Mathematical Model for Dial a Ride Problem with Time Windows, case: hemodialysis patients transportation

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Abstract— **End stage renal disease (ESRD) has become a very common disease nowadays as the number of patients is increasing. As a result, an increase in the number of dialysis treatment is noticed. Many dialysis patients treated at dialysis facilities require six trips each week, three trips to dialysis centersand three trips from the dialysis centers. This comes to more than 300 trips each year per patient. This paper focuses on the transportation of patients undergoing the hemodialysis, as they request rides to their treatments. The transportation problem for hemodialysis patients comes with challenges such as the scheduling problems, extra care and support for the patients andthe increased cost of the trips. This paper focuses only on reducingthe total cost of transportation. We consider the hemodialysis patients' transportation problem as the problem of determining an optimal routing for a fleet of vehicles used to cover the regular trips requested. The rides must respect some real-life constraints such as the maximum vehicle capacity, availabilityof the vehicle, patient's schedule… etc. The study focuses on the dial a ride problem with a time window (DARPTW), which is a type of Vehicle Routing Problem (VRP) that is known in the literatureas a complex combinatorial optimization problem related to transportation. Mainly used for transporting individuals with requests. In which a set of users must be picked up from an originlocation and they must be delivered to a destination location. The problem is modeled mathematically as a static dial a ride problem (DARP) where the aim or the objectif function is to minimize the total cost of the transportation.**

Keywords—transportation, dial-a-ride problem, vehicle routing problem, hemodialysis.

I. Introduction

One of the variant of VRPs is Dial-a-Ride Problem with Time Window, which is mainly used for transporting individuals with requests. That is the case studied in this work, as our goal is to satisfy patients undergoing hemodialysis, theirs requests are received as trips to theirs dialysis facilities. The objective is to mathematically model our problem where the objectif function minimizes the total distance travelled, while respecting some constraints.

The remaining of this paper is as follows: the next section presents the literature review on DARP. Then, Section III describes the problem under study. Section IV is reserved to the mathematical model. The computational results are shown in section V. Finally, section VI is dedicated to the conclusion and recommendations for future works.

II. Literature review on DARP

Surprisingly only few publications have been released on the PADPTW. The interest of the reader should be referred to [1] and [2] for their interesting researches on VRPs with a special consideration of pickup and delivery problems. Moreover, Savelsbergh and Sol presented a survey of the problem types and solution methods found in literature about general pickup and delivery problem (PDP) [3].

In literature review, there is a distinguish between exact and approximate methods in solving PDPs. Psaraftis developed the first exact dynamic programming algorithms to solve single vehicle dial a-ride problems (DARP). He developed a backward dynamic programming method where he imposes a maximum profit shift with respect to the order of the pickup and delivery requested times. Psaraftis then modified his algorithm to take into account the time windows at every pickup and delivery points. The main difference between the two is the use of forward dynamic programming instead of backward. These algorithms could solve only small problem instances involving 10 or fewer customers [4]. Kalantari et al. used a branch and bound algorithm for the pickup and delivery problem with time windows (PDPTW) with finite and infinite vehicle capacity, his method solves up to 37 customer instances [5].

For the approximate algorithms, literature has more focusedon insertion algorithms. These algorithms try to insert customers in a partial route, while conserving its feasibility. Sexton and Bodin presented an insertion method for the single vehicle PDPTW [6]. In other papers, the same authors investigated the DARP in which every customer specifies his pickup or delivery time, they separate the routing and scheduling parts ([7] and [8]).

Nanry and Barnes presented a reactive tabu search for the PDPTW where each transportation request has a single origin and a single destination [9]. Cordeau and Laporte were among the first onesto presented a tabu search algorithm for the DARP. Besides using a simple neighborhood operator to generate the neighborhood, this heuristic has shown to be effective and efficient [10]. For that reason, many of the recent studies of tabu search application on DARPs ([11], [12], [13], [14], [15] and [16]) are in fact inspired from Cordeau and Laporte algorithm presented in

[17]. These studies are typically on DARPs with more complicated and real-life constraints.

In [17], the authors adapted tabu search to handle the more complex DARPs. Usually, the most time consuming task with the tabu search algorithm is the evaluation of the neighborhood. To speed up the evaluation, some may only consider moves within a certain threshold [15] while others do a random sampling [16]. Tabu search works well as a standalone method, but it also shows to work well when incorporating into a multi-start heuristic [17] or a multi-criteria framework [14].

III. Problem Description

One of the variants of VRPs is Dial-a-Ride Problem with Time Window, which is mainly used for transporting individuals with requests. That is the case studied in this work, as our goal is to satisfy patients undergoing hemodialysis, their requests are received as trips to their dialysis facilities.

The problem is composed of multiple vehicles where each vehicle needs a planned routing; each vehicle/ambulance has a starting point and an ending point (generally, they have the same one). In Addition, the vehicles have the mission of satisfying a set of requests fixed by the patients. Every request has two nodes; an origin node and a destination node. The patients are to be picked from their origins in order to be transported to their destinations.

The aim is to optimize door-to-door transportation for a group of patients, in which they specify their origins and destinations and their time windows. To attain the proposed aim, a mathematical model was developed.

The problem is then solved using LINGO optimizer for a case study and the optimization results are presented by the end of the paper. Figure 1 shows an example of how each patient has two trips; one to the dialysis center (from origin to destination) andthe second back home (from destination to origin). Ideally, the patient transport service would carry one patient at a time on each route. That would not give an optimal solution since it requires many resources both human and in capital (vehicles for instance). In order to reduce the cost of transport, weshould consider carpooling as a solution for patients who do not require individual transportation. From the NEMT's point of view, vehicle routing problem aims for the determination of the optimal set of routes to be performed by a fleetof ambulances to serve a given set of patients where no ambulance can service more patients than its capacity permits.

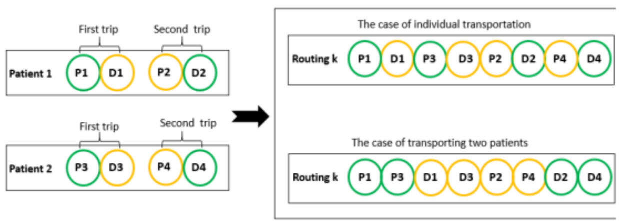

Fig. 1. Routing of patient 1 and patient 2 in two cases

IV. Mathematical Model

We suppose that m vehicles of the same type pick up the n patients. Each vehicle can give a ride to a maximum ofthree patients at the same time. Since we are working on a static model, we will model the problem for just one period. The model formulated involves a series of, sets, parameters and decision variables that will be used to define the model constraints and the objectif function.

The problem is defined on a complete graph $\boldsymbol{G = (V, A)}$ with: $\boldsymbol{V = \{v_0, v_1, \ldots, v_{2n}\}}$ represents the vertex set. $\boldsymbol{A = \{(v_i, v_j): v_i, v_j \in V, i \neq j\}}$ represents the arc set. Vertex $\boldsymbol{v_0}$ represents the depot of the fleet of $\boldsymbol{m}$ vehicles, and the rest of the $\boldsymbol{2n}$ vertices represent origins and destinations for the transportation requests.

Each pair of vertices $\boldsymbol{(v_i, v_{i+n})}$ represents a request $\boldsymbol{r}$ for transportation from origin $\boldsymbol{v_i}$ to destination $\boldsymbol{v_{i+n}}$. Then back to his origin. Being $\boldsymbol{i}$ the index for vertex. The set $\boldsymbol{N}$ composed by the entire vertex formed by three sub-sets: $\boldsymbol{N = P \cup D \cup \{0\}}$.

Sub-set $\boldsymbol{\{0\}}$ represents the vehicles points of departure and arrival known as depots. Sub-set $\boldsymbol{P}$ represents the pick-up vertex, where $\boldsymbol{P = \{1, \ldots, n\}}$. Sub-set $\boldsymbol{D}$ represents the delivery vertex where $\boldsymbol{D = \{1 + n, \ldots, 2n\}}$. Each vertex $\boldsymbol{v_i \in V}$ is associated with a Time window: $\boldsymbol{TW_i = [TME_i, TML_i]}$ where:

$\boldsymbol{TME}$: is the lower limit of time window defined for node i, it represents the earliest time at which to pick up or deliver the patient.

$\boldsymbol{TML}$: The upper limit of time window defined for node i, it represents the latest time at which to pick up the patient. If the vehicle arrives to the patient before the time window opens, it must wait for an extra time until the opening of the time window, the service starts right away if the vehicle arrives during the time window. We suppose that all vehicles are obliged to the service to start within the time window, so waiting times before the opening of the time window are neglected.

- $\boldsymbol{s_i}$**:** Service time.

Note that $\boldsymbol{s_0 = 0}$, the necessary time for a patient to get in the vehicle or to leave the vehicle, we fix $\boldsymbol{s_i = 5\ min}$ for all the patients.

- $\boldsymbol{A_{ik}}$**:** The arriving time of the vehicle it is also the start of service time since no waiting before time window is allowed.

Furthermore, each couple of vertices $\boldsymbol{(v_i, v_j)}$ has:

- $\boldsymbol{C_i}$**:** The routing cost between two nodes.
- $\boldsymbol{t_i}$**:** The travel time between two nodes.
- $\boldsymbol{d_{ij}}$**:** Distance between nodes $i$and j.
- $\boldsymbol{r_{ij}}$**:** A request, $\boldsymbol{r_{ij} = 1}$ when j is the destination of the origin i.

Each vehicle or ambulance as in this model is represented by k, where K is the set of available ambulances. $K = \{1, \dots, k\}$. Note that every patient must return from his destination to his origin after his treatment. This only means that patients make two requests or two trips (outward trip and return trip): (from origin to destination) and after a specific amount of time (from destination back to origin). Therefore, our mathematical model needs some modifications.

1. Each patient at node v now has four time windows:
   - For outward trip:

     $TW_i = [TME_i, TML_i]$ at the first pickup stop (origin) v_i
     $TW_{i+n} = [TME_{i+n}, TML_{i+n}]$ at the first delivery stop (destination) v_{i+n}

   - For return trip:

     $TW'_{i+n} = [TME'_{i+n}, TML'_{i+n}]$ at the second pickup stop (destination) v_{i+n}
     $TW'_i = [TME'_i, TML'_i]$ at the second delivery stop (origin) v_i

2. Each vehicle has a load q_i^k that changes at every node $i \in N$ (with $q_0^k == 0$):
   - For outward trip:

$$q_i^k = \begin{cases} \mathbf{1} & \text{if } v_i \in \{v_1, \dots, v_n\} \text{ a pickup} \\ \mathbf{-1} & \text{if } v_i \in \{v_{n+1}, \dots, v_{2n}\} \text{ a delivery} \end{cases}$$

   - For return trip:

$$q_i^k = \begin{cases} \mathbf{1} & \text{if } v_i \in \{v_{n+1}, \dots, v_{2n}\} \text{ a pickup} \\ \mathbf{-1} & \text{if } v_i \in \{v_1, \dots, v_n\} \text{ a delivery} \end{cases}$$

In this mathematical model, we deleted the return trip to reduce the complexity of the model, yet and in order to receive the same results we multiplied the objectif function by two, since the vehicles are going to perform the same exact routings twice. To resume, below we have the element sets, parameters, decision variables, the objectif function of the model and the constraints.

A. Elements sets

(i, j): Pair of nodes.
k: Index of vehicles.

B. Sets

P: Set of origin vertex $P = \{1, \dots, n\}$.
D: Set of destination vertex $D = \{n + 1, \dots, 2n\}$.
$N = P \cup D \cup \{0\}$: Set of all the nodes.
K: Vehicle.

C. Parameters

n: Number of requests.
r (origin and destination nodes).
m: Number of vehicles.
$C_{ij}^k = C_{ji}^k$: Cost per kilometer traversed by vehicle k.
$d_{ij} = d_{ji}$: Distance between nodes i and j.
r_{ij}: Request of nodes $i$and to be transported to j.
TME_i: Earliest allowed arrival time at node i for the first trip.
TML_i: Latest allowed arrival time at node i for the first trip.
s_i: Time of service at node i.
q_i^k: indicates if a patient at node $i$is boarding or leaving the vehicle k.

A solution to this problem is a set of feasible routes obtained by satisfying all the requests of the patients. The solution can be mathematically obtained by the following decision variables:

$$X_{ijk} = \begin{cases} \mathbf{1} & \text{if some vehicle k travels from i to j} \\ \mathbf{0} & \text{otherwise} \end{cases}$$

$$Y_{ik} = \begin{cases} \mathbf{1} & \text{patient at node i is visited by vehicule k} \\ \mathbf{0} & \text{otherwise} \end{cases}$$

A_{ik} : instance of beginning of service at node i of vehicule k

D. Objectif function

The final solution proposed provide a complete schedule for every medical vehicle with all the stops it must perform during a period. The solution determines the routing of the vehicles in order to minimize the total trip costs. Therefore, the considered objective function is presented as follow:

$$\min Z = 2 * \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} C_{ij} * d_{ij} * X_{ijk} \quad (1)$$

E. Constraints

The proposed mathematical model considers the following constraints:

$$\sum_{k \in K} \sum_{j \in N, j \neq i} X_{ijk} = 1 \quad \forall\, i \in P \quad (2)$$

$$\sum_{k \in K} \sum_{i \in P, i \neq j} X_{ijk} = 1 \quad \forall\, j \in D$$

$$\sum_{j \in D} r_{ij} = 1 \quad \forall\, i \in P \quad (3)$$

$$\sum_{i \in P} r_{ij} \geq 1 \quad \forall\, j \in D$$

$$\sum_{k \in K} Y_{ik} = 1 \quad \forall\, i \in P \quad (4)$$

$$\sum_{k \in K} Y_{ik} \geq 1 \quad \forall\, i \in D$$

$$\sum_{k \in K} Y_{0k} = 3$$

$$\sum_{j\in N, j\neq i,0} X_{ijk} = Y_{ik} \quad \forall\, i\, \in P, \forall\, k\, \in K \qquad (5)$$
$$\sum_{i\in P, ij\neq j} X_{ijk} = Y_{jk} \quad \forall\, j\, \in P, \forall\, k\, \in K$$
$$\sum_{i\in P} X_{ijk} = Y_{jk} \quad \forall\, j\, \in D, \forall\, k\, \in K$$

$$\sum_{i\in P} X_{ijk} = 0 \quad \forall\, j\, \in D, \forall\, k\, \in K \qquad (6)$$

$$\sum_{j\in N} Y_{jk} * r_{ij} = Y_{ik} \quad \forall\, i\, \in P, \forall\, k\, \in K \qquad (7)$$

$$\sum_{i\in P} X_{0ik} = Y_{0k} \quad \forall\, k\, \in K \qquad (8)$$

$$\sum_{i\in P} X_{0ik} = 1 \quad \forall\, k\, \in K \qquad (9)$$

$$q_i^k = \sum_{j\in D} Y_{jk} * r_{ij} \quad \forall\, i\, \in P, \forall\, k\, \in K \qquad (10)$$
$$q_j^k = -Y_{jk} \quad \forall\, j\, \in D, \forall\, k\, \in K$$

$$A_j^k \geq (A_i^k + s_i + t_{ij}) * X_{ijk} \quad \forall\, i,j\, \in N, \forall\, k\, \in K \qquad (11)$$

$$\sum_{i\in P\cup D} q_i^k \leq 3 \quad \forall\, k\, \in K \qquad (12)$$

$$TME_i \leq A_{ik} \leq TML_i \quad \forall\, i \in N, \forall\, k\, \in K \qquad (13)$$

$$X_{ijk}, Y_{ik} \in \{0, 1\} \quad \forall\, i,j\, \in N, \forall\, k\, \in K \qquad (14)$$
$$A_{ik} \in \mathbb{R} \quad \forall\, i\, \in N, \forall\, k\, \in K$$

Constraints (2) ensure that arc (i, j) is traversed once from i to j.

Constraints (3) ensure that a pickup node has one request, but a delivery node can receive multiple requests.

Constraints (4) is for making sure that every origin point is visited exactly once and a destination point is visited more than once, and the depot is visited exactly three times with the three vehicles.

Constraints (5) establish a relationship between the two decision variables X and Y, they ensure that if arc (i, j) is traversed, then node i and node j are visited. Since we assumed there is no return trips so **Constraint (6)** guarantees that the routes are ending at a delivery node.

Constraint (7) specifies that for a request to be satisfied, the same vehicle must visit both the pickup and the delivery node.

Constraint (8) ensures that there is an arc between the depot and some pick-up nodes if these pickup nodes are visited after departing from depot.

Constraint (9) establishes that every vehicle visits only one pick-up node from depot.

Constraints (10) indicate that if a patient is boarding the vehicle then the load increases by one and if he is leaving the vehicle, the load decreases by one. **Constraint (12)** guarantees the consistency in the vehicle's load and that the capacity of three patients is not violated.

Constraint (11) establishes that the start service in a certain request depends on the sum of the start service in the previous request and the service time in that node plus the travelling time between the two nodes. Obligation of arriving within the time window **[TME, TML]** is established in **constraint (13)**.

The last **constraints (14)** define the domain and the nature of the variables and parameters used in this problem.

V. COMPUTATIONAL RESULTS

In order to solve our mathematical model, we have chosen LINGO as our optimization modeling software.

The input data of our model are presented as follow:

- Number of patients: 10 patients.
- Number of hemodialysis centers: 3 centers.
- Number of used vehicles: 3 vehicles.
- Service time is 5 minutes (0.08h).

Table 1 and 2 summarize the destination of each patient and the corresponding time window to pick up or deliver each patient.

Table 1. Request table

Origins/destinations	D1	D2	D3
P1	0	1	0
P2	0	1	0
P3	0	0	1
P4	0	0	1
P5	1	0	0
P6	0	1	0
P7	0	0	1
P8	1	0	0
P9	1	0	0
P10	1	0	0

Table 2. Time Windows data

Patients/time windows	TME	TMI
P1	8:05	8:30
P2	8:00	8:20
P3	8:00	8:20

P4	8:30	8:50
P5	9:00	9:25
P6	9:05	9:25
P7	9:30	10:00
P8	9:30	9:50
P9	10:05	10:30
P10	10:30	10:55

Cost matrix: in order to estimate the cost matrix, we have to fix a cost for each distance travelled by the vehicles. (These costs are not the ones fixed by the Algerian government; they are just an example to test our mathematical model).

	DEPOT	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	D1	D2	D3
DEPOT	0	9	9	9	9	9	9	12	9	9	9	9	9	12
P1	9	0	9	9	9	9	9	12	9	9	9	9	9	9
P2	9	9	0	9	9	9	9	9	9	9	9	9	9	9
P3	9	9	9	0	9	9	9	9	9	9	9	9	9	9
P4	9	9	9	9	0	9	9	9	9	9	9	9	9	9
P5	9	9	9	9	9	0	12	9	9	9	9	9	12	9
P6	9	9	9	9	9	12	0	12	12	12	9	9	9	9
P7	12	12	9	9	9	9	12	0	9	12	9	9	12	9
P8	9	9	9	9	9	9	12	9	0	9	9	9	9	9
P9	9	9	9	9	9	9	12	12	9	0	9	9	9	12
P10	9	9	9	9	9	9	9	9	9	9	0	9	9	9
D1	9	9	9	9	9	9	9	9	9	9	9	0	9	9
D2	9	9	9	9	9	12	9	12	9	9	9	9	0	9
D3	12	9	9	9	9	9	9	9	9	12	9	9	9	0

After inserting all the input data and the model into LINGO, we have obtained the following results, figure 2 summarizes the obtained routing for the three vehicles, based on the results of the decision variables.

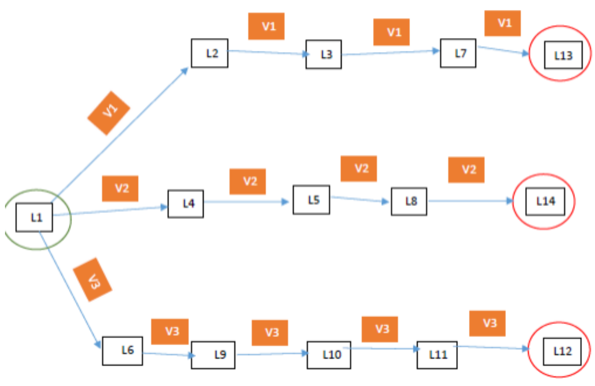

Fig. 2. Final routing results

The obtained results by LINGO optimizer gave a cost of 504 as an objective function value with a logical order of visits. The vehicles respect the requests of patients while performing their routes. The choice of the next stop is based on the shortest path between nodes since the objectif function minimizes the total cost and distance travelled. The time windows constraints are also respected for every node. However, in the problem description we ignored the waiting time before the opening of time windows, so the vehicles spend a lot of time waiting for a time window to open in order to perform a pick-up. Sometimes they even perform the pickups at the end of the time window, which is not the optimal case since we are transporting individuals.

VI. Conclusion and Perspectives

The DARP is an important and a very difficult routing problem encountered in several contexts and likely to gain an importance in the upcoming years. It shares several features with pickup and delivery problems arising in courier services, but since it is concerned with the transportation of people, levelof service criteria becomes more important. Thus, punctuality and route directness are more critical in DARP. To conclude this paper, we believe that more emphasis should now be puton the dynamic version of the problem. This involves taking the full request of patients into consideration, as they need three treatments per week. And how existing routes should be modified every day to accommodate the requests.

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