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Demi-flexible operating policies to promote the performance of public transit in low-demand areas

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ABSTRACT

The efforts of providing attractive transport service to residents in sparse communities have previously focused on operating flexible transit services. This paper identifies a new category of transit policies, called demi-flexible operating policies, to fill the gap between flexible transit services and conventional fixed-route systems. The passenger cost function is defined as the performance measure of transit systems and the analytic work is performed based on a real-world flag-stop transit service, in which we compare its system performance with another two comparable systems, the fixed-route and flex-route services, at expected and unexpected demand levels in order to be closer to reality. In addition, the dynamic-station policy is introduced to assist the flex-route service to better deal with unexpectedly high demand. Experiments demonstrate the unique advantages of demi-flexible operating policies in providing affordable, efficient, and reliable transport service in low-demand operating environments and this work is helpful to optimize the unifying framework for designing public transit in suburban and rural areas.

1. Introduction

Recent economic growth patterns and social changes have resulted in more sparse residential districts, which do not favor conventional fixed-route, fixed-schedule forms of transit services. Demand responsive transit might be an option in low-demand areas, which provides curb-to-curb services to all customers entirely based on the actual travel demand, without fixed stations or routes. However, due to the high operating cost, demand responsive transit is generally limited to special systems, such as paratransit, and it seems unpractical to provide this kind of costly personalized transport service to the general public.

In the past decades, planners have been trying to introduce flexible operating policies to the public transit market in suburban and rural areas, to encourage residents to leave their cars in the trips to nearby destinations. Similar to Koffman (2004), in this paper we use "flexible transit services" to name all types of hybrid services that are not pure demand responsive service or conventional fixed-route, fixed-schedule transit, but could offer some degree of complete curb-to-curb services to requests in designated service areas. In other words, flexible transit services provide curb-to-curb services on the basis of serving regular station-to-station passengers. In Koffman's survey, flexible operating policies mainly include six

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types: flex-route transit (also called route deviation transit), point deviation transit, demand-responsive connector, request stops, flexible-route segments and zone route.

The investigation by Potts et al. (2010) revealed that although the concept of flexible transit services has been proposed for over 40 years, these innovative operating policies have been applied only by a small percentage of transit agencies. The survey in Great Britain by Davison et al. (2014) also indicated that the future of these new services remains uncertain due to financial sustainability and operational issues. The development of flexible transit systems seems to be far behind prior expectation, especially considering the convenience of provided curb-to-curb services. Recently, researchers have begun to realize that applying flexible operating policies is more complex than it looks. Velaga et al. (2012a) summarized the challenges of operating flexible transit services, besides the lack of the support of public policies and appropriate planning and evaluation tools, another two primary bottlenecks are the uncertainties of travel demand in low-demand areas, and the short of sophisticated real-time communication systems between transit providers and customers.

In operation, flexible transit services are generally operated with predetermined slack time for curb-to-curb services, which is allotted based on the expected demand level, while the uncertain travel demand in low-demand areas makes it difficult for operators to forecast the actual passenger flow. In addition, the costs, staffing and training in the use of sophisticated support systems make transit divisions more hesitant to attempt at these emerging flexible operating policies. These challenges inevitably limit the popularity of flexible transit services, and explain why some transit agencies abandoned the efforts after several months or years.

In order to fill the gap between flexible transit services and conventional fixed-route systems, here we define another category of services named demi-flexible transit policies, which means that policies in this group do not provide complete curb-to-curb services, but still offer some kind of flexibility to transit customers. In reality, the concept of demi-flexible operating policies is not strange, and the most common form of services in this category is the flag-stop operating policy, which can be found in many transit systems operated in rural and suburban areas.

In this paper our analysis is performed by implementing comparable operating policies in a real-life transit service. Besides demi-flexible operating policies, appropriate flexible and fixed-route policies have also been examined in the same system. The system performance under different operating policies has been compared, considering the uncertainties of travel demand, to explore the advantages of demi-flexible operating policies in promoting public transit services in low-demand areas.

2. Literature review

Different from Koffman (2004) and our work, Errico et al. (2013) used "semi-flexible transit systems" to name all kinds of transit services combining on-demand service adjustment capabilities and schedule characteristics of conventional transit. In our framework, these services can be divided into two groups: flexible and demi-flexible operating policies. To our knowledge, demi-flexible operating policies have been seldom examined in existing studies. Almost all the efforts have been made on optimizing flexible transit systems and a limited number of related literatures have primarily focused on two common forms of flexible transit services: flex-route transit and demand-responsive connector.

Flex-route transit is regarded as an innovative combination of fixed-route transit and demand responsive service, and was by far the most popular form of flexible transit services (Potts et al., 2010). Daganzo (1984) proved that flex-route transit could possibly become cost-effective compared with demand responsive service. Fu (2002) revealed the fundamental relationships between system performance and design parameters in flex-route services. Quadrifoglio et al. (2006, 2008a) developed bounds on the maximum longitudinal velocity of service vehicles, and proposed a static scheduling formulation for a flex-route service called mobility allowance shuttle transit. Alshalalfah and Shalaby (2012) investigated the feasibility of applying flex-route policy as feeder transit in suburban areas. Nourbakhsh and Ouyang (2012) proposed a flex-route transit network that has a system advantage under low-to-moderate demand levels. Qiu et al. (2014a, 2014b) proposed a novel scheduling system for flex-route services and investigated the feasibility of utilizing accepted curb-to-curb stops to promote the operating reliability of flex-route services. Qiu et al. (2014c) developed a two-stage scheduling model for flex-route services. Furthermore, Qiu et al. (2015a,b) explored the choice modeling between fixed-route and flex-route operating policies in transit systems.

Demand-responsive connector is generally operated as feeder transit to collect passengers from their houses to the transfer terminal linked with the major transit network. Aldaihani et al. (2004) developed an analytical model to design a hybrid grid network that integrates demand-responsive connectors with a fixed-route major transit service. Li and Quadrifoglio (2009, 2010), and Quadrifoglio and Li (2009) investigated the zone design problem and service mode choice between demand-responsive connector and conventional fixed-route service. Chandra and Quadrifoglio (2013) estimated the optimal service cycle in operating demand-responsive connector.

In contrast, demand responsive transit has been extensively studied in the past decades. Most efforts were made on the efficient routing and scheduling of demand responsive service (Fu, 1999; Horn, 2002; Dessouky et al., 2003; Cremers et al., 2009). In addition, some other studies dealt with planning and operation problems in demand responsive service. Daganzo (1978) evaluated the system performance of many-to-many demand responsive service. A new high-coverage point-to-point transit system was proposed by Cortes and Jayakrishnan (2002). Palmer et al. (2004) and Quadrifoglio et al. (2008b) examined the effect of advanced technologies and specific operating practices on the productivity of demand responsive transit systems. The travel behavior issues were considered in designing and operating integrated flexible and demand responsive transit systems (Nelson et al., 2010; Velaga et al., 2012b).

Previous researches were mostly conducted in operating environments with steady and predictable demand, which actually underestimated the difficulty of implementing flexible operating policies. In reality, all of flexible transit services have some fixed operating schedule (Koffman, 2004), typically limited to departure and arrival times at checkpoints, and the uncertain travel demand in low-demand areas makes it difficult to design reliable deviation services to meet all curb-to-curb requests, which explains why nowadays flexible operating policies are mostly limited to extreme low-demand areas.

In this paper, continuous approximations are utilized to evaluate the system performance under comparable operating policies. These approximation approaches could provide reasonable solutions with as little information as possible, and are quite common in existing works, such as Li and Quadrifoglio (2010) and Qiu et al. (2015b), that developed support tools for strategic planning and management in transit services.

3. Model description

3.1. Service area and demand

The analytic work in our analysis is built primarily based on the Route 289 service in the suburban area of Zhengzhou City in China, which is currently operated under flag-stop policy. The service area can be modeled as a rectangle of width W and length L, delimited by two terminal checkpoints located at connection centers (see Fig. 1). Route 289 serves as community buses to transport passengers between their houses and connection centers, where they could transfer to the major transit network. Passengers of Route 289 are part of the general public and special needs passengers are not considered.

Because the number of transit lines passing by is very high (more than 10 lines at each connection center in Route 289), the coordination at connection centers is not considered and we assume a temporal Poisson distribution for the arrivals of demand from the major transit network. It is also reasonable to assume a temporal Poisson distribution for travel demand with starting points outside checkpoints in transit services without advance notice. In addition, demand outside checkpoints is assumed to be uniformly distributed in the service area, and in operation there are mainly three types of passengers in the system with proportions η_1, η_2, η_3 ($\eta_1 + \eta_2 + \eta_3 = 1$) as follows[1]:

Type I: Starting point and destination both at checkpoints.
Type II: Starting point at checkpoints, and destination not at checkpoints.
Type III: Starting point not at checkpoints, and destination at checkpoints.

3.2. Operating policies

Besides flag-stop policy, another two comparable operating policies, fixed-route and flex-route policies, are also applied in the same system. We define fleet size M and maintain the same service vehicle fleet under different operating policies. Moreover, it is assumed that headway control strategies, such as the adaptive bus holding strategy (Daganzo, 2009) and bus cruising speed control (Daganzo and Pilachowski, 2011), are implemented in this transit service, so the variation of travel demand, especially the curb-to-curb requests in the flex-route service, can be negligible.

3.2.1. Flag-stop policy

In Route 289, lots of marked flag-stop stations are distributed along the base route. Because the distance between adjacent flag-stop stations can be as short as 0.03 mile, it is possible to simplify this system to a flag-stop service without specific marked stations. In operation, passengers have a vertical walk between the base route and their houses (see Fig. 1) and service vehicles will stop to board or discharge passengers anywhere on request along the base route. It is obvious that flag-stop policy falls into the category of demi-flexible operating policies.

A no-rejection policy is applied in the real-life operation of Route 289, considering that passengers need to hail buses and advance notice is not required. Checkpoints feature a soft time window. If the actual demand is higher than the expected demand level, service vehicles might arrive later than the predetermined departure time at the downstream checkpoint. Conversely, if the actual demand is lower than the expected demand level, there will be idle time at the downstream checkpoint until its scheduled departure time. The further discussion of the soft time window in the flag-stop service can be found in Section 5.3.

3.2.2. Fixed-route policy

We assume that there are N stations including two terminal checkpoints along the base route, and adjacent stations have the same distance d (see Fig. 2). Service vehicles move forth and back along the base route between two terminals. Passengers are not required to make advance notice and can only be picked up or dropped off at stations.

[1] Theoretically, there might be another type of passengers whose starting point and destination are both not at checkpoints, while in Route 289 this type of passengers occupies a very small fraction and can be disregarded. In reality, this has a very limited influence on the validity of our analysis.

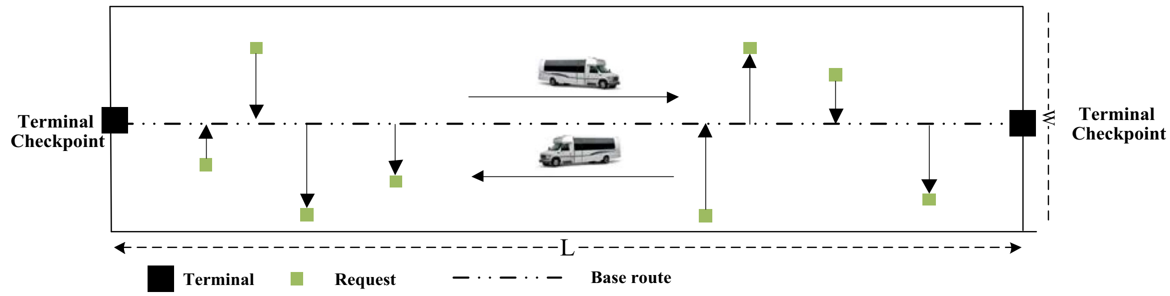


Fig. 1. The route 289 flag-stop transit service.

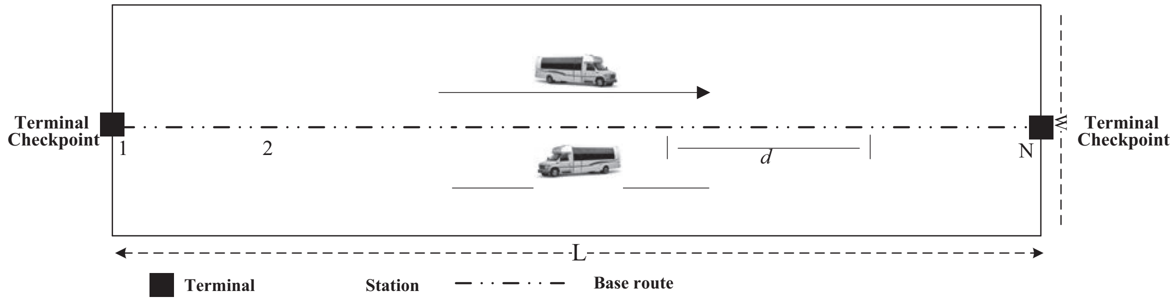


Fig. 2. Fixed-route operating policy.

3.2.3. Flex-route policy

As a kind of flexible transit operating policies, service vehicles in the flex-route system are allowed to deviate from the base route to serve curb-to-curb requests (see Fig. 3). The slack time for deviation services is allotted based on the expected demand level and curb-to-curb passengers accepted by the system have no walking in their trips. In real-life operation, because curb-to-curb requests are required to make advance notice under flex-route policy, some of them might be rejected by the system if the actual demand exceeds the expected demand level. Deviation services are available on a first-come, first-available basis. If deviation services cannot use up all of allotted slack time, there will be idle time at the downstream checkpoint.

4. Modeling

Here we define the average speed V_b of service vehicles, walking speed V_{wk} of passengers, dwelling time T_d^r of a stop on request, and dwelling time T_d^f of a fixed stop. We may expect $T_d^r \leqslant T_d^f$, because there might be more passengers picked up or dropped off at fixed stations. Similar to Quadrifoglio et al. (2006), we assume rectilinear movement because it is a good approximation of reality. λ and λ_c represent the expected demand rate and the actual travel demand in the service area, respectively.

Due to the uncertainties of travel demand in low-demand areas, there is probably a difference between the expected demand level and the actual travel demand in operation (similar to Fig. 4). In order to investigate the reliability of operating

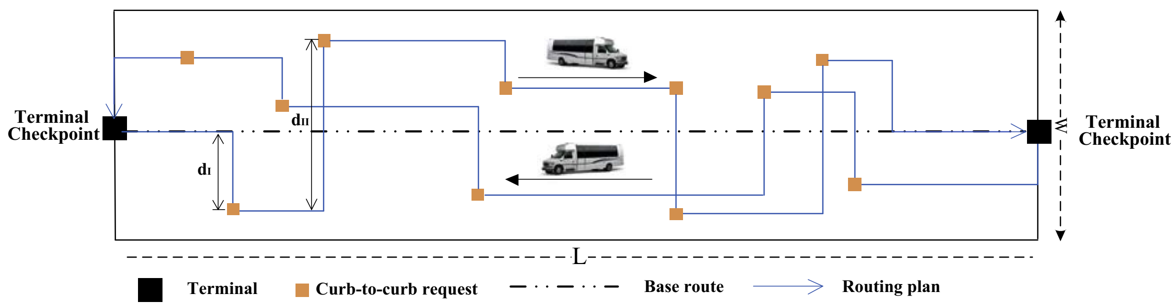


Fig. 3. Flex-route operating policy.

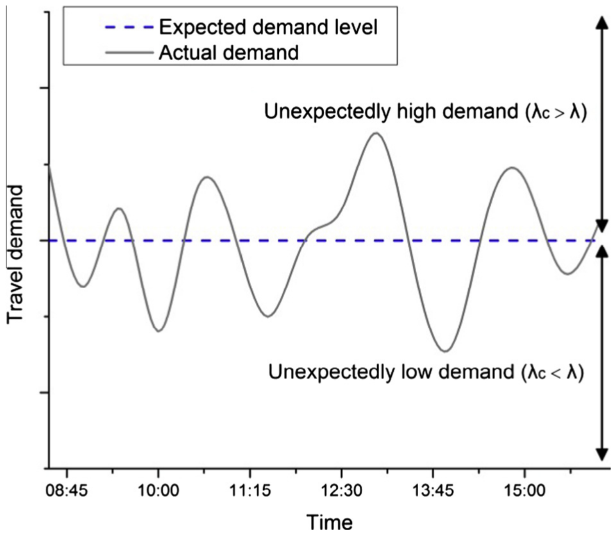


Fig. 4. Possible travel demand in the system.

policies, we intend to examine the system performance under the three transit policies at expected and unexpected demand levels in the following sections.

4.1. System performance measure

There is no designed idle time between trips under the three operating policies. In general, transit system performance is identified as a combination of operating cost and passenger cost. If we disregard the infrastructure investments for different services, which actually occupy a small fraction of long-term transit operating cost, it is possible to assume that the three transit systems have the same operating cost because the same fleet keeps running in operation. This assumption is reasonable especially considering that there are systems where the drivers take the bookings and no dispatch center is required.

Thus, in our analysis the system performance measure is defined as the transit passenger cost function $U = K + A + R$, which is built as the sum of the expected walking time K, the expected waiting time A, and the expected riding time R. A lower value of passenger cost function definitely represents a better system performance.

4.2. System performance under flag-stop policy

4.2.1. At expected demand levels

Type I passengers have no walking time in their trips, and thus $K_I = 0$. Types II and III passengers walk vertically between their houses and the base route, and their expected walking time is

$$K_{\mathrm{II}} = K_{\mathrm{III}} = \frac{W}{4V_{wk}} \tag{1}$$

Thus the expected walking time of passengers is

$$K = \eta_1 K_{\mathrm{I}} + \eta_2 K_{\mathrm{II}} + \eta_3 K_{\mathrm{III}} = \frac{(\eta_2 + \eta_3)W}{4V_{wk}} \tag{2}$$

If a vehicle serves k passengers in a service cycle T_c, moving from a terminal checkpoint and back, the following are obtained:

$$T_c = \frac{2L}{V_b} + k(\eta_2 + \eta_3)T_d^r + 2T_d^f \tag{3}$$

$$k \times M = \lambda \times T_c \tag{4}$$

Combining Eqs. (3) and (4), it can be derived as

$$T_c = \frac{2ML + 2MV_b T_d^f}{MV_b - \lambda V_b T_d^r (\eta_2 + \eta_3)} \tag{5}$$

In the system, the headway is T_c/M, and the expected waiting time of the three passenger groups is

$$A_{\mathrm{I}} = A_{\mathrm{II}} = A_{\mathrm{III}} = \frac{T_c}{2M} \tag{6}$$

Thus, the expected waiting time per passenger in the flag-stop service is

$$A = \eta_1 A_{\mathrm{I}} + \eta_2 A_{\mathrm{II}} + \eta_3 A_{\mathrm{III}} = \frac{L + V_b T_d^f}{MV_b - M\lambda V_b T_d^r(\eta_2 + \eta_3)} \tag{7}$$

Type I passengers travel between two terminals, and their expected riding time is

$$R_I = \frac{T_c}{2} \tag{8}$$

Types II and III passengers can be dropped off or picked up uniformly anytime in the trip between two terminals, and their expected riding time can be expressed as

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{T_c}{4} \tag{9}$$

Then the expected riding time per passenger is

$$R = \eta_1 R_{\mathrm{I}} + \eta_2 R_{\mathrm{II}} + \eta_3 R_{\mathrm{III}} = \frac{(ML + MV_b T_d^f)(1 + \eta_1)}{2MV_b - 2\lambda V_b T_d^r(\eta_2 + \eta_3)} \tag{10}$$

4.2.2. At unexpected demand levels

Because of the no-rejection policy, when the actual demand λ_c is higher than the expected demand λ, passenger cost indicators can be calculated by replacing λ with λ_c in Eqs. 2, 7 and 10.

Conversely, when $\lambda_c < \lambda$, there will be idle time at the downstream checkpoint. At this condition, the expected walking time is the same as the result from Eq. (2), and the expected waiting time can be derived using Eq. (7) with expected demand λ.

For each vehicle, the actual number k_c of accepted customers in a service cycle is

$$k_c = \frac{\lambda_c \times T_c}{M} = \frac{2\lambda_c L + 2\lambda_c V_b T_d^f}{MV_b - \lambda V_b T_d^r(\eta_2 + \eta_3)} \tag{11}$$

The expected riding time of different passenger groups can be obtained:

$$R_{\mathrm{I}} = \frac{L}{V_b} + \frac{k_c(\eta_2 + \eta_3)}{2} T_d^r + T_d^f \tag{12}$$

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{L}{2V_b} + \frac{k_c(\eta_2 + \eta_3)}{4} T_d^r + \frac{T_d^f}{2} \tag{13}$$

Thus, the expected riding time per passenger at unexpectedly low demand levels can be expressed as

$$R = \eta_1 R_{\mathrm{I}} + \eta_2 R_{\mathrm{II}} + \eta_2 R_{\mathrm{III}} = \left[\frac{L + V_b T_d^f}{2V_b} + \frac{T_d^r\left(\lambda_c L + \lambda_c V_b T_d^f\right)(\eta_2 + \eta_3)}{2MV_b - 2\lambda V_b T_d^r(\eta_2 + \eta_3)}\right](1 + \eta_1) \tag{14}$$

4.3. System performance under fixed-route policy

Similar to Qiu et al. (2014b), we can derive analytical models for the fixed-route service. Type I passengers have no walking in their trips ($K_I = 0$) and the expected walking time of types II and III passengers can be derived as

$$K_{\mathrm{II}} = K_{\mathrm{III}} = \frac{1}{4V_{wk}}\left[\frac{L}{N-1} + W\right] \tag{15}$$

Thus the expected walking time per passenger is

$$K = \eta_1 K_{\mathrm{I}} + \eta_2 K_{\mathrm{II}} + \eta_3 K_{\mathrm{III}} = \frac{\eta_2 + \eta_3}{4V_{wk}}\left[\frac{L}{N-1} + W\right] \tag{16}$$

In the fixed-route service, the service cycle T_c is

$$T_c = \frac{2L}{V_b} + 2(N-1)T_d^f \tag{17}$$

The expected waiting time of passengers is

$$A_{\mathrm{I}} = \frac{T_c}{2M} \tag{18}$$

$$A_{\mathrm{II}} = A_{\mathrm{III}} = \left[1 - \frac{1}{2(N-1)}\right]\frac{T_c}{2M} = \frac{T_c}{2M} - \frac{T_c}{4M(N-1)} \tag{19}$$

$$A = \eta_1 A_{\mathrm{I}} + \eta_2 A_{\mathrm{II}} + \eta_3 A_{\mathrm{III}} = \frac{L}{MV_b} + \frac{T_d^f(N-1)}{M} - \frac{L(\eta_2+\eta_3)}{2MV_b(N-1)} - \frac{T_d^f(\eta_2+\eta_3)}{2M} \tag{20}$$

The expected riding time of passengers is

$$R_{\mathrm{I}} = \frac{T_c}{2} = \frac{L}{V_b} + (N-1)T_d^f \tag{21}$$

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{1}{2(N-1)} \times 0 + \frac{1}{N-1}\sum_{i=1}^{N-2}\frac{iT_c}{2(N-1)} + \frac{1}{2(N-1)} \times \frac{T_c}{2} = \frac{L}{2V_b} + \frac{(N-1)T_d^f}{2} \tag{22}$$

$$R = \eta_1 R_{\mathrm{I}} + \eta_2 R_{\mathrm{II}} + \eta_3 R_{\mathrm{III}} = \left[\frac{L}{2V_b} + \frac{(N-1)T_d^f}{2}\right](1+\eta_1) \tag{23}$$

Because the calculation of passenger cost indicators in the fixed-route service is not related with the demand level in the service area, there is no difference between passenger cost function values at expected and unexpected demand levels.

4.4. System performance under flex-route policy

Deviation services in flex-route systems mostly require customers to make advance notice before the beginning of a ride (Potts et al., 2010), and thus the real-time insertion is not considered in our research. A no-backtracking constraint is applied in constructing vehicle routing plans, which forces service vehicles to move only in the forward direction (similar to the routing plan in Fig. 3).

4.4.1. At expected demand levels

At expected demand levels, all requests can be accepted without walking ($K = 0$). In stops of a ride between two terminals, the expected distance d_{I} between the first/last stop and the terminal (see Fig. 3) is

$$d_{\mathrm{I}} = \frac{W}{4} \tag{24}$$

The expected distance d_{II} between each pair of stops is

$$d_{\mathrm{II}} = \frac{W}{3} \tag{25}$$

Thus the travel distance D of a vehicle in a service cycle is

$$D = 2 \times L + 4 \times d_{\mathrm{I}} + [k(\eta_2+\eta_3) - 2] \times d_{\mathrm{II}} = 2L + \frac{W}{3} + \frac{Wk(\eta_2+\eta_3)}{3} \tag{26}$$

In the flex-route service, another two relationships can be obtained:

$$T_c = \frac{D}{V_b} + k(\eta_2+\eta_3)T_d^r + 2T_d^f \tag{27}$$

$$k \times M = \lambda \times T_c \tag{28}$$

Combining Eqs. (26)–(28), it is possible to derive the service cycle T_c as follows

$$T_c = \frac{6ML + MW + 6MV_bT_d^f}{3MV_b - W\lambda(\eta_2+\eta_3) - 3V_bT_d^r\lambda(\eta_2+\eta_3)} \tag{29}$$

The expected waiting time of types I and II passengers is

$$A_{\mathrm{I}} = A_{\mathrm{II}} = \frac{T_c}{2M} \tag{30}$$

Passengers can receive the feedback from the scheduling system, when they book curb-to-curb services. For type III passengers, the waiting time is defined as the interval between the scheduled pick-up time and actual pick-up time, because before the scheduled pick-up time, they are more likely to spend their time at houses instead of waiting outside. For a type III passenger j, waiting time occurs only when there are follow-up customers, who make reservation later than j and result in additional stops before the vehicle picks up j. Based on mathematical probability theory, the expected value of waiting time can be expressed as

$$\begin{aligned}A_{\mathrm{III}} &= \frac{k(\eta_2+\eta_3)/2-1}{k(\eta_2+\eta_3)/2}\left(\frac{W}{3V_b}+T_d^r\right)\frac{1}{2}+\frac{k(\eta_2+\eta_3)/2-2}{k(\eta_2+\eta_3)/2}\left(\frac{W}{3V_b}+T_d^r\right)\frac{1}{2}+\cdots+\frac{1}{k(\eta_2+\eta_3)/2}\left(\frac{W}{3V_b}+T_d^r\right)\frac{1}{2}\\ &= \frac{Wk(\eta_2+\eta_3)}{24V_b}+\frac{T_d^r k(\eta_2+\eta_3)}{8}-\frac{W}{12V_b}-\frac{T_d^r}{4}\end{aligned} \tag{31}$$

Thus, the expected waiting time per passenger in the flex-route service is

$$\begin{aligned}A &= \eta_1 A_{\mathrm{I}}+\eta_2 A_{\mathrm{II}}+\eta_3 A_{\mathrm{III}}\\ &= \frac{(6L+W+6V_bT_d^f)(\eta_1+\eta_2)}{6MV_b-2W\lambda(\eta_2+\eta_3)-6V_bT_d^r\lambda(\eta_2+\eta_3)}+\frac{\eta_3 Wk(\eta_2+\eta_3)-2\eta_3 W}{24V_b}+\frac{T_d^r\eta_3 k(\eta_2+\eta_3)-2\eta_3 T_d^r}{8}\end{aligned} \tag{32}$$

Similarly, the expected riding time of type I passengers is

$$R_{\mathrm{I}} = \frac{T_c}{2} \tag{33}$$

The expected riding time of types II and III passengers is

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{T_c}{4} \tag{34}$$

Thus, the expected riding time per passenger can be obtained:

$$R = \eta_1 R_{\mathrm{I}}+\eta_2 R_{\mathrm{II}}+\eta_3 R_{\mathrm{III}} = \frac{(6ML+MW+6MV_bT_d^f)(1+\eta_1)}{12MV_b-4W\lambda(\eta_2+\eta_3)-12V_bT_d^r\lambda(\eta_2+\eta_3)} \tag{35}$$

4.4.2. At unexpectedly high demand levels

In the flex-route service, most customers (commuters) would subscribe to the system for guaranteed services, and other occasional curb-to-curb customers might be rejected due to the limitation of slack time when the actual demand exceeds the service capacity of deviation services, which is defined as the maximum feasible number of stops under predetermined slack time. Similar to Qiu et al. (2014b, 2015b), we use the assumption that rejected customers utilize the nearest checkpoint for transit service, to estimate the system performance at unexpectedly high demand levels, because passengers prefer fast and efficient travel and the system cannot guarantee that rejected customers can be served in the right next ride. If they intend to wait for the next available service vehicle, passenger cost function might have a much greater value. Because of advance notice requirement and a larger service cycle, rejected passengers in the flex-route service are considered to have sufficient time to get to checkpoints before pick-up.

When $\lambda_c > \lambda$, the first λ passengers can be accepted and the values of their passenger cost indicators are the same as the results in Section 4.4.1. For the last $\lambda_c-\lambda$ passengers, type I passengers can be accepted and the actual number of rejected curb-to-curb requests is $(\lambda_c-\lambda)(\eta_2+\eta_3)$. For these $\lambda_c-\lambda$ passengers, $K_{\mathrm{I}}^{\lambda_c-\lambda}=0$, and the expected walking time of types II and III passengers is

$$K_{\mathrm{II}}^{\lambda_c-\lambda} = K_{\mathrm{III}}^{\lambda_c-\lambda} = \frac{L}{4V_{wk}}+\frac{W}{4V_{wk}} \tag{36}$$

Thus, the expected walking time of all passengers can be derived as

$$K = \frac{(\lambda_c-\lambda)(\eta_2+\eta_3)(L+W)}{4V_{wk}\lambda_c} \tag{37}$$

Rejected requests have access to the transit schedule. For the last $\lambda_c-\lambda$ passengers, we can obtain the waiting time of type I passengers $A_{\mathrm{I}}^{\lambda_c-\lambda}=T_c/2M$ and type III passengers $A_{\mathrm{III}}^{\lambda_c-\lambda}=0$. In addition, the expected waiting time of type II passengers is

$$A_{\mathrm{II}}^{\lambda_c-\lambda} = \frac{1}{2}\times 0+\frac{1}{2}\times\frac{T_c}{2M} = \frac{T_c}{4M} \tag{38}$$

We define $a = A$ in Eq. (32), and the expected waiting time of all passengers is

$$A = \frac{\lambda}{\lambda_c}\alpha+\frac{(\lambda_c-\lambda)\eta_1 T_c}{2M\lambda_c}+\frac{(\lambda_c-\lambda)\eta_2 T_c}{4M\lambda_c} \tag{39}$$

Similarly, in the $\lambda_c-\lambda$ passengers, the expected riding time of type I passengers is $R_{\mathrm{I}}^{\lambda_c-\lambda}=T_c/2$ and for types II and III passengers, the results are

$$R_{\mathrm{II}}^{\lambda_c-\lambda} = R_{\mathrm{III}}^{\lambda_c-\lambda} = \frac{1}{2}\times 0+\frac{1}{2}\times\frac{T_c}{2} = \frac{T_c}{4} \tag{40}$$

So the expected riding time per passenger is actually equal to the result of Eq. (35).

4.4.3. At unexpectedly low demand levels

If $\lambda_c < \lambda$, all customers can be accepted and have no walking ($K = 0$). The expected waiting time of types I and II passengers can be calculated by Eq. (30). The actual number of customers in a service cycle is $k_c = \lambda_c T_c / M$ and the expected waiting time of type III passengers can be obtained by replacing k with k_c in Eq. (31):

$$A_{\mathrm{III}} = \frac{W\lambda_c T_c(\eta_2+\eta_3)}{24MV_b} + \frac{T_d^r \lambda_c T_c(\eta_2+\eta_3)}{8M} - \frac{W}{12V_b} - \frac{T_d^r}{4} \tag{41}$$

Then we can directly obtain the expected waiting time by $A = \eta_1 A_{\mathrm{I}} + \eta_2 A_{\mathrm{II}} + \eta_3 A_{\mathrm{III}}$. At unexpectedly low demand levels, there will be idle time at terminal checkpoints, and it is possible to derive the expected riding time as follows:

$$R_{\mathrm{I}} = \frac{L}{V_b} + \frac{W}{6V_b} + \frac{k_c W(\eta_2+\eta_3)}{6V_b} + \frac{k_c T_d^r(\eta_2+\eta_3)}{2} + T_d^f \tag{42}$$

$$R_{\mathrm{II}} = R_{\mathrm{III}} = \frac{L}{2V_b} + \frac{W}{12V_b} + \frac{k_c W(\eta_2+\eta_3)}{12V_b} + \frac{k_c T_d^r(\eta_2+\eta_3)}{4} + \frac{T_d^f}{2} \tag{43}$$

$$R = \left[\frac{L}{2V_b} + \frac{W}{12V_b} + \frac{\lambda_c T_c W(\eta_2+\eta_3)}{12MV_b} + \frac{\lambda_c T_c T_d^r(\eta_2+\eta_3)}{4M} + \frac{T_d^f}{2}\right](1+\eta_1) \tag{44}$$

4.5. Dynamic-station policy

To deal with the uncertain travel demand, another kind of demi-flexible operating policies, named dynamic-station policy, is proposed. It is not an independent operation fashion and can be applied along with flexible transit services to handle rejected curb-to-curb requests (Qiu et al., 2014b). In this policy, passengers rejected by deviation services can make use of accepted curb-to-curb stops for their pick-up and drop-off (see Fig. 5). This policy can be easily implemented by sending the schedule of suggested pick-up or drop-off points to rejected requests from the system.

The slack time of a flex-route system is generally designed based on the expected demand level, which directly determines the maximum number of deviation stops. The dynamic-station policy will be active if rejection of curb-to-curb requests occurs. This happens when the actual demand in some rides is higher than the expected demand level ($\lambda_c > \lambda$, unexpectedly high demand in Fig. 4).

Walking is most uncomfortable in trips and quite sensitive to external surroundings. Thus we assume that under dynamic-station policy, the system will choose the nearest stop for each rejected request. The expected walking distance of a rejected curb-to-curb request is the distance between his house and the nearest point among all available stops. In an area, the rectilinear distance $E(d_f)$ from an arbitrary location to the fth nearest point distributed uniformly can be obtained (Miyagawa, 2012):

$$E(d_f) = \frac{(2f-1)!!}{(2f-2)!!}\frac{\sqrt{\pi}}{2\sqrt{2\rho}} \tag{45}$$

where ρ is the density of points.

Thus we get an approximation of the expected walking distance D_r of rejected curb-to-curb passengers under dynamic-station policy:

$$D_r \approx \frac{\sqrt{\pi}}{2\sqrt{2[k(\eta_2+\eta_3)/2M]/LW}} \tag{46}$$

Because Eq. (46) does not consider the influence of fixed checkpoints, a simulation modeling is developed for the flex-route service including checkpoints to verify the validity of the theoretical approximation. This experiment is performed with a service area 1 mile^2, and results from Eq. (46) and the simulation are displayed in Fig. 6. The outcomes indicate that the difference between theoretical modeling and simulation is clear only at a very low density of stops. As the density increases, the two results match very well and the influence of fixed checkpoints can be negligible.

Then the expected walking time of rejected types II and III passengers is

$$K_{\mathrm{II}}^r = K_{\mathrm{III}}^r = \frac{D_r}{V_{wk}} \approx \frac{\sqrt{\pi}}{2V_{wk}\sqrt{k(\eta_2+\eta_3)/MLW}} \tag{47}$$

Thus in the flex-route service with unexpectedly high demand levels, the expected walking time per passenger under dynamic-station policy is

$$K = \frac{(\lambda_c-\lambda)(\eta_2+\eta_3)}{\lambda_c}\times\frac{D_r}{V_{wk}} = \frac{\sqrt{\pi}(\lambda_c-\lambda)(\eta_2+\eta_3)}{2V_{wk}\lambda_c\sqrt{k(\eta_2+\eta_3)/MLW}} \tag{48}$$

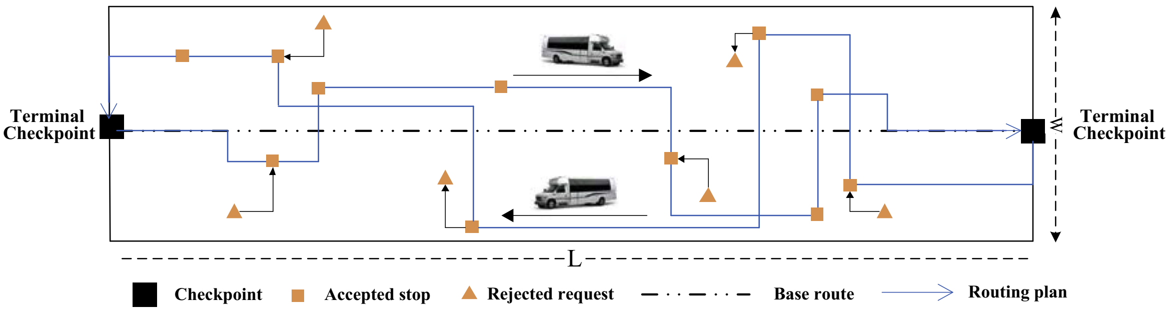


Fig. 5. The dynamic-station policy.

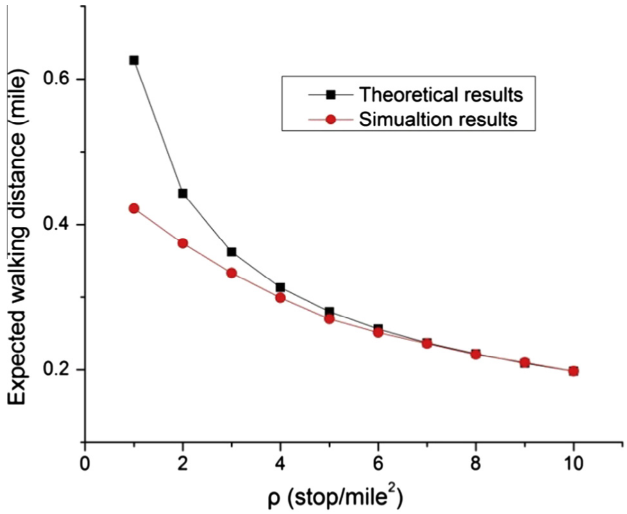


Fig. 6. Theoretical modeling VS simulation for the expected walking distance.

The expected waiting time of rejected requests can be estimated as $A_{\mathrm{I}}^{\lambda_c-\lambda} = A_{\mathrm{II}}^{\lambda_c-\lambda} = T_c/2M$ and $A_{\mathrm{III}}^{\lambda_c-\lambda} = 0$, and thus the expected waiting time of all passengers is

$$A = \frac{\lambda}{\lambda_c}\alpha + \frac{(\lambda_c - \lambda)(\eta_1 + \eta_2)T_c}{2M\lambda_c} \tag{49}$$

The expected riding time of the three passenger groups in the last $\lambda_c - \lambda$ requests can be calculated using Eqs. (33) and (34), and the expected riding time of all passengers is equal to the result of Eq. (35).

5. Result analysis

5.1. Parameter values

The default system parameter values are as follows: $L = 3$ mile; $W = 1$ mile; $d = 0.5$ mile; $V_{wk} = 3$ mile/h; $V_b = 25$ mile/h; $T_d^f = 15$s; $T_d^r = 12$s. Based on the real-life data in Route 289, the proportions of passengers are set as $\eta_1 = 0.2$, $\eta_2 = 0.4$, $\eta_3 = 0.4$. There is only one service vehicle in the Route 289 system, and thus we set $M = 1$. The values of walking and waiting time can be expected to vary depending on a wide range of socio-economic and situational factors (Wardman, 2004). Because walking and waiting are more uncomfortable than in-vehicle time, we assume that one unit of walking and waiting time is respectively 3 times and 2 times as costly as in-vehicle riding time.

5.2. System performance at expected demand levels

At expected demand levels, passenger cost function values under the three operating policies can be calculated using the above theoretical modeling. The cost indicators in the fixed-route service remain stable at different demand levels, and their values are $K = 6$ min, $A = 8.12$ min, $R = 5.22$ min, $U = 39.46$. The results in the flag-stop and flex-route services are displayed in Tables 1 and 2, respectively.

The time of serving a demand on request in the flag-stop service is much shorter than in the flex-route service. As the demand increases, the service cycle T_c experiences a very small rise in Table 1, and is enlarged obviously in the flex-route service (see Table 2). In the flag-stop service, as the demand increases, the expected walking time K remains the same and the values of A and R rise slightly, which suggests that the demand level has a limited influence on the system performance.

Although passengers have no walking ($K = 0$) in the flex-route service, A and R have a significant rise as the demand increases, which indicates that the system performance under flex-route policy is quite sensitive to the demand level. When the travel demand rises to 50 passenger/h, the passenger cost function value of the flex-route service has already exceeded those of the fixed-route and flag-stop services.

The system performance under the three operating policies is displayed in Fig. 7. The results indicate that in the designed demand range, flag-stop policy always has a better performance than fixed-route policy. Flex-route policy is preferable at low demand levels, and the passenger cost function of the flex-route service has a significant rise with the increase of travel demand. It is observed that in this base case, flag-stop policy has the best performance when $\lambda > 42$ passenger/h. The difference between flag-stop and fixed-route policies narrows with the rise of demand, and thus an intersection between the two policies can be expected at a sufficiently high demand level.

Sensitivity analysis over several system parameters is conducted and the results are presented in Fig. 8. The walking speed might vary in different passenger groups and we define another walking speed V_{wk} = 1.5 mile/h (Fig. 8(a)). The results

Table 1
Passenger cost indicators in the flag-stop service.

Indicator (min)	Demand (passenger/h)						
	26	30	34	38	42	46	50
T_c	16.01	16.20	16.39	16.58	16.78	16.98	17.19
K	4.00	4.00	4.00	4.00	4.00	4.00	4.00
A	8.00	8.10	8.19	8.29	8.39	8.49	8.60
R	4.80	4.86	4.92	4.97	5.03	5.09	5.16
U	32.81	33.05	33.30	33.55	33.81	34.08	34.35

Table 2
Passenger cost indicators in the flex-route service.

Indicator (min)	Demand (passenger/h)						
	26	30	34	38	42	46	50
T_c	24.03	26.17	28.72	31.82	35.68	40.60	47.10
K	0	0	0	0	0	0	0
A	7.53	8.27	9.17	10.25	11.60	13.33	15.60
R	7.21	7.85	8.62	9.55	10.70	12.18	14.13
U	22.26	24.40	26.95	30.05	33.91	38.83	45.33

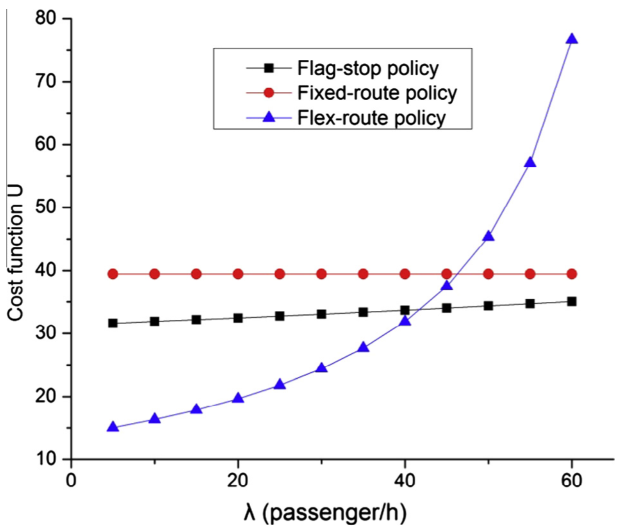


Fig. 7. System performance at expected demand levels.

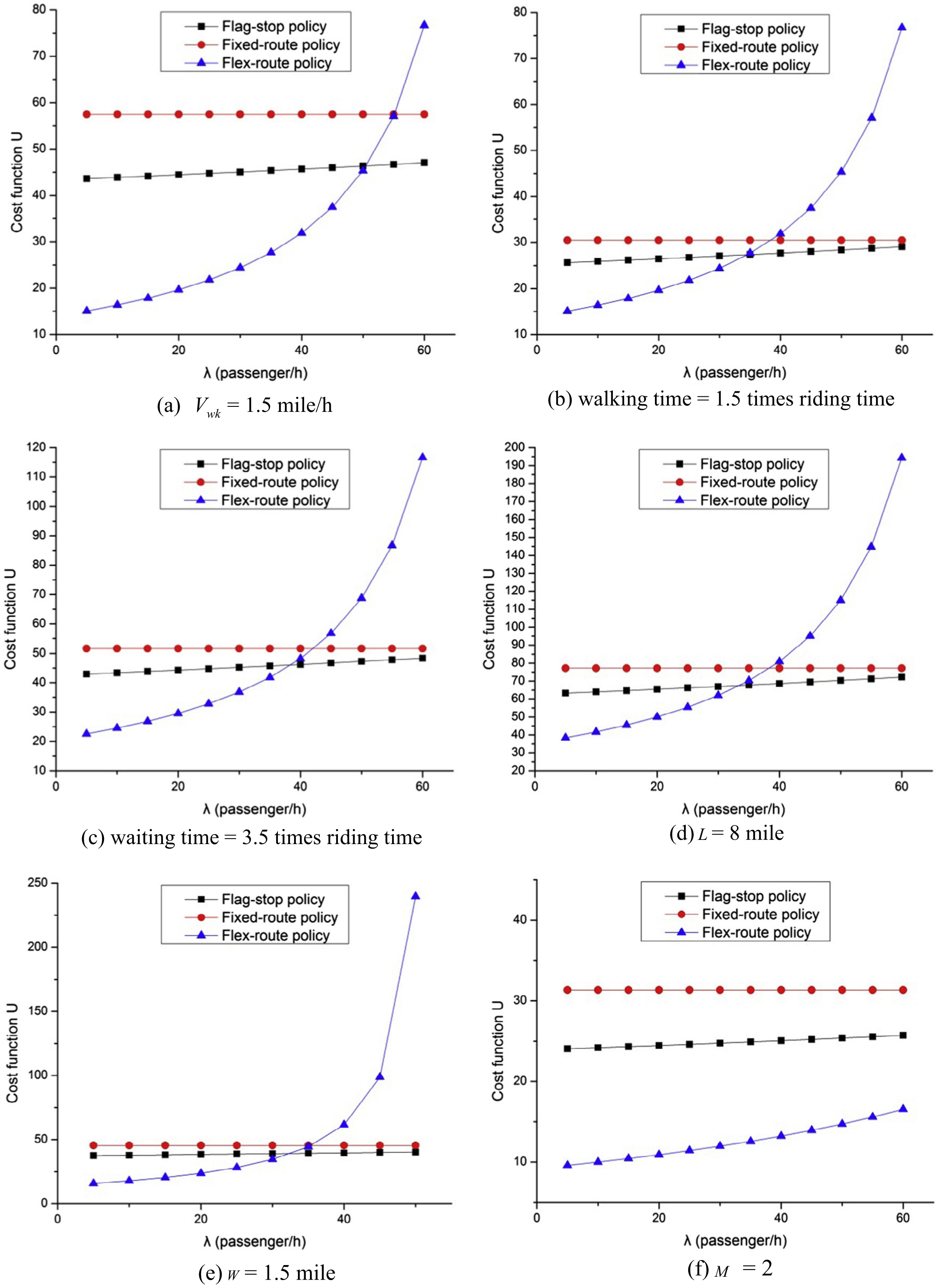


Fig. 8. Sensitivity analysis over (a) walking speed; (b) weight of walking time; (c) weight of waiting time; (d) length; (e) width; (f) vehicle fleet size.

indicate that with a lower walking speed, the passenger cost function values under flag-stop and fixed-route policies both have a distinct rise, and the line representing flex-route policy intersects with another two lines at higher demand levels. In addition, the difference between flag-stop and fixed-route policies becomes more remarkable than the base case. These findings suggest that flag-stop and flex-route policies are more preferred by the passenger group with a lower walking speed, e.g. elderly or less mobile passengers. It is also found that the demand range, where flag-stop policy has the best system performance, is smaller than the base case.

The weight of walking time can be affected by external operating environments and we define one unit of walking time to be 1.5 times as costly as riding time (see Fig. 8(b)). This has no influence on the performance of the flex-route service. The difference between fixed-route and flag-stop policies is smaller than the base case, and the fixed-route system is most sensitive to the weight of walking time. The results also suggest that there will be a smaller demand range fitting flag-stop policy with a larger weight of walking time, which often represents extreme operating environments, such as bad weather and unsafe surroundings.

We consider another case where one unit of waiting time is set as 3.5 times as costly as riding time (see Fig. 8(c)). This case would occur when passengers have to wait possibly at unsafe locations or at night. A rise of passenger cost function under the three operating policies is observed, and the rise in the flex-route service is much greater than in the other two services, particularly at high demand levels, which suggests that flex-route policy is most sensitive to the weight of waiting time. The demand range suitable for flag-stop policy is slightly larger than that the base case.

To study the influence of the service area shape on the system performance, a greater length $L = 8$ mile is set for the Route 289 service (see Fig. 8(d)). This definitely enlarges the service cycle and inevitably results in the increase in waiting and riding time. The passenger cost function values of the flag-stop and fixed-route systems are approximately twice as much as the base case, and the rise is even greater for the flex-route service. It is also noticed that the difference between fixed-route and flag-stop polices is slightly more obvious than the base case. In this scenario, the demand range for the flag-stop service to be optimal is distinctly larger.

Another experiment with a larger value of area width $W = 1.5$ mile is conducted (see Fig. 8(e)). In this case, fixed-route and flag-stop policies have a similar rise in the passenger cost function, and the passenger cost of the flex-route service increases much more dramatically, due to a greater travel distance of serving curb-to-curb requests. The line of flex-route policy intersects with another two lines at significantly lower demand levels, which demonstrates that flex-route policy is quite sensitive to the area width and more preferable in narrow service areas. The results also indicate that the flag-stop service becomes favorable for a larger travel demand range, compared with the base case.

Furthermore, a two-vehicle case ($M = 2$) is investigated and the results are displayed in Fig. 8(f). The two service vehicles actually run along the base route in the opposite directions, and they leave the two terminals at the same time. The outcomes indicate that more available vehicles result in lower values of passenger cost function under the three operating policies, especially for the flex-route service. In this case, the flex-route service always has a much better system performance in the designed demand range, and it is obvious that flex-route policy is most sensitive to the vehicle fleet size.

The above analysis confirms that in the designed demand range, flag-stop policy always performs better than the conventional fixed-route policy. In Fig. 8(a) and (f), if the fixed-route service turns to flag-stop policy, the passenger cost function will decrease by up to approximately 25%. Although flex-route policy has the best system performance at low demand levels, flag-stop policy still has its unique advantage for it is quite easy for transit divisions to implement this kind of service. The high-efficiency operation of flex-route services cannot be achieved without sophisticated support systems, and the required costs, staffing and training unavoidably raise the bar for the application of the emerging flex-route policy.

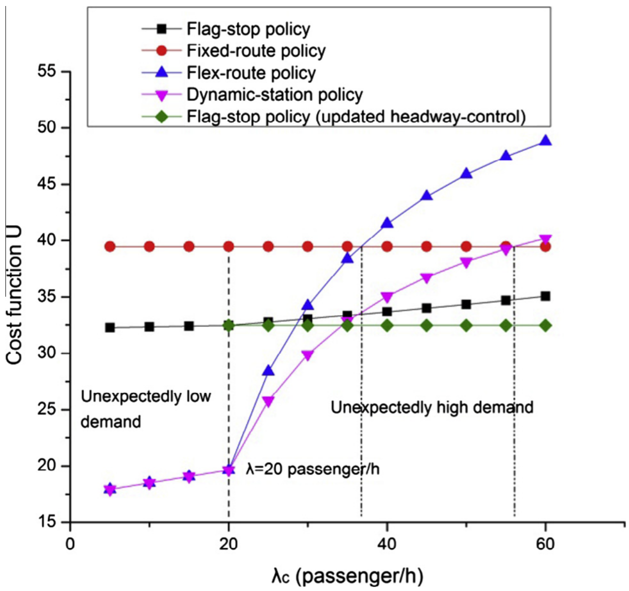


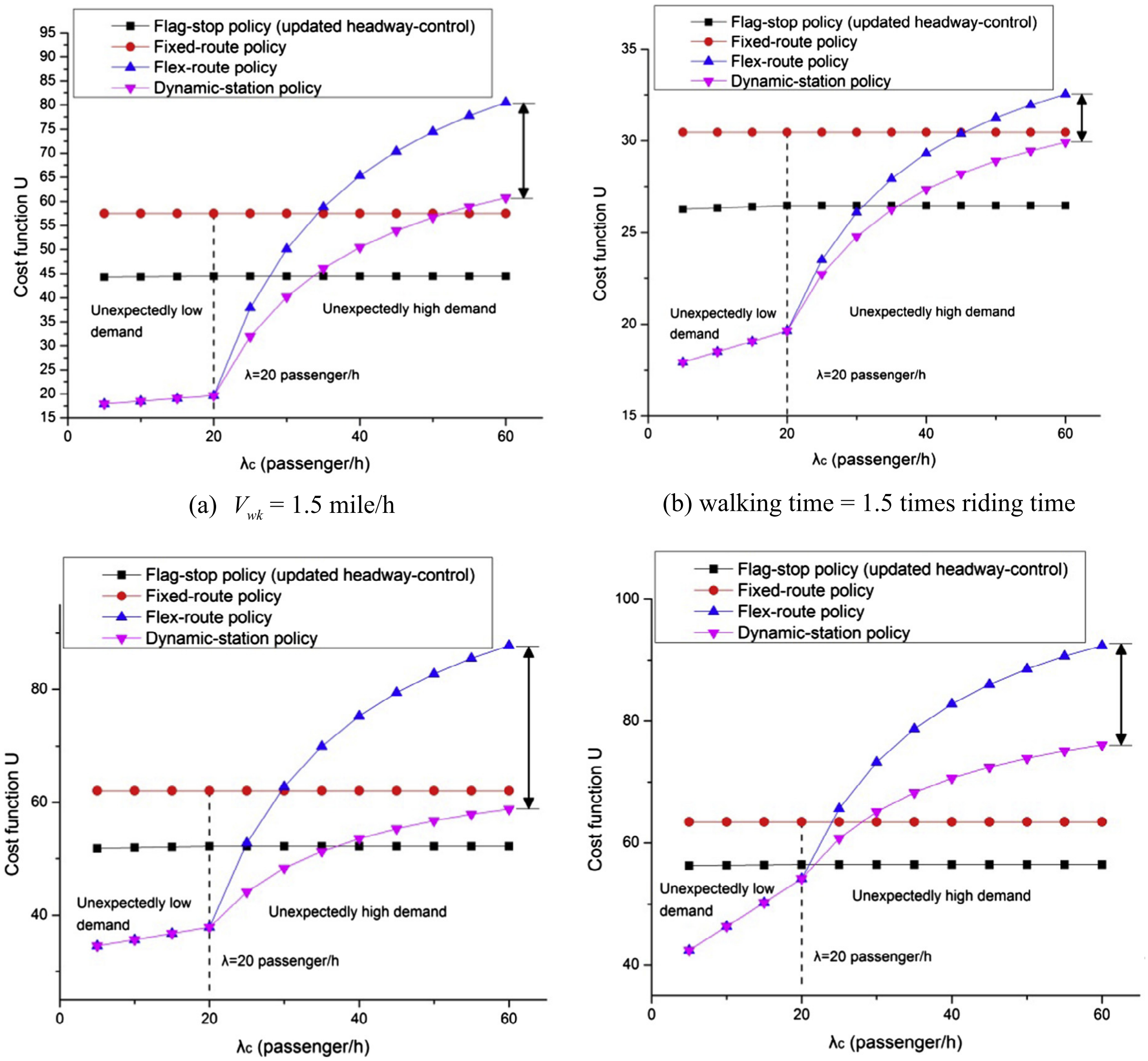
Fig. 9. System performance at unexpected demand levels.

In our analysis the capacity constraint of service vehicles is not considered because flexible and demi-flexible operating policies are generally implemented in low-demand areas. Flex-route policy is most sensitive to the capacity constraint for its longest service cycle. The simulation built for the flex-route service reveals that in the designed demand range, the maximum number of passengers on board is 21, which implies that a 25-seat service vehicle could sufficiently accommodate current travel demand. The capacity constraint needs to be considered if there is a higher demand level in the service area.

5.3. System performance at unexpected demand levels

In this section, the system is designed with expected demand rate $\lambda = 20$ passenger/h and the system performance at unexpected demand levels is presented in Fig. 9. The results show that when $\lambda_c < \lambda$, flag-stop and flex-route policies experience a decrease of passenger cost function and fixed-route policy is not affected, which suggests that the three operating policies work well at unexpectedly low demand levels.

At unexpectedly high demand levels ($\lambda_c > \lambda$), the system performance of the fixed-route service still remains the same. Flag-stop policy seems not to be quite sensitive to the rise of actual demand λ_c, and its passenger cost function increases slightly. Conversely, the passenger cost function of the flex-route service surges with the increase of actual demand, and it seems that flex-route policy might not be suitable to apply in operating environments with uncertain travel demand.



(a) $V_{wk} = 1.5$ mile/h (b) walking time = 1.5 times riding time (c) $L = 6$ mile (d) $W = 3$ mile

Fig. 10. Sensitivity analysis over (a) walking speed; (b) weight of walking time; (c) length; (d) width.

In the system, the dynamic-station policy will be active if $\lambda_c > 20$ passenger/h. After implementing the dynamic-station policy for rejected requests, the system performance of the flex-route service has been greatly improved at unexpectedly high demand levels (see Fig. 9). For instance, the line signifying flex-route policy intersects with the line representing fixed-route policy at the demand level $\lambda_c = 37$ passenger/h, and with the assistance of the dynamic-station policy, the intersection occurs at $\lambda_c = 56$ passenger/h. The results demonstrate that the dynamic-station policy enables the flex-route service to better adjust to operating environments in low-demand areas.

The results in Table 1 show that the service cycle in the flag-stop service is not sensitive to the demand level, and thus the soft time window setting actually has a very limited influence on the transit operation under flag-stop policy. For instance, in the scenario with $\lambda = 30$ passenger/h and $\lambda_c = 40$ passenger/h, the service cycle is only enlarged by approximately 0.48 min in the flag-stop service, while if the soft time window setting is implemented in the flex-route service, the service cycle will be prolonged by nearly 7.5 min, which actually results in unacceptable variations in the transit schedule.

Considering that in the flag-stop service, the variations of service cycle caused by the soft time window setting are quite small at unexpectedly high demand levels, it is technically feasible to update the headway control strategies to eliminate them, such as by assigning a little higher traveling speed to service vehicles. If so, it is possible for the flag-stop service to maintain the same system performance at expected and unexpectedly high demand levels (see Fig. 9). Conversely, the updated headway control strategies might not be able to support the flex-route service to be operated with a soft time window, because of the wild swings of service cycle at unexpectedly high demand levels.

Sensitivity analysis has been conducted to study the performance of the dynamic-station policy in variable scenarios. We consider a case with a lower walking speed $V_{wk} = 1.5$ mile/h (see Fig. 10(a)). The results indicate that at unexpectedly high demand levels, there is a greater decline of passenger cost function in the flex-route system than the base case after using the dynamic-station policy. For instance, at a high demand of $\lambda_c = 60$ passenger/h, the passenger cost in the base case will decrease by 17% through the dynamic-station policy and the decrease increases to approximately 25% in the case of Fig. 10(a). Thus the proposed dynamic-station policy will be probably more preferred by the passenger group with a lower walking speed.

If we set one unit of walking time as 1.5 times as costly as in-vehicle riding time (see Fig. 10(b)), the cost function of the flex-route service decreases less than the base case after using the dynamic-station policy. At the demand level $\lambda_c = 60$ passenger/h, the decrease of passenger cost through the dynamic-station policy in the flex-route system will decline from 17% (the base case) to almost 8% (Fig. 10(b)). This suggests that the dynamic-station policy is more preferable when a larger value is allocated to the weight of walking time, which often represents uncomfortable operating environments.

Another two cases, larger values of the area length $L = 6$ mile (Fig. 10(c)) and width $W = 3$ mile (Fig. 10(d)), have been examined to study the influence of service area shape on the performance of the dynamic-station policy. The results both suggest that the passenger cost function will decrease more drastically after using the dynamic-station policy, compared with the base case. When $\lambda_c = 60$ passenger/h, the decrease of passenger cost function will rise from 17% (the base case) to approximately 35% (Fig. 10(c)) or 20% (Fig. 10(d)). In a larger service area, rejected curb-to-curb passengers in the flex-route service will have to spend more time on the way to nearby checkpoints and the dynamic-station policy makes it easier for them to get on the service vehicles.

6. Conclusions

This paper addresses a problem faced by planners in designing attractive public transit services in low-demand areas. In the past decades, the efforts of serving passengers in sparse residential districts have primarily focused on developing various kinds of flexible transit services. However, operating flexible transit policies is confronted with lots of challenges, and the development of these emerging services is far behind expectation. In our work, a new category of transit policies, named demi-flexible operating policies, is identified for their unique advantages of providing affordable and efficient transport services to residents in low-demand communities.

By modeling a real-life transit service, the system performance of flag-stop policy, which is the most common kind of demi-flexible operating policy, is compared with another two comparable operating policies, the fixed-route and flex-route systems, at expected and unexpected travel demand levels. The results suggest that it is actually difficult to provide reliable flexible transit services in operating environments with uncertain travel demand. Thus another kind of demi-flexible operating policy, called dynamic-station policy, is introduced to assist flexible transit services to handle unexpectedly high demand in daily operation.

The outcomes demonstrate that the group of demi-flexible operating policies is actually an indispensable part of the unifying framework for designing public transit services in low-demand areas. If there is lack of sophisticated support systems for the flex-route service, flag-stop policy will be a better choice at low demand levels for its better performance than the conventional fixed-route service. If the transit service can be equipped with required support systems: flex-route policy plus dynamic-station policy definitely has a clear system advantage at low demand levels; flag-stop policy is a better choice at low-to-moderate demand levels; fixed-route policy will be more preferred at high demand levels.

The purpose of this work is to help planners choose appropriate transit operating policies and increase the attractiveness of public transit in low-demand areas. Some limitations in our research need to be admitted. For instance, the fixed dwelling time T_d^f is expected to be a function of the number of passengers boarding/discharging, but we consider that this has limited influence on the validity of our research in low-demand scenarios. In practical use, simulation modeling can be constructed to assist transit planners to make better decisions because it can accurately reproduce transit operation and is easier to update with real-life operational data than analytic modeling.

In operation, demi-flexible operating policies can be applied singly in a transit system, or along with flexible transit policies and conventional fixed-route services to guarantee their adaptation to particular settings and improve passengers' travel experience. Future work includes exploring more demi-flexible operating policies to plug the gap between fixed-route and flexible transit services in various operating scenarios, and investigating the influence of complex land use plan and roadway networks on the performance of demi-flexible transit policies with different system compositions.

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