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Flexible feeder transit route design to enhance service accessibility in urban area

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SUMMARY

To improve the accessibility of transit system in urban areas, this paper presents a flexible feeder transit routing model that can serve irregular-shaped networks. By integrating the cost efficiency of fixed-route transit system and the flexibility of demand responsive transit system, the proposed model is capable of letting operating feeder busses temporarily deviate from their current route so as to serve the reported demand locations. With an objective of minimizing total bus travel time, a new operational mode is then proposed to allow busses to serve passengers on both street sides. In addition, when multiple feeder busses are operating in the target service area, the proposed model can provide an optimal plan to locate the nearest one to response to the demands. A three-stage solution algorithm is also developed to yield meta-optimal solutions to the problem in a reasonable amount of time by transforming the problem into a traveling salesman problem. Numerical studies have demonstrated the effectiveness of the proposed model as well as the heuristic solution approach. Copyright © 2015 John Wiley & Sons, Ltd.

KEY WORDS: public transit; flexible service; demand responsive; rural postman problem; genetic algorithm

1. INTRODUCTION

In most cities, the common feeder bus systems are mainly operated in a fixed-route transit (FRT) mode with pre-determined schedules and stop [1]. However, because of the lack of accessibility and flexibility, such FRT mode is inconvenient for some passengers, such as children, disabled, or senior passengers, to reach the bus stops [2]. Recognizing the aforementioned limitations of FRT, transit agencies have also promoted the demand responsive transit (DRT) mode [3], such as dial-a-ride services [4], to improve the accessibility of transit systems. However, because users' requests will vary from day to day with vehicle routes varying accordingly [5], the high operational cost of DRT can inevitably limit its application in practice.

To better design a flexible feeder transit system, this study proposes a new operational mode named flexible transit service. Notably, such operational mode can take advantage of regular FRT's cost efficiency and DRT's flexibility. In review of literature, two types of flexible transit service modes have been promoted by the transit system designers [6]. The first type is the so-called demand responsive connector, which dispatches feeder busses to transfer passengers between demand points and transfer hubs within a pre-defined service area [7, 8]. The second type is route deviation, which allows the bus to deviate from the pre-determined route, serves the requested nearby passengers, and returns to its original route before arriving to the next stop [9]. In both service modes, passengers who need a flexible transit service are required to contact the transit operators at least 1 or 2 h in advance to reserve the

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service. Once the scheduling system has confirmed the service, a text message or phone call will be made by the representatives to inform the estimated pick-up time.

During the past decades, the flex-transit service has been recognized as one of the most efficient solutions to medium/low demand conditions toward a sustainable mobility in newly developed urban/suburban areas (e.g., industrial parks and new suburban residential communities) [5, 10]. Regarding to the route deviation service type, Malucelli (1999) presented several transportation models and associated solutions that were characterized by the management of the service requests [11]. Quadrifoglio *et al.* solved the route design and scheduling problem of mobility allowance shuttle transit system by developing an integer programming formulation [12], insertion heuristics [13–15], and a mixed integer programming formulation [16]. Quadrifoglio also studied the bounds on the maximum longitudinal velocity to evaluate the performance and to design some key parameters, such as slack time and headway [17]. For the demand responsive connector, Li (2009) developed analytical models validated by simulation to design the system [7].

This study will focus on the integration of both aforementioned flexible systems, which is mainly a demand responsive connector system but with route deviation operations. The vehicles can deviate from the base line to service the passenger with special demand and feed them to the transfer hubs/stations. The potential application area of this system will be the one without high enough demand for the fixed mode but sufficient number of passengers for the demand responsive mode, such as shuttle busses of the supermarket. For the route design or scheduling problem of the flexible system, most existing studies in the literature have developed analytical models that require a uniform demand distribution in the service area. Moreover, the feeder busses are assumed to be operated on the grid road network. Obviously, those assumptions in existing studies may prevent the methodologies from real-world applications, especially for the gated communities that prevent feeder busses from serving customers at the designated pick-up/drop-off points, rather than in front of the buildings.

Hence, the transit operation agencies may require an efficient model to generate the bus routing plans to traverse each demand point and return to the original defined route in response to individual service requests from passengers. Therefore, it is practically impossible to apply the existing continuous analytical models in practical applications because of the discrete demand points and the irregular shape of road network. The operation mechanism of the flexible transit system is depending on the requested demands every day, so the scheduling and route plans have to vary accordingly and have some regular pattern to follow. An even more important factor is that most of the demands will be reserved only 1 or 2 h in advance. So the scheduling and dispatching module of the system should be sufficiently efficient to obtain a nearly optimal solution in few minutes. At the same time, the road attribute should be considered during the procedure of route plan. Some roads with median barrier have to be traversed twice to serve the demand distributed at both sides of the road, and the one-way road can only be traveled in the assigned direction.

Recognizing those critical operational issues to be solved, this study aims to develop a flexible feeder transit system with efficient optimization and design functions. The primary contributions of this study is four-fold: (i) designed a new service mechanism that let flexible feeder busses temporally leave their routes and serve the reported demand points in the target region; (ii) optimized the route plan of the each feeder bus in response to those irregularly urban networks; (iii) defined an effective operational mode that allows the busses to serve passengers on both sides of streets; and (iv) developed a quick-responsive solution heuristic to generate meta-optimal bus routing plans.

The paper is organized as follows: Section 2 presents the research background along with the problem nature; then Section 3 provides the brief introduction of problem transformation; Section 4 illustrates a three-stage solution heuristic based on the genetic algorithm (GA); Section 5 uses a network consisting of over 50 nodes and edges for system demonstration; and Section 6 summarizes the key findings and conclusion of this study.

2. PROBLEM NATURE

The proposed flexible feeder transit system can be illustrated in Figure 1. In the service area, the feeder routes with base lines take the responsibility to transfer the passengers to the hubs. To enhance the

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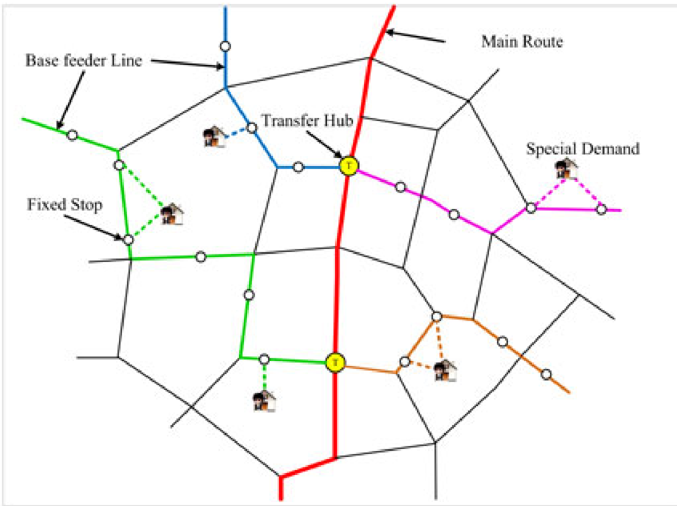

Figure 1. An illustrative example of the mixed system.

accessibility of the feeder system, the flexible feeder system allows some passengers to send requests for special flexible services. If there are no special demands reserved, the feeder busses will be operated along their pre-determined routes and only serve the stops. Otherwise, the busses would leave the route, pick up the reserved passengers, and move back to their routes before arriving the next stop. Therefore, in such route deviation operation mode, the feeder busses have to traverse both fixed stops and requested demand points when running within the target service area.

The operation of the proposed system shall be carried out according to the following procedures. After the passengers sent their service requests, the operational agency will notify the passengers whether their requests are approved or not. Then with those approved requests and confirmed route of each feeder bus, those passengers must be picked-up at their reported locations [18]. To reduce the operational cost and total travel time, the system shall also design an optimal routing plan with an objective of minimizing the total time cost for both operators and passengers.

There are some important attributes in the actual road network that should be considered when solving the routing design problem of the flexible system. For some streets with barriers in the middle, feeder busses must traverse both sides of the road to serve the demands distributed on the two sides. In contrast, the bus vehicles can only travel in one direction to pick up passengers on those one-way streets. Hence, the road network needs to be represented with a directed graph with the roads as arcs and the intersections as nodes.

By using directed arcs to represent each direction of the streets, a connected directed network G = (V, A) with nodes (V) and arcs (A) can be set for analysis. Specifically, for those two-way streets, arcs are built in pair (with opposite directions) to connect nodes (intersections or end of street). Because turning is usually prohibited between nodes, bus vehicles are required to traverse the entire arc (street) until they arrive at another node.

Based on the geometric features of those two-way streets, two types of attributes are defined in this study: "meander", and "non-meander". "Meander" roads can be found in the residential communities where street-crossing is safe for passengers. On the other hand, if the geometric feature of a street (e.g., major arterials) is not safe for crossing or it is separated by continuous barriers, the street will be defined as the "non-meander". To improve the operational efficiency and reduce the total travel time, the "meander" streets allow the feeder busses to serve both sides of the street and pick up all passengers when traveling in either direction (Figure 2 (A)). In contrast, as shown in Figure 2(B), on those "non-meander" streets, feeder busses are required to traverse in both directions if passengers are located on both sides of street.

Denoting s_{ij} as the total service time on the arc from node i to node j (including the on-vehicle time and the service time) and α as a ratio of increased time, the meander total travel time MC_{ij} with the traveling direction $i \to j$ is defined as follows:

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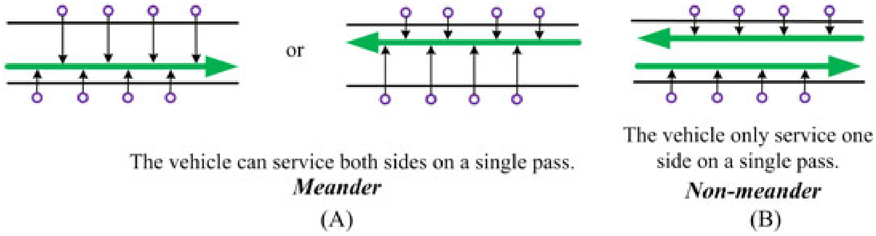

Figure 2. Two attributes of the road.

$$MC_{ij} = s_{ij} + (1+\alpha)s_{ji}; \qquad 1 \geq \alpha \geq 0 \tag{1}$$

As shown in Eq. (1), the service time on opposite street side will be increased by a ratio of α if the "meander" plan is selected. This is reasonable because a feeder bus has to spend more time to serve those passengers who have to across the street. However, it should also be noted that such "meander" service strategy can effectively reduce the total travel time because travel time on the opposite arc has been eliminated.

In summary, given the reported demand points and the location of those fixed stops, the three critical issues that remained to be solved are identified in this problem: (i) how to divide the target network into several sub-areas served by different feeder busses; (ii) how to effectively design the optimal routing plan for each bus to serve both pre-determined stops and reserved passengers; and (iii) how to assign the reserved passengers to different feeder busses so as to balance the increasing of travel time along each bus route.

Also, to facilitate the model formulation, some important assumptions are listed as follows:

1 All special service requested by passengers shall be confirmed at least 1 or 2 h in advance, which offers sufficient time for designing bus routes.
2 Pick-up service is only available to special passengers (e.g., elderly, disabled) within the service area, and each feeder bus only serves part of them to avoid extremely long travel time.
3 The original route of each bus is pre-determined as the model's input.
4 The travel time between each pair of nodes (intersections) is pre-estimated for planning purpose.

3. PROBLEM REFORMULATION

In review of the literature, the rural postman problem (RPP) is a practical extension of the well-known Chinese postman problem, in which a subset of the edges (streets) from the road network is required to be traversed at a minimal cost [19]. Hence, the flexible feeder bus routing problem could be treated as an extension of the RPP with the additional concerns of meander service strategies. In addition, this is a type of the non-deterministic polynomial hard (NP-hard) arc routing problems that cannot be solved in polynomial time for global optimality. Hence, a possible and efficient way to solve the proposed problem is to transform it into a RPP.

Note that several types of the RPP with respect to different network structures have been discussed in literature, including the RPP on an undirected graph [19], the RPP on a directed graph [20–22], and the RPP on a mixed graph [23]. To transform the proposed problem into a RPP, the first stage is to re-construct the network and make it fit in one type of the RPPs. For the "meander" pair arcs, the busses can serve both arcs on a single traverse in either direction, while those "non-meander" arcs are required to be traversed separately. Hence, the problem should be re-defined as a RPP on mixed graphs. This study provides the following steps to complete the network re-construction process:

- For those arcs with passengers or connected with stops, define them as the required arcs that must be traversed by the feeder busses.
- For those meander arcs, combine the pair-arcs into one undirected edge.

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- For those arcs without customers, combine their pair-arcs into undirected edges.

The RPP on mixed graph is one of the most difficult arc routing problems. To simplify the solution process, the problem is further transformed into a traveling salesman problem (TSP) by defining artificial nodes on each edge or arc required to be traversed, and the distance between two artificial nodes is the shortest path between their adjacent real nodes. The transformation process is illustrated in Figure 3.

As shown in Figure 3(A), the solid links (distributed with passengers) indicate the edges or arcs required to be traversed; the dash links means they are optional arcs for travel, and the black nodes represent the real nodes (intersections). Two artificial nodes are created on each solid edge or arc, and the distance between the artificial nodes and the associated adjacent real nodes is assumed to be one-fourth of the link length. Hence, if those artificial nodes are sequentially visited, the resulting tour could be used as one feasible solution to the RPP.

Another important issue of problem transformation is to define the distance between those artificial nodes. As shown in Figure 3(B), for any two artificial nodes on edges, the distance is computed by

$$Dis(m,n) = \left\{ spl(N_i,,N_k) + 0.25L\left(N_i + N_j\right) + 0.25L\left(N_k + N_l\right)\right) \quad \text{if } i{\neq}j \, and \, Cell_m{\neq}Cell_n 0.5L\left(N_i + N_j\right) \tag{2}$$

i

where N_i is the adjacent real node of i, L_i is the length of edge contains i, $Cell_i$ is the node pair including i, and *spl* indicates the shortest path length between two real nodes, which is calculated by the Dijkstra algorithm [24].

Particularly, the distance from (or to) the artificial nodes on arcs is defined in different ways because of the restricted traverse direction. Considering two nodes m and $m+1$, where m is the upstream node, the distance related to these two nodes are given by

$$Dis(m+1,m) = Inf \tag{3}$$

$$Dis(m,j) = Inf \ \text{for} \forall j{\neq}m, m+1 \tag{4}$$

$$Dis(j,m+1) = Inf \ \text{for} \forall j{\neq}m, m+1 \tag{5}$$

where variable *Inf* means infinite.

Then the new network will be constructed as illustrated in Figure 3, and feeder busses will visit each artificial node. Based on the aforementioned rules, the following steps can be applied when one needs to transfer a realistic RPP network to a TSP network.

Step 1 *Obtain the shortest path and travel time between each node.*

In the realistic RPP network, each road is illustrated by an edge with two nodes at each end of it. So the network can be transformed into a node network. The travel time between each node is calculated by following rules:

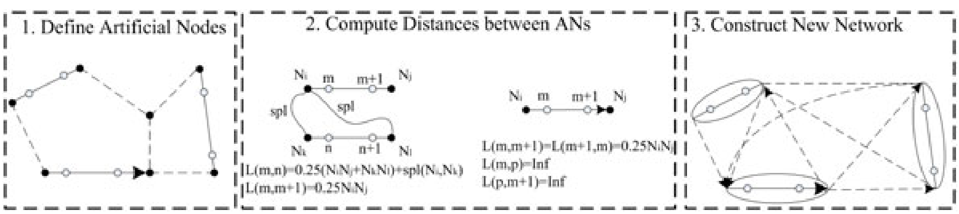

Figure 3. Transformation from rural postman problem to traveling salesman problem.

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- For the nodes of two-way road, the travel time is the actual travel time of the road. And the travel time of each direction is the same.
- For the nodes of the one-way roads, only the same direction with the road is the actual travel time. The travel time of opposite direction is set to a very big integer M, such as 10,000.
- For the main line roads, the same rule of one-way roads is used to avoid the opposite traverse.
- If two nodes are not connected directly, the travel time of both directions between them is also set to a very big integer M, such as 10,000.

Following those rules, the travel time matrix of all the nodes is generated. Then the shortest path and travel time between any two nodes in the network can be calculated by shortest path algorithm, such as Dijkstra algorithm.

Step 2 *Transform the actual network into a mixed graph.*

In order to construct the network into a mixed graph, the following steps are developed.

Step 2.1 Transform each road into a pair arcs with opposite direction.
Step 2.2 Select those meander arcs, and combine the pair-arcs into one undirected edge.
Step 2.3 For the arcs transformed by the main feeder line roads and the one-way road, select those arcs with the same direction of main line and the allowed one-way as the required arcs. All the demands at the opposite direction arcs must cross the road.
Step 2.4 For those non-meander arcs, the pair arcs are all selected as the required arcs.

The mixed graph is constructed with all the required arcs by using the aforementioned steps. Then the travel time matrix of the graph is abstracted from the matrix solved in Step 1.

Step 3 *Transform the mixed graph into a TSP network.*

In this step, the mixed graph is transformed into a TSP network by using the algorithm presented in Figure 3. The distance between each node can be calculated by Eq. (2–5).

4. GENETIC ALGORITHM-BASED SOLUTION HEURISTIC

Given the transformed TSP, this section presents a GA-based solution heuristic to find meta-optimal bus routing plans. As the destination of all busses is the transfer hub and there are directed main lines to be traveled, this problem can be viewed as a fixed open multiple traveling salesmen problem (MTSPOF). In addition, because one of the objectives is to balance the travel time of each bus route, this study further transform the MTSPOF problem into a fixed open traveling salesmen problem. Also, considering multiple bus vehicles may be operated in the service area, those demand points are intended to be served by their adjacent bus lines. More specifically, the entire heuristic consists of the following three primary steps:

Step 1 *Service area division*

In most existing studies of the flexible transit services [5, 7, 9], the entire service area is usually divided into a set of sub ones. Each sub area is usually served by one vehicle running roundtrips to transfer passengers. Hence, to design the optimal routes of busses, one essential step is to split the entire service area and assign those demand points to proper bus routes. Taking a network with two bus lines, for example, and the division of the service area is illustrated in Figure 4.

As shown in Figure 4, the entire service region is divided into three sub-areas, where Area 1 will be served by the left bus line and Area 2 will be served by the right bus line. The two lines illustrated in red arrows can be regarded as the base lines of the busses that are running in Areas 1 and 2. The service Area 3 is defined as the common service area, and its demand points can be served by either of two lines.

In general, if there exist N base lines in the service area, the entire service area will be divided into $N+1$ small areas. And the demands in the $N-2$ service area at the middle of the entire area will be served by either the left or the right lines. Also, how to balance the increased travel times of different busses when designing the route should be also considered.

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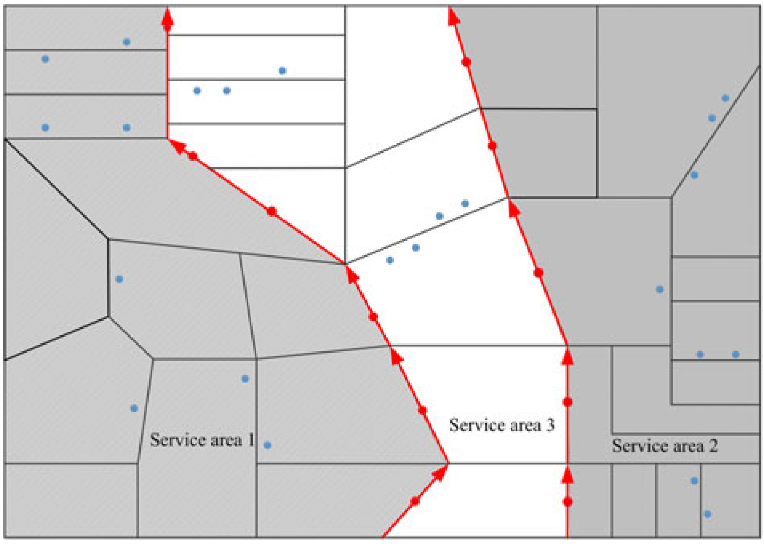

Figure 4. Service area divisions.

Step 2 *Solve the problem with genetic algorithm*

In this step, the GA is applied to solve the transformed TSP problem from the passenger assignment process.

Step 2.1 *GA encoding schemes*

To ensure that two artificial nodes of one road are sequentially traversed, the cell-based permutation chromosomes are defined for convenience. For example, as shown in Figure 5, there are five edges or arcs needed to be traversed from 1 to 10 by one bus, the chromosome can be encoded as a cell-based string of numbers, such as ((1, 2) (3, 4) (5, 6), (7, 8) (9, 10)). The size of the string depends on the nodes needed to be traversed, and the sequence of the string represents the bus route. And the first and the last positions are filled with fixed numbers (the starting and ending nodes of the final tour are fixed).

Step 2.2 *Initialization*

The endpoints of each road are labeled as nodes in the TSP problem. The population is initialized as 60.

Step 2.3 *Crossover and mutation*

This study uses three mutation operators (flip, swap, and slide between nodes) to generate new solution populations. The proposed model randomly group four nodes at a time and select the best one of those four to the next generation until termination criteria are reached.

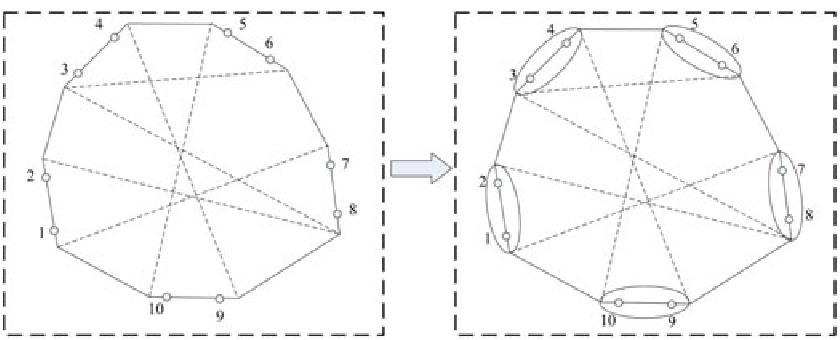

Figure 5. Illustration of cell-based string of numbers.

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Step 2.4 *Fitness evaluation*

The fitness evaluation function is derived from the objective function and used in successive genetic operators. For this problem, the shortest route (least travel time to travel from the fixed start to the fixed end while visiting the other nodes exactly once) is selected as the fitness function.

Step 2.5 *Stopping criteria*

Once the fitness function values of all the strings in a particular generation are obtained, the minimum fitness values of the strings in a population are calculated. Then the termination criterion will be checked for stopping the solution procedure. When the minimum fitness of all the strings in a population is nearly equal to the best fitness, the population is said to be converged, and the GA is terminated. Note that the termination of GA can also be performed by setting the maximum number of generations. Considering further improvement may be found after the convergence, this study sets the maximum number of generations as 10,000.

Because more than one bus line may exist in the network, to optimally assign the demand points to the proper bus lines, this study follows the key steps shown in Figure 6 to solve the problem.

Step 3 *Revert the routes to the actual network*

The routes of each feeder bus are solved in the mixed graph in Step 2. Then the routes must be reverted to the actual network using the following steps:

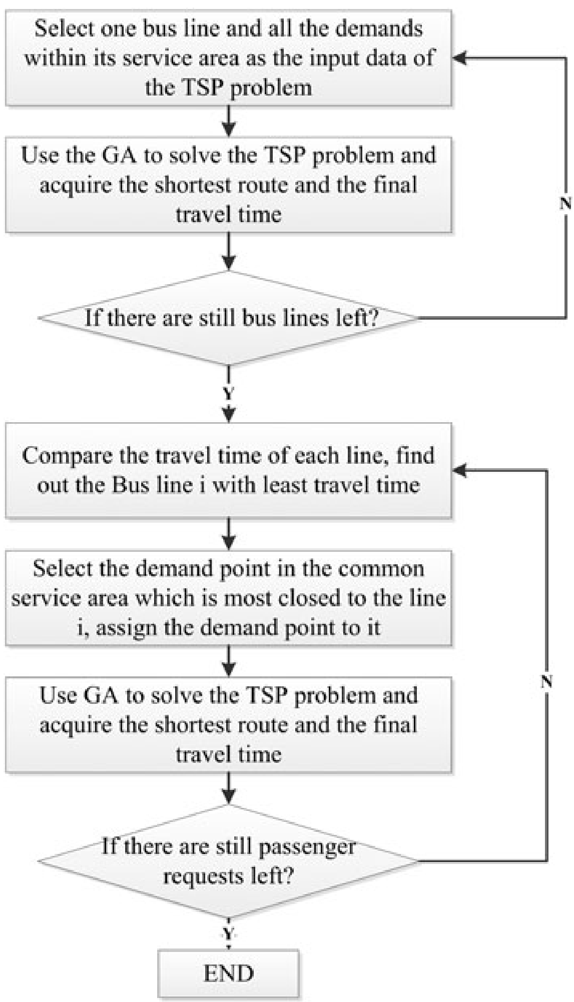

Figure 6. Genetic algorithm-based algorithm to solve the transformed traveling salesman problem.

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Step 3.1 Replace the artificial nodes with the real nodes in the network.

Step 3.2 Replace the connection of two nodes belonging to different roads with the shortest path acquired in the problem transformation step.

Through the aforementioned three-step solution algorithm, the routes of each bus can be obtained.

5. NUMERICAL EXAMPLES

To test the proposed optimization model and evaluate the effectiveness of the heuristic, this study selects the network around the Hi-tech District in Jinan, China for case studies. Two main feeder bus lines are provided for normal feeder service within the service area. Also, to facilitate the numerical experiment, this section lists some key parameters as follows:

- The service time of one demand without road crossing is 0.5 min.
- The service time of one demand with road crossing is 1.5 min ($\alpha = 2$).
- The average operating speed of feeder busses is 15 km/h.
- There are two base lines existing in the network, and there is one bus operating along each line.

The selected road network is shown in Figure 7. The ends of all links (nodes) are numbered from 1 to 110 (shown as black or red points). All streets in this network are two-way streets and can be traversed by busses from either direction. Two feeder bus lines along with the stops are shown in red. And there are two terminals in the road network, which are located at nodes 6 and 10. If no individual request is reported, the feeder busses will be operating along their pre-determined routes back and forth. Otherwise, busses can deviate from the route and service the special demand requested by passengers.

Firstly, the shortest path between each pair of nodes in the network is computed. Particularly, for the circling route along nodes 59, 50, and 60, the actual distance between each point is used to obtain the shortest travel time. For example, the shortest path from 1 to 30 is 1→2→3→4→5→6→22→29→30,

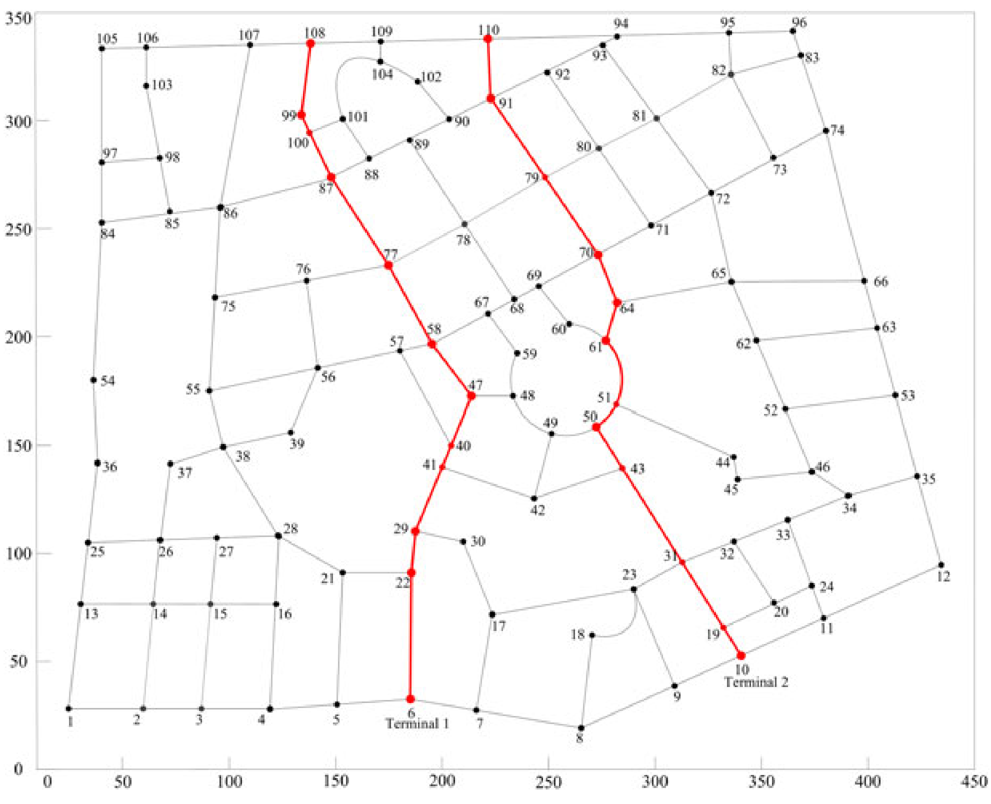

Figure 7. The test road network.

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and the shortest travel time is 18 min. After re-constructing the actual network as a mixed graph based on the criterions introduced earlier, the mixed graph is further transformed into the TSP network by using the aforementioned transformation approach.

Following the solution approach stated in Step 2, the resulting assignment plan for all the demand points is listed in Table I. Note that “Area 1” represents the service area at the left side; “Area 2” is the one at the right side, and “Area 3” is the common area between the two bus lines. In addition, this study classifies the streets into three types, whereas type 1 is illustrated for the meander street and type 2 is shown as the non-meander street. For type 3, busses can select either service plan (meander or non-meander) when traveling on the streets.

The next step is to solve the shortest routes to serve all the demands with the shortest travel time. The route of main line 1 is solved by serving all the demands in Area 1 and its original fixed stops. The GA is used to solve the problem. The result is shown in the solid line in Figure 8. The bus deviates from the main line to serve the special demands after the road (87, 77) had been traversed. The shortest travel time is 69 min. We have also solved the shortest routes for the situation of operating two busses, where one bus is operated along the main line and the other is used to pick up all the passengers with special demands. The results show that the shortest travel time of the first bus is 29 min, and the shortest travel time of the second one is 53.5 min. The total travel time of two busses is 82.5 min, which increases travel time by 20% relative to the deviation route. The routes of those two busses are illustrated in dotted lines in Figure 8.

The same procedure is applied on bus line 2 in Area 2, and the result is shown with solid lines in Figure 9. In order to serve all the special demands, the bus deviates from the main line after the road (79, 70) was traversed. The shortest travel time is 55 min.

Similarly, the scenario of two operating busses is also resolved for comparison. One bus is used to run along the main line of Area 2, and the other one is used to serve all the special demands. In this case, the results show that the shortest travel time of main line is 29.5 min, and the shortest travel time of the other bus is 46.5 min. Thus, the operation time of two busses is 76 min, which increases the travel time by 38% relative to the deviation route. The routes of those two busses are illustrated in dotted lines in Figure 9.

At the current stage, two special demands on link (67, 59) remain to be served in Area 3, which is between the two bus lines. To balance the increased travel time of the two bus lines, we solve the route and shortest travel time using aforementioned method. If the demands are added into the route of main line 1, the total travel time increases from 69 to 75 min. Meanwhile, the total travel time is 67 min when the demands are serviced by main line 2, which is increasing by 12 min. Therefore, the demands on road (67, 59) are added to the route of main line 1. So the final route results with all demands served are shown in Figure 10, and the total travel time is 130 min.

Finally, this study also tests the case of using three busses to serve all the demands, whereas two busses are operating along the main line without deviation and the third bus is used to service all the special demands in the total area. The final route results with all demands served are shown in Figure 11, and route of the third bus is illustrated in pink dotted arrows. The travel time of the three

Table I. Demands distribution.

No.	Road	Demand	Area	Type	No.	Road	Demand	Area	Type
1	(108, 99)	2	1	1	17	(110, 91)	2	2	1
2	(100, 87)	2	1	1	18	(91, 79)	1	2	1
3	(87, 77)	2	1	1	19	(79, 70)	2	2	1
4	(77, 58)	1	1	1	20	(64, 61)	1	2	1
5	(47, 40)	3	1	1	21	(51, 50)	2	2	1
6	(41, 29)	1	1	1	22	(43, 31)	2	2	1
7	(22, 6)	1	1	1	23	(31, 19)	2	2	1
8	(75, 55)	1	1	2	24	(19, 10)	1	2	1
9	(39, 56)	2	1	2	25	(80, 81)	1	2	2
10	(27, 28)	1	1	3	26	(83, 74)	1	2	3
11	(14, 15)	2	1	3	27	(71, 80)	2	2	3
12	(28, 38)	2	1	3	28	(67, 59)	2	3	3

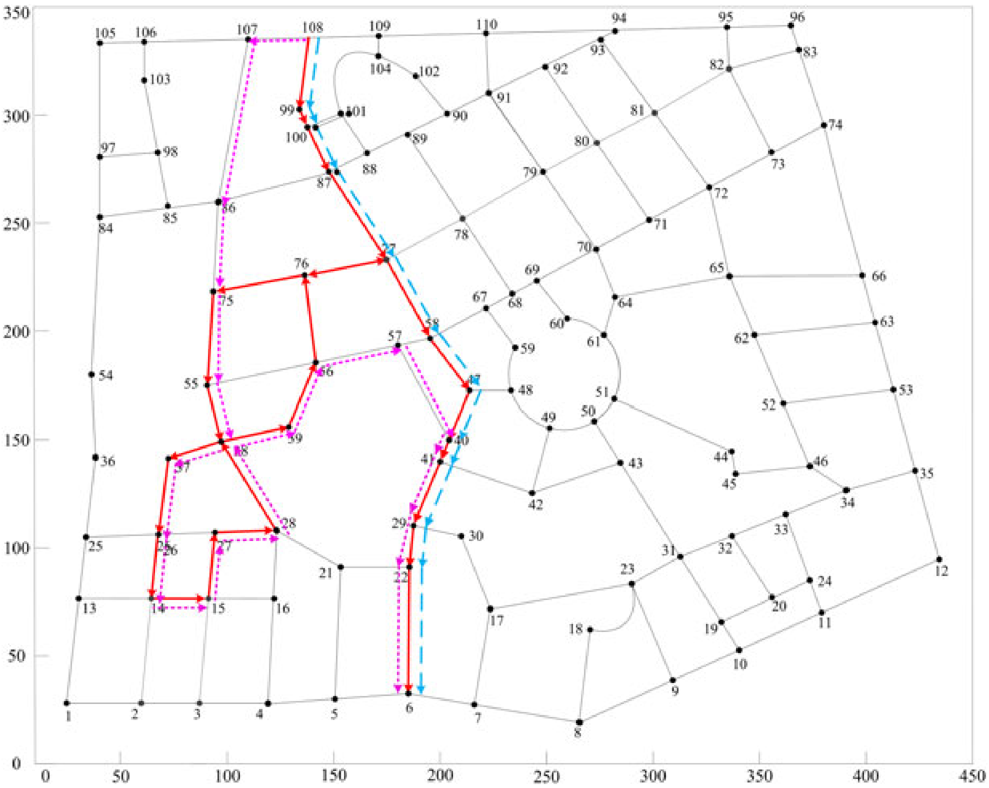

Figure 8. The routing plan in Area 1.

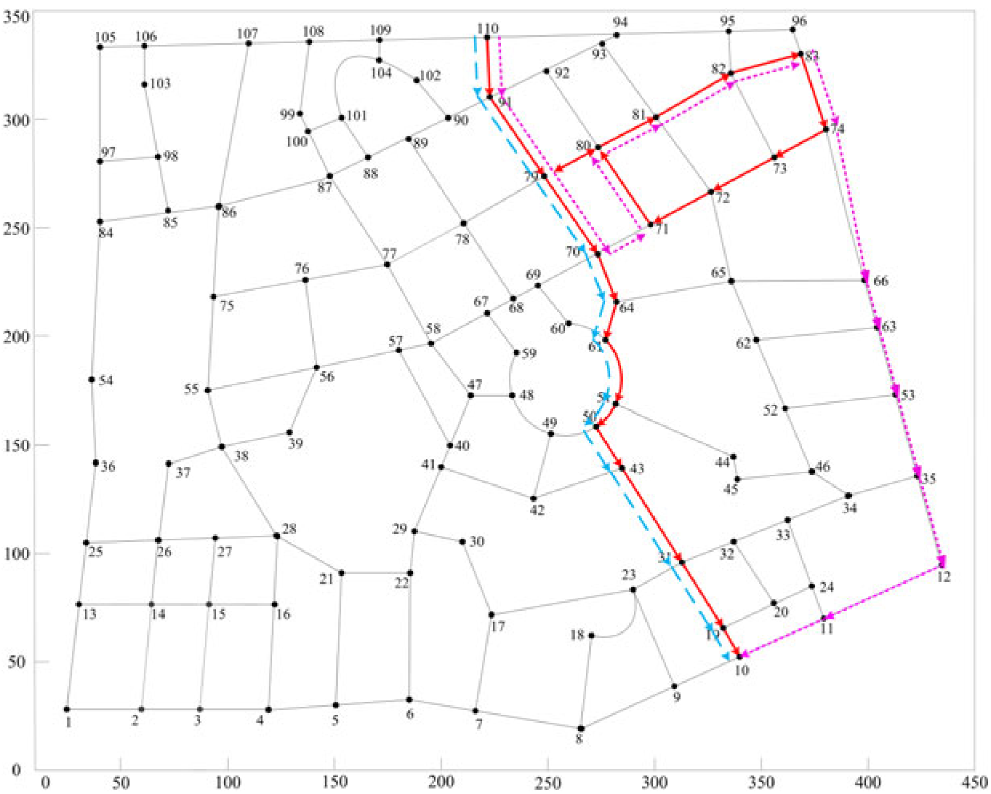

Figure 9. The routing plan in Area 2.

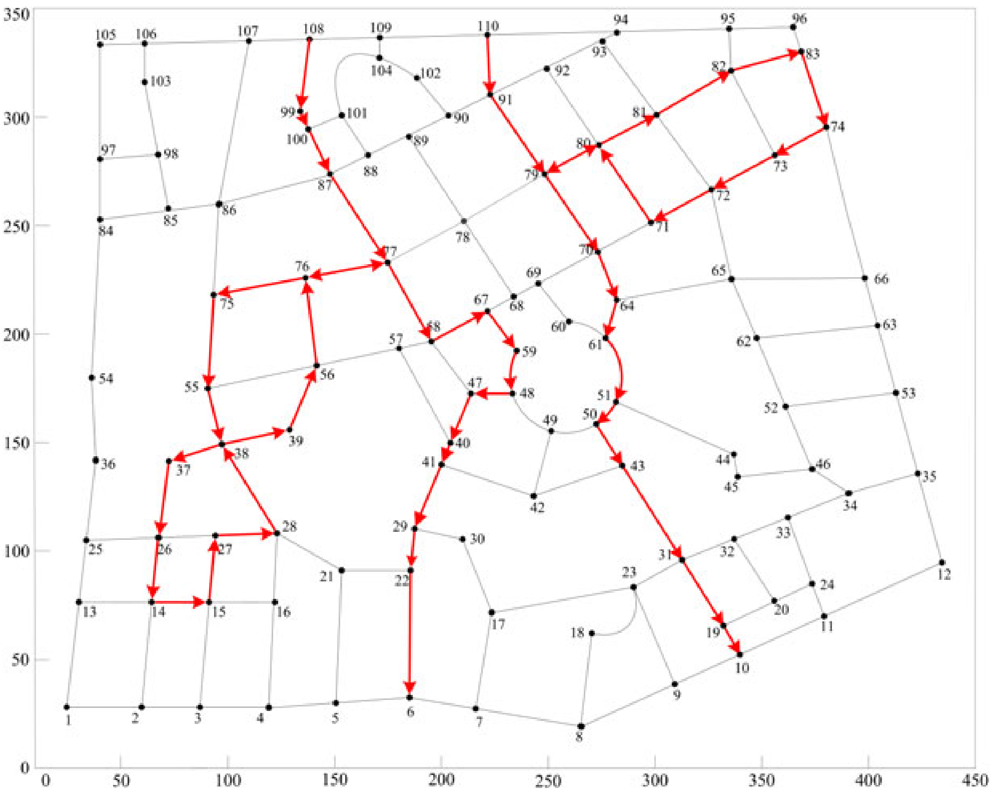

Figure 10. The deviation routes of two busses in the entire area.

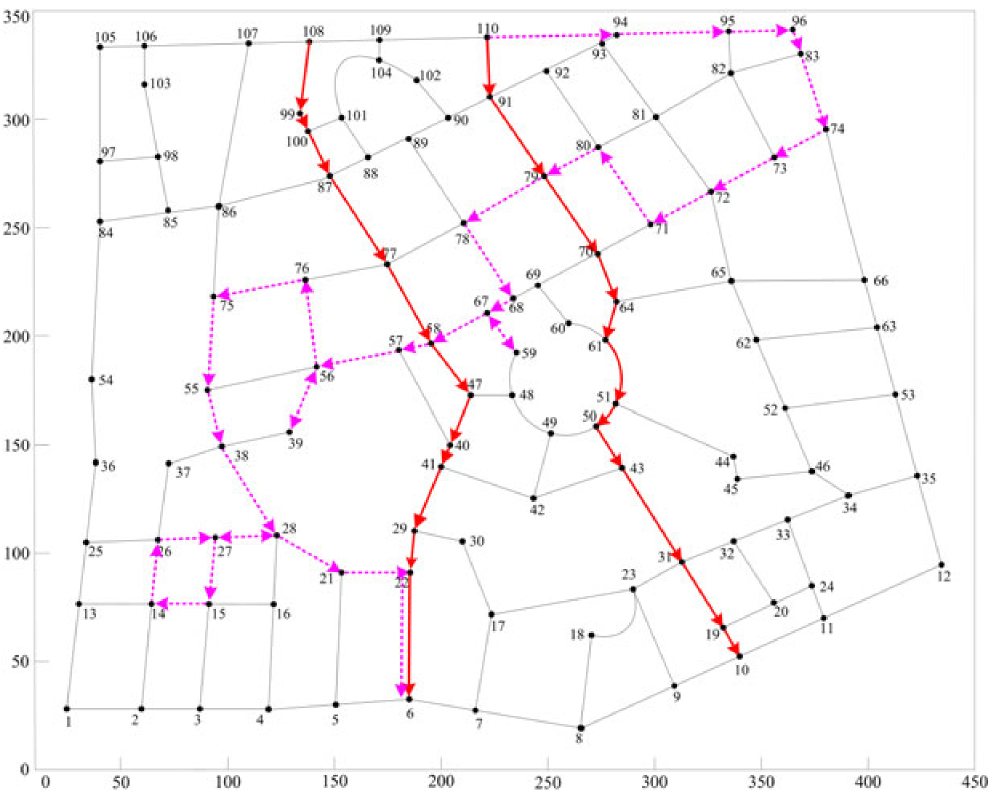

Figure 11. The routes of three busses in the entire area.

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Table II. Computational results of different scenarios.

	Flexible mode (2 busses)		Fixed + demand responsive mode (3 busses)		Difference of travel time (%)
Scenarios	TTT (mins)	Computational time (mins)	TTT (mins)	Computational time (mins)	
1	112.0	6.4	125.5	3.5	12.05%
2	138.0	7.8	153.5	4.1	11.23%
3	148.5	8.2	153.0	5.2	3.03%
4	146.0	9.0	160.5	5.5	9.93%
5	146.0	8.5	168.0	5.3	15.07%
6	120.5	7.4	133.0	4.9	10.37%
7	133.5	8.6	152.0	4.1	13.86%
8	149.5	8.5	165.0	3.9	10.37%
9	156.0	9.2	168.5	5.8	8.01%
10	142.5	8.7	171.0	4.6	20.00%

TTT, total travel time.

busses is 29, 29.5, and 87.5 min, respectively. And the total travel time is 146 min, which increases the travel time by 12% relative to the deviation route operated by two busses.

And through comparing the results between two busses and three busses cases, one can find that the travel time of two busses without deviation to pick up disabled and elderly passengers is 58.5 min. And the travel time of deviation is 130 min, which is over two times of the situation without deviation. So in order to guarantee the service quality for the passengers along the fixed part of the route, it is crucial to set up a reasonable slack time or maximal total travel time.

In order to assess the effectiveness of the proposed model, this study has further run 10 more scenarios (with different demand levels but the same network), and results are summarized in Table II. The differences of total travel time between two different modes (flexible mode with two busses versus fixed + demand responsive mode with three busses) range from 3% to 20%, indicating the proposed flexible mode is more effective to deal with the complicated demand distribution patterns. The computational times of all scenarios are less than 10 min, which is practically applicable considering the usual 1–2 h time window before dispatching busses. Also note that the bus travel time and model's computation time will be increased when more special service requests are confirmed. Hence, this study only allows a specified group of passengers to request such service.

To further test the computational efficiency of the proposed GA-based algorithm, this study further tested scenarios with different sizes of network, and the results are summarized in Table III.

As shown in Table III, the proposed algorithm can yield solutions for all large-scale scenarios within 20 min, which is owing to the capability of the proposed heuristic to divide the large service area into small ones to reduce the computational load. The proposed algorithm is expected to achieve its best performance if run in parallel. In addition, it should be noted that all aforementioned case studies are completed with pre-estimated link travel time. However, the actual travel time may be affected by traffic condition in practice. And such travel time variation can directly impact the optimality of the routing plan produced by the proposed model. To overcome this vital issue, one of our future research works is to dynamically adjust the bus routing plans when most recent network traffic information is obtained.

Table III. Computational results of different problem sizes.

Nodes	Service area size (km^2)	Computational time (mins)
50	4.5	5.5
110	16.5	8.6
150	21.0	14.8
220	35.5	18.5

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6. CONCLUSIONS

This paper presents a service mechanism of the flexible feeder transit that allows busses to pick up passengers at both sides of street by a single traverse to enhance service accessibility in urban areas. A network optimization model minimizing the total travel time of busses and balancing the travel time of each bus is developed to create the routing plans of busses in the target service area. By transforming the proposed model into a TSP, the proposed methodology can deal with the route design problem with discrete demand points and irregular shape of road network. A three-stage solution heuristic is developed to yield meta-optimal solutions to the problem in a reasonable amount of time and can be used in real-world operations. Case studies have demonstrated the effectiveness of the proposed methodology as well as the heuristic solution approach. Future work along the line is to check the impact of different number and locations of the special demand on the solutions and model performance.

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