node *i*, it must exit from that node. Constraint (23) in scenario *s* guarantees that only when selected combinations of customers *i* and *j* contain day *d*, path *i-j* has the possibility of selection. Constraint (24) is the maximum time of the vehicles accessibility. Constraint (25) calculate reaching time to the customer *i* in day *d*. Constraint (26) is about the maximum capacity of the vehicle in scenario *s* that is if for customer *i* in day *d* vehicle's reaching time occurs after an upper bound of rival's reaching, time dependent demand of customer *i* will not be carried from the beginning. Constraints (27) to (33) are for calculation of the profit in each scenario. Constraint (34) is used to linearization expression of absolute value in the first objective function. Constraint (35) eliminates the sub-tours, and finally Constraint (36) defines variables of the model.

4. Problem-solving Approach

The use of a DE algorithm in solving a problem is presented in Sub-section 4.1. A mutation operator is described in Sub-section 4.2, and a crossover operator is explained in Sub-section 4.3. In Sub-section 4.4, the DE strategies used for the proposed model are demonstrated. In Sub-section 4.5, a creative solution representation is presented.

4.1 Differential Evolution Algorithm

A Differential Evolution (DE) algorithm, which was first proposed by Storn and Price (1997), is an operational meta-heuristic method for optimizing problems. Because of the continuous nature of DE, research on the use of this algorithm for discrete optimization problems is limited. This limitation drove researchers to develop different ways of converting a continuous problem to optimize a VRP through the DE algorithm. An example of such methods is to find a solution to a generalized multi-depot VRP with pickup and delivery or a VRP that considers phase-based demands (Kunnapapdeelert and Kachitvichyanukul, 2015).

4.2 Mutation Operator

In each generation ($g = 1,\dots,g_{max}$), trial vector $v_{i,g}$ is generated for each individual (denoted by i, where = 1, ..., n_s) via mutation. The formulation of trial vector $v_{i,g}$ for random and current-to-best strategy proceeds by:

$$v_i, g = y_{i,g} + \alpha(y_{i1,g} - y_{i2,g}) \tag{37}$$

$$v_{i,g} = y_{i,g} + \alpha(y_{best,g} - y_{i,g}) + \alpha(y_{i1,g} - y_{i2,g}) \tag{38}$$

where $y_{i,g} \neq y_{i1,g} \neq y_{i2,g}$ and $y_{i1,g}, y_{i2,g} U(1,\dots,n_s)$. Term $y_{best,g}$ is denoted as the best individual in the generation of g, and $\alpha \in (0,\infty)$ is used to adjust the intensity of differential variations.

4.3 Binominal Crossover Operator

To produce offspring $y'_{ij,g}$, parent vector $y_{i,g}$ and trial vector $v_{i,g}$ are combined through crossover as follows:

$$y'_{ij,g} = \begin{cases} v_{ij,g} & \text{if} \quad r\ \text{and}(0,1) \leq CR \\ y_{ij,g} & \quad\text{otherwise} \end{cases} \tag{39}$$

where $y'_{ij,g}$ denotes the j-th $j \in \{1,\dots,n_x\}$ generation of a chromosome, and nx is the problem dimension. Finally, $CR \in (0,1)$ is initialized by the decision maker in DE.

4.4 Differential Evolution Strategies

DE involves different strategies, each used for optimization in different functions. The *DE/current–to best/2/bin* strategy also converges answers. To ensure the optimal use of these strategies, an optimized DE is used in such a way that each of the strategies dynamically displaces the other. Each of the strategies is applied on the basis of probability. If $p_{s,1}$ is defined as the probable use of the *DE/rand/1/bin* strategy for choosing next-generation members, then $p_{s,2} = 1 - p_{s,1}$ is defined as the probable use of the *DE/current − to best/2/bin strategy*. The possibility of $p_{s,1}$ is defined by:

$$p_{s,1} = \frac{n_{s,1}(n_{s,2}+n_{f,2})}{n_{s,2}(n_{s,1}+n_{f,1})+n_{s,1}(n_{s,2}+n_{f,2})} \tag{40}$$

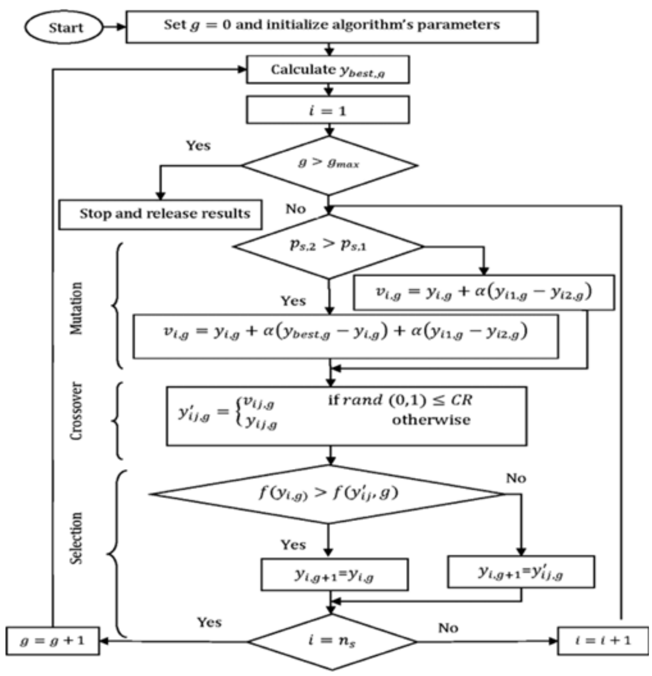

Fig. 1. Diagram of the Proposed Algorithm

*	Day1						Day2						Day3					
**	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
***	1.7	2.6	1.2	3.5	3.7	2.8	0.5	1.3	1.6	3.4	2.5	2.4	1.2	2.2	0.3	3.8	3.6	2.4
****	1	2	1	3	3	2	-	1	1	3	2	2	1	2	-	4	5	2
* Service time **Customer *** Continuous presentation of solutions **** Allocation of customers to vehicles																		

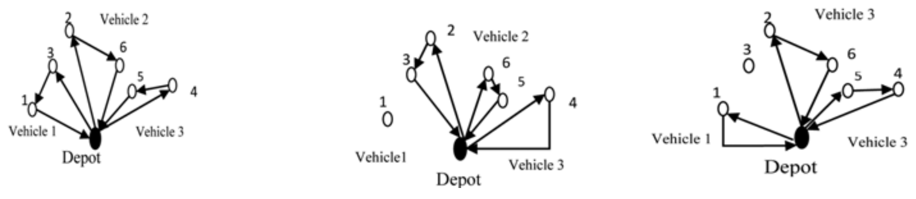

Fig. 2. Example of a Problem Representation

where $n_{s,1}$ and $n_{s,2}$ indicate a number of children $y_i'(g)$ that are selected using *DE/rand/1/bin* and *DE/current − to best/2/bin* in next generation $c(g + 1)$, respectively. $n_{f,1}$ and $n_{f,2}$ represent the number of children that are not transferred to the next generation in each strategy. The higher the number of selected children for transfer to the next generation in one strategy, the greater the probability that this strategy will be chosen in the next generation (Qin and Suganthan, 2005). The steps of the improved IDE are shown in Fig. 1.

4.5 Solution Display

To display solutions in a VRP continuously as the proposed model is solved using the IDE algorithm, we define vector $y_i(g)$ at the size of τn_y gene in such a way that n_x indicates the number of customers, and τ indicates the number of servicing days. For each gene $j \in \{1, \ldots, \tau n_y\}$, one random integer in interval $(0, kv + 1 - \varepsilon]$ shows the number of specified vehicles for customer j. Such option is specified if only all the vehicle's capacity restrictions, service times, combinations of servicing days, and other restrictions are followed. Otherwise, a vehicle number less than 1 is specified for customer j. To define the sequence of customer-passing vehicles for each gene in vector $y_i(g)$, a decimal value is added to the integer. By arranging the decimal value assigned in ascending order, the sequencing of passing vehicles in a tour is determined. Fig. 2 indicates a display solution for vector $y_i(g)$ with six customers (i.e., genes), three vehicles, and three days of servicing.

5. Computational Results

This section compares the performance of the IDE algorithm for small- and large-scale problems. To transform the objective functions into one section, a weight factor was used. That is, the weight of each objective function was set at 0.5 and was used to maximize the objective functions. The experiments were conducted using MATLAB, and the programs were executed on a Core i7 computer with a capacity of 2/3 GHz and an internal memory of 2 GB.

5.1 Parameter Setting Method

On the basis of the Taguchi method (Taguchi, 1986), the parameter setting for proposed algorithm is examined through experimentation on real-time PVRPTW problems. The least and most sensitive IDE parameter settings are selected using the signal-to-noise (*S*/*N*) ratio. The Taguchi orthogonal array size is defined on the basis of the number of important parameters to be considered. In the Taguchi method, an S/N ratio is called, where *S* and *N* are the levels of desirability and undesirability (i.e., this value defines a standard deviation of answers), respectively. The Taguchi method attempts to exceed the *S*/*N* ratio. The function of the *S*/*N* ration in the proposed model is presented by:

$$\frac{s}{N} = -10log\left(\frac{1}{m'}\sum_{i=1}^{m'} q_i^2\right) \tag{41}$$

where $q_i(i = 1, \ldots, m')$ represents the values of the objective function answers. With consideration for consequences, $\alpha = 0.5$, $n_s = 200$, and $CR = 0.6$ are calculated; these values are assumed to be the same across DE strategies. The trial-and-error method is used to ascertain the number of iterations in the IDE algorithm. The results indicates that $g_{Max} = 150$ reflects suitable performance in solving the proposed model.

Table 1. Numerical Data in Small Sizes

Applied distribution	Parameter
~ Uniform (0,25)	Coordinates
~ Uniform (1,12)	Customer demand
~ Uniform (1,6)	Servicing time
~ Uniform (5,20)	Lower bound of rival's reaching time
~ Uniform (e_i + 5, e_i + 20)	Upper bound of rival's reaching time
~ Uniform (2,5)	Number of scenarios
~ Uniform (2,4)	Number of periodic days (*D*)
~ Uniform (1, *D*)	Required customers' visit

Table 2. Characteristics of samples

No. of problems	Small sizes	Large sizes
	N/*K*/*D*/*Ω*	*N*/*K*/*D*/*Ω*
1	5/2/2/2	48/2/4/3
2	5/3/3/3	96/4/4/3
3	6/3/3/2	144/6/4/4
4	7/3/4/3	192/8/4/4
5	10/3/4/3	240/10/4/5
6	12/3/4/3	288/12/4/5
7	12/4/5/4	72/3/6/6
8	14/3/5/4	144/6/6/5
9	14/4/5/4	216/9/6/4
10	-	288/12/6/4

5.2 Performance of IDE on Small-scale Problems

To examine the effectiveness of the DE strategies, nine small-scale problems were defined and solved. The results were compared with an optimal solution obtained using the CPLEX solver embedded in GAMS 23.6. Table 1, shows the numerical data required to create small-scale problems were considered to have uniform distribution. Table 2 presents the characteristics of the small and large-sized problems. The number of customers, vehicles, and days and the set of scenarios for each sample are denoted by *N*, *K*, *D*, and *Ω*, respectively.

Table 3 shows the computational results for the solutions to the small-sized problems. As shown in this table, all the algorithms were used 150 times. With the exception of one, all the other sample tests indicated that the IDE algorithm can calculate the optimal solution without errors. Only sample 4 registers error of more than 0.4%. In two sample tests, the *DE/current − to best/2/bin* and *DE/rand/1/bin* strategies exhibit average errors of 0.09% and 0.13%, respectively, confirming the effectiveness of the strategies. The average computational time of the exact solution method is 27.4 seconds, whereas that of the *DE/current − to best/2/bin* strategy is 4.9 seconds, which is the lowest average run time among all the strategies.

Table 3. Comparison of Results in CPLEX and DE Strategies in Small Sizes

#	CPLEX		IDE			*DE/current − to best/2/bin*			*DE/rand/1/bin*		
	OFV	Time (s)	OFV	Time (s)	Gap %	OFV	Time (s)	Gap %	OFV	Time (s)	Gap %
1	289.3	25.3	289.3	4.9	0.00	289.3	3.2	0.00	289.3	4.5	0.00
2	357.9	26.2	357.9	5.6	0.00	357.9	6.3	0.00	357.9	5.1	0.00
3	382.2	25.5	382.2	4.6	0.00	380.5	3.9	0.45	382.2	3.7	0.00
4	401.6	27.8	399.8	6.5	0.45	401.6	5.1	0.00	401.6	4.8	0.00
5	406.9	24.1	406.9	3.9	0.00	406.9	2.9	0.00	403.2	3.5	0.92
6	529.6	28.4	529.6	7.1	0.00	527.6	7	0.38	529.6	6.8	0.00
7	483.5	30.2	483.5	6.9	0.00	483.5	5.3	0.00	483.5	5.9	0.00
8	624.5	31.3	624.5	6.8	0.00	624.5	6.3	0.00	622.8	6.5	0.27
9	723.8	27.8	723.8	5.9	0.00	723.8	4.8	0.00	723.8	4.3	0.00
AVG	466.59	27.40	466.39	5.80	0.05	466.18	4.98	0.09	465.99	5.01	0.13

OFV: Objective function value #:Number of problems

Table 4. Comparison of the Performance of the Proposed DE Strategies

#	IDE			*DE/current − to best/2/bin*			*DE/rand/1/bin*		
	OFV	Time (s)	Gap %	OFV	Time (s)	Gap %	OFV	Time (s)	Gap %
1	1,025.9	290.8	0.0	1,018.5	285.3	0.7	1,018.3	983.1	0.7
2	2,367.3	589.3	0.0	2,337.6	550.6	1.2	2,300.8	503.9	2.9
3	3,752.9	1,023.6	0.0	3,728.5	1,149.6	0.6	3,690.7	1,002.9	1.7
4	4,689.0	1,439.5	0.0	4,632.6	1,523.9	1.2	4,635.3	1,801.6	1.1
5	1,069.5	1,763.8	0.0	1,052.8	1,836.3	1.5	1,059.3	1,124.6	0.9
6	6,395.4	2,564.7	0.1	6,392.3	2,495.3	0.1	6,401.5	2,500.3	0.0
7	3,426.8	689.6	0.0	3,396.3	583.6	0.9	3,395.7	1,423.8	0.9
8	3,406.6	1,438.6	0.0	3,400.6	1,425.8	0.1	3,386.3	1,420.6	0.6
9	7,609.3	1,979.2	0.0	7,592.7	1,913.5	0.2	7,599.8	1,921.3	0.1
10	6,539.6	2,798.6	0.0	6,498.3	2,698.3	0.6	6,456.9	2,625.3	1.3
AVG	4,028.2	1,457.7	0.0	4,005.0	1,446.2	0.7	3,994.4	1,530.7	1.0

OFV: Objective function value #:Number of problems

5.3 Performance of IDE on Large-scale

To examine performance on large-scale problems, the standard PVRPTW samples presented by Cordeau *et al*. (1997) were used. With respect to the difference between the proposed model and other PVRPTWs, some changes were applied in standard sampling. That is, the upper and lower limits of time of reach were examined in scenarios t_{uis} and t_{lis} at a uniform distribution in intervals [15, 60] and [10, 40], respectively. Table 4 presents the results on the performance of the DE strategies. Most of the errors in sample 3 are produced by the *DE/rand/1/bin* strategy. Only in sample 6 does the IDE algorithm present a performance weaker than that of the other strategies. The average error of the IDE in this regard is 0.01%, which is the lowest computational error among the strategies. The average computational time of the *DE/current − to best/2/bin* strategy is 1,446.2 seconds, which is better than those of the other strategies. The computational results showed despite the longer computational time taken by the IDE algorithm, its solutions are of high quality.

5.4 Sensitivity Analysis

To analyze the sensitivity of the proposed model, a numerical example in a small size, is solved for one day, and the results of the robust approach are compared with the exact solution for each of the scenarios. The experiment involved nine customers, one depot, and three vehicles. All known parameters were selected randomly. Uncertain parameters, including the upper and lower limits of a distributor's time of reach to customers (per minute), were established in two scenarios as a uniformly distributed function. The associated probabilities of the first and second scenarios were 0.6 and 0.4, respectively. Results show that, the profit obtained from service to customers through the robust approach is 16.257 units; those obtained with the use of the exact approach for scenarios 1 and 2 are 15.593 and 15.028 units, respectively. Put differently, the profit derived using the

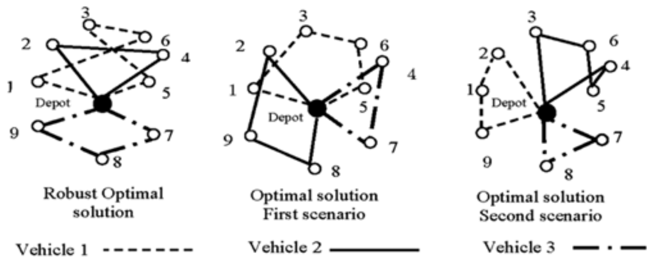

Fig. 3. How to Service Customers

robust approach is 8% and 6% less than that obtained with the exact approach for scenarios 1 and 2, respectively. This finding highlights the critical nature of robustness cost from a decision maker's viewpoint. Fig. 3 illustrates how customers can be serviced by vehicles in each aforementioned approach.

6. Conclusions

In this paper, a robust mathematical model for competitive periodic routing problem has been presented under uncertainty of customers' visit time by competitors. For the first time, the uncertainty of competitors' starting time of service has been considered in the proposed model based on our best knowledge. Additionally, a set of scenarios as for starting time of service have been considered in modeling, instead of an estimated time of starting time of service. Considering the problem as an NP-hard, a meta-heuristic algorithm based on the Deferential Evolution (DE) algorithm has been proposed in order to solve large-sized problems. To examine the Improved Differential Evolution (IDE) algorithm, a number of tests on small-sized problems were carried out with three DE strategies, and the results were compared to those derived using the exact solution algorithm. The findings indicated the appropriateness of the presented DE strategies for solving such problems. To further delve into the performance of the strategies, large-size sample problems were established. The computational results revealed that the solutions obtained by the IDE algorithm are of very high quality despite the fact that it entails a longer computational time than that required in the other strategies. In addition to considering competitors' time of service to customers, we regarded customer demand as uncertain. Considering competition in the design of other routing modes can be an attractive direction for future research.

Notations

Indices and parameters:

Ca = Maximum capacity of vehicles

Co_{ij} = Cost of travel from customers i to j

D = Set of days in the period, where d is the index of days in the period

de_{tdi} = Time dependent demand of customer i

de_is = Demand of customer i in each day of combination $de_i = de_{tdi} + de_{idi}$

de_{idi} = Time independent demand of customer i

K = Set of available vehicles and k is the index of the vehicles

l_{ids} = Lower bound of competitor's arrival time to customer i under scenario s on day d.

M = A large number, (say $M >> \infty$)

MT = Maximum travel time of vehicles

N = Set of all nodes also i and j are its indices, and node 0 is depot

p_s = Incidence probability of Scenario s

st_i = Service time of customer i

tr_{ij} = Required time to travel from customers i to j

u_{ids} = Upper bound of competitor's arrival time to customer i under scenario s on day d

V_i = Set of the i-th customer combinations, where v is its index

Ω = Set of scenarios, whose indices are s *and* s'

λ = is a positive constant value. indicates the weight placed on the variance of a solution

Variables

ad_{ivd} = 1, if day d is existed in the combination v of customer i; 0, otherwise.

a_{iv} = 1, if the v-th combination of customer i is selected; 0, otherwise

am_{id} = 1, if day d selected to visit customer i; 0, otherwise.

γ_{ids} = A binary variable, 0, if the distributer company in scenario s, in day d reaches the customer i after the upper bound of the competitor; free in value, otherwise.

η_{ids} = A binary variable, 0, if the distributer company in scenario s, in day d reaches the customer i after the lower bound of the competitor; free in value, otherwise.

θ_s = A positive variable used to linearize the absolute function.

t_{id} = Reaching time to customer i on day d.

ϖ_{ids} = A binary variable, 0 if the distributer in scenario s, in day d begins customer service before competitors' upper bound; free in value, otherwise.

x_{ijk}^{d} = 1, if vehicle k travels through edge i-j in day d; 0, otherwise

References

Alinaghian, M., Ghazanfari, M., Salamatbakhsh, A., and Norouzi, N. (2012). "A new competitive approach on multi-objective periodic vehicle routing problem." *International Journal Applied Operational Research*, Vol. 1, No. 3, pp. 33-41, http://ijorlu.liau.ac.ir/article-1-83-en.html.

Archetti, C., Savelsbergh, M., and Speranza, M. (2016). "Vehicle routing problem with occasional drivers." *European Journal of Operational Research*, Vol. 254, Issue 2, pp. 472-480, DOI: 10.1016/j.ejor.2016.03.049.

Coene, S., Arnout, A., and Spieksma, F. (2010). "On a periodic vehicle routing problem." *Operations Research*, Vol. 61, Issue 12, pp. 1719-1728, DOI: 0.1057/jors.2009.154.

Cordeau, J. F., Gendreau, M., and Laporte, G. (1997). "A tabu search heuristic for periodic and multi-depot vehicle routing problems." *An International Journal Networks*, Vol. 30, Issue 2, pp. 105-119, DOI: 10.1002/(SICI)1097-0037(199709)30:2<105::AID-NET5>3.0.CO;2-G.

Dantzig, G. and Ramser, J. H. (1959). "The truck dispatching problem." *Management Science,* Vol. 6, Issue 1, pp. 80-91.

Das, S. and Suganthan, P. N. (2011). "Differential evolution: A survey of the state-of-the-art." *IEEE Transaction on Evolutionary Computation*, Vol. 15, Issue 1, pp. 4-31, DOI: 10.1109/TEVC.2010.2059031.

Fazel Zarandi, M. H., Hemmati, A., Davari, S., and Turksen, I. B.

(2014). “A simulated annealing algorithm for routing problems with fuzzy constrains.” *Journal of Intelligent & Fuzzy Systems*, Vol. 26, Issue 6, pp. 2649-2660, DOI: 10.3233/IFS-130935.

Gounaris, C. E., Wiseman, W., and Floudas, C. A. (2013). “The robust capacitated vehicle routing problem under demand uncertainty.” *Operations Research*, Vol. 61, Issue 3, pp. 677-693, DOI: 10.1287/opre.1120.1136.

Goksal, F. P., Karaoglan, I., and Altiparmak, F. (2013). “A hybrid discrete particle swarm optimization for vehicle routing problem with simultaneous pickup and delivery.” *Computers & Industrial Engineering*, Vol. 65, Issue 1, pp. 39-53, DOI: 10.1016/j.cie.2012.01.005.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Rath, S. (2011). “A heuristic solution method for node routing based solid waste collection problems.” *Journal of Heuristics*, Vol. 19, Issue 2, pp. 129-156, DOI: 10.1007/s10732-011-9188-9.

Jia, H., Li, Y., Dong, B., and Ya, H. (2013). “An improved tabu search approach to vehicle routing problem.” *Procedia – Social and Behavioral Sciences*, Vol. 96, pp. 1208-1217, DOI: 10.1016/j.sbspro. 2013.08.138.

Kohl, N. and Madsen, O. B. G. (1997). “An optimization algorithm for the vehicle routing problem with time windows based on Lagrangian relaxation. “*Operations Research*, Vol. 45, Issue 3, pp. 395-406, DOI: 10.1287/opre.45.3.395.

Kos, C. and Karaoglan, I. (2016). “The green vehicle routing problem: A heuristic based exact solution approach.” *Applied Soft Computing*, Vol. 39, pp. 154-164, DOI: 10.1155/2016/8461857.

Kunnapapdeelert, S., and Kachitvichyanukul, V. (2015). *Modified DE algorithms for solving multi-depot vehicle routing problem with multiple pickup and delivery requests*, In: Toward Sustainable Operations of Supply Chain and Logistics Systems, Eco Production, springer nature, USA. DOI: 10.1007/978-3-319-19006-8_25.

Lee, C., Lee, K., and Park, S. (2012). “Robust vehicle routing problem with deadlines and travel time/demand uncertainty.” *The Journal of the Operational Research Society*, Vol. 63, Issue 9, pp. 1294-1306, DOI: 10,1057/jors.2011.136.

Lenstra, J. K. and Rinnooy Kan, A. H. G. (1981). “Complexity of vehicle and scheduling problem.” *An International Journal Networks*, Vol. 11, Issue 2, pp. 221-227, DOI: 10.1002/net.3230110211.

Leung, S. C. H. and Chan, S. S. W. (2009). “A goal programming model for aggregate production planning with resource utilization constraint.” *Computers & Industrial Engineering*, Vol. 56, Issue 3, pp. 1053-1064, DOI: 10.1016/j.cie.2008.09.017.

List, B. F., Wood, B., and Nozick, L. K. (2003). “Robust optimization for fleet planning under uncertainty.” *Transportation research part E: Logistics and transportation review*, Vol. 39, Issue 3, pp. 209-227, DOI: 10.1016/S1366-5545(02)00026-1.

Li, X., Cao, Y., Zhai, X., and Xie, D. (2015). “Drivers’ diversion from expressway under real traffic condition information shown on variable message signs.” *KSCE Journal of Civil Engineering*, Vol. 19, No. 7, pp. 2262-2270, DOI: 10.1007/s12205-014-0692-y.

Montgomery, D. C. (2001). *Design and analysis of experiments. (5th Ed.)*, Anderson Wayne, USA.

Mulvey, J. M., Vanderbei, R. J., and Zenios, S. A. (1995). “Robust optimization of large-scale systems.” *Operations Research*, Vol. 43, Issue 2, pp. 264-281, DOI: 10.1287/opre.43.2.264.

Norouzi, N., Tavakkoli-Moghaddam, R., Gazanfari, M., Alinaghian, M., and Salamatbakhsh, A. (2012). “A new multi-objective competitive open vehicle routing problem solved by particle swarm optimization.” *Networks and Spatial Economics*, Vol. 12, Issue 4, pp. 609-633, DOI: 10.1007/s11067-011-9169-4.

Pan, F. and Nagi, R. (2010). “Robust supply chain design under uncertain demand in agile manufacturing.” *Computers & Operations Research*, Vol. 37, Issue 4, pp. 668-683, DOI: 10.1016/j.cor.2009.06.017.

Pourghaderi, A. R., Tavakkoli-Moghaddam, R., Alinaghian, M., and Beheshti-Pour, B. (2008, December). A simple and effective heuristic for periodic vehicle routing problem, *IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pp. 133-137.

Qin, A. K. and Suganthan, P. N. (2005). “Self-Adaptive differential evolution algorithm for numerical optimization.” *Proceedings of the IEEE Congress on Evolutionary Computation*, IEEE, Scotland, UK, Vol. 2, Issue 3, pp. 1785-1791, DOI: 10.1109/CEC.2005.1554904.

Storn, R. and Price, K. (1997). “Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces.” *Journal of Global Optimization*, Vol. 4, Issue 4, pp. 359-431, DOI: 10.1023/A: 1008202821328.

Sungur, I., Ordonez, F., and Dessouky, M. (2008). “A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty.” *IIE Transactions*, Vol. 40, Issue 5, pp. 509-523, DOI: 10.1080/07408170701745378.

Taguchi, G. (1986). *Introduction to quality engineering: Designing quality into products and processes*, Asian productivity organization, Japan, p. 191, DOI: 10.1002/qre.4680040216.

Tavakkoli-Moghaddam, R., Gazanfari, M., Alinaghian, M., Salamatbakhsh, A., and Norouzi, N. (2011). “New mathematical model for a competitive vehicle routing problem with time windows solved by simulated annealing.” *Journal of Manufacturing Systems*, Vol. 30, Issue 2, pp. 83-92, DOI: 10.1016/j.jmsy.2011.04.005.

Wang, Y., Ma, X., Xu, M., Wang, Y., and Liu, Y. (2015). “Vehicle routing problem based on a fuzzy customer clustering approach for logistics network optimization.” *Journal of Intelligent & Fuzzy Systems*, Vol. 29, Issue 4, pp. 1447-1442, DOI: 10.3233/IFS-151578.

Yan, F., Xu, J., and Han, B. T. (2015). “Material transportation problems in construction projects under an uncertain environment.” *KSCE Journal of Civil Engineering*, Vol. 19, No. 7, pp. 2240-2251, DOI: 10.1007/s12205-015-0204-8.

Yu, C. S. and Li, H. L. (2000). “A robust optimization model for stochastic logistic problems.” *International Journal of Production Economics*, Vol. 64, Issues 1-3, pp. 385-397, DOI: 10.1016/S0925-5273(99)00074-2.