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# Real-Time Optimal Scheduling Model for Transit System with Flexible Bus Line Length

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## Abstract

We propose a new structured transit system with flexible bus line length based on the real-time requests of passengers. This operation mode can meet the travel demands of passengers (lower travel-time cost) and reduce the number of unnecessary stops (without boarding or alighting activities). We then compare the performance of the proposed transit system with that of fixed-route transit service and show the pros and cons of each system for several passenger demand levels. It is found that at low-to-moderate demand levels, the proposed flexible-route system tends to provide the lowest total travel time. In comparing a real-world deployment of the system in Guangzhou with comparable alternative system states, the optimization of the former is evident in idle and tidal modes, in which the total travel times reduced by more than 10%. Moreover, this operating system can achieve enhanced performance with the development of automated vehicle technology in the future.

Urban public transport has the advantages of being efficient, reducing energy consumption, and being environmentally friendly. Urban traffic congestion is a severe problem in city development, with which urbanization and traffic mobility are developing rapidly. Thus, it is significant for citizens to choose public transit (PT) and live a green lifestyle.

According to their service mode, urban public transportation systems can be roughly divided into fixed and flexible transit systems. Fixed transit services, which have fixed routes and fixed service schedules, and flexible bus services, which provide doorstep services, have different advantages and disadvantages (*1*). Fixed routes in a transit system provide clarity and regularity to transit services and are known to work very well for cities with high passenger demand in general (*2*). However, in low-demand areas (such as the suburbs of a city, industrial parks, etc.), the demand for public transport is relatively low and distributed; thus, it is expensive to support a fixed transit service with a high degree of passenger satisfaction in such areas (*1*). In such cases, adding flexibility to transit routes and schedules seems desirable, which leads to a type of flexible transit system. By selecting a flexible system, the operator can change one or more characteristics, such as bus routes, bus schedule, bus stops, or bus types. The flexible percentage will rise as the number of flexible characteristics increases. Moreover, by effectively integrating conventional and flexible services the overall system cost may be significantly reduced in comparison with conventional or flexible services (*3–6*).

The flexible transit system was initially designed to serve people with reduced mobility (*7*) and was later extended to general customer service, constituting the first attempt in this direction (*8*). Compared with fixed-line buses, flexible buses make it possible to provide personalized service by modifying routes, schedules, and parking areas according to the traffic demand of passengers at a particular time. Moreover, such buses ensure a certain degree of resource sharing through common service requests. Thus, there is a need for a transit system that provides flexible service at a low price (*9*). Table 1 summarizes some applications and attempts to develop flexible systems in recent years. The findings can be briefly summarized as follows:

1. System cost minimization has been widely used as an optimization objective to operate flexible transit systems in recent years (*2*, *9–14*). Most studies have incorporated total user costs and agency costs (*2*, *10*, *12*, *14*). Others have sought to

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**Table 1.** Summary of Previous Studies on Flexible Transit Systems

| Year | Study area | Transit network | Operation mode | Problem description | Study purpose | Optimization objective | Model types | Solving method | Influence factors | Sensitivity analysis | Result analysis | Case study |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2006 (*10*) | Square area (10*10) | A fleet of vehicles | Demand-responsive connector | Static | Determine the number of vehicles needed | Minimize system cost | Probabilistic modeland continuous approximation models | Parallel regret insertion algorithm | Time window | Yes | Actual and simulated | Computational experiments |
| 2006 (9) | Square district | Single bus line | Deviate from the fixed path | Static | Maximum longitudinal velocity | Minimize system cost | Continuous approximation models | Successive approximation (compute lower and upper bounds) | Slack time | Yes | Actual and simulated | Computational experiments (a two-vehicle system) |
| 2007 (*11*) | Slim service area | Single bus line | Deviate from the fixed path | Dynamic | Scheduling | Minimize system cost | Mixed-integer programming | Insertion heuristic algorithm | Usable slack time | Yes | Actual and simulated | Line 646 in Los Angeles |
| 2008 (*16*) | Loop between terminals | A single vehicle | Deviate from the fixed path | Static | Scheduling | Minimize weighted sum of three factors | Mixed-integer programming | Delete invalid solutionand sequential insertion algorithm | Slack time | Yes | Logic and illogic cuts | Data for DRT service in Los Angeles County |
| 2008 (*12*) | Zone district | A fleet of vehicles | Deviate from the fixed path | Static | Scheduling | Minimizes additional distance | Simulation model | Heuristic algorithm and simulation | Time window | Yes | Actual and simulated | Los Angeles County |
| 2008 (*15*) | Slim service area | A single vehicle | Deviate from the fixed path | Static | Evaluate the shape of service area | Minimize system cost | Simulation model | Insertion heuristic algorithm | Slack time | Yes | MAST/Fixed-Route | Line 646 in Los Angeles |
| 2009 (*13*) | Square district | Single bus line | Demand-responsive connector | Static | Determine the number of zones | Minimize system cost | Simulation model | Insertion heuristic algorithm | Dwelling time | Yes | FRT and DRC | Computational experiments |
| 2012 (2) | Area around terminals | Single bus line | Demand-responsive connector | Static | Scheduling | Minimize system cost | Simulation model | Simulation | Slack time | Yes | Flex-route and fixed-routes | City of Oakville (Toronto) |
| 2012 (*14*) | Hybrid network structure | Bus fleet | Zone route | Static | Find the best decision variables | Minimize system cost | Constrained nonlinear optimization problem | Steepest descent method | Time window | Yes | Alternative systems (fixed-route and taxi) | Numerical examples |
| 2016 (*17*) | Irregularly shaped networks | Multi buses | Deviate from the fixed path | Dynamic | Routing | Minimize total time | TSP problem | Three-stage solution algorithm | Time window | Yes | Fixed + demand responsive mode and flexible mode | Numerical examples |
| This paper | Slim service area | A fleet of vehicles | Demand-responsive (flexible line length) | Dynamic | Routing and scheduling | Minimize total time | 0–1 integer programming | Accurate solutionand simulation | Time window | Yes | Fixed-route system | Line 15 in Guangzhou |

minimize total time as another way to describe model performance. In this paper, we have selected total time as a vital index for optimizing a flexible system.

2. The time windows of buses and passengers, which means an available period (*10*), are essential in a flexible system and have been considered in nearly all of the papers reviewed (*2*, *9–17*). To improve service reliability, some researchers have included significant amounts of slack in the schedule (*9*, *11*, *12*, *14*, *15*). This addition can render the model much closer to the real case. Hence, this key factor is included in the proposed model.
3. Because of technological limitations, over the past few years, most studies (*2*, *9*, *10*, *12–16*) have explored static problems and have lacked ways of establishing real-time communication with passengers and bus information processing centers (IPCs). To better design a flexible transit system, this study proposes a new operational mode under a new information system (instant messaging, real-time location, "pay-by-mobile" technology and autonomous vehicles).

With the increased availability of mobile internet, a few flexible transit options have appeared. Table 2 presents a list of flexible transit operator companies in China. Flexible PT has numerous operating modes, two of which are already common in the Chinese market. One mode, which composes most of the market share, is a type of flexible transit operation system in which bus companies determine passenger demands and design bus lines; then, passengers vote to decide which bus line will operate in the subsequent month. The whole process is performed monthly, and the system does not frequently change, which means that the system cannot adapt to changes in demand. Another operating mode is fully flexible transit, such as the DIDI minibus system in Beijing and Chengdu. The DIDI minibus is a seven-seat car without a fixed route and is similar to a succession of passenger carpools (DIDI express). This minibus cannot pick up many passengers or provide transit over long ranges (its range is generally less than 4 Km).

Flexible transit is also becoming popular in the U.S.A—both Uber and Lyft are trying to make some attempts with it. Uber put up a service called UberHop, a service that debuted in Seattle a few years ago, which was designed around fixed pickup and drop-off points, just like a bus (similar to the minibus service provided by DIDI in China). Lyft works with PT agencies across the country to eliminate transportation barriers, for example it launched a new model for suburban mobility, enabling residents and visitors to access a Lyft ride

**Table 2.** List of Flexible Transit Operators in China

| | Name | Pricing strategy | Seating and capacity | Bus line length | Route | Book strategy | Route update period | City | Since |
|---|---|---|---|---|---|---|---|---|---|
| I | Ruyue bus | Flat fee (¥6–20) | 37–55 seats (normal bus) | Fixed | Static[b] | Book in advance | 1 month | Guangzhou/Foshan | 2015 |
| | DaDa bus[a] | Flat fee (¥6–15) | 37–55 seats (normal bus) | Fixed | Static[b] | Book in advance | 1 month | Shenzhen/Beijing/Hangzhou | 2015 |
| | Hollo bus | Flat fee (¥2–10 for distances < 5 km) | 11–19 seats (minibus) | Fixed | Static[b] | Book in advance | 1 month | Beijing | 2016 |
| | Udian bus | Flat fee (¥6–20) | No seats | Fixed | Static[b] | Book in advance | 1 month | Shenzhen | 2016 |
| II | DIDI mini bus | Dynamic pricing (¥2.99–12.99 for distances < 4 km) | 5–7 seats (MPV) | Flexible | Dynamic changing | Real-time response | 1 second | Beijing/Chengdu | 2016 |
| III | This paper | Flat fee (future study could include passengers' WTP) | 19–37 seats (normal bus) | Flexible | Dynamic changing | Real-time response | 1 second | – | – |

[a]There are approximately 47 other operators of the same mode: DuDu bus, E bus, CC bus, etc.
[b]These bus companies collect passenger demands and design bus lines; then, passengers vote to decide which bus line will operate in the next month.

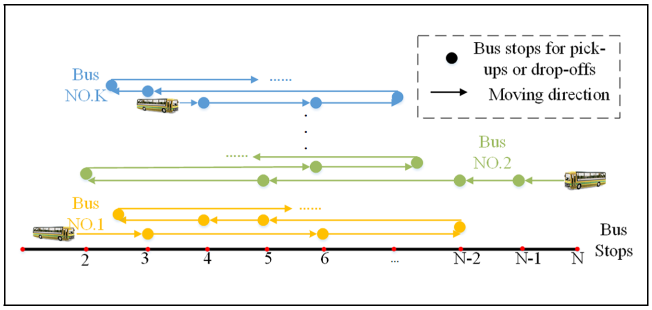


**Figure 1.** General scheme of the structured flexible transit system.

anywhere in the Monrovia service area for just $ 0.50 (temporarily, owing to the financial supports of Lyft), and to easily connect to the Los Angeles Metro Gold Line. Unfortunately, both these market operations above lack a method for addressing low-demand areas with a distance of approximately 4–10 Km at a low price.

Traditionally, transit systems are designed to contain fixed bus routes with a fixed bus line length, and passengers move to predetermined stations to gain access to the system with a fixed bus schedule. Operators serve every bus stop according to the order of bus stops, even if there are several unnecessary stops (without boarding or alighting activities). This paper aims to integrate these ideas (e.g., flexible routes, U-turns, uninterrupted operation) into the design of a new structured "flexible-line-length system." The system features a bus line with several buses operating without interruption at the same time, for which bus stops are ordered in turn and are served only when there is a request from a passenger (either pick-ups or drop-offs). The route and schedule are updated every time there is a new request, and the IPCs system calculates the farthest U-turn at the end of the line in both the up and down directions. The system then determines the optimal U-turn point for buses. We express the system's operating performance as an analytical function with a few key constraints and use a method of real-time control for simulation. When the latest route and schedule are obtained, a message is sent to the bus driver (bus brain if it is an automatic vehicle). Automated buses can respond to real-time instructions from IPCs (including changes to routes and schedules) faster than bus drivers (*18*). With the incorporation of automated buses, this flexible system can respond in a timely fashion and without interruption.

This paper provides two main contributions. First, we propose a new structured transit system with flexible bus-line length based on the real-time requests of passengers. This operation mode can meet the travel demands of passengers (lower travel-time cost) and reduce the number of unnecessary stops (no boarding or alighting the bus) to some extent. Limited-stop services, which omit some stops from a corridor to provide a faster journey, can be beneficial to both operators and users when adequately designed (*19*). The second contribution is the proposal and analysis of a real-world scenario for this type of operation mode.

The remainder of this paper is organized as follows. The Methodology section introduces the operation characteristics, notation, concept, and formulation of the reciprocating automated, flexible transit system. The Alternative Systems section compares this system with a fixed-route transit system. We then describe a real-world case set in Guangzhou and compare it with other systems at different demand levels. Finally, conclusions are provided.

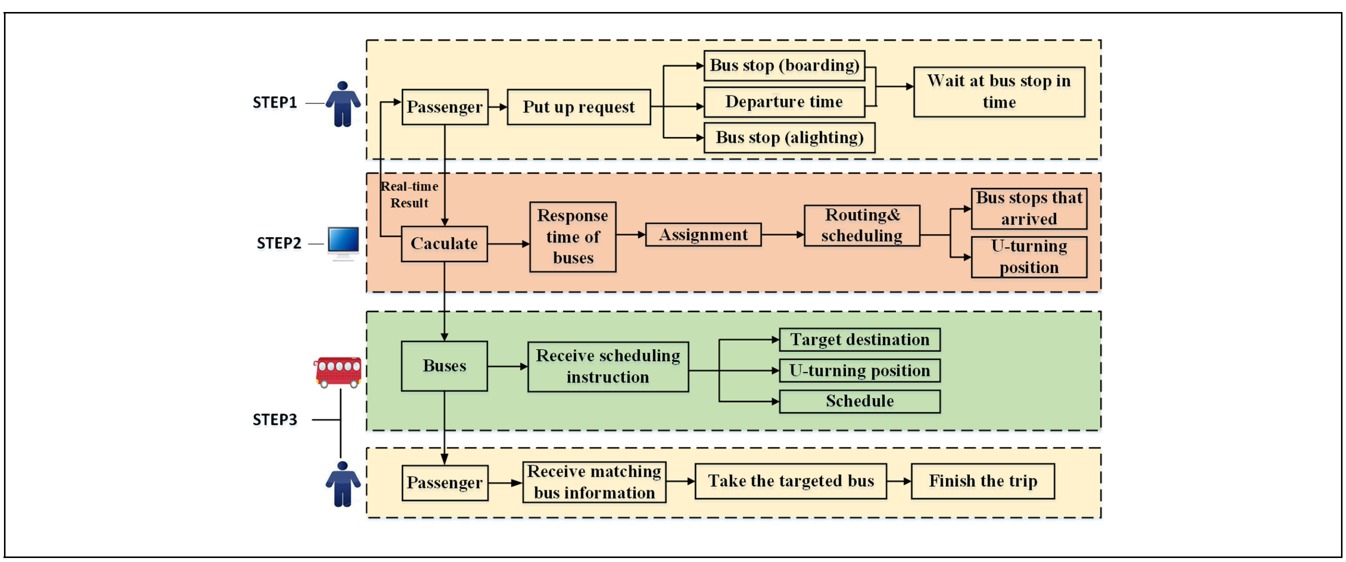


**Figure 2.** Full flow of the flexible-route operation process.

**Table 3.** Notation

| Symbol | Definition |
|---|---|
| Set and indices | |
| $V$ | Set of buses in operation, $k \in V$ |
| $N$ | Set of bus stops, $i \in N$ |
| $L$ | Set of bus line connections, $L_{ij} \in L$ |
| $R$ | Set of passengers, $r \in R$ |
| Parameters and functions | |
| $i, j$ | Bus stops, $i \in N, j \in N$ |
| $\alpha(k)$ | Bus $k$'s real-time location, $0 \leqslant \alpha(k) \leqslant L_{1N}$ |
| $L_{ij}$ | The actual distance from $i$ to $j$ (m), $L_{ij} \in L$ |
| $k$ | Number of buses in operation, $k \in V$ |
| $K$ | Total number of buses in operation |
| $v$ | The average speed of buses (m/s) |
| $t_s$ | Average dwell time at a bus stop |
| $r$ | Passenger, $r \in R$ |
| $s(r)$ | The boarding bus stop for passenger $r$, $s(r) \in N$ |
| $d(r)$ | The alighting bus stop for passenger $r$, $d(r) \in N$ |
| $n_k^{ij}$ | Number of bus stops that bus $k$ serves from bus stop $i$ to bus stop $j$, $k \in V, i \in N,\ j \in N$ |
| $T_k^i$ | Arrival time for bus $k$ at bus stop $i$, $k \in V, i \in N$ |
| $t_{sr}$ | Starting travel time of passenger $r$, $r \in R$ |
| $t_{dr}$ | The arrival time of passenger $r$, $r \in R$ |
| $t_{wk}^F(r)$ | Waiting time for passenger $r$ if served by bus $k$ in the flexible transit system (min) |
| $t_w^F(r)$ | Waiting time for passenger $r$ in the flexible transit system (min) |
| $t_{sw}(r)$ | Waiting time saved for passenger $r$ (min) |
| $t_w^T(r)$ | Waiting time for passenger $r$ in the fixed transit system (min) |
| $t_w^{TS}(i)$ | Arrival time for bus stop $i$ in the fixed transit schedule (min) |
| $T^F$ | Total time for all passengers in the flexible transit system (min) |
| $T^T$ | Total time for all passengers in the fixed transit system (min) |
| $\alpha$ | Total time optimization effect (%) |
| $\alpha_w$ | Waiting time optimization effect (%) |
| $t_{tw}(r)$ | Riding time saved for passenger $r$ (min) |
| $t_t^T(r)$ | Riding time for passenger $r$ in the fixed transit system (min) |
| $t_t^F(r)$ | Riding time for passenger $r$ in the flexible transit system (min) |
| $\alpha_t$ | Riding time optimization effect (%) |
| $K_D$ | Direction imbalance coefficient |
| $mm$ | Farthest U-turn end of the line (up direction) |
| $nn$ | Farthest U-turn end of the line (down direction) |
| Variables | |
| $y_{kj}^+$ | Whether bus $k$ along up direction serves bus stop $i$. If served, $y_{kj}^+ = 1$; otherwise, $y_{kj}^+ = 0$ |
| $y_{kj}^-$ | Whether bus $k$ along down direction serves bus stop $i$. If served, $y_{kj}^- = 1$; otherwise, $y_{kj}^- = 0$ |
| $\tilde{y}_{kj}^+$ | Time at which bus $k$ along up direction serves bus stop $i$. If served, $\tilde{y}_{kj}^+ =$ the time; otherwise, $\tilde{y}_{kj}^+ = -1$ |
| $\tilde{y}_{kj}^-$ | Time at which bus $k$ along down direction serves bus stop $i$. If served, $\tilde{y}_{kj}^- =$ the time; otherwise, $\tilde{y}_{kj}^- = -1$ |

## Methodology

### *Operation Characteristics*

Figure 1 shows a general overview of the structured flexible-route transit system. The system features several buses operating at the same time, and all buses are independent and follow the scheduling determined by the system. There are $N$ potential bus stops, and passengers can make stop requests with smart buttons on the platform, apps on their mobile phone, etc. The requests include origin and destination bus stops and departure time. The system calculates the requests in real time and determines the operating state of each bus (location, direction, farthest end, the number of passengers on the bus, etc.). Then, the system assigns each bus the minimum waiting time to pick up passengers based on the model described in this paper. Because of the real-time requests, the routes and schedules of the buses are updated continuously to meet the minimum total system time. Whenever there is a change of assignment, the automated buses

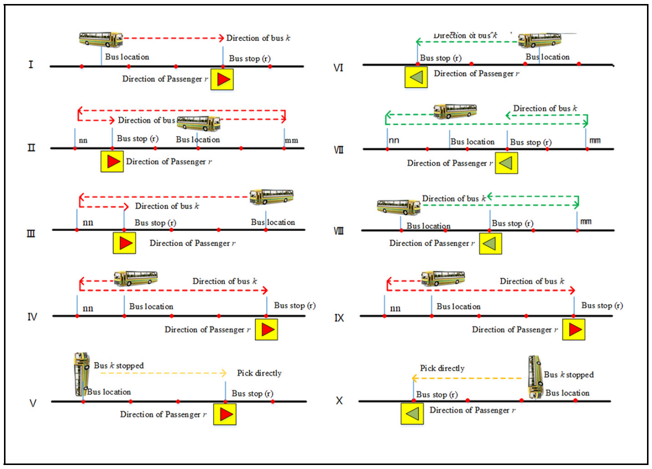


**Figure 3.** Diagrams of waiting time calculation in different situations.

follow the new route and schedule promptly, and passengers receive a new message indicating their pick-up bus number, estimated arrival time and GPS location of the bus. Passengers are required to arrive at the bus stop no later than their departure time on their reservation lists. Passengers then take the target bus and finish their trips. Figure 2 shows the full flow of this operation process.

## Notation and Definition

The proposed model considers a single service region composed of only one bus line and generates $R$ trip demands per hour. The proposed flexible transit system provides service to a relatively small number of passengers. To simplify the model, we assume that all passengers in the system travel rationally and that the automated buses operate at the same speed. Table 3 defines the notation used in this paper.

## Formulation

During operation, the buses run continuously, and the cost of bus operation remains constant with driving speed. The time cost significantly affects the operation of the system. Therefore, the objective of this model is to minimize the total travel time of all passengers. For passenger $r$, the total travel time is the period between the required departure time $t_{sr}$ and the time at which the passenger exits the bus and finishes the trip $t_{dr}$. For a passenger, as stated in Equation 1, the total travel time consists of two parts: waiting times and riding times. In the following subsections, we present formulas for the waiting time and riding time:

$$min \sum_{r \in R} \left( t_w^F(r) + t_t^F(r) \right) \tag{1}$$

*Service Constraints.* In the proposed system, all passenger requests should be considered and served, without the passengers transferring between buses. Equations 2, 3 and 4 state that all passengers with travel demands are picked up and dropped off, and the same passenger can only be served by one bus at a time:

$$\begin{cases} \sum_{k \in V} y^+_{k,s(r)} = 1, \forall |d(r) \rangle s(r), r \in R \\ \sum_{k \in V} y^-_{k,s(r)} = 1, \forall |d(r) < s(r), r \in R \end{cases} \tag{2}$$

$$\begin{cases} \sum_{k \in V} y^+_{k,d(r)} = 1, \forall |d(r) \rangle s(r), r \in R \\ \sum_{k \in V} y^-_{k,d(r)} = 1, \forall |d(r) < s(r), r \in R \end{cases} \tag{3}$$

$$\begin{cases} y^+_{k,s(r)} = y^+_{k,d(r)}, \forall |d(r) \rangle s(r), r \in R \\ y^-_{k,s(r)} = y^-_{k,d(r)}, \forall |d(r) < s(r), r \in R \end{cases} \tag{4}$$

*Waiting Time Function.* To minimize the waiting time of each passenger, we search all buses in operation and calculate the waiting time of each bus based on its operation situation. Figure 3 shows diagrams of various situations. Then, the chosen bus is assigned for service.

Equations 5 and 6 describe the optimization search process. The waiting time for each passenger includes two parts in Equation 5: the time spent driving from bus$k$'s real-time location to the requested starting bus stop of passenger $r$ and the total dwell time associated with the stopping of the bus:

$$t_w^F(r) = min\left\{ t_{w1}^F(r), t_{w2}^F(r), \ldots t_{wk}^F(r), \ldots, t_{wK}^F(r) \right\}, k \in V, , r \in R \tag{5}$$

$$t_{wk}^F(r) = \frac{L_{\alpha(k),s(r)}}{v} + n_k^{\alpha(r),s(r)} * t_s, k \in V, , r \in R \tag{6}$$

For passenger $r$, the distance from bus $k$'s real-time location $\alpha(r)$ to passenger $r$'s starting bus stop $\alpha(r)$ can involve ten types of situations at most. According to the relative position of each bus and passenger, the distance can be calculated by Equation 7:

$$L_{\alpha(k),s(r)} = \begin{cases} \left| L_{s(r)} - L_{\alpha(k)} \right|, situation I, V, IX, X \\ 2^* L_{mm} - 2^* L_{nn} + \left| L_{s(r)} - L_{\alpha(k)} \right|, situation\ II, VI \\ L_{\alpha(k)} + L_{s(r)} - 2^* L_{mm}, situation\ III, V \\ 2^* L_{mm} - L_{\alpha(k)} - L_{s(r)}, situation\ VIII, IX \end{cases} \tag{7}$$

To compute the total dwell time associated with the stopping of a bus, we should count the number of bus stops that bus$k$ serves ($n_k^{\alpha(k),s(r)}$) during this period. Equation 8 describes this process. Because of the low-demand flow of passengers, the number of passengers waiting at a single bus stop at a given time is also small (generally one or two passengers at a bus stop); thus, it is acceptable to use the average dwell time at a bus stop ($t_s$):

$$n_k^{\alpha(r),s(r)} = \begin{cases} \sum\limits_{s=\alpha(k)}^{s(r)} y_{ks}^{+}, \textit{situation } I, IX \\ \sum\limits_{s=s(r)}^{\alpha(k)} y_{ks}^{-}, \textit{situation } V, X \\ \sum\limits_{s=\alpha(k)}^{mm} y_{ks}^{+} + \sum\limits_{s=nn}^{mm} y_{ks}^{-} + \sum\limits_{s=nn}^{s(r)} y_{ks}^{+}, \textit{situation } II \\ \sum\limits_{s=nn}^{\alpha(k)} y_{ks}^{-} + \sum\limits_{s=nn}^{mm} y_{ks}^{+} \sum\limits_{s=s(r)}^{mm} y_{ks}^{-}, \textit{situation } VI \\ \sum\limits_{s=nn}^{s(r)} y_{ks}^{+} \sum\limits_{s=nn}^{\alpha(k)} y_{ks}^{-}, \textit{situation } III, IV \\ \sum\limits_{s=\alpha(k)}^{mm} y_{ks}^{+} + \sum\limits_{s=mm}^{s(r)} y_{ks}^{-}, \textit{situation } VII, VIII \end{cases} \tag{8}$$

*Riding Time Function.* Equation 9, which calculates the riding time, also includes two parts: the time spent driving from the starting bus stop to the destination bus stop of passenger$r$ and the total dwell time associated with stopping during this period. In addition, the distance between the origin and destination is determined by Equation 10, and the number of bus stops served is given by Equation 11:

$$t_t^F(r) = \frac{L_{s(r),d(r)}}{v} + n_k^{s(r),d(r)} * t_s, r \in R \tag{9}$$

$$L_{s(r),d(r)} = \left| L_{d(r)} - L_{s(r)} \right|, r \in R \tag{10}$$

$$n_k^{s(r),d(r)} = \begin{cases} \sum\limits_{s=s(r)}^{d(r)} y_{ks}^{+}, \textit{situation } I, II, III, IV, IX \\ \sum\limits_{s=d(r)}^{s(r)} y_{ks}^{-}, \textit{situation } V, VI, VII, VIII, X \end{cases} \tag{11}$$

Passengers make real-time reservations at or near a bus stop, and they should arrive at the bus stop no later than $t_{sr}$, as stated in Equation 12:

$$t_w^F(r) + t_t^F(r) \geqslant t_{sr}, r \in R \tag{12}$$

## Alternative Systems

In this subsection, we compare the performance of other service providing systems that are comparable with the proposed flexible-route transit system.

For the fixed-route system, a passenger can be in the following states: walking between their destination and the nearest bus station, waiting for a fixed-line bus, and riding a fixed-line bus. To be consistent with our flexible system and to be conservative (i.e., favoring the fixed-route system), we use the same automated buses and the same number of buses in service and disregard any extraneous effects on the fixed-route system. The optimization objective can be expressed using similar notation (while replacing each bus route with a fixed bus route) as in Equation 13. For a passenger, the total travel time in the fixed-line bus system consists of two parts: waiting times and riding times. Equation 14 describes the waiting time, and Equation 15 gives the riding time for passenger $r$ at bus stop $i$ on a fixed transit schedule:

$$T^T = \sum_{r \in R} \left( t_w^T(r) + t_t^T(r) \right) \tag{13}$$

$$t_w^T(r) = t_w^{TS}(i) - t_{sr}, i \in N, r \in R \tag{14}$$

$$t_t^T(r) = t_w^{TS}(j) - t_w^{TS}(i), i \in N, j \in N, r \in R \tag{15}$$

To calculate the economic efficiency of the model, indexes $t_{sw}(r)$, $t_{tw}(r)$, $\alpha_w$ and $\alpha_t$ are introduced. Equations

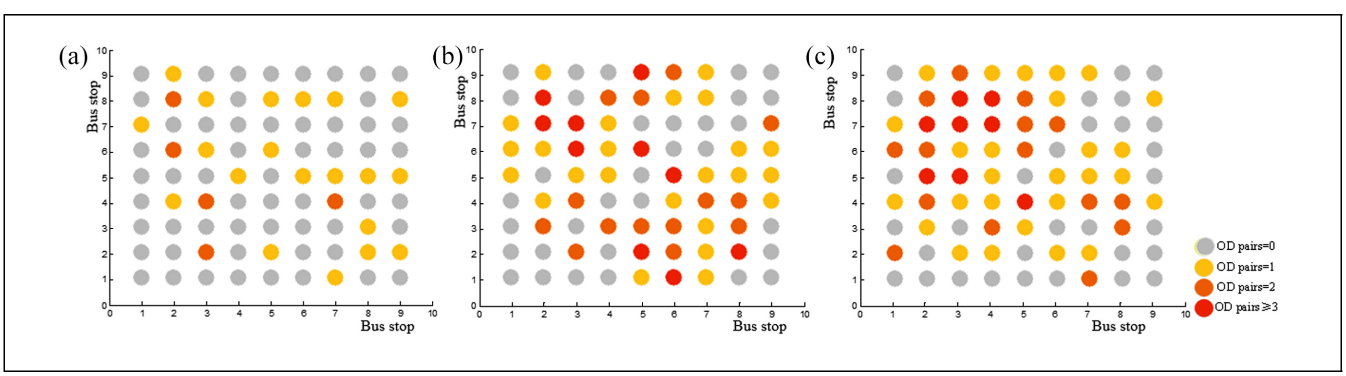


**Figure 4.** Distribution of passengers' origin–destination: (a) R=30, equilibrium mode; (b) R=90, equilibrium mode; (c) R=90, disequilibrium mode.

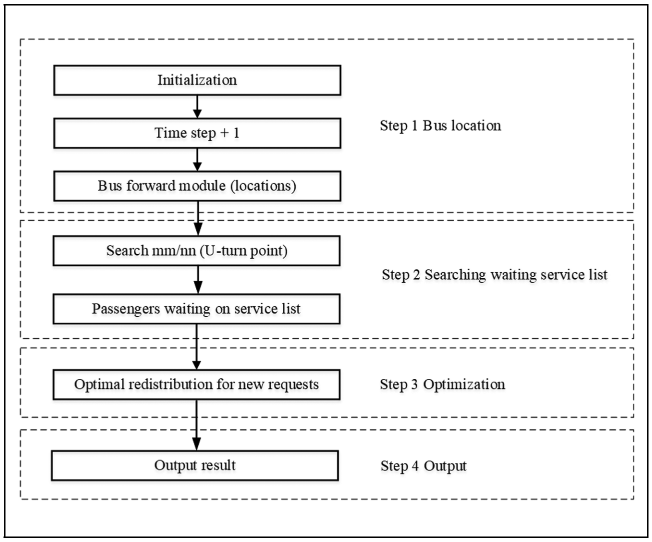


**Figure 5.** Program flow chart for simulation.

16 and 17 provide the waiting-time optimization effect (%) of the flexible bus-line-length model proposed in this paper, and Equations 18 and 19 provide the riding-time optimization effect (%) observed when servicing only the required bus stops:

$$t_{sw} = t_w^T(r) - t_w^F(r), r \in R \quad (16)$$

$$\alpha_w = \frac{\sum_{r \in R} t_{sw}(r)}{\sum_{r \in R} t_w^T(r)} * 100\% \quad (17)$$

$$t_{tw}(r) = t_t^T(r) - t_t^F(r), r \in R \quad (18)$$

$$\alpha_t = \frac{\sum_{r \in R} t_{tw}(r)}{\sum_{r \in R} t_t^T(r)} * 100\% \quad (19)$$

## Numerical Example

We select the No. 14 bus line in Nansha District, Guangzhou, which is a low-density bus line in an industrial park. The entire bus line is approximately 8.0 Km long, includes nine bus stops and requires approximately 30 min (each station stop requires approximately 20 s) to service from the first bus stop to the last bus stop, which means there usually are two buses in service ($K$ = 2). This bus line has an active commuting feature, for many passengers ride from Xianfeng New Town (the first bus stop) to the Pearl River Industrial Park (the 6th bus stop) in the morning, which causes idle and tidal modes.

To validate the model, we choose three passenger flow patterns: equilibrium mode of low passenger density ($R$ = 30), equilibrium mode of high passenger density ($R$ = 90), and disequilibrium mode of high passenger density ($R$ = 90, morning peak). In addition, we compare the flexible transit system in this paper with the fixed-line bus currently on the market. The data are derived from vehicle surveys. Bus passenger flow exhibits a tidal phenomenon, which affects the results of the model. Thus, Figure 4 shows the distribution of passengers' origin–destination and their direction imbalance coefficient $K_D$ considered in this paper.

We use MATLAB to design the solution program (Figures 5 and 6), which can simulate the bus operation process. In this simulation process, we simulate requests in real time and address them on a second-by-second basis. Once every second, if there is a new request, we determine the locations of buses and simulate their real routes (include their updated service schedule). Then, U-turn points of each bus can be calculated according to their service lists. We compare the total time for each service strategy to determine the optimized route. Then, the optimal bus route and schedule are output. The routes and schedules update with the new demand, and finally, we obtain the real-time bus route second by second.

The details of this process are as follows:

**Step 1: Find the real-time location of each bus.**

We set a forward bus module in this step to simulate the operation of the buses (including moving, stopping,

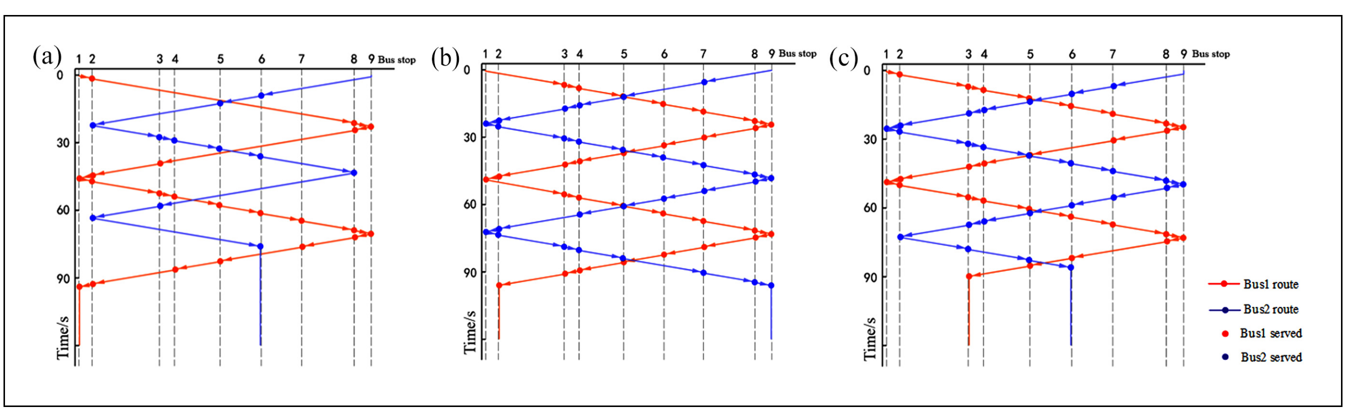


**Figure 6.** The bus route and bus stops served ($K$ = 2): (a) R=30, equilibrium mode; (b) R=90, equilibrium mode; (c) R=90, disequilibrium mode.

**Table 4.** Optimization Effect Analyses

| | Flexible $\text{Max}\{t_w^F(r)\}$ | Fixed $\text{Max}\{t_w^T(r)\}$ | $\text{Max}\{\alpha_w(r)\}$ | $R^*/R$ | Flexible $\sum_{r\in R} t_w^F(r)$ | Fixed $\sum_{r\in R} t_w^T(r)$ | $\alpha_w$ | |
|---|---|---|---|---|---|---|---|---|
| $R = 30$, equilibrium mode | 25.5 | 29.5 | 13.68% | 19/30 | 348.5 | 441.8 | 21.12% | |
| $R = 90$, equilibrium mode | 24.1 | 29.4 | 18.06% | 63/90 | 1122.1 | 1123.1 | 0.09% | |
| $R = 90$, disequilibrium mode | 24.3 | 30.0 | 18.85% | 65/90 | 1098.4 | 1356.2 | 19.01% | |
| | Flexible $\text{Max}\{t_t^F(r)\}$ | Fixed $\text{Max}\{t_t^T(r)\}$ | $\text{Max}\{\alpha_t(r)\}$ | $R^*/R$ | Flexible $\sum_{r\in R} t_t^F(r)$ | Fixed $\sum_{r\in R} t_t^T(r)$ | $\alpha_t$ | $\alpha$ |
| $R = 30$, equilibrium mode | 39.6 | 48.8 | 18.85% | 20/30 | 671.6 | 774.7 | 13.31% | 19.25% |
| $R = 90$, equilibrium mode | 40.45 | 45.4 | 12.23% | 63/90 | 2010.9 | 2014.2 | 0.16% | 0.14% |
| $R = 90$, disequilibrium mode | 39.9 | 47.9 | 16.67% | 65/90 | 2033.9 | 2296.2 | 11.42% | 16.60% |

*Note*: $R^*$ = number of passengers served. Direction imbalance coefficient $K_D$ = 0.533, 0.511, 0.733. Red shading scales according to percentage.

**Table 5.** Parameter Sensitivity Analysis

| | $K = 1$ | | | $K = 2$ | | | $K = 3$ | | | $K = 4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r$ | $T^F$ | $T^T$ | $\alpha$ | $T^F$ | $T^T$ | $\alpha$ | $T^F$ | $T^T$ | $\alpha$ | $T^F$ | $T^T$ | $\alpha$ |
| 20 | 652.1 | 712.2 | 8.44% | 421.6 | 438.5 | 3.87% | 356.6 | 373.7 | 4.58% | 349.7 | 328.9 | –6.32% |
| 40 | 1309.3 | 1434.3 | 8.72% | 784.2 | 871.6 | 10.03% | 709.9 | 701.3 | –1.23% | 615.0 | 640.2 | 3.93% |
| 60 | 2137.4 | 2169.9 | 1.50% | 1275.5 | 1309.2 | 2.57% | 1179.2 | 1065.9 | –10.63% | 1030.2 | 968.0 | –6.43% |
| 80 | 2871.5 | 2923.3 | 1.77% | 1738.9 | 1752.6 | 0.78% | 1627.5 | 1452.5 | –12.05% | 1470.9 | 1301.8 | –12.99% |
| 100 | 3558.5 | 3620.8 | 1.72% | 2200.9 | 2176.3 | –1.13% | 1899.6 | 1811.4 | –4.87% | 1738.2 | 1652.4 | –5.19% |
| 120 | 4294.3 | 4411.0 | 2.64% | 2631.5 | 2674.5 | 1.61% | 2481.1 | 2204.2 | –12.57% | 2111.7 | 2004.4 | –5.35% |

*Note*: Red shading scales according to percentage.

and taking U-turns) and calculate the locations at each second. The location of each bus is based on the product of the average speed and the moving times and directions.

**Step 2: Search the waiting list of each bus and determine the U-turn points for each bus.**

Every bus has a service list, which records the passengers to board. By calculating the farthest point, the future route can be determined. This information can help optimize new requests.

**Step 3: Optimize real-time routes and add new requests to the waiting list of the bus.**

The total time optimization is performed by the traversing method and is updated second by second.

**Step 4: Output the new route and schedule.**

### *Time Comparisons*

For further comparison, we use the same buses in the fixed mode to maintain the same number of buses. The buses in the fixed system also operate without interruption. As the system shares the same buses and covers the same distance (uninterrupted vehicles with the same average speed), the objective of this model is to minimize the total travel time of all passengers. Then, we compare the waiting time and riding time of each passenger and calculate the total time of all passengers (Table 4).

As shown in Table 4, all times (waiting time and riding time) are optimized. When the passenger density is relatively low, the total time optimization effect shows the best performance; this situation mainly occurs during off-peak hours and when passenger flow is in an equilibrium mode. During peak hours, a tide phenomenon consistently occurs, and the flexible transit system also operates well in this disequilibrium mode. Encouragingly, the system performs well in reducing the maximum waiting time and riding time of passengers in all three modes, which should help satisfy even the most annoyed passenger and gain better passenger satisfaction overall.

### Sensitivity Analysis

During the analysis, the selected number of passengers was $R = 20, 40, 60, 80, 100, 120$/hour, and the number of buses $K = 1, 2, 3, 4$. Because a single run takes 1462 s, the departure intervals of the corresponding fixed-route buses are 2924, 1462, 975, and 731 s (i.e., 48.7, 24.4, 16.3, and 12.2 min). Thus, the total time of each parameter mode is as shown in Table 5.

Table 5 illustrates the advantages of the proposed system when the number of buses is relatively small ($K = 1$ or $K = 2$); moreover, the table shows that the total time decreases with a decrease in passenger flow. When the passenger flow exceeds 60 people per hour, the model is still optimal, but the total travel time saved is not appreciable, only approximately 2%. As the number of buses increases ($K \geqslant 3$), the departure interval decreases, which leads to a greater time savings for a traditional fixed service compared with the flexible bus system proposed in this paper. Moreover, as the number of vehicles and passenger flow increase, the flexible bus system proposed in this paper becomes inapplicable because it has no predictive component; in future research, we aim to improve the model in this aspect.

## Conclusion

This paper proposed a flexible transit system that operates vehicles without interruption. In this mode, all buses operate at the same time with a flexible choice of passengers to pick up, and the bus line length is flexible according to the objective of minimizing the total travel time of all passengers. Thus, a real-time routing and scheduling model for this flexible bus mode was proposed, and a corresponding simulation program was designed in MATLAB. We applied this model to the No. 14 bus line of Guangzhou and compared the optimal results of three modes of operation. The optimization effect was distinct in the idle and tidal modes, in which total travel times could reduce by more than 10%. Moreover, through a sensitivity analysis of vehicle number and passenger flow density, the optimization system shows improvements over fixed-route service when the fixed-route service is less frequent than every 16 min and the passenger demand is 40 passengers per hour or less. Through a comparative analysis, we concluded that the proposed flexible system suits situations involving a small number of buses, a vast departure interval, and low passenger demand. Also, in low-density areas, we can use this flexible system to reach a win-win situation by reducing both the total waiting time and riding time of passengers, increasing the satisfaction of PT riders.

With increasing competition in the vehicle market and the rise of intelligent transportation system research, automated vehicles will revolutionize urban traffic (*20–24*). The driverless operation of flexible bus systems could be a future trend in PT systems. Automated buses could respond to real-time instructions from IPCs (including changes to routes and schedules) faster than bus drivers. With the help of this new technology, this flexible transit system could perform even better.

We would continue doing research in this field. Future research could be conducted to explore more parameters (e.g., willingness to pay, and price discrimination) or any hybrid scheduling scheme combining a flexible bus line with a fixed line (e.g., hybrid scheduling targeting the percentage of passengers who do not make requests in advance). Other complex bus routes (e.g., circular or x-shaped) or heterogeneous demand levels (e.g., a strict time window or bus type) could be interesting to analyze, although such conditions may increase the complexity of the model. Moreover, a forward-looking optimization system that anticipates future passenger demands would also be considered in future research, which would render the model more applicable to high passenger density.

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## Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: MP; data collection: JO; analysis and interpretation of results: MP, PL; draft manuscript preparation: MP, PL, JO. All authors reviewed the results and approved the final version of the manuscript.

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