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Integrating bus services with mixed fleets

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ABSTRACT

Conventional bus service (with fixed routes and schedules) has lower average cost than flexible bus service (with demand-responsive routes) at high demand densities. At low demand densities flexible bus service has lower average costs and provides convenient door-to-door service. Bus size and operation type are related since larger buses have lower average cost per passenger at higher demand densities. The operation type and other decisions are jointly optimized here for a bus transit system connecting a major terminal to local regions. Conventional and flexible bus sizes, conventional bus route spacings, areas of service zones for flexible buses, headways, and fleet sizes are jointly optimized in multi-dimensional nonlinear mixed integer optimization problems. To solve them, we propose a hybrid approach, which combines analytic optimization with a Genetic Algorithm. Numerical analysis confirms that the proposed method provides near-optimal solutions and shows how the proposed Mixed Fleet Variable Type Bus Operation (MFV) can reduce total cost compared to alternative operations such as Single Fleet Conventional Bus (SFC), Single Fleet Flexible Bus (SFF), Mixed Fleet Conventional Bus (MFC) and Mixed Fleet Flexible Bus (MFF). With consistent system-wide bus sizes, capital costs are reduced by sharing fleets over times and over regions. The sensitivity of results to several important parameters is also explored.

1. Introduction

Conventional public transit services (which include most bus and rail transit services) are characterized by their fixed routes and schedules. They can provide relatively high passenger-carrying capacities at relatively low average operating costs. However, their service quality is limited since passengers must somehow reach some predetermined stations, wait for a vehicle, possibly transfer several times, and then move from their exit stations to their destinations. Thus, conventional transit services are least disadvantaged in areas and time periods with high demand densities, which can sustain high network densities and service frequencies. Some paratransit services can provide more flexible routes and schedules, including the possibility of door-to-door service. For instance, taxis provide great service flexibility, but at high unit costs (especially in labor cost per passenger-mile). Since their service quality does not depend on network density or service frequency, flexible services may be preferable in low demand areas. In heterogeneous regions with substantial variability in demand across time and space we seek to better match the demand and supply by integrating conventional and flexible services.

In the literature on bus transit systems several studies focused on operation types (such as conventional bus and/or subscription bus) and fleet assignment problems (e.g. multiple fleet assignments for conventional and flexible bus operation) (Lee et al., 1995; Chang and Schonfeld, 1991a, 1991b, 1991c). Optimization models were developed for conventional and subscription bus operations (Chang, 1990; Chang and Schonfeld, 1991a). These studies confirmed that conventional bus

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(with fixed routes and fixed schedule) is preferable to subscription bus (which has demand responsive routes and flexible schedule) at high demand densities, and vice versa.

They also explored optimal dimensions of bus service areas (Chang and Schonfeld, 1993), multiple period optimization of public bus systems (Chang and Schonfeld, 1991c), and temporal integration of fixed and flexible bus systems (Chang and Schonfeld, 1991b). Zhou et al. (2008) maximized welfare under financial constraints for various bus transit service types and determined conditions under which subsidies may be justifiable. The above studies provided various approaches for improving the performance of bus transit systems. However, most of them considered only one local region.

Public bus operations with flexible routes and schedules, have attracted considerable interest from researchers, especially in recent years (Diana et al., 2009; Horn, 2002; Fu and Ishkhanov, 2004; Quadrifoglio et al., 2006, 2008; Quadrifoglio and Li, 2009; Luo and Schonfeld, 2011a, 2011b; Shen and Quadrifoglio, 2011; Jung and Jayakrishnan, 2011; Becker and Teal, 2011; Kim and Haghani, 2011; Baumgartner and Schofer, 2011; Nourbakhsh and Ouyang, 2011). Li and Quadrifoglio (2009) developed an analytic model for optimizing zones for feeder transit services. They developed closed form solutions for fixed routes and demand responsive services, and they utilized a simulation to verify their analytic solutions. However, that model was solved for one vehicle and one local region. They then applied two vehicles for the analysis (Li and Quadrifoglio, 2011). Chandra and Quadrifoglio (2013) developed an analytic queuing model for estimating the tour length for demand responsive feeder transit services. Several researchers have also solved feeder bus network design problems (Kuan et al., 2006; Ciaffi et al., 2012). Kuan et al. (2006) used a genetic algorithm and ant colony optimization while Ciaffi et al. (2012) developed a two phased solution method, which uses a heuristic and genetic algorithm, for realistically sized applications.

Lee et al. (1995) and Fu and Ishkhanov (2004) analyzed the assignment of buses with dissimilar sizes (i.e. "mixed fleets") to public transit operations. Lee et al. (1995) studied mixed bus fleet operations in conventional urban public transit systems. Fu and Ishkhanov (2004) studied mixed fleet bus operation for paratransit services. When demand densities differ considerably over time or space, mixed fleets can reduce total system cost compared to single fleets because vehicles of different sizes may be matched to the operations for which they are most suited.

Kim and Schonfeld (2012) considered bus size and service type jointly, confirming that variable-type operation can reduce system cost by changing the operation type (or "mode") as demand density changes. However, their study optimized decision variables and minimized total cost only between one terminal and one local region. Kim and Schonfeld (2012) analyzed solutions with a simple heuristic, which is inadequate for large problems with multiple regions and multiple vehicle types.

The potential benefits of using variable operation types (or "modes") and multiple fleets should theoretically increase when multiple dissimilar regions are considered, due to the increased variability of demand densities. To explore these potential benefits we analyze in this paper the concept of Mixed Fleet Variable Type Bus Operation (MFV) in multiple regions, which was not considered in Kim and Schonfeld (2012). No other studies were found that integrated service types and fleets jointly. This study also explores the benefits (i.e., capital cost savings) of sharing mixed bus fleets (i.e., fleets with vehicles of different sizes) among regions and time periods. For overall system efficiency, we optimize here decision variables for bus sizes (i.e. large and small) and decision variables for bus operation characteristics (i.e. route spacing in each region for conventional bus, service area within each region for flexible bus).

2. Bus operation descriptions

The analyzed bus system provides service from a major terminal (or CBD) to multiple regions. In Fig. 1, a public bus system serves multiple regions connected to a central terminal. For each region, either conventional bus or flexible bus can be

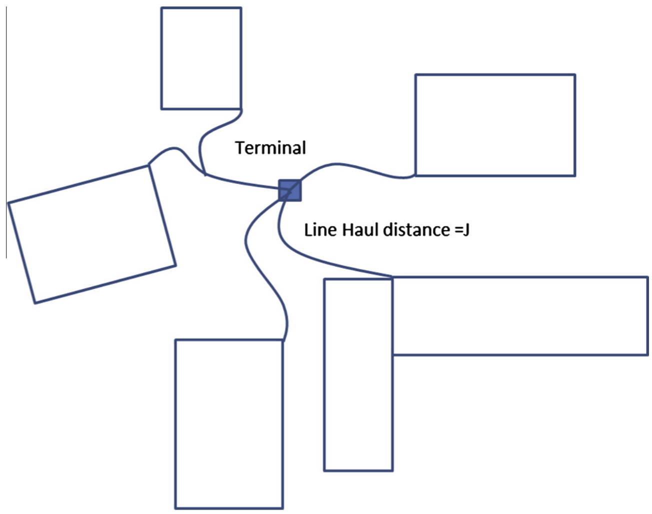

Fig. 1. Terminal and local regions.

provided. To analyze this local bus system, we specify some simplifying assumptions before explaining the cost functions and their optimization.

2.1. Assumptions

A previous study (Kim and Schonfeld, 2012) addressed assumptions for analyzing a one route service (i.e. connecting one terminal to one region). Here, we modify some assumptions and notation to analyze a more general system with multiple local regions and multiple bus sizes as well as multiple periods. Henceforth, superscripts k and i correspond to route and time period, respectively, while subscripts c and f represent conventional and flexible service, respectively. Definitions, units and default values of variables are presented in Table 1.

2.1.1. Assumptions for both conventional and flexible buses

All service regions, $1,\ldots,k$, are rectangular, with lengths L^k and widths W^k. These regions may have different line haul distances J^k (miles, in route k) connecting a terminal and each region's nearest corner.

(a) The demand is fixed with respect to service quality and price.
(b) The demand is uniformly distributed over space within each region and over time within each specified period.
(c) The optimized bus sizes (S_c *for conventional,* S_f *for flexible)* are common for all periods and regions in the system; however, different buses (among the available small and large ones) may be idle in different periods.
(d) The average waiting time of passengers is approximated as half the headway (h_c *for conventional,* h_f *for flexible*).
(e) Bus layover time is negligible.
(f) Within each local region k, the average speed (V_c^i *for conventional bus,* V_f^i *for flexible bus*) includes stopping times.
(g) External costs are assumed to be negligible.

2.1.2. Assumptions for conventional bus only

(a) The region k is divided into N^k parallel zones with a width $r^k = W^k/N^k$ for conventional bus, as shown in Fig. 2. Local routes branch from the line haul route segment to run along the middle of each zone, at a route spacing $r^k = W^k/N^k$.
(b) Q^{ki} trips/mile^2/h, entirely channeled to (or through) the single terminal, are uniformly distributed over the service area.
(c) In each round trip, as shown in Fig. 2, buses travel from the terminal a line haul distance J^k at non-stop speed yV_c^i to a corner of the local regions, then travel an average of $W^k/2$ miles at local non-stop speed zV_c^i from the corner to the assigned zone, then run a local route of length L^k at local speed V_c^i along the central axis of the zone while stopping for passengers every d miles, and then reverse the above process in returning to the terminal.

2.1.3. Assumptions for flexible bus only

(a) To simplify the flexible bus formulation, region k is divided into N'^k equal zones, each having an optimizable zone area $A^k = L^kW^k/N'^k$. The zones should be "fairly compact and fairly convex" (Stein, 1978).
(b) Buses travel from the terminal line haul distance J^k at non-stop speed yV_f^i and an average distance $(L^k + W^k)/2$ miles at local non-stop speed zV_f^i to the center of each zone. They collect (or distribute) passengers at their door steps through an efficiently routed tour of n stops and length D_c^{ki} at local speed V_f^i. D_c^{ki} is approximated according to Stein (1978), in which $D_c^{ki} = \varnothing\sqrt{\mathrm{n}A^k}$, and $\varnothing = 1.15$ for the rectilinear space assumed here (Daganzo, 1984). The values of n and D_c^{ki} are endogenously determined. To return to their starting point the buses retrace an average of $(L^k + W^k)/2$ miles at zV_f^i miles per hour and J^k miles at yV_f^i miles per hour.
(c) Buses operate on schedules with preset headways and with flexible routing designed to minimize each tour distance D_c^{ki}.
(d) Tour departure headways are equal for all zones in each region and uniform within each period.

3. Bus operation cost and optimized headways

For the operation cost of conventional and flexible bus, we consider bus operating cost, user in-vehicle cost, user waiting cost, and user access cost. Since flexible bus provides door-to-door service, its user access cost is negligible. Detailed formulation derivations regarding conventional bus and flexible bus can be found in Kim and Schonfeld (2012).

3.1. Conventional bus cost formulation

Conventional bus cost for route k and period i, SC_c^{ki}, includes operating cost, user in-vehicle cost, user waiting cost, and user access cost, as shown in Eq. (1):

Table 1
Notation.

Variable	Definition	Baseline value
a	hourly fixed cost coefficient for operating bus ($/bus h)	30.0
a_c	fixed cost coefficient for bus ownership (capital cost) ($/bus day)	100.0
A^k	service zone area (mile^2) = L^kW^k/N'	–
b	hourly variable cost coefficient for bus operation ($/seat h)	0.2
b_c	variable cost coefficient for owning bus (capital cost) ($/day)	0.5
d	bus stop spacing (miles)	0.2
D_c^{ki}	distance of one flexible bus tour in local region k and period i (miles)	–
D_f^k	equivalent line haul distance for flexible bus on route k (=$(L^k + W^k)/z + 2J^k/y$) (miles)	–
D^k	equivalent average bus round trip distance for conventional bus on route k (=$2J^k/y + W^k/z + 2L^k$) (miles)	–
f	directional demand split factor	1.0
F^{ki}	fleet size for route k and period i (buses) subscript corresponds to (c = conventional, f = flexible)	–
h_c, h_c^{ki}	headway for conventional bus; for route k and period i (h/bus)	–
h_f, h_f^{ki}	headway for flexible bus; for route k period i (h/bus)	–
$h_{c\,max}^{ki}, h_{f\,max}^{ki}$	maximum allowable headway for route k and period i subscript: c = conventional, f = flexible	–
$h_{c\,min}^{ki}, h_{f\,min}^{ki}$	minimum cost headway for route k and period i subscript: c = conventional, f = flexible	–
$h_{c\,opt}^{ki}, h_{f\,opt}^{ki}$	optimized headway for route k and period i subscript: c = conventional, f = flexible	–
k, i	index (k: route, i: period)	–
J^k	line haul distance of route k (miles)	–
l_c, l_f	load factor for conventional and flexible bus (passengers/seat)	1.0
L^k, W^k	length and width of local region k (miles)	–
M^k	equivalent average trip distance for route k (=$J^k/y_c + W^k/2z_c + L^k/2$)	–
n	number of passengers in one flexible bus tour	–
N, N'	number of zones in local region for conventional and flexible bus	–
Q^{ki}	round trip demand density (trips/mile^2/h)	–
Q_t^{ki}	threshold demand density between conventional and flexible service (trips/mile^2/h)	–
r^k	route spacing for conventional bus at region k (miles)	–
R_c^{ki}	round trip time of conventional bus for route k and period i (h)	–
R_f^{ki}	round trip time of flexible bus for route k and period i (h)	–
S_c, S_f	sizes for conventional and flexible bus (seats/bus)	–
S_l, S_s	sizes of larger and smaller buses in MFC and MFF service formulation	–
S_c^{ki}, S_f^{ki}	conventional and flexible bus sizes for route k and period i (seats/bus)	–
SC_c^{ki}, SC_f^{ki}	service cost for route k and period i subscript: c = conventional, f = flexible	–
TSC_c, TSC_f	total service cost over all routes and periods subscript: c = conventional, f = flexible	–
TC	total cost = service cost + capital cost over times and over regions	
t^{ki}	time duration for route k and period i	–
u	average number of passengers per stop for flexible bus	1.2
V_c^i	local service speed for conventional bus in period i (miles/h)	20 at $i = 1$ 30 at $i = 2,3,4$
V_f^i	local service speed for flexible bus in period i (miles/h)	18 at $i = 1$ 25 at i = 2,3,4
V_x	average passenger access speed (mile/h)	2.5
v_v, v_w, v_x	value of in-vehicle time, wait time and access time ($/passenger h)	5, 12, 12
y	express speed/local speed ratio for conventional bus	conventional bus = 1.8 flexible bus = 2.0
z	non-stop ratio = local non-stop speed/local speed; same values as y	–
Ø	constant in the flexible bus tour equation (Daganzo, 1984) for flexible bus	1.15
*	superscript indicating optimal value; subscript: c = conventional, f = flexible	–

$$SC_c^{ki} = \frac{D^kW^k(a + bS_c)}{r^kV_c^ih_c^{ki}} + \frac{v_vL^kW^kQ^{ki}M^k}{V_c^i} + \frac{v_wL^kW^kQ^{ki}h_c^{ki}}{2} + \frac{v_xL^kW^kQ^{ki}(r^k + d)}{4V_x} \quad (1)$$

3.2. Conventional bus optimized headway

Since we consider multiple periods for bus operations, the optimized headway should be the maximum allowable headway or the minimum cost headway, whichever is smaller. The maximum allowable headway for route k and period i is:

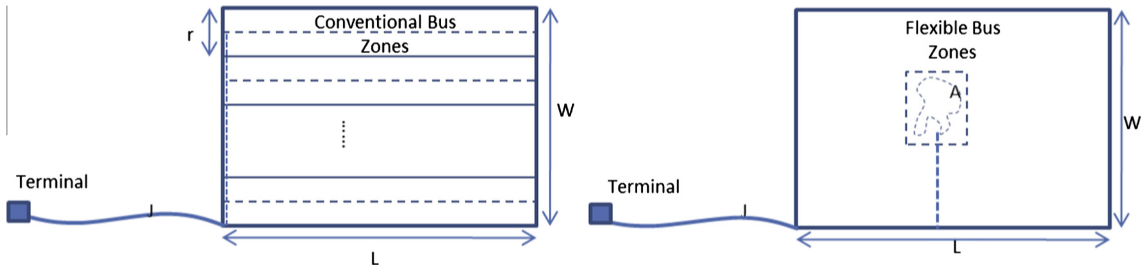

Fig. 2. Conventional and flexible service descriptions.

$$h^{ki}_{c\,max} = \frac{S_c l_c}{rL^k f Q^{ki}} \tag{2}$$

The minimum cost headway can be obtained from the partial derivative of Eq. (1) with respect to headway;

$$h^{ki}_{c\,min} = \sqrt{\frac{2D^k(a+bS_c)}{v_w L^k r^k Q^{ki} V^i_c}} \tag{3}$$

The optimal headway is then:

$$h^{ki}_{c\,opt} = \min\left\{\frac{S_c l_c}{r^k L^k f Q^{ki}}, \sqrt{\frac{2D^k(a+bS_c)}{v_w L^k r^k Q^{ki} V^i_c}}\right\} \tag{4}$$

The optimized headway obtained in Eq. (4) applies for optimizing the conventional bus fleet size for route k and period i $\left(F^{ki}_c = \frac{D^k W^k}{r^k h^{ki}_{c\,opt} V^i_c}\right)$. However, the resulting fleet size must be rounded off to an integer value. The modified headway h^{ki*}_c can be obtained with an integer value of fleet size ($h^{ki*}_c = \frac{D^k W^k}{r^k F^{ki}_{c\,opt} V^i_c}$).

The service cost for route k and in period i, is finally formulated by substituting the modified headway into Eq. (1):

$$SC^{ki*}_c = \frac{D^k W^k}{r h^{ki*}_c V^i_c}(a+bS_c) + v_v L^k W^k Q^{ki}\frac{M^k}{V^i_c} + v_w L^k W^k Q^{ki}\frac{h^{ki*}_c}{2} + \frac{v_x L^k W^k Q^{ki}(r+d)}{4V_x} \tag{5}$$

3.3. Flexible bus cost formulation

Similarly, flexible bus cost consists of bus operating cost, user in-vehicle cost and user waiting cost. Service cost for route k, in period i, SC^{ki}_f, is formulated as follows:

$$SC^{ki}_f = \frac{L^k W^k (a+bS_f)\left(D^k_f + \varnothing A^k\sqrt{\frac{Q^{ki}h^{ki}_f}{u}}\right)}{A^k V^i_f h^{ki}_f} + \frac{v_v L^k W^k Q^{ki}\left(D^k_f + \varnothing A^k\sqrt{\frac{Q^{ki}h^{ki}_f}{u}}\right)}{2V^i_f} + \frac{v_w L^k W^k Q^{ki} h^{ki}_f}{2} \tag{6}$$

3.4. Flexible bus optimized headway

Since we consider multiple periods, the optimized headway should be the maximum allowable headway or the minimum cost headway, whichever is smaller. The maximum allowable headway for route k and period i is:

$$h^{ki}_{f\,max} = \frac{S_f l_f}{A^k Q^{ki}} \tag{7}$$

The minimum cost headway can be obtained from the partial derivative Eq. (6) with respect to headway h^{ki}_f. An analytically optimized solution with respect to headway for a one route bus service is provided in Kim and Schonfeld (2012). However, since the partial derivative of Eq. (6) is analytically intractable, we find it numerically using existing computing software (i.e., MATLAB). The minimum cost headway $h^{ki}_{f\,min}$ can easily be obtained using a function called *fminbnd* in MATLAB (version R2011b). Thus, the optimized headway for flexible bus is:

$$h^{ki}_{f\,opt} = min\left\{\frac{S_f l_f}{A^k Q^{ki}}, h^{ki}_{f\,min}\right\} \tag{8}$$

The optimized fleet size for flexible bus is:

$$F^{ki}_{f\,opt} = \frac{L^k W^k \left(D^k_f + \varnothing A^k \sqrt{Q^{ki} h^{ki}_{f\,opt}/u} \right)}{V_s A^k h^{ki}_{f\,opt}} \tag{9}$$

Eq. (9), similarly to conventional bus fleet size ($F^{ki}_{c\,opt} = \frac{D^k W^k}{r^k h^{ki}_{c\,opt} V^i_c}$), must yield an integer value. The number of zones for flexible bus, $\frac{L^k W^k}{A^k}$, must have an integer value. Therefore, the remaining part of Eq. (9), $\frac{(D^k_f + \varnothing A^k \sqrt{Q^{ki} h^{ki}_{f\,opt}/u})}{V_s h^{ki}_{f\,opt}}$, should have an integer value. Since this part of the equation is a function of headway, we round off fleet size to an integer value, and then check if the modified headway violates the maximum allowable headway. The modified headway corresponding to an integer fleet size should not exceed the maximum allowable headway. The modified headway denoted as h^{ki*}_f provides minimum total service cost with an integer fleet size.

Minimum service cost for flexible bus operation with an integer fleet is obtained by substituting the modified headway into Eq. (6):

$$SC^{ki*}_f = \frac{L^k W^k (a + bS_f) \left(D^k_f + \varnothing A^k \sqrt{Q^{ki} h^{ki*}_f / u} \right)}{V^{ki}_f A^k h^{ki*}_f} + \frac{v_v L^k W^k Q^{ki}}{2} \frac{D^k_f + \varnothing A^k \sqrt{Q^{ki} h^{ki*}_f / u})}{V^{ki}_f} + v_w L^k W^k Q^{ki} \frac{h^{ki*}_f}{2} \tag{10}$$

3.5. Capital cost

After headways are optimized for each period, they and the round trip times determine fleet size. Thus, with optimized bus sizes, we have the required fleet matrix for each route and period. For capital cost, which is our fixed cost component, the required fleet size is the largest of the fleet sizes that are needed to serve any local region in any period (i.e. largest value among $\sum_{k=1}^{K} F^{k1}, \sum_{k=1}^{K} F^{k2}, \ldots, \sum_{k=1}^{K} F^{kI}$). Here, the capital cost units are \$/day.

4. Bus operation alternatives

4.1. Single Fleet Conventional Bus (SFC)

For SFC, a single conventional bus size covers all regions. Since the number of zones can differ by regions, there are $k + 1$ unknown variables (k = the number of regions). This bus size and the number of zones for each region must be optimized. Then, this integer number of zones for each region yields the route spacing in each region:

$$r^k = \frac{W^k}{N^k} \tag{11}$$

After vehicle size and route spacings are determined, headways and required fleets are optimized using Eqs. (2)–(6).

Total cost for SFC is formulated as:

$$TC = f(S_c, N, r, F, h) = (a_c + b_c S_c)F + \sum_k \sum_i SC^{ki}_c t^{ki} \tag{12}$$

Subject to

$$S_c = \text{integer} \quad \forall 1, \ldots S^{max}$$
$$N^k = \frac{W^k}{r^k} = \text{integer} \quad \forall 1, \ldots \frac{W^k}{r^{min}}$$
$$F^{ki} = \text{integer}$$
$$F^i = \sum_k F^{ki} \quad \forall \mathrm{k} = 1, \ldots, \mathrm{K}$$
$$\mathrm{F} \geqslant F^i \quad \forall \mathrm{i} = 1, \ldots, \mathrm{I}$$
$$0 \leqslant h^{ki} \leqslant \frac{S_c l_c}{r^k L^k f Q^{ki}}$$

SC^{ki}_c is provided in Eq. (5)

4.2. Single Fleet Flexible Bus (SFF)

Similarly to SFC, SFF has the same number of decision variables, flexible bus size and the number of zones for each region. The number of zones can be converted into service area (mi^2/zone) served by each flexible route tour:

$$A^k = \frac{L^k W^k}{N'^k} \tag{13}$$

With vehicle size and service area values for regions, headway and fleet size are optimized with Eqs. (7)–(10).

Total cost for SFF is formulated as:

$$TC = f(S_f, N', A, F, h) = (a_c + b_c S_f)F + \sum_k \sum_i SC_f^{ki} t^{ki} \tag{14}$$

Subject to

$S_f = \text{integer} \quad \forall 1, \ldots S^{max}$

$N'^k = \frac{L^k W^k}{A^k} = \text{integer} \quad \forall 1, \ldots \frac{L^k W^k}{A^{min}}$

$F^{ki} = \text{integer}$

$F^i = \sum_k F^{ki} \quad \forall k = 1, \ldots, \text{K}$

$\text{F} \geqslant F^i \quad \forall i = 1, \ldots, \text{I}$

$0 \leqslant h^{ki} \leqslant \frac{S_f l_f}{A^k Q^{ki}}$

SC_f^{ki} is provided in Eq. (10)

4.3. Mixed Fleets Conventional Bus (MFC)

For MFC, large and small conventional buses are used. To efficiently allocate demand between large and small buses, our approach identifies the threshold demand at which their costs are equal, using Eq. (1). Then, we obtain:

$$Q_t^{ki} = \frac{v_w l^2 S_l S_s V_c^i}{2aD^k r^k L^k f^2} \tag{15}$$

Thus, we use large buses when demand, Q^{ki}, exceeds the threshold, Q_t^{ki}, and small buses otherwise. After bus sizes are selected, the analytically optimized headway can also be found using Eqs. (2)–(5). One interesting finding from Eq. (15) is that the value of the passengers' in-vehicle time, v_v, does not affect the threshold demand.

4.4. Mixed Fleets Flexible Bus (MFF)

For MFF, we operate two sizes of flexible buses. To find the demand threshold, Q_t^{ki}, between these two, we first substitute the maximum allowable headway in Eq. (7) into flexible bus formulation in Eq. (6) to ensure acceptable headways, and then set the cost of large and small flexible buses to be equal, using Eq. (6).

$$Q_t^{ki} = \frac{v_w l}{2A^k}(S_s - S_l) \Bigg/ \left\{ \frac{(a + bS_l)\left(D_f^k + \varnothing\sqrt{\frac{S_l l A^k}{u}}\right)}{V_f^i l S_l} + \frac{v_v\left(D_f + \varnothing\sqrt{\frac{A^k S_l l}{u}}\right)}{2V_f^i} - \frac{(a + bS_s)\left(D_f^k + \varnothing\sqrt{\frac{S_s l A^k}{u}}\right)}{V_f^i l S_s} - \frac{v_v\left(D_f + \varnothing\sqrt{\frac{A^k S_s l}{u}}\right)}{2V_f^i} \right\} \tag{16}$$

For finding headways of MFF in each time period, Eqs. (7)–(10) are still applicable.

4.5. Mixed Fleets Variable Type Bus (MFV)

4.5.1. Demand threshold matrix over routes and periods

Anticipating that conventional bus has lower average cost than flexible bus at high demand densities, and vice versa, we must find the threshold demand for route k in period i, above which conventional bus is preferable and below which flexible bus is preferable. This threshold is obtained in Eq. (17) by setting Eqs. (1) and (6) to be equal:

$$Q_t^{ki} = \frac{\frac{v_w}{2}\left\{\frac{S_f l_f}{A^k} - \frac{S_c l_c}{r^k f L^k}\right\}}{\left\{\frac{D^k f(a + bS_c)}{V_c^i S_c l_c} - \frac{(a + bS_f)(D_f + \varnothing\sqrt{A^k S_f l_f / u})}{V_f^i S_f l_f} + \frac{v_v M^k}{V_c^i} - \frac{v_v(D_f + \varnothing\sqrt{A^k S_f l_f / u})}{2V_f^i} + \frac{v_x(r^k + d)}{4V_x}\right\}} \tag{17}$$

Using Eq. (17), for any combination of decision variables (S_c, S_f, r^k and A^k), we determine the demand threshold matrix for selecting the conventional or flexible mode.

5. Solutions for bus operations

5.1. Number of integer variables

The number of integer variables varies based on the types of bus operations. SFC, for instance, has $k + 1$ integer variables which are the vehicle size and number of zones for each of k regions. Thus, if we consider 4 regions, we have 5 integer variables. Then, these integer variables are used to analytically optimize headways and required fleet sizes over time. Similarly to SFC, SFF also has $k + 1$ integer variables.

However, mixed fleet operations with two bus sizes, such as MFC and MFF, have $k + 2$ integer variables. Lastly, MFV requires up to $2k + 2$ integer variables because two different bus sizes (i.e. large conventional bus size and small flexible bus size) as well as the numbers of zones for both conventional bus and flexible bus are needed.

5.2. Solution approach

As noted for various bus operation alternatives, the problem formulations are non-linear mixed-integer problems, which are known to be NP-hard. For optimizing our five alternatives, we propose a solution approach which combines analytic optimization with a genetic algorithm. To find a solution efficiently, we split variables into two groups. We have between $k + 1$ to $2k + 2$ integer decision variables (i.e., vehicle sizes, the number of zones for conventional bus, and the number of zones for flexible bus) that are optimized by the GA, depending on the type of bus operations. Then, analytic optimization determines headways and required fleet sizes based on the values of decision variables provided by the GA. Thus, the GA and analytic optimization work iteratively in this hybrid solution approach. The detailed interactions between GA and analytic optimization are shown in Fig. 3.

In this study, the role of GA is to find integer values of decision variables. To provide integer solutions, we use an Integer Genetic Algorithm (IGA), which is described below after a brief overview of genetic algorithms.

5.3. Integer Genetic Algorithm

Genetic Algorithms (GAs) are widely used for optimization problems. The GA concept was introduced by Holland (1975). A detailed background of GA may be found in Goldberg (1989). The way in which variables are coded can greatly affect a GA's efficiency. Real Coded Genetic Algorithms (RCGAs), which use real numbers for encoding, have faster convergence towards optimal than binary and gray coded GAs (Deb, 2000; Deep et al., 2009). The details, such as Laplace crossover, Power mutation, truncation procedure for integer restrictions and constraint handling techniques, can be found in Deep et al. (2009). Since this RCGA handles integer variables efficiently, we call this "Integer Genetic Algorithm (IGA), and use it to solve our nonlinear mixed integer formulations (MathWorks, 2011).

RCGAs attempt to minimize a penalty function, which includes a penalty term for infeasibility, rather than a normal fitness function. This penalty function is combined with binary tournament selection to select individual solutions for subsequent generations (Deb, 2000). According to Deb (2000), if the solution is feasible, the penalty function is the fitness function; however, if the solution is infeasible, the penalty function is the maximum fitness function among feasible solutions in the population, plus a sum of the constraint violations.

Since the method used here, which combines GA and analytic optimization, is partially heuristic, it does not guarantee a global optimum. However, this hybrid solution approach can provide a near-optimal solution quickly. We evaluate the proposed method with numerical examples in the following section.

6. Numerical analysis

To confirm that the proposed hybrid method minimizes cost efficiently, we solve a numerical example and compare its cost for various bus operations. Furthermore, we analyze sensitivity to important input parameters. In the following sections, a numerical case study and sensitivity analyses are presented.

6.1. Base Case

6.1.1. Inputs values

In the base numerical case, we consider four distinct local regions, each with four periods (i.e. $K = 4$ and $I = 4$). Demand, service time and line-haul distance are presented in Table 2. All other required input parameters are presented in Table 1.

6.1.2. Results of Base Case study

The detailed results obtained with our hybrid approach, combining IGA and analytic optimization, are shown in Table 3, including vehicle sizes, route spacings, optimal headways, required fleets, and corresponding costs. For SFC capital cost is \$9085/day and operation cost is \$ 145289.27/day; thus, total cost is \$154374.27/day.

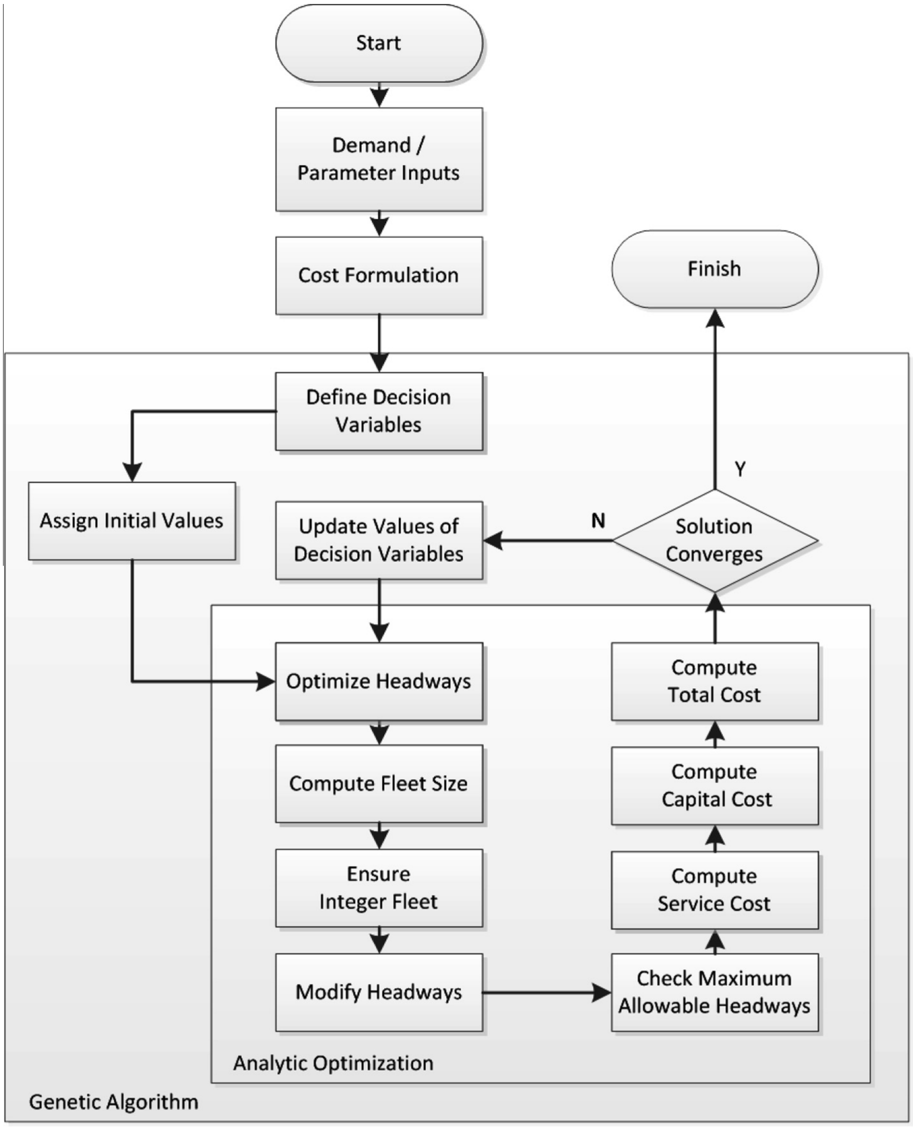

Fig. 3. Graphical description of solution approach.

For SFF, detailed results are shown in Table 4. The optimized flexible bus size is 19 seats/bus, and optimized service areas are 3.0, 2.5, 3.0, and 3.0 mile^2/bus for regions A, B, C, and D, respectively. This 19 seat bus serves all regions as well as all time periods. Total cost is \$151654.96/day, which is slightly lower than for SFC, mainly because input parameters for line-haul distance and length of local region are relatively small. This is further explored in the sensitivity analysis section.

For Mixed Fleet Conventional Bus (MFC), two bus sizes are optimized with 40 and 27 seats/bus. We also note that MFC's total cost (\$153640.08/day) is below that of SFC (\$154374.27/day, in Table 3). This result implies that, given significant demand variations, operating multiple sizes of buses can reduce capital cost and operation cost. With current input parameters, large conventional buses serve region D only in period 1 while all the other periods and regions are served by small conventional buses.

As noted for Table 6, sizes for flexible buses are below those of mixed conventional bus service (in Table 5) because flexible buses are preferred for lower demand areas. Vehicle sizes are optimized with 22 and 17 seats/bus for larger and smaller flexible bus, respectively. In this MFF operation, large flexible bus is preferable for period 1 in regions A and B. MFF's total cost is below that of Single Fleet Flexible Bus (SFF) in Table 4.

MFV operation has up to 10 decision variables, namely conventional bus and flexible bus sizes, four route spacings and four service areas. Optimized vehicle sizes are somewhere in between mixed fleet conventional buses and mixed fleet flexible buses. Except for MFV, Mixed Fleet Flexible Bus (MFF) operation is the least cost alternative. However, by considering different types of bus operations as well as different sizes of vehicles, MFV reduces total cost compared to MFF. Detailed results are presented in Table 7.

Table 2
Demand, service time, and line-haul distance.

	Region			
Period	A	B	C	D
Demand (trips/mile^2/h)				
1	70	80	60	55
2	30	35	40	40
3	10	15	30	15
4	5	7.5	10	5
Time (h)				
1	4	4	4	4
2	6	6	6	6
3	8	8	8	8
4	6	6	6	6
Line-haul distance (miles)	4	5	3	5
Length of region (miles)	3	2	4	5
Width of region (miles)	4	5	3	3

Table 3
SFC results.

	SFC Vehicle Size				Route Spacing for Conventional Bus			
					A	B	C	D
	30				1.00	1.00	0.75	0.75
	Conventional Bus Headway (h) Region				Conventional Bus Fleet Assignment (buses)			
Period	A	B	C	D	A	B	C	D
1	0.141	0.154	0.153	0.144	18	20	17	24
2	0.169	0.206	0.158	0.153	10	10	11	15
3	0.338	0.294	0.173	0.255	5	7	10	9
4	0.422	0.411	0.347	0.459	4	5	5	5
	Conventional Bus Cost ($/h)				Operation Cost × Time			
1	3581.93	3645.33	2903.51	3775.33	14327.73	14581.33	11614.02	15101.33
2	1533.20	1597.06	1757.02	2386.22	9199.20	9582.33	10542.11	14317.33
3	692.67	861.45	1414.80	1154.11	5541.33	6891.62	11318.40	9232.89
4	430.73	537.58	656.40	548.56	2584.40	3225.50	3938.40	3291.33

Total Operation Cost ($/day) = 145289.27, Total Capital Cost ($/day) = 9085, Total Cost ($/day) = 154374.27.

Table 4
SFF results.

	SFF Vehicle Size				Service Area for Flexible Bus			
					A	B	C	D
	19				3.00	2.50	3.00	3.00
	Flexible Bus Headway (h) Region				Flexible Bus Fleet Assignment (buses)			
Period	A	B	C	D	A	B	C	D
1	0.090	0.094	0.098	0.115	38	37	32	41
2	0.139	0.156	0.119	0.129	16	15	18	25
3	0.295	0.240	0.138	0.228	7	9	15	13
4	0.379	0.421	0.266	0.459	5	5	7	6
	Flexible Bus Cost ($/h)				Operation Cost × Time			
1	3536.44	3449.17	2920.60	3889.67	14145.75	13796.68	11682.39	15558.68
2	1343.78	1347.03	1592.10	2280.22	8062.695	8082.155	9552.63	13681.3
3	603.98	721.93	1268.52	1080.88	4831.873	5775.41	10148.17	8647.04
4	376.32	457.32	567.73	512.66	2257.946	2743.901	3406.406	3075.932

Total Operation Cost ($/day) = 135448.96, Total Capital Cost ($/day) = 16,206, Total Cost ($/day) = 151654.96.

6.1.3. Benefits of sharing fleets

When demands vary over time and among local regions, fleets can be shared among regions as well as time periods. For instance, fleets that are used for peak periods can also be used for other periods or other regions without additional capital costs. This sharing of fleets can significantly reduce capital costs; to realize such savings, we constrain vehicle size(s) to be consistent throughout regions and times. Table 8 shows that the cost of our integrated multi-zone approach can be significantly lower than the sum of four separately optimized costs.

6.2. Solution reliability

Since a GA is heuristic and does not guarantee global optimality, we check here the solutions' reliability. We run the same instance 20 times. MFV is shown in Fig. 4 since it is the most complex and computationally demanding among our five alter-

Table 5
MFC results.

	Vehicle Size				Route Spacing for Conventional Bus			
	Large Conv. Bus		Small Conv. Bus		A	B	C	D
	40		27		1.00	1.00	0.75	1.00
	Large Conventional Bus Headway (h) Region				Small Conventional Bus Headway (h)			
Period	A	B	C	D	A	B	C	D
1	0.000	0.000	0.000	0.144	0.127	0.154	0.144	0.000
2	0.000	0.000	0.000	0.000	0.169	0.187	0.158	0.132
3	0.000	0.000	0.000	0.000	0.338	0.294	0.173	0.215
4	0.000	0.000	0.000	0.000	0.422	0.411	0.347	0.431
	Large Conventional Bus Fleet Assignment (buses)				Small Conventional Bus Fleet Assignment (buses)			
1	0	0	0	18	20	20	18	0
2	0	0	0	0	10	11	11	13
3	0	0	0	0	5	7	10	8
4	0	0	0	0	4	5	5	4
	Mixed Fleet Conventional Bus Service Cost ($/h)				Operation Cost × Time			
1	3571.00	3633.33	2892.00	3842.83	14284.00	14533.33	11568.00	15371.33
2	1527.20	1587.21	1750.42	2412.23	9163.20	9523.28	10502.51	14473.41
3	689.67	857.25	1408.80	1126.99	5517.33	6858.02	11270.40	9015.93
4	428.33	534.58	653.40	519.74	2570.00	3207.50	3920.40	3118.43

Total Operation Cost ($/day) = 144897.08, Total Capital Cost ($/day) = 8743, Total Cost ($/day) = 153640.08.

Table 6
MFF results.

	Vehicle Size				Service Area for Flexible Bus			
	Large Flex.Bus		Small Flex. Bus		A	B	C	D
	22		17		3.00	2.50	3.00	3.00
	Large Flexible Bus Headway (h) Region				Small Flexible Bus Headway (h)			
Period	A	B	C	D	A	B	C	D
1	0.097	0.105	0.000	0.000	0.000	0.000	0.094	0.101
2	0.000	0.000	0.000	0.000	0.139	0.156	0.110	0.129
3	0.000	0.000	0.000	0.000	0.295	0.240	0.138	0.228
4	0.000	0.000	0.000	0.000	0.379	0.338	0.266	0.459
	Large Flexible Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
1	36	34	0	0	0	0	33	45
2	0	0	0	0	16	15	19	25
3	0	0	0	0	7	9	15	13
4	0	0	0	0	5	6	7	6
	Mixed Fleet Conventional Bus Service Cost ($/h)				Operation Cost × Time			
1	3559.10	3466.35	2907.78	3889.18	14236.40	13865.41	11631.12	15556.72
2	1337.38	1341.03	1582.63	2270.22	8024.29	8046.15	9495.77	13621.30
3	601.18	718.33	1262.52	1075.68	4809.47	5746.61	10100.17	8605.44
4	374.32	447.65	564.93	510.26	2245.95	2685.89	3389.61	3061.53

Total Operation Cost ($/day) = 135121.84, Total Capital Cost ($/day) = 16,233, Total Cost ($/day) = 151354.84.

Table 7
MFV results.

	Vehicle Size		Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Large Conv. Bus	Small Flex. Bus	A	B	C	D	A	B	C	D
	31	16	1.00	–	0.75	0.75	4.00	3.33	4.00	7.50
	Large Conventional Bus Headway (h) Region				Small Flexible Bus Headway (h)					
Period	A	B	C	D	A	B	C	D		
1	0.141	0.000	0.153	0.150	0.000	0.060	0.000	0.000		
2	0.000	0.000	0.000	0.153	0.125	0.127	0.092	0.000		
3	0.000	0.000	0.000	0.000	0.240	0.224	0.114	0.135		
4	0.000	0.000	0.000	0.000	0.404	0.338	0.218	0.298		
	Large Conventional Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)					
1	18	0	17	23	0	45	0	0		
2	0	0	0	15	15	15	19	0		
3	0	0	0	0	7	8	15	12		
4	0	0	0	0	4	5	7	5		
	Mixed Fleet Bus Service Cost ($/h)				Operation Cost × Time					
1	3585.53	3576.37	2906.91	3774.82	14342.13	14305.48	11627.62	15099.28		
2	1330.98	1320.51	1593.94	2389.22	7985.91	7923.05	9563.62	14335.33		
3	573.37	690.51	1258.39	1034.28	4586.97	5524.10	10067.11	8274.24		
4	359.07	423.46	541.43	439.51	2154.44	2540.74	3248.55	2637.04		

Total Operation Cost ($/day) = 134215.62, Total Capital Cost ($/day) = 11,991, Total Cost ($/day) = 146206.62.

natives. 17 of 20 runs yield the same consistent minimized value while the remaining 3 runs yield a slightly costlier value (by less than 0.3%). SFC, SFF, MFC, and MFF yield results faster because their search boundaries are much smaller. Their results (not shown here) are also more consistent than those for MFV. Therefore, we apply IGA so that the hybrid approach can find near-optimal solutions for these five alternative models.

Fig. 5 shows the convergence of IGA. MFV, the most complex alternative, converges relatively quickly (i.e. in less than 50 generations). However, we run every instance up to 250 generations to carefully check cost variations. The detailed IGA setting is provided in the Appendix B.

Fig. 5 shows that solutions converge quickly. To assess the quality of the solution obtained from our partially heuristic hybrid approach without knowing the actual globally optimal solution we use the following statistical approach (Jong and Schonfeld 2003; Wang and Schonfeld 2012). We generate one million random candidate solutions, and then compare them to the solution provided by the hybrid algorithm. The computation time for generating a million random solutions was about 12.8 h with a quad core processor (Intel(R) Core™ i7-3610QM CPU @ 2.30 GHz). The best of the million candidate solutions is $147563.32/day, which is 0.92% costlier than our hybrid solution. The average of the 10 best random solutions is $147941.15/day, which is 1.17% costlier than our solution.

We additionally design a small problem (a terminal connecting two local regions with four time periods) to check the solution quality by comparing solutions obtained with our method with the optimal solution obtained through complete enumeration. Since complete enumeration is only used to validate the solution quality of our approach, its computational time is not a great concern in this test. Input values for this complete enumeration are shown in Table 9. For the other input values, the Notation Table 1 is still applicable, and the same units are also applied.

For this verification, a total of 185,856 candidate solutions (7–50 conventional bus seats * 7–50 flexible bus seats * up to 2 conventional bus zones for zone A * up to 2 conventional bus zones for region B * up to 6 flexible bus zones for region A * up to 4 flexible bus zones for region B) are compared.

Comparison of results with those of complete enumeration shows that they are identical. We only compare here the MVF results (in Tables 10 and 11) because MVF has more optimizable variables and is more general than the other alternatives.

With two local regions, we confirm that our hybrid method finds identical solutions to those from complete enumeration. Thus, depending on the problem sizes, our proposed solution can provide optimal or near optimal solutions. Since MFV has a more complex formulation than the other alternatives, repeating such tests seems unnecessary for SFC, SFF, MFC, or MFF.

Table 8
Comparison of Integrated and Separately Optimized Costs.

	Vehicle Size		Total Cost	
Total Cost	Large Conv. Bus	Small Flex. Bus	Regional Cost	Total Cost
Integrated System	31	16	–	146206.6
Region A only	30	16	32745.0	149437.8
Region B only	25	17	34179.2	
Region C only	32	17	38197.2	
Region D only	31	16	44298.4	

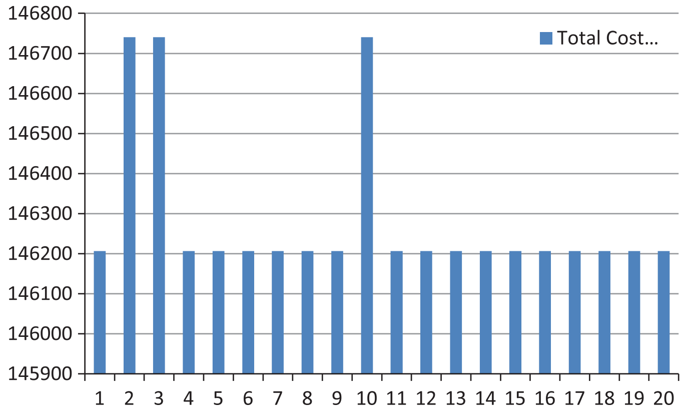

Fig. 4. Reliability of IGA.

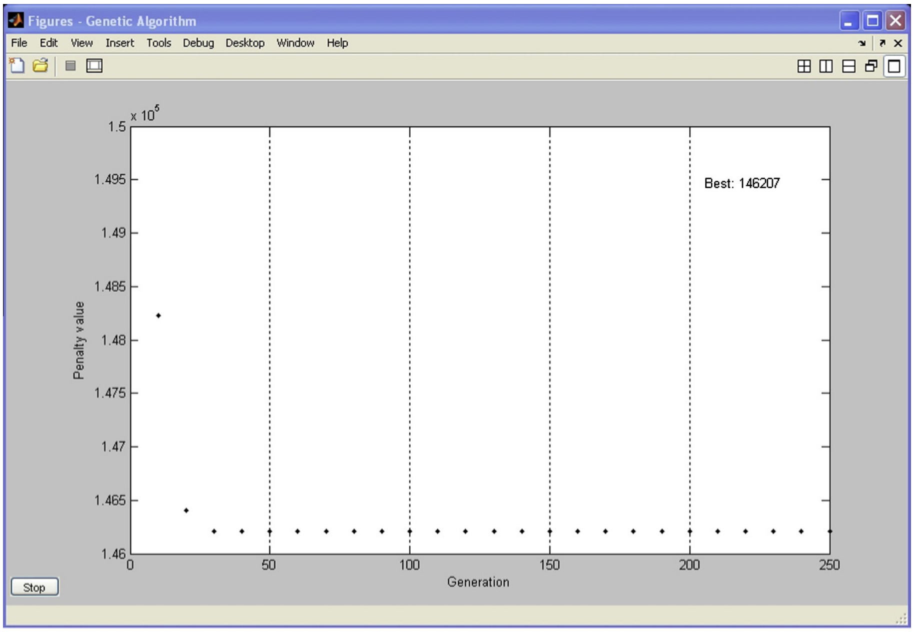

Fig. 5. Convergence of IGA to the MFV.

6.3. Sensitivity analysis

In this section, we analyze the sensitivity of results to important input factors. From this analysis, we can note how total cost and other optimized characteristics change from baseline values and vary among alternatives. Here we mainly summarize the cost comparisons among the five alternatives, (SFC, SFF, MFC, MFF and MFV).

Table 9
Input values for a complete enumeration.

Parameter		Region A	Region B
J		4	5
L		3	2
W		2	2
Q	Period 1	100	80
	Period 2	50	40
	Period 3	10	30
	Period 4	5	5

6.3.1. Sensitivity analysis inputs

6.3.1.1. Demand. To explore how bus operations change mainly with demand density, we multiply demand inputs by 10, as shown in Table 12.

6.3.1.2. Line-haul distance. First, we increase line-haul distance by five miles, as shown in Table 13. In this case, we explore how distances between the terminal (or CBD) and local regions affect system costs for five alternative operations.

6.3.1.3. Value of waiting time. Waiting time value is important in this study because if affects the optimized headway, user cost and the threshold demands for mixed fleets operations such as MFC, MFF, and MFV. In this sensitivity analysis, the value of waiting time is increased by 40% from 12 to $16.8/h.

6.3.2. Sensitivity analysis results

For three important parameters, namely demand, line-haul distance, and value of waiting time, Fig. 6 shows total cost variations. Sensitivity analysis results in Fig. 6 shows that the MFV provide the minimum total cost among five operations. Table 14 also summarizes sensitivity results.

When we analyze Base Case results, MFV reduces total cost by 5.29, 3.59, 4.84, and 3.40% compared to SFC, SFF, MFC, and MFF, respectively. Since Base Case input demands are relatively favorable to flexible bus operations, SFC has the highest cost here.

Demand is an obviously important factor for public transit analysis. Therefore, we first multiply all demands by 10 to explore resulting changes in system characteristics. Consequently, total costs increase by 532–568% compared to the corresponding Base Case. MFV provides cost reductions about 4.32–4.10% compared to flexible bus operations such as SFF and MFF. It is also notable that MFC does not provide any cost reduction because all demands are assigned to the larger conventional bus based on the threshold demand in Eq. (15). This implies that when demand density exceeds some level, mixed conventional bus operation is no longer beneficial, unless perhaps unusually large buses could be operated. MFV and SFC differ very slightly (0.70%) in total cost. This implies that flexible bus operation may not be very attractive when demand is high. Nonetheless, providing different bus sizes and different types of operations reduces cost when demand densities vary greatly over time and over regions.

Line-haul distance directly affects travel time. When line-haul distance increases by 5 miles for all local regions, we note that total costs increase by about 21–25% compared to the corresponding Base Case operations. More specifically, the costs of conventional bus operations such as SFC or MFC increase by 21.45–21.56% while the costs of flexible bus operations by SFF or MFF increase by 24.95–24.99%. This line-haul distance change increases total cost significantly, and implies that conven-

Table 10
Result comparison.

	Complete enumeration	Our method
Vehicle Size (Sc, Sf)	37,15	37,15
Conventional Bus Zones (A, B)	(2,2)	(2,2)
Flexible Bus Zones (A, B)	(2,1)	(2,1)
Total Cost	33452.64	33452.64

Table 11
Resulting fleet sizes.

(Conv, Flex)		Region A	Region B
Periods	1	(10,0)	(0,18)
	2	(0,11)	(0,7)
	3	(0,3)	(0,5)
	4	(0,2)	(0,1)

Table 12
Demand input.

	Region			
Period	A	B	C	D
Demand (trips/mile^2/h)				
1	700	800	600	550
2	300	350	400	400
3	100	150	300	150
4	50	75	100	50

Table 13
Line-haul distance by region.

Region	A	B	C	D
Line-haul distance (miles)	9	10	8	10

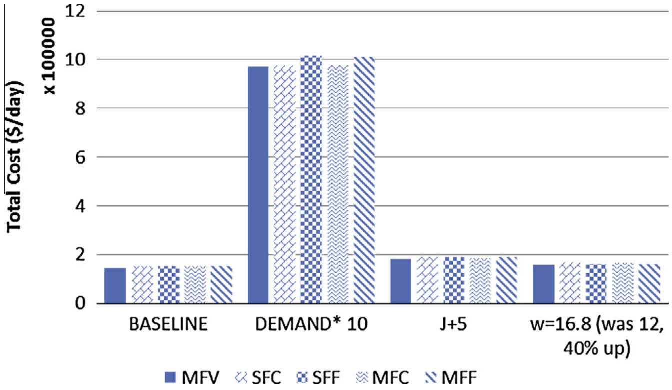

Fig. 6. Sensitivity analysis results.

Table 14
Results comparison among various sensitivity inputs.

	MFV	SFC	SFF	MFC	MFF
BASELINE	146206.6	154374.3	151655.0	153640.1	151354.8
Savings bet. MFV and alternatives		5.29%	3.59%	4.84%	3.40%
DEMAND* 10	970303.3	977175.0	1014112.7		1011816.9
Savings bet. MFV and alternatives		0.70%	4.32%		4.10%
Savings bet. Same Services	563.65%	532.99%	568.70%		568.51%
$J+5$	180483.5	187482.2	189549.9	186763.2	189123.3
Savings bet. MFV and alternatives		3.73%	4.78%	3.36%	4.57%
Savings bet. Same Services	23.44%	21.45%	24.99%	21.56%	24.95%
$w = 16.8$ (was 12, 40% up)	156887.3	166984.0	161966.6	166226.8	161532.3
Savings bet. MFV and alternatives		6.05%	3.14%	5.62%	2.88%
Savings bet. Same Services	7.31%	8.17%	6.80%	8.19%	6.72%

** Savings bet. MFV and alternatives = (Alternative-MFV)/alternative.
** Savings bet. Same Services = (Sensitivity − BASELINE)/BASELINE, for each service.

tional bus operations may be preferable with long line-haul distances. Still, MFV operation is the most promising alternative among MFV, SFC, SFF, MFC, and MFF. MFV reduces total cost about 3.36–4.78% compared to the other alternatives.

With a waiting time value change, MFV reduces total cost by 2.88–6.05%., especially in comparison with conventional bus (SFC, MFC). As shown in Tables 20–24, a higher value of waiting time results in higher bus frequencies, larger fleets, increased bus size(s), and increased route spacings (or service areas). Compared to the Base Case results, the total costs increase by 6.72–8.19% when the waiting time value increases by 40%.

7. Conclusions

The primary purpose of this paper is to investigate the merits of integrating different types of bus services and fleets over different regions and periods and explore how the variability of demand over time and across regions affects the desirability of integration. We found no comparable studies that explored the possibility of such integration, even macroscopically.

To reduce total costs when demand and other factors vary considerably over times and over regions, the Mixed Fleet Variable Type Bus (MFV) operation is preferable to the other alternatives. In order to compare the performances of MFV, we also formulate four other types of bus operations, namely Single Fleet Conventional Bus (SFC), Single Fleet Flexible Bus (SFF), Mixed Fleet Conventional Bus (MFC), and Mixed Fleet Flexible Bus (MFF). For mixed fleet operations (i.e. MFC, MFF, and MFV), the demand thresholds between using large or small buses are analytically formulated using bus operation cost functions.

To solve these five different problems (nonlinear mixed integer problem formulations) efficiently, we propose a hybrid solution approach combines an Integer Genetic Algorithm (IGA) and analytic optimization. Such a hybrid algorithm helps reduce the computation time because some variables (i.e. headways and resulting fleets) are optimized analytically.

To examine the quality of solutions, we randomly generate one million candidate solutions and compare them to the best solution found by the hybrid algorithm. We found that the solution from our approach is superior to any of the million random solutions. We also generate a small problem (i.e., two regions with four periods) to obtain complete enumeration solutions. Our hybrid method finds the identical solution obtained through complete enumeration. Thus, we find that our proposed hybrid method yields solutions that are at least near-optimal.

As shown in Table 8, we explore the benefits of sharing fleets throughout the system. To do that, we optimize common vehicle size(s) over regions as well as periods. Through numerical evaluation, we find that the cost of an integrated multi-zone system is more economical than the sum of separately optimized results. Numerical evaluation also shows that MFV can yield significantly lower costs than the other four alternatives. Other numerical cases and sensitivity analyses confirm that the proposed approach finds good solution quickly.

Acknowledgements

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Appendix A. Demand threshold derivations on mixed fleet services

A.1. Mixed Fleets Conventional Bus Service threshold

For MFC, we operate two sizes of conventional buses. To find the demand threshold, Q_t^{ki}, we first substitute the maximum allowable headway in Eq. (A.2) into conventional bus formulation in Eq. (A.1) to ensure acceptable headways. Then, Eq. (A.1) becomes Eq. (A.3).

$$SC_c^{ki} = \frac{D^k W^k (a + bS_c)}{r^k V_c^i h_c^{ki}} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w L^k W^k Q^{ki} h_c^{ki}}{2} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} \tag{A.1}$$

$$h_{c\,max}^{ki} = \frac{S_c l_c}{r L^k f Q^{ki}} \tag{A.2}$$

$$SC_c^{ki} = \frac{D^k L^k W^k f Q^{ki} (a + bS_c)}{V_c^i S_c l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_c l_c}{2rf} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} \tag{A.3}$$

The costs of large and small conventional bus are formulated in Eqs. (A.4) and (A.5), respectively:

$$SC_l^{ki} = \frac{D^k L^k W^k f Q^{ki} (a + bS_l)}{V_c^i S_l l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_l l_c}{2rf} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} \tag{A.4}$$

$$SC_s^{ki} = \frac{D^k L^k W^k f Q^{ki} (a + bS_s)}{V_c^i S_s l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_s l_c}{2rf} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} \tag{A.5}$$

Then, we set Eqs. (A.4) and (A.5) to be equal, and solve them in terms of demand. The equation we obtain here is same as Eq. (15):

$$\begin{aligned}
&\frac{D^k L^k W^k f Q^{ki} (a + bS_l)}{V_c^i S_l l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_l l_c}{2rf} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} = \frac{D^k L^k W^k f Q^{ki} (a + bS_s)}{V_c^i S_s l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_s l_c}{2rf} + \frac{v_x L^k W^k Q^{ki} (r^k + d)}{4V_x} \\
&\rightarrow \frac{D^k f Q^{ki} (a + bS_l)}{V_c^i S_l l_c} + \frac{v_w S_l l_c}{2r^k f L^k} = \frac{D^k f Q^{ki} (a + bS_s)}{V_c^i S_s l_c} + \frac{v_w S_s l_c}{2r^k f L^k} \\
&\rightarrow Q^{ki} \left\{ \frac{D^k f (a + bS_l)}{V_c^i S_l l_c} - \frac{D^k f (a + bS_s)}{V_c^i S_s l_c} \right\} = \frac{v_w l_c}{2r^k f L^k} (S_s - S_l) \\
&\rightarrow Q_t^{ki} = \frac{v_w l_c^2 V_c^i S_l S_s}{2ar^k f^2 L^k D^k}
\end{aligned} \tag{A.6, 15}$$

A.2. Mixed Fleets Flexible Bus service threshold

Similarly to MFC, MFF uses two sizes of flexible buses. We substitute the maximum allowable headway in Eq. (A.8) into the flexible service cost function in Eq. (A.7) to ensure acceptable headways:

$$SC_f^{ki} = \frac{L^k W^k (a + bS_f)\left(D_f^k + ØA^k\sqrt{\frac{Q^{ki}h_f^{ki}}{u}}\right)}{A^k V_f^i h_f^{ki}} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + ØA^k\sqrt{\frac{Q^{ki}h_f^{ki}}{u}}\right)}{2V_f^i} + \frac{v_w L^k W^k Q^{ki} h_f^{ki}}{2} \tag{A.7}$$

$$h_{f\,max}^{ki} = \frac{S_f l_f}{A^k Q^{ki}} \tag{A.8}$$

$$SC_f^{ki} = \frac{Q^{ki} L^k W^k (a + bS_f)\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{V_f^i S_f l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_f l_f}{2A^k} \tag{A.9}$$

Then, the costs of large and small flexible services are formulated in Eqs. (A.10) and (A.11), respectively:

$$SC_l^{ki} = \frac{Q^{ki} L^k W^k (a + bS_l)\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{V_f^i S_l l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_l l_f}{2A^k} \tag{A.10}$$

$$SC_s^{ki} = \frac{Q^{ki} L^k W^k (a + bS_s)\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{V_f^i S_s l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_s l_f}{2A^k} \tag{A.11}$$

Now, we set Eqs. (A.10) and (A.11) to be equal, and find the threshold demand in Eq. (A.12):

$$\frac{Q^{ki} L^k W^k (a + bS_l)\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{V_f^i S_l l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_l l_f}{2A^k} = \frac{Q^{ki} L^k W^k (a + bS_s)\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{V_f^i S_s l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_s l_f}{2A^k}$$

$$\rightarrow Q^{ki}\left\{\frac{(a + bS_l)\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{V_f^i S_l l_f} - \frac{(a + bS_s)\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{V_f^i S_s l_f} + \frac{v_v\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{2V_f^i} - \frac{v_v\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{2V_f^i}\right\} = \frac{v_w l_f}{2A^k}(S_s - S_l)$$

$$\rightarrow Q_t^{ki} = \frac{v_w l_f}{2A^k}(S_s - S_l) \Bigg/ \left\{\frac{(a + bS_l)\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{V_f^i S_l l_f} - \frac{(a + bS_s)\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{V_f^i S_s l_f} + \frac{v_v\left(D_f^k + Ø\sqrt{\frac{A^k S_l l_f}{u}}\right)}{2V_f^i} - \frac{v_v\left(D_f^k + Ø\sqrt{\frac{A^k S_s l_f}{u}}\right)}{2V_f^i}\right\} \tag{A.11}$$

A.3. Mixed Fleets Variable Type Bus service threshold

For the threshold demand between large conventional bus and small flexible bus services, we set the Eqs. (A.3) and (A.9) to be equal. Then the threshold Eq. (A.12) is:

$$\frac{D^k L^k W^k f Q^{ki}(a + bS_c)}{V_c^i S_c l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_c l_c}{2rf} + \frac{v_x L^k W^k Q^{ki}(r^k + d)}{4V_x} = \frac{Q^{ki} L^k W^k (a + bS_f)\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{V_f^i S_f l_f} + \frac{v_v L^k W^k Q^{ki}\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{2V_f^i} + \frac{v_w L^k W^k S_f l_f}{2A^k}$$

$$\rightarrow \frac{D^k f Q^{ki}(a + bS_c)}{V_c^i S_c l_c} + \frac{v_v Q^{ki} M^k}{V_c^i} + \frac{v_w S_c l_c}{2rfL^k} + \frac{v_x Q^{ki}(r^k + d)}{4V_x} = \frac{Q^{ki}(a + bS_f)\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{V_f^i S_f l_f} + \frac{v_v Q^{ki}\left(D_f^k + ØA^k\sqrt{\frac{S_f l_f}{uA^k}}\right)}{2V_f^i} + \frac{v_w S_f l_f}{2A^k}$$

$$\rightarrow Q^{ki}\left\{\frac{D^k f(a + bS_c)}{V_c^i S_c l_c} + \frac{v_v M^k}{V_c^i} + \frac{v_x Q(r^k + d)}{4V_x} - \frac{(a + bS_f)\left(D_f^k + Ø\sqrt{\frac{S_f l_f A^k}{u}}\right)}{V_f^i S_f l_f} - \frac{v_v\left(D_f^k + Ø\sqrt{\frac{S_f l_f A^k}{u}}\right)}{2V_f^i}\right\} = \frac{v_w S_f l_f}{2A^k} - \frac{v_w S_c l_c}{2r^k f L^k}$$

$$\rightarrow Q^{ki} = \frac{\frac{v_w}{2}\left(\frac{S_f l_f}{A^k} - \frac{S_c l_c}{r^k f L^k}\right)}{\left\{\frac{D^k f(a+bS_c)}{V_c^i S_c l_c} + \frac{v_v M^k}{V_c^i} + \frac{v_x Q(r^k+d)}{4V_x} - \frac{(a+bS_f)\left(D_f^k + Ø\sqrt{\frac{S_f l_f A^k}{u}}\right)}{V_f^i S_f l_f} - \frac{v_v\left(D_f^k + Ø\sqrt{\frac{S_f l_f A^k}{u}}\right)}{2V_f^i}\right\}} \tag{A.12}$$

Appendix B. Integer Genetic Algorithm (IGA) setting in global optimization toolbox in MATLAB

As mentioned previously, IGA finds a solution using special creation, crossover, and mutation functions to enforce integer values (Deep et al, 2009). To insure integer decision variables, the population type should be ''double vector'' rather than ''bit string'' or ''custom setting'' (MathWorks, 2011).

For optimizing our five bus operation alternatives, we must set a population size, elite counts, and the number of generations. Here we set a population size of 100, 10 elite counts, and up to 250 generations. To use this IGA algorithm, we must provide bounds for each decision variable. For both conventional and flexible buses we specify a range of 1 to 50 seats/bus.

To optimize route spacings for conventional bus, we must first optimize the number of zones in each region. These optimized numbers of zones are convertible into route spacings. The minimum number of zones is set to be one; in this case, one conventional bus serves an entire local region. The minimum specified route spacing (0.5 miles here) determines the maximum number of zones for each region. Bounds are also needed for the service area of flexible bus. A minimum service area (one mile^2 is assumed here for all regions) determines the maximum number of zones for the various regions. The minimum number of zones is one, similarly to conventional bus; in this case, flexible buses serve the entire region (undivided into zones).

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