A Branch-and-Price Algorithm for Large-Scale Multidepot Electric Bus Scheduling

Mengyan Jiang, Yi Zhang, *Member, IEEE*, and Yi Zhang

***Abstract*— Electric buses (e-buses) are increasingly adopted in the transit systems for their benefits of reduced roadside pollution and better onboard experience. E-bus scheduling is a critical problem in the operation planning stage of transit management to ensure efficient and reliable transit service. In Chinese mega cities, the e-bus networks are operated with many bus routes characterized by high service frequency and long operation time, making the e-bus scheduling a large-scale problem. Besides, the transit agency requires the vehicle-depot constraint to limit the e-bus reposition since it is not cost-efficient. An efficient method to generate optimized schedules that meet the above requirements is desired by the transit agencies. In this paper, we address a large-scale multi-depot electric bus scheduling problem considering the vehicle-depot constraint and partial recharging policy. A mixed integer programming model and an efficient branch-and-price (BP) algorithm are developed to solve the problem. In the BP algorithm, we devise a heuristic method to generate good initial solutions and adopted heuristic decisions in the label setting algorithm to solve the pricing problem. In this way, the efficiency of the BP algorithm is achieved and the large-sized problem instances can be solved. We conduct extensive numerical experiments based on the fixed-route and multi-route operation cases in Shenzhen. The results show that the BP algorithm can generate provable high-quality solutions. Sensitivity analysis indicates that increasing battery capacity and charging rate can reduce the operational cost. The optimal charging schedules can also provide guide in determining the capacity of the charging facilities.**

***Index Terms*— Electric bus, scheduling, partial recharging, branch-and-price, resource-constrained shortest path.**

I. Introduction

Recent years have seen an increasing deployment of electric buses (e-buses) globally aiming to reduce on-road pollution and providing high-quality public transit service. Buses can have significant impacts on the environment of a city owing to their long operation time and travel distance. As such, despite the higher purchase cost of an e-bus than that of a conventional one, public transit is a pioneer of the Electric Vehicle (EV) transformation with government support. The number of e-buses in operation worldwide has reached approximately 500,000 in 2020, which is predicted to account for over 67% of the total share by 2040 [1]. Shenzhen, a major city in southern China, switched all the 16359 diesel buses to electric ones by the end of 2017 [2].

Vehicle scheduling is a critical problem in the operation planning stage of transit management to ensure efficient and reliable transit service. With the electrification of the transit system, new challenges arise regarding the e-bus scheduling. Firstly, e-buses have a shorter driving range per charge compared with their diesel counterparts. The maximum driving range of most of the diesel buses is larger than 300 km in urban driving conditions; while, the driving range for e-buses currently in the operation varies from less than 100 to over 300 km depending on the battery capacity and driving condition [3]. Characterized by the long operation time and travel distance, e-buses need daytime recharging to ensure continuous operation. Secondly, in the transit system operated by large number of e-buses, the resource of charging facilities is always limited regarding the location, charging power and number of charging piles. For example in Shenzhen, the e-buses have been adopted in large scale whereas there is still a lack of charging infrastructure especially in depots [4]. As such, decisions on the location, time and amount for e-bus to get charged should be made carefully to reduce congestion and achieve efficient operation.

The limited driving range with charging demand during the operation and restricted charging resource made the scheduling problem for e-buses more complex than that for the conventional ones. To cope with these challenges and optimize operation schedules, researchers proposed e-bus scheduling models and solution methods including heuristics such as Large Neighborhood Search (LNS) and exact algorithm based on Column Generation (CG). However, e-bus scheduling problem considering the characteristics of the transit networks in China is not well addressed in the literature due to its scale and complexity. In Chinese mega cities, with the growing scale of urban transit network and electrification process, e-bus fleets are managed in large-scale on multiple routes to fulfill hundreds to thousands of timetabled trips in a day. It becomes increasingly difficult to guarantee high service level regarding punctuality and charging availability. Besides,

Manuscript received 17 September 2021; revised 15 December 2021 and 16 February 2022; accepted 4 April 2022. Date of publication 4 May 2022; date of current version 29 November 2023. This work was supported in part by the Basic Research Program of Shenzhen Science and Technology Innovation Committee under Grant JCYJ20180307123910003, in part by the Scientific Research Start-Up Funds of Tsinghua Shenzhen International Graduate School under Grant QD2021007N, and in part by the National Natural Science Foundation of China under Grant 61673233. The Associate Editor for this article was Z. Lv. *(Corresponding author: Yi Zhang. e-mail: zy1214@sz.tsinghua.edu.cn)*

Mengyan Jiang is with the Center of Environmental Science and New Energy Technology, Tsinghua-Berkeley Shenzhen Institute, Tsinghua University, Shenzhen 518055, China (e-mail: jiangmy17@mails.tsinghua.edu.cn).

Yi Zhang is with the Tsinghua National Laboratory for Information Science and Technology (TNList), Department of Automation, Tsinghua University, Beijing 100084, China (e-mail: zhyi@mail.tsinghua.edu.cn).

Yi Zhang is with the Institute of Future Human Habitat, Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, China (e-mail: zy1214@sz.tsinghua.edu.cn).

Digital Object Identifier 10.1109/TITS.2022.3165876

the transit agency requires the vehicle-depot constraint which means that the e-buses should begin and end the operation of the day at their base depots. This requirement means to limit the long-distance non-service travel of the e-buses which is not cost cost-effective. The vehicle-depot constraint is often not considered in the existing studies and it adds further restrictions to feasible schedules, making the problem more complex to solve.

The above conditions make the e-bus scheduling a large-scale problem with multiple depots and vehicle-depot constraints which is not addressed in existing studies. To ensure efficient use of the e-buses, an algorithm is needed to generate the applicable schedules to ensure a dynamic operation with high efficiency and strong reliability.

Motivated by the above issues, we proposed an efficient branch-and-price (BP) algorithm to address the large-scale multi-depot e-bus scheduling problem considering the vehicle-depot constraint and partial recharging policy. Enabled by the advanced communication systems and information technologies, Intelligent Public Transport Systems (IPTS) ensure the public transit operation to be organized efficiently and safely [5]. As an essential part of the IPTS, Decision Support Systems (DSS) assist transit agency to make optimized pre-hand operation plan, real-time control strategy and decision under emergency for the e-bus fleets with the application of optimization and control methods. The proposed MD-EVSP model and BP algorithm can serve as part of the DSS in IPTS to assist the transit agency in generating high-quality e-bus schedules and charging plans for large-scale e-bus operation. It can greatly improve the quality and efficiency of the operation schedules and reduce manual labor. The main contributions of this study are as follows:

- A mixed-integer programming (MIP) model is developed with trip start time window and partial recharging being allowed.
- An efficient BP algorithm enhanced by a heuristic is devised that can generate provably high-quality solutions for large-scale problem instances with hundreds of trips scheduled on multiple bus routes.
- Comprehensive numerical experiments are conducted based on the real-world fixed-route and multi-route transit operation cases in Shenzhen to demonstrate the effectiveness of the BP algorithm. Sensitivity analysis has been conducted to investigate the proper choice of battery size, charging rate and the capacity of the charging facilities.

The reminder of the paper is organized as follows. Section 2 gives a review on the works related to e-bus scheduling. Section 3 introduces the MIP model for the problem. Section 4 presents the details of our BP algorithm to achieve its efficiency. In Section 5, we present the results of the numerical experiments and discuss the insights on e-bus operation. Section 6 concludes with some final remarks.

II. Literature Review

As a critical part of transit operation planning, Vehicle Scheduling Problem (VSP) aims at allocating transit vehicles to carry out timetabled trips. A common objective is to minimize the total number of vehicles used. Based on the number of depots involved, the VSP problems can be categorized into the single-depot and multi-depot VSP, with the latter proven to be NP-hard by [6]. For the models and algorithms for the VSP, we refer interested readers to the comprehensive reviews by [7] and [8]. Electric Vehicle Scheduling Problem (EVSP) draws wide research interests with the increasing adoption of e-buses in recent years. The commonly used charging technologies nowadays include battery swapping, fast opportunity charging and regular charging [4]. Battery swapping can be completed in a short period thus having a minor influence on the daytime operation. However, establishing a charging station is costly owing to the land resource and battery backup required. Fast charging and regular charging are plug-in charging technologies that differ in their charging rates and charging locations. Considering battery swapping and a single depot, a model with independent objective functions to minimize both the purchasing cost of e-buses and total charging demand was developed in [9]. A heuristic framework based on a non-dominated sorting genetic algorithm was proposed. A column-generation algorithm to solve the problem is presented in [10]. Models and algorithms for the EVSP under battery swapping and fast charging modes are presented in [11].

Under the plug-in charging mode, Wen *et al.* [12] developed a multi-depot EVSP and an Adaptive LNS (ALNS) heuristic to solve instances with customized bus trips. van Kooten Niekerk *et al.* [13] proposed two models for single-depot scenarios with a different level of detail resembling the nonlinear charging process. A CG algorithm was developed to solve the problem. A multi-objective bi-level model to optimize the vehicle schedule and charging schedule of the mixed bus fleet considering a single depot was introduced in [14]. The time-of-use tariff was considered to optimize the charging schedule. An integrated heuristic method was developed to solve the problem. An optimal fleet composition and scheduling problem to minimize the total operational cost was studied in [15]. A set partition model was solved by an IP-column-generation (ICG) algorithm where the pricing problem was solved by a Dynamic Programming (DP) approach. An EVSP considering the degradation of battery and nonlinear charging process adopting full charging was addressed in [16]. A tailored BP algorithm was devised to solve the problem.

A proper decision on the capacity of the charging facility is critical in achieving efficient e-bus charging. In [17], a linear MIP model for a single-depot EVSP considering the capacity of the charging facility and the assignment of chargers to the vehicles was proposed. The model can be solved by the standard solver for medium-scaled instances. In [18], an EVSP model was developed based on the deficit function theory with the aim to minimize the number of vehicles and chargers. As the e-bus fleet may be composed of different types of vehicles, some studies considered the mixed fleet EVSP [18]-[22]. The integrated e-bus and crew scheduling problem is introduced in [23] where e-bus scheduling problem considered a single depot and full battery recharging policy. An ALNS heuristic was devised to tackle the problem.

TABLE I
A SUMMARY OF THE LITERATURE RELATED TO EVSP

	# Depots	Charging policy	Nonlinear charging	Charger capacity	Mixed-fleet	Vehicle-depot constraint	Solution method
Wen et al. (2016) [12]	multiple	partial				√	ALNS heuristics
van Kooten Niekerk et al. (2017) [13]	single	partial	√				CG
Tang et al. (2019) [26]	single	full					DP and BP
Rinaldi et al. (2019) [19]	single	full			√		standard solver
Li et al. (2019) [20]	multiple	full	√		√		standard solver
Janoveca and Kohánia (2019) [17]	single	partial		√			standard solver
Yao et al. (2020) [21]	multiple	full			√		GA
Liu and Ceder (2020) [18]	multiple	full	√	√		√	Deficit Function
Zhou et al. (2020) [14]	single	partial			√		Local search-based
Lu et al. (2021) [22]	multiple	full			√		GA
Perumal et al. (2021) [23]	single	full					ALNS heuristics
Yıldırım and Yıldız (2021) [15]	multiple	partial			√		CG
Zhang et al. (2021) [16]	single	full	√				BP
Our work	multiple	partial				√	BP

To tackle the travel time uncertainty of the buses, stochastic and dynamic rescheduling methods were proposed in the literature. Shen *et al.* [24] proposed a probabilistic network flow model assuming certain trip time distributions and developed a hybrid heuristic solution method. He *et al.* [25] formulated a stochastic dynamic VSP and adopted an approximate DP approach where the objective function is approximated by a feed-forward Neural Network. A stochastic model and a dynamic rescheduling paradigm for a single depot EVSP was developed in [26].

Closely related to the EVSP is the Electric Vehicle Routing Problem (EVRP) arising in the logistics and distribution domain. In fact, the service trips in the EVSP can be regarded as customers with very tight time windows in the EVRP. An EVRP considering explicitly the intermediate nodes is studied in [27]. An MIP model with multi-objectives was formulated. In the numerical experiments, the model was solved by the optimization solver and the results suggest that a trade-off between the actuality of the network topology and the computational efficiency should be considered to obtain solutions. An EV location routing problem with intermediate nodes to jointly optimize the vehicle routing plan and the location of the charging station is addressed in [28]. An MIP model was formulated considering the characteristics of battery discharging and recovering the braking energy.

In the modification of searching algorithms applied to routing problems, De *et al.* [29] studied a ship routing problem considering a time window and bunker fuel arrangement. A hybrid algorithm, basic variable neighborhood search with particle swarm optimization algorithm, was proposed to solve the model. In comparison with the exact solution and other popular algorithms, the proposed heuristic was able to provide high-quality solutions. Ji *et al.* [30] addressed the generalized lock scheduling problem in the marine-time transportation. A novel ALNS heuristic was proposed to solve the large-scale problem instances with the performance that generally outperformed the exact method. Some studies considered the joint routing and scheduling problem for EV and their charging operation. A joint fleet sizing and charging system planning problem for a fleet of autonomous electric vehicles is studied in [31]. An MIP model is formulated for intercity scenarios. Zhang and Leung [32] addressed the problem of jointly optimize the power flow routing and Vehicle to Grid scheduling for providing regulation service. A hierarchical system model was developed to solve the problem.

Table I gives a summary of the works related to EVSP. Each column of the table shows a critical aspect that affects the problem complexity. As shown in Table I, [12] and [15] are the few studies that have considered the multi-depot and partial charging policy. The problem that we considered has different requirements compared with those in the existing studies. Firstly, the e-buses are operated on the bus routes instead of customized random trips as introduced in [12]. Secondly, vehicle-depot constraint is required which in not considered in [15]. We proposed an efficient BP algorithm to address the problem under consideration.

III. Mathematical Formulation

A. Problem Description

We consider a fleet of e-buses carrying out service trips on bus routes according to the precompiled timetable. A bus route consists of two end stops and a series of intermediate stops. E-buses are allowed to park and get recharged at the bus depots established close to the end stops. A service trip on a route starts from one end stop (depot) at the scheduled start time, passing through a series of intermediate stops on the route and ends at the other end stop (depot). An e-bus will carry out a series of trips throughout the day which is called a trip chain. An operation schedule is made up of the trip chains for each e-bus. Based on this setting, the multi-depot EVSP

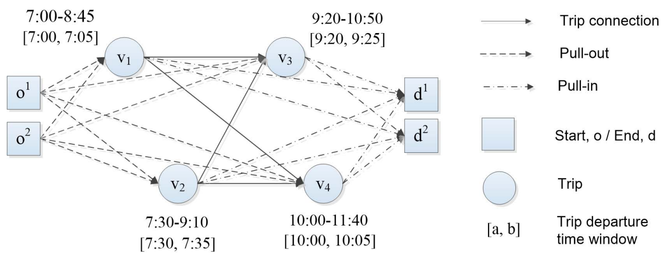


Fig. 1. An illustrative example of the graph for the robust multi-depot EVSP.

(MD-EVSP) is formulated as follows. The list of notations is provided in Table II.

Let $G = (V, A)$ be an acyclic direct graph. Denote K as the set of depots. Each depot $k \in K$ is created with two nodes in the graph: an operation start node o^k and an operation end node d^k, representing the e-bus begins/ends the operation from/at depot k, respectively. Each node o^k and d^k, $k \in K$ is assigned with a time window $[a_0, b_0]$, where a_o is the earliest operation start time and b_0 is the latest operation end time. The e-buses can begin and end the operation at any time within time window $[a_0, b_0]$. Each trip in the timetable is created with a trip node in graph G. Denote $T \subseteq V$ as the set of trip nodes. For a trip node $i \in T$, s_i and e_i represent the scheduled trip time and trip energy consumption, respectively. Each trip node $i \in T$ is associated with a trip start time window $[a_i, b_i]$, where a_i and b_i are the planned and latest trip start time, respectively. This means that the start time of the trip corresponding to trip node i should be within $[a_i, b_i]$. We calculate b_i as $b_i = a_i + \alpha h_i$ where h_i is the headway between trip node i and the next trip node starting from the same depot and on the same bus route; $\alpha < 1$ is a parameter that controls the length of the time window.

The above two kinds of nodes are connected by three categories of arcs: (i) The pull-out arc connects an operation start node o^k, $k \in K$ and a trip node $i \in T$, representing that an e-bus begins its first trip i from depot k; (ii) the pull-in arc connects a trip node i and an operation end node d^k, $k \in K$, representing that an e-bus completes its last trip i and return to depot k at the end of the planning horizon; (iii) the trip connection arc connects two trip nodes, representing that two trips are carried out successively. Each arc $(i, j) \in A$ is associated with a non-service travel time t_{ij} and energy consumption e_{ij}. To ensure that the trip start time window is respected, trip nodes i and j are connected only if the time compatible condition $a_i + s_i + t_{ij} \leq b_j$ is satisfied. Each arc is associated with a cost c_{ij} including the non-service travel cost and vehicle usage cost c^f on the pull-out arcs.

Fig. 1 gives an example of graph G for an instance with two depots and four trips. Each depot is created with an operation start node o^k and an operation end node d^k, $k = 1, 2$. Each trip is represented by a trip node i associated with a trip start time window and a scheduled trip time, $i = 1, 2, 3, 4$.

We assume that the e-buses begin the operation with a fully charged battery and can get recharged at any depots anytime during the day. Define g_{ij}, $(i, j) \in A$ as the unit charging cost in the time interval between the end of trip node i and the start of trip node j; g_{ij} is calculated as the weighted average

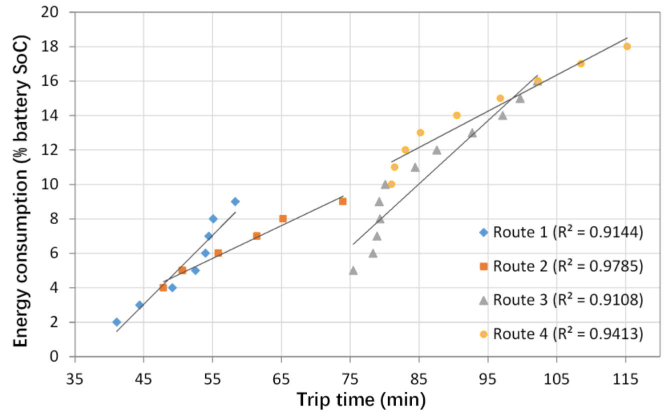


Fig. 2. Relationship between the average trip time and energy consumption for the selected bus routes.

of the time-of use tariff in this time interval. To estimate the energy consumption of a trip, we selected four bus routes in Shenzhen and analyzed the historical running data in five months. Fig. 2 shows the relationship between the average trip time and energy consumption. The solid lines are the linear regression results with the R-square values given in the brackets next to the labels. The energy consumption of a trip can be described as a linear function of the trip time. Therefore, we assume that the energy consumption of a trip is proportional to the trip time with a rate of r_u (kWh/min). A safety range of the battery SoC is $[u^{min}, u^{max}]$. Denote t^s as the unit charging time and t^p as the charging preparation time. The e-buses should charge integer times of t^s with a rate charging rate of r_s (kWh/min).

Based on the above settings, the problem aims to obtain minimum-cost schedule satisfying the following constraints: (i) Each trip is carried out exactly once; (ii) the departure time window of each trip is respected; (iii) each vehicle begins and ends its operation at the same depot (the vehicle-depot constraint); (iv) the vehicle battery SoC is always kept within the safety range $[u^{min}, u^{max}]$.

B. MIP Model

Let X_{ijk} be the binary variables that take value 1 if a vehicle housed at terminal k traverses arc $(i, j) \in A$. At each trip node $i \in T$, denote Z_i as the trip start time and Y_i as the lowest battery SoC of the vehicle at the trip start time. The lower bound of Y_i is denoted as $u_i^{min} = u^{min} + e_i$. At each operation start (end) node $o^k(d^k)$, $k \in K$, denote Z_i as the operation start (end) time and Y_i as the lowest battery SoC of the vehicle at the operation start (end) time. On each arc $(i, j) \in A$, $i, j \in T$, denote W_{ij} as the amount of energy to be charged after completing trip i and before starting trip j. According to our assumption, the charging place is at the ending depot of trip node i. R_{ij} are the auxiliary variables defined corresponding to Z_i for constraints linearization. Denote E_{ij} as the binary variables that equals 1 if the e-buses can be charged on arc $(i, j) \in A$. Denote U_{ij} as the integer variables that specify the number of time units spent charging on arc $(i, j) \in A$. The problem is formulated as follows:

(MD-EVSP)

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}X_{ijk} + \sum_{(i,j)\in A} g_{ij}W_{ij} \quad (1)$$

$$\text{s.t.} \sum_{k\in K}\sum_{j:(i,j)\in A} X_{ijk} = 1, \quad i \in T \quad (2)$$

$$\sum_{j:(i,j)\in A} X_{ijk} - \sum_{j:(j,i)\in A} X_{jik} = 0, \quad i \in T, \quad k \in K \quad (3)$$

$$\sum_{\beta\in K\setminus\{k\}}\sum_{i\in V} X_{o^\beta ik} = 0, \quad k \in K \quad (4)$$

$$\sum_{\beta\in K\setminus\{k\}}\sum_{i\in V} X_{id^\beta k} = 0, \quad k \in K \quad (5)$$

$$a_i \le Z_i \le b_i, \quad i \in V \quad (6)$$

$$Z_j - Z_i - s_i - t_{ij} + M(1 - \sum_{k\in K} X_{ijk}) \ge 0, \quad (i,j) \in A \quad (7)$$

$$Y_i - e_i + W_{ij} - e_{ij} + M(1 - \sum_{k\in K} X_{ijk}) \ge Y_j, \quad (i,j) \in A \quad (8)$$

$$E_{ij} - \sum_{k\in K} X_{ijk} \le 0, \quad (i,j) \in A \quad (9)$$

$$\left(a_j - Z_i - s_i - t_{ij} - t^p\right) + M(1 - E_{ij}) \ge 0, \quad (i,j) \in A \quad (10)$$

$$W_{ij} + R_{ij}r_s - \left(a_j - s_i - t_{ij} - t^p\right) r_s E_{ij} \le 0, \quad (i,j) \in A \quad (11)$$

$$W_{ij} + Y_i - e_i - u^{max} \le 0, \quad (i,j) \in A \quad (12)$$

$$W_{ij} - r_s t^s U_{ij} = 0, \quad (i,j) \in A \quad (13)$$

$$R_{ij} - b_i E_{ij} \le 0, \quad (i,j) \in A \quad (14)$$

$$a_i E_{ij} - R_{ij} \le 0, \quad (i,j) \in A \quad (15)$$

$$R_{ij} - Z_i + a_i(1 - E_{ij}) \le 0, \quad (i,j) \in A \quad (16)$$

$$Z_i - b_i(1 - E_{ij}) - R_{ij} \le 0, \quad (i,j) \in A \quad (17)$$

$$u_i^{min} \le Y_i \le u^{max}, \quad i \in V \quad (18)$$

$$W_{ij} \ge 0, \quad (i,j) \in A \quad (19)$$

$$X_{ijk} \in \{0,1\}, (i,j) \in A, \quad k \in K \quad (20)$$

$$E_{ij} \in \{0,1\}, \quad (i,j) \in A \quad (21)$$

$$R_{ij} \ge 0, \quad (i,j) \in A \quad (22)$$

$$U_{ij} \in Z, \quad (i,j) \in A \quad (23)$$

Objective (1) minimizes the sum of vehicle usage, non-service travel and charging cost. Constraints (2) ensure that each trip is performed only once. Constraints (3) are the flow conservation constraints. Constraints (4) and (5) require that each vehicle begins and ends its operation at its base depot. Constraints (6) limit the start time of each trip to be within the trip start time window. Constraints (7) ensure the time consistency of two successive trips. Constraints (8) ensure the vehicle battery energy consistency of two successive trips.

Constraints (9) require that charging on arc (i,j) is performed only if trip j is succeeded by trip i. Constraints (10) restrict that charging on arc (i,j) can be performed only if time is available. Constraints (11) define the upper bound of W_{ij} on arc $(i,j) \in A$ according to the time available. Originally, constraints (11) are formulated as follows:

$$W_{ij} + (Z_i - a_j - t_i - t_{ij} - t^p)r_s E_{ij} \le 0, \quad (i,j) \in A \quad (24)$$

To linearize quadratic terms $Z_i E_{ij}$, auxiliary variables R_{ij} are introduced where $R_{ij} = Z_i E_{ij}$ is satisfied by constrains (14)-(17). Constraints (12) require the battery SoC after charging not to exceed u^{max}. Constraints (13) ensure that the charging time can only be integer times of the minimum charging time unit. Constraints (18) keep the vehicle battery SoC within the safety range. Constraints (19)-(23) define the domains of the variables.

IV. A Branch-and-Price Algorithm

A. Set Partitioning Formulation

The MD-EVSP problem can also be formulated as a set partitioning problem. Let Ω be a set of feasible trip chains. Define a_{pi}, $i \in T$, a binary parameter that equals 1 if trip i is carried out in p. Denote θ_p, $p \in \Omega$ as a binary variable that takes value 1 if trip chain p is selected as part of the solution. The set partitioning formulation of the MD-EVSP is as follows:

$$\min \sum_{p\in\Omega} c_p\theta_p \quad (25)$$

$$\sum_{k\in K} a_{pi}\theta_p = 1, \quad i \in T \quad (26)$$

$$\theta_p \in \{0,1\}, \quad p \in \Omega \quad (27)$$

Objective (25) minimizes the total operation cost of the schedule where c_p is the total operational cost of trip chain $p \in \Omega$. Constraints (26) ensure that each trip is carried out once by a vehicle. Constraints (27) define the domain of the binary variables θ_p.

As set Ω includes a large number of feasible trip chains, it is impractical to solve the model directly. Therefore, we develop a column generation (CG) algorithm to solve the linear relaxation of model (25)-(27). The CG algorithm starts by solving the linear relaxation of model (25)-(27) with an initial set of feasible trip chains, called the restricted master problem (RMP). In each iteration, a pricing problem is solved for each sub-network involving depot $k \in K$ to generate columns with negative reduced cost to be added to the RMP. The algorithm alternates between the optimization of the RMP and the pricing problems until no more columns with negative reduced cost are generated, implying the master problem has been solved to optimality. Let π_i, $i \in T$ be the dual variables associated with constraints (26). Let $\bar{c}_p$ be the reduced cost of trip chain $p \in \Omega$ with respect to π_i, $i \in T$, i.e. $\bar{c}_p = c_p - \sum_{i\in T} a_{pi}\pi_i$. The pricing problem is defined as follows.

$$\min \sum_{p\in\Omega} \bar{c}_p\theta_p \quad (28)$$

Each *o-d* path in graph G represent a feasible trip chain $p \in \Omega$. The pricing problem aims at finding a feasible *o-d* path with the minimum reduced cost. We modify the cost of each arc $(i,j) \in A$ by a modified cost $\bar{c}_{ij} = c_{ij} - \pi_i$ where $\pi_i = 0$ for $i \in \left\{o^k, d^k \mid k \in K\right\}$. The reduced cost of an *o-d* path p equals the sum of the $\bar{c}_{ij}$, $(i,j) \in p$.

B. Solving the Pricing Problem

The pricing problem is a resource-constrained shortest path problem (RCSPP) which is time consuming to solve by the standard optimization software. We solve the pricing problem by a label setting algorithm proposed by [33]. Label setting is a DP approach to generate feasible *o-d* paths from the origin o to destination d on a graph. The labels of a node i are used to record the partial paths origin from o to i and the resource accumulated along it. New labels are generated by extending the existing labels on graph G according to the resource extension functions (REF). New labels are checked for resource feasibility and only feasible labels are retained.

In graph G, a trip chain is represented by a path $P = \{o^k, v_1, v_2, \ldots, v_n, d^k\}$ starting from the operation start node o^k, visiting a sequence of trip nodes $v_i, i = 1, 2, \ldots, n$, and end at the operation end node d^k. Denote $L_i = (C_i, R_i^t, R_i^e)$ as a label representing a partial path from the start node o^k to a node $i \in V$ with three kinds of resource defined as follows: C_i is the reduced cost of the path, R_i^t the earliest start time of node i and R_i^e the lowest battery SoC of the vehicle at the start time of node i. Let $P_j = \{o^k, 1, 2, \ldots, i, j\}$ be a partial path from node o^k to a trip node j. Denote E_j as the minimum charging amount required on P_j. c_j^e is the minimum charging cost for charging E_j amount on P_j. The forward extension of a label L_i to a label L_j along an arc $(i, j) \in A$ is performed by REF (29)-(32). A label at node j is feasible only if $R_j^t \in [a_j, b_j]$ and $R_j^e \in \left[u_j^{min}, u^{max}\right]$.

$$C_j = C_i + c_{ij} + c_j^e - \pi_i \tag{29}$$

$$R_j^t = \max\{a_j, R_i^t + s_i + \frac{W_{ij}}{r_s} + t_{ij}\} \tag{30}$$

$$R_j^e = \min\{u^{max}, R_i^e - e_i + W_{ij} - e_{ij}\} \tag{31}$$

$$W_{ij} = \max\{0, \min\{u^{max}, \left\lfloor \left(a_j - Z_i - s_i - t_{ij} - t^p\right) / t^s \right\rfloor t^s r_s\}\} \tag{32}$$

The efficiency of a label setting algorithm relies on the dominance rule to reduce the number of labels with the label extension going on. Let L_i^1 and L_i^2 be the two labels associated with the paths ending at node i. L_i^1 dominates L_i^2 if (i) $C_i^1 \leq C_i^2$; (ii) $R_i^{e1} \geq R_i^{e2}$; (iii) $R_i^{t1} \leq R_i^{t2}$.

C. Initial Trip Chain Set Generation

The CG algorithm starts by solving the RMP with an initial set of feasible trip chains $\bar{\Omega}$. The quality of this solution often has a significant impact on the performance of the BP algorithm, especially for large-sized instances [34]. We adopted a heuristic procedure to construct the initial solution of good quality [35].

Given a set of trips, a complete schedule is generated by a construction heuristic, **Algorithm 1**, in two steps. In the first step, a set of short trip chains named Trip Chain Segments (TCS) are generated, each of which starts from and ends at the same depot without long-distance empty travel. In the second step, the TCSs are merged with each other to formulate complete trip chains. Such procedure is designed to satisfy the vehicle-depot constraint and avoid unnecessary empty travel of the e-buses. The outline of **Algorithm 1** is described in Fig. 3.

The complexity of heuristic Algorithm 1 is $O\left(N^3\right)$ which is dependent on the number of timetabled trips N. Denote

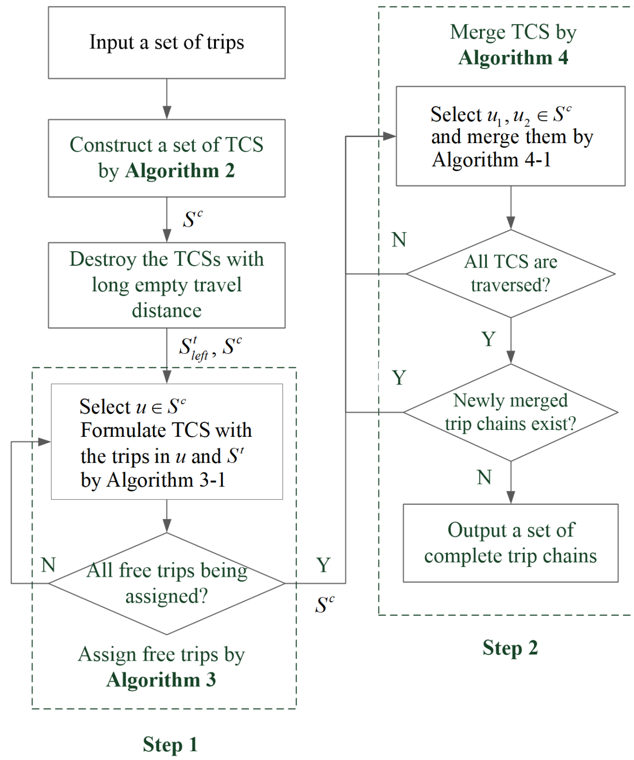


Fig. 3. Flow chart of Algorithm 1 to generate a set of trip chains.

$C(\cdot)$ as the complexity of an algorithm. In Algorithm 2, firstly the N service trip need to be sorted, and then each of the first n trips is created with a new TCS; the remaining $(N - n)$ are either appended to the existing TCS or created with a new TCS; then the infeasible TCSs are discarded and the unassigned are reformulated into new TCSs by Algorithm 3. The above gives the complexity of Algorithm 2 as $O\left(NlogN\right)+C(\text{Algorithm3})$. In Algorithm 3, the outside loop repeats at most two times; while the inside loop repeats at most $\max\{N, |S^c|\}$ times where $|S^c|$ is the number of TCSs formulated by Algorithm 2. Each of the iteration needs $O\left(N\right) + C\left(\text{Algorithm3} - 1\right)$ operations. As such, Algorithm 3 requires $\max\{N, |S^c|\} \cdot \left(O\left(N\right) + C\left(\text{Algorithm3} - 1\right)\right)$ computations which are dependent on Algorithm 3-1. Algorithm 3-1 requires $O\left(N^2\right)$ computations since it is a *Regret-k* Insertion Heuristic that searches for possible trip insertion positions among all TCSs. Based on the above analysis, Algorithm 2 requires $O\left(N^3\right)$ computations. After the set of TCSs S^c are generated by Algorithm 2, they are input into Algorithm 4 to generate complete trip chains. Algorithm 4 requires $O\left(|S^c|^2\right)$ computations. In Algorithm 4, firstly, the $|S^c|$ TCSs need to be sorted, then $|S^c|$ iterations are performed where the selection of possible merging choices for each TCS is performed by Algorithm 4-1. This gives the complexity of Algorithm 4 as $O\left(|S^c|\log|S^c|\right)+|S^c|\cdot C\left(\text{Algorithm4} - 1\right)$. In Algorithm 4-1, we try to select a TCS in S^c to merge with the selected TCS. This process can be completed in $O\left(|S^c|\right)$. Based on the above analysis, Algorithm 4 requires $O\left(|S^c|^2\right)$ computations. Combing the complexity of Algorithm 2 and Algorithm 4, the overall complexity of the trip chain construction heuristic Algorithm 1 is $O\left(N^3 + |S^c|^2\right)$. As the number of TCSs $|S^c|$

is always no larger than the number of timetabled trips N, the computational complexity of Algorithm 1 is $O\left(N^3\right)$.

Algorithm 2 Formulate TC

Input: A set of trips S^t
Output: A set of trip chain segments S^c
Sort the trips in S^t in increasing order of the start time
Select the first n trips from S^t
Create a new trip chain for each of the n trips, $n \sim G(p_1)$, and add them into S^c;
Repeat
  Select the next trip $t \in S^t$, and append it to the trip chains in S^c
   If t cannot be appended, create a new trip chain for t
Until S^t is empty
Remove all the TCSs in S^c that violate the vehicle reposition constraint, and add the trips of these TCSs into set S^t_{left}
Generate a new set of trip chains S^c_{new} by **Algorithm 3**
$S^c \leftarrow S^c_{new}$

Algorithm 3

Input: Set of trips S^t_{left}; set of trip chain segments S^c
Output: Set of trip chain segments S^c_{new}
Repeat
  Repeat
   Select the trip chain $u \in S^c$ with the latest operation start time
   Add the trips in u and S^t_{left} into a new empty set S^t_1;
   Generate a set of TCSs S^c_u with unassigned trips S^t_u by
Algorithm 3-1 (S^t_1)
   If $\left|S^t_u\right| < \left|S^t_{left}\right|$
   $S^c \leftarrow S^c u \cup S^c_u$ and $S^t_{left} \leftarrow S^t_u$
  Until $S^t_{left} = \emptyset$ or S^c is traversed
  Relax the vehicle reposition constraint
Until $S^t_{left} = \emptyset$

D. Heuristic Pricing

The pricing problem solved by the label setting algorithm can be time consuming, especially on the large-scaled graphs. The efficiency of the GC procedure is dependent on the number of iterations and pricing problem solving. We do not need to a most-negative reduced cost column by solving the pricing problem to optimal at each iteration of the CG. To seed up the problem solving, we adopt heuristic decisions in the pricing problem solving before invoking the exact label setting algorithm. We only maintain a pre-specified number of non-dominant columns instead of storing all of them in the label setting. When a new non-dominant label is added into the list and the total number of labels has reached the limit, the label with the largest cost in the list will be discarded. This setting can effectively reduce the number of labels stored and extended at each node, speeding up the detection of path with negative reduced cost. Note that this heuristic may lead to more CG iterations and a trade-off should be made to decide on the number of labels maintained.

E. Branching

At the end of the CG procedure, branches are created in the branch-and-bound tree by fixing the flow variables $(x_{ij})_{(i,j)\in A}$ on the arcs. If not all the variables $(x_{ij})_{(i,j)\in A}$ are integers, we choose to branch on the x_{ij} that has the highest fractional

Algorithm 3-1 Regret-k Insertion Heuristi

Input: Set of trips S^t_1
Output: Set of TCSs S^c_u and set of unassigned trips S^t_u
Initialize set $S^c_u = \emptyset$
Repeat
  Select a trip $t \in S^t_1$
  Search for feasible insertion positions for t into the TCSs in S^c_u
  If the insertion positions exist
   Generate a set of candidate TCSs $S^c_{u,t}$ for all the feasible insertion positions of t
   Calculate the k criteria $k_t = \sum_{i=1}^{k} \left(f^i(u) - f^1(u)\right)$,
   where $f^i(u)$ is the i-th best evaluation value of $u \in S^c_{u,t}$
   Retain the best candidate TCS u^* in $\bigcup_{t \in S^t_1} S^c_{u,t}$ and the corresponding trip t^*
   $S^c_u \leftarrow S^c_u \cup u^*$; $S^t_1 \leftarrow S^t_1 t^*$
  Else
   Create a new TCS u for trip t
   $S^c_u \leftarrow S^c_u \cup u$; $S^t_1 \leftarrow S^t_1 t$
Until $S^t_1 = \emptyset$
Remove all TCSs $u \in S^c_u$ with vehicle reposition and add the trips of u into set S^t_u;

Algorithm 4 Merge Trip Chain Segment

Input: A set of TCSs S^c
Output: Set of merged trip chain segments S^c_{merge}
$S^c_{merge} \leftarrow S^c$
Repeat
  Sort the TCSs in S^c in increasing order of the operation start time
  Repeat
   Select the first trip chain $u \in S^c$
   Attempt to merge u with the remaining trip chains in S^c by **Algorithm 4-1**
   If merge succeeds
    Add the merged trip chain into S^c_{merge}
    Remove the two original trip chains in S^c and S^c_{merge}
  Until $S^c = \emptyset$
  Add all the trips chains in S^c_{merge} into S^c
Until no feasible merge exists

value that is less than 0.99, creating two nodes, one with $x_{ij} = 1$ and the other with $x_{ij} = 0$. Besides, the x_{ij} that have the values larger than 0.99 are set to 1; while the x_{ij} that have the values less than 0.01 are set to 0. Numerical experiments show that this setting works well on improving the computational efficiency while maintaining high solution quality. We adopted the depth-first search strategy to explore the branch-and-bound tree.

F. Obtain Optimal Charging Plan

The optimal charging plan for each individual trip chain is obtained after the optimal schedule is generated by the BP algorithm. The problem is solved as a fixed route electric vehicle refueling problem (FRVRP) to determine the optimal charging location and amount for a given trip chain of an e-bus.

The overall method is described in the flow chart shown in Fig. 4. Firstly, a set of feasible trip chains are initialized by the construction heuristic Algorithm 1 and provided to the master problem; then, the dual variables of the constraints are obtained by solving the RMP. The pricing problems are solved

Algorithm4-1 Merge One Trip Chain Segmen

Input: TCS u and set of TCSs S^c
Output: Merged trip chain u^m
$\tilde{S^c} \leftarrow S^c u$; $S^c_{candid,u} = \emptyset$
Repeat
  Select the next $u' \in \tilde{S^c}$
  Generate a candidate TCS u'' by merging u with u'
  $S^c_{candid,u} = S^c_{candid,u} \cup \{u''\}$
Until the TCSs in $\tilde{S^c}$ are traversed
If $S^c_{candid,u} \neq \emptyset$
  Select the TCS u^m with the i-th lowest evaluation value from $S^c_{candid,u}$ by evaluation function $f^i(u)$ where $i \sim G(p_2)$;
Else
  $u^m = null$

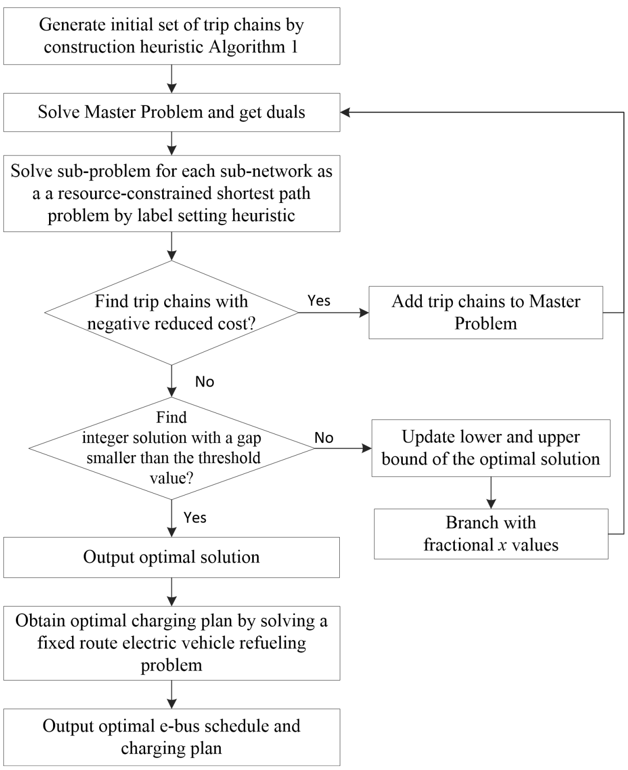


Fig. 4. Framework of the solution approach.

to generate a set of trips chains with negative reduced cost. Each sub-network corresponding to a depot is considered in a separate pricing problem. The newly generated trips chains are then added to the RMP and the RMP is solved again. If there is no trip chain with negative reduced cost being generated, then the current CG procedure will end and the branching strategy will be applied to branch on the fractional variables x_{ij}. If an integer solution has been obtained with a gap smaller than the threshold value, the BP procedure will end and the optimal solution is obtained. Then, the optimal charging plan of the optimal solution is obtained by solving a FRVRP problem.

V. Numerical Experiments

A. Data Preparation

We conducted numerical experiments based on the transit operation cases in Shenzhen. Regarding the operation management of the transit systems in China, two major modes can

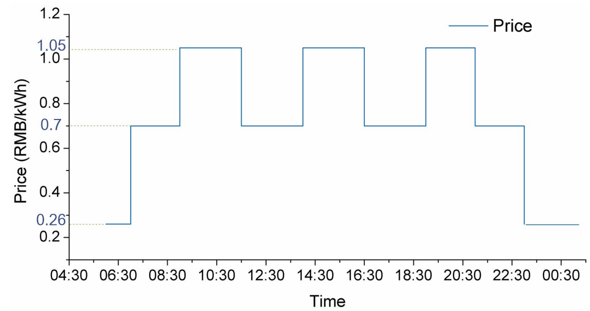


Fig. 5. Time-of-use tariff in Shenzhen.

be distinguished: fixed route and multi-route operation. Under the fixed-route operation mode, a bus fleet serves only one fixed bus route while under the multi-route operation mode, a bus fleet can serve multiple bus routes. Both of the two modes have their suitable scenarios for application depending on the timetables and local conditions. We aim to demonstrate the applicability and benefits of our algorithm in solving the real-world problems.

In the following numerical experiments, the e-bus schedules are generated with all the constraints introduced in Section 2.1 being satisfied. The required operational data for the experiments includes the headway requirements at different times of the day, travel time of the historical trips, parameters related to the charging technology and the type of the e-bus used. Based on these data, we firstly generated one day's timetables of the routes. Each trip is given the information of its start time from the starting depot and expected end time at the ending depot. The trip times are estimated according to the procedure described in [23]. Firstly, trip time samples are collected from the historical data set; then the whole operation period is divided into homogeneous running periods within which the trip time follows the same distribution. The trip time is calculated according to a rule-of-thumb [36]: The average value plus standard deviation of trip times.

In the numerical experiments, the BYD e-buses with a battery capacity (B) of 260 kWh operate on the routes; the safety range of the battery SoC is set between $0.3B$ and $1.0B$. In Shenzhen, plug-in DC chargers are usually established in the depots the majority of which have the maximum charging power of 100 kW. The charging rate is estimated as 1.6 kWh/min. The charging preparation time t^p and minimum charging time t^s are set as 2 min and 5 minutes, respectively. Parameter α takes the value of 0.5. The daily usage cost of an e-bus c^f is set as 1000 CNY. Fig. 5 shows the time-of-use tariff in Shenzhen, which is used to set the values of g_{ij}, $(i, j) \in A$.

We used the historical running data of 150 bus routes in five month in Shenzhen to analyze the relationship between the trip time and trip energy consumption. The average energy consumption rate is estimated to be 1.1 kWh/km. We observed that the travel time of the trips starting during the morning and evening peak period is a bit higher than those during the other time due to the busy traffic and higher passenger demand. At the same time, As Fig. 2 shows, the energy consumption of a trip and trip travel time can be expressed

TABLE II
DEFINITIONS OF THE SETS AND PARAMETERS FOR THE MD-EVSP MODEL

Sets	
K	Set of depots with index k
T	Set of trip nodes
V	Set of nodes
A	Set of arcs
Parameters	
B	Battery capacity of the e-bus
a_0	Earliest operation start time
b_0	Latest operation end time
s_i	Trip time of trip node i
e_i	Energy consumption of trip node i
h_i	Headway between trip node i and the next trip node starting from the same depot and one the same bus route
α	A parameter that controls the length of the time window
t_{ij}	Empty travel time from node i to node j
e_{ij}	Energy consumption of the non-service travel from node i to node j
c_{ij}	Cost of arc $(i,j) \in A$
g_{ij}	Unit charging cost on arc $(i,j) \in A$
c^f	Daily vehicle usage cost
t^p	Charging preparation time
t^s	Unit charging time
r_s	Energy charging rate
r_u	Energy consumption rate
u^{max}	Upper bound of the vehicle battery SoC
u^{min}	Lower bound of the vehicle battery SoC
u_i^{min}	Lower bound of variable Y_i

as an linear relationship in our cases. Therefore, to account for the variation of energy consumption in different time of the day, we set the energy consumption rate as 0.45 kWh/min for the e-buses.

The MIP model was solved by the standard optimization solver Cplex 12.6 with a time limit of 3600 s and the BP framework was coded in Java using Cplex 12.6 to solve the RMP. All the experiments were run on a PC with Windows 10, Intel Core i5-8250U, 1.80GHz and 8GB RAM.

B. Fixed-Route Scheduling

We analyze the performance of the MIP model and BP algorithm based on three bus routes: No. 727, 81 and M133, the average trip time of which is 45, 80 and 110 minutes, respectively. The layout of these routes is shown in Fig. 6. We compiled timetables with around 100, 200, 300 and 400 trips for each route, creating 12 problem instances in total. These instances represent different route configurations and problem scale that may appear in real-world fixed-route scheduling cases. The instances are named in the format “route name/number of trips”. For example, the instance for route M133 with 300 trips is named as M133/300.

1) Results: The computational results of the MIP model and BP algorithm are presented in Table III. The number of e-buses used (#V), objective of the MIP model solved by Cplex and BP algorithm (Obj), computational time and the solution gap are reported. The MIP gap is the gap between the solution and lower bound value of the MIP model solved by the Cplex. The BP gap is the gap between the solution and root relaxation value obtained by the BP algorithm. If the MIP gap is higher than 95% within 3600s of computational time, we report the lower bound value obtained by Cplex in the bracket.

TABLE III
COMPUTATIONAL RESULTS OF THE FIXED-ROUTE INSTANCES

Instance Name	#V	Obj		Run time (s)		Gap (%)	
		MIP	BP	MIP	BP	MIP	BP
727/100	14	14000.0	14000.0	0.01	12	0	0
81/100	20	20258.9	20278.0	3600	11	1.28	0
M133/100	26	26354.1	26403.7	3600	6	1.34	0
727/200	16	16786.7	16825.7	3600	240	4.69	0
81/200	24	25554.8	25610.2	3600	89	6.08	0.12
M133/200	32	35225.3	34433.2	3600	22	9.16	0.16
727/300	26	(26000)	27323.9	3600	273	—	0.09
81/300	38	(38000)	40799.1	3600	599	—	0.22
M133/300	51	(51000)	54937.0	3600	104	—	0.19
727/400	42	(42000)	42784.7	3600	958	—	0.28
81/400	64	(64000)	65742.4	3600	810	—	0.16
M133/400	83	(83000)	85048.0	3600	229	—	0.32

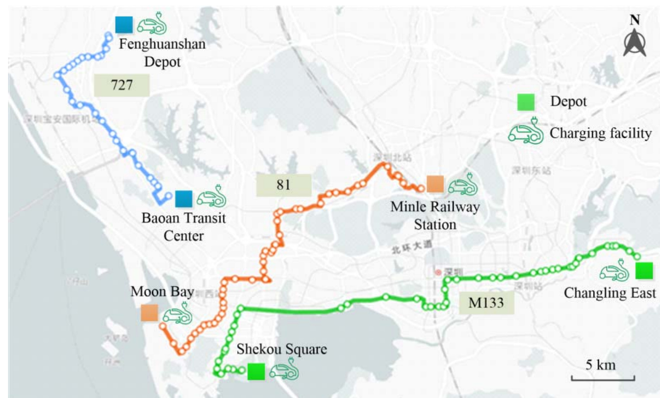


Fig. 6. A display of bus routes No. 727, 81 and M133 in Shenzhen.

The results show that the MIP model can be solved by Cplex for instances with 100 trips. The instances with 300 and 400 trips cannot be solved by Cplex in 3600s. The BP algorithm is able to generate high-quality solutions in a short time for all the instances. We can observe that with the increase of the route length, the computational time becomes shorter under the same instance size. This is because a shorter route indicates a longer trip chain for an individual vehicle. While in the pricing problem of the CG, label setting is used to generate candidate trip chain paths, the computational efficiency of which is negatively related to the length of a trip chain path. As such, generating longer paths are more time-consuming, leading to a longer run time of the BP algorithm.

We further compare the computational time of the BP with the initial solution generated Algorithm 1 (IS-BP) with that of the BP without using a good-quality initial solution (BP). The results are shown in Fig. 7. We can observe that IS-BP has computational times that are constantly much shorter than that of the BP. Also note that the computational time of the IS-BP is dependent on the quality of the initial solution. We can

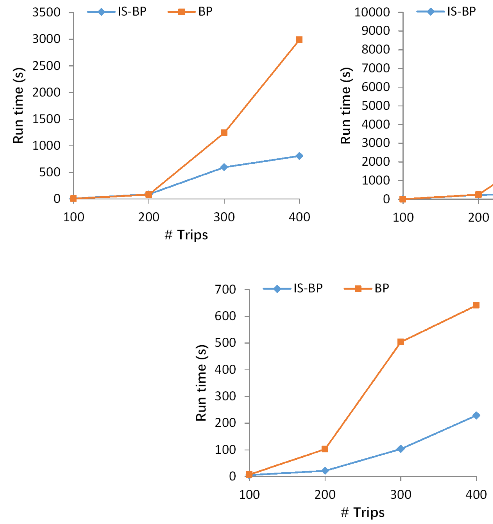


Fig. 7. A comparison between the computational time of the BP with and without good initial solution.

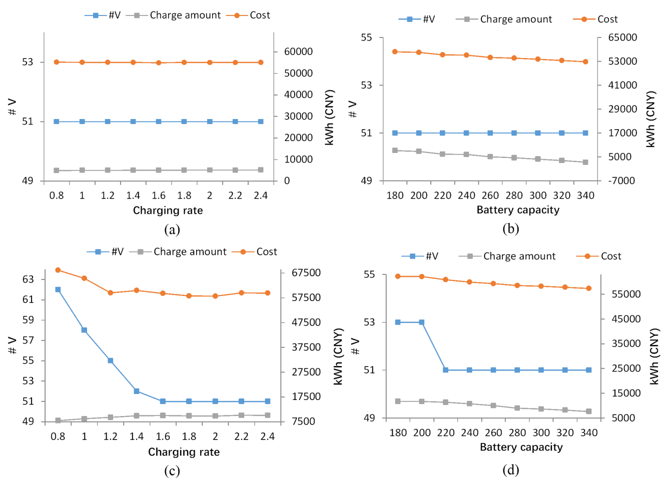


Fig. 8. Sensitivity analysis of the battery capacity and charging rate under energy consumption of 0.45 kWh/min (a), (b) and 0.6 kWh/min (c), (d).

conclude that using a good initial solution can greatly improve the computational efficiency of the BP, thus enabling instances of larger sizes to be solved.

2) Sensitivity Analysis:

a) Battery capacity and charging rate: We analyze the impact of the vehicle battery capacity and charging rate on the number of vehicles used and charging amount on instances. Instances for route M133 with 300 trips are used in the experiments. The battery capacity and charging rate take values in the intervals [180, 340] kWh and [0.8, 2.4] kWh/min, respectively. We set two levels of energy consumption rate, 0.45 kWh/min and 0.6 kWh/min, to explore the impact of battery capacity and charging rate under different of energy consumption rate.

Fig. 8 display the results of the sensitivity analysis. As shown in Fig. 8(a) and 8(b) under energy consumption rate of 0.45 kWh/min, the fleet size remains 51 e-buses and the charging rate has a minor impact on the operational cost and charging amount. However, with the increase in the battery capacity, the charging amount is reduced, resulting in

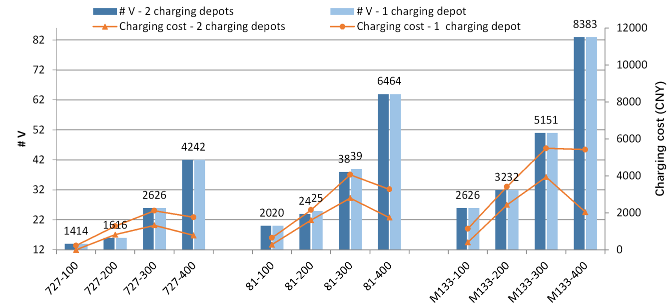


Fig. 9. A comparison of the number of e-buses used and charging cost for the instances with one or two end depots equipped with charging facilities.

a decrease in the operational cost. Under energy consumption rate of 0.6 kWh/min as shown in Fig. 8(c) and 8(d), with the increase in the charging rate, the fleet size decreases from 62 to 51 and then keeps constant. Since the charging efficiency is improved, e-buses operate more efficiently. We can observe that the fleet size decreases at the expense of a larger charging amount since to objective is to optimize the total cost. With the increase in the battery capacity, the fleet size decreases from 53 to 51 and then keeps constant. The longer driving range per charge and less time spent in charging make the e-buses operate more efficiently. Although a smaller fleet size may lead to an increase in daytime recharging, the charging amount actually decreases because the impact of the longer driving range is more significant.

To summarize, the impact of battery capacity and charging rate on the operational cost can have a great difference under different energy consumption rate. When energy consumption rate is higher, the charging demand of the e-buses is higher so that the operational cost is more sensitive to the changes of battery capacity and charging rate. With a certain number of timetabled trips, increasing battery capacity can reduce the daytime charging amount and fleet size to some extent. Increasing charging rate can reduce the time spent on charging thus reducing the fleet size. However, the charging amount is increased and a trade-off between the fleet size and charging cost should be considered to optimize to total operational cost.

b) Availability of the charging facilities: As transit electrification is still in the developing stage, the location and capacity of the charging stations can be limited. To analyze the impact of the availability of charging facilities on the e-bus operational cost, we assume that in the above problem instances, one of the end depots is unavailable for e-bus charging. The e-buses can be moved to out-of-depot charging stations for recharging during the off time. We assume that $0.1B$ of energy will be consumed traveling from the depot to the charging station. Therefore, the e-buses will start the operation from the depot with $0.9B$ of battery SoC and should maintain at least $0.4B$ of battery SoC at the end of the operation. The results are shown in Fig. 9 and Fig. 10. The instances are names by “route name-number of trips”.

Fig. 9. shows a comparison of the number of e-buses used and charging cost for the instances with one or two end depots equipped with charging facilities. From Fig. 9, we can see that when only one depot is charging available, more e-buses are used in the instances of route 81; in the

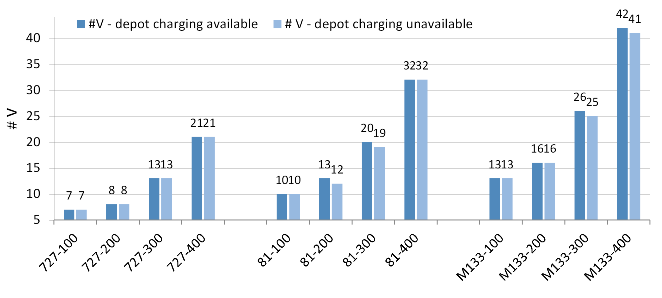


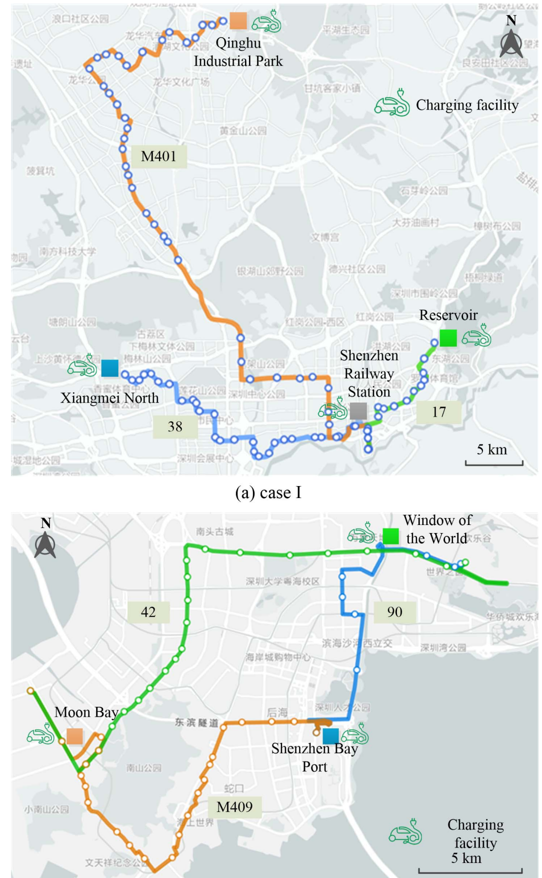
Fig. 10. A comparison of the number of e-buses used at the charging-available depot and that at the charging-unavailable depot in the instances with only one depot equipped with charging facilities.

instances of route 727 and M133, the number of e-buses used remains unchanged. We can observe an increase in the charging cost in all the instances. This is because for the e-buses housed at the charging unavailable depot, the battery SoC at the start of the operation decreases from $1.0B$ to $0.9B$ and that at the end of the operation is required to be $0.4B$ instead of $0.3B$. As such, when operating for a same length of time, the e-buses housed in the charging-unavailable depots have higher recharging demand than those housed in the charging-available ones. Fig. 10 presents a comparison of the number of e-buses used at the charging-available depots and that at the charging-unavailable depots in the instances with only one depot equipped with charging facilities. We can see that more e-buses are housed in the charging-available depots than in the charging-unavailable ones. This is also due to that the e-buses housed at the charging-unavailable depot have a higher charging demand thus being less efficient in operation. With the aim to optimize the total cost, we tend to reduce the number of e-bused housed at the charging-unavailable depot. In conclusion, the lack of charging facilities in the depots can cause an increase in the overall operational cost.

C. Multi-Route Scheduling

We carry out multi-route scheduling on two cases, the characteristics and layouts of which are shown in Table IV and Fig. 11. In case I, three bus routes M401, 38 and 17 share one end depot, the Shenzhen Railway Station Depot, and formulate a hub-and-spoke topology. In case II, the three bus routes M409, 42 and 90 share ending depots with each other and formulate a circle. The bus routes share one bus fleet and the e-buses are allowed to operate on different routes to improve the service efficiency. In this case, all the depots are equipped with charging facilities; a total number of 460 and 356 trips are scheduled in case I and II, respectively.

We obtained the optimal schedule and charging plan for the three bus routes by BP algorithm. The computational results are shown in Table V. In case I, 73 e-buses are used for operation, including 36, 26, 7 and 4 housed at the Shenzhen Railway Station, Qinghu Industrial Park, Xiangmei North and Reservoir, respectively. The total daily charging cost is 4063.6 and the total operational cost is 77063.6. In case II, 27 e-buses are used for operation, including 6, 11, and 9 housed at the Shenzhen Bay Port, Moon Bay, and



(a) case I

(b) case II

Fig. 11. A display of the route layout of case I and II.

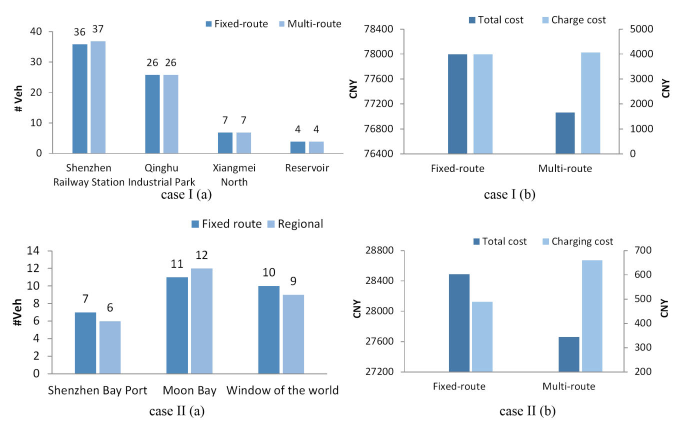


Fig. 12. A comparison of the fixed-route and multi-route scheduling on (a) the number of vehicles used at each depot; (b) the operational cost and daily charging amount.

Window of the world, respectively. The total daily charging cost is 660.2 and the total operational cost is 27660.2.

We further applied the fixed-route scheduling mode, the results of which are shown in Fig. 12. Compared with fixed-route scheduling, multi-route scheduling saves one e-bus in case I and two e-buses in case II. The total operational cost is also significantly reduced. In case I, the difference of daytime charging cost between the fixed-route and multi-route mode is not significant; while in case II, the daytime charging cost

TABLE IV
CHARACTERISTICS OF THE ROUTES IN THE MULTI-ROUTE CASES

Case	Line	Aver. trip time (min)	Operation period	# scheduled trips
Case I	M401	120	6:10-22:00	232
	38	80	7: 00-22:00	124
	17	45	7: 00-22:00	104
Case II	42	30	7:00-22:00	136
	M409	40	7:00-22:00	92
	90	12	7:00-21:00	128

TABLE V
COMPUTATIONAL RESULTS OF THE FIXED-ROUTE INSTANCES

Case	Obj.	#V	Charging cost (CNY)	Run time (s)	BP gap (%)
case I	77063.6	73	4063.6	1038	1.06
case II	27660.2	27	660.2	14203	1.58

under fixed-route mode is 200 CNY less than that under multi-route mode. This result indicates the benefits of multi-route operation in saving cost. However, from the management viewpoint, the e-bus operation system becomes larger with the schedule being more complicated for the drivers to follow.

We calculated the number of e-buses charging at the depots during different time of the day, as shown in Fig. 13. We can observe that in case I, a maximum of 7, 4, 2 and 1 e-bus is charging at the Shenzhen Railway Station, Qinghu Industrial Park, Xiangmei North and Reservoir, respectively. In case II, a maximum of 2, 1 and 3 e-buses are charging at the Shenzhen Bay Port, Moon Bay, and Window of the world, respectively. This result can be a useful reference to the decision on the capacity of the charging facilities at the depots. Fig.13 also shows that in both case I and II, the number of e-buses chagrining during the off-peak time periods is higher than that in the peak periods. Such charging arrangements will reduce the charging cost significantly.

D. A Comparative Evaluation of the Algorithm Performance

As listed in Table I, several algorithms were proposed in the literature to solve the e-bus scheduling problems including the BP algorithm ([14], [15]) Adaptive Large Neighborhood Search (ALNS) [10], Genetic algorithm (GA) ([18], [19]) and local search based heuristic [12]. These algorithms fit for problem scenarios with different route configurations including the number of depots, charging policy and vehicle-depot constraint etc. However, existing algorithms are rarely designed to generate e-bus schedules involving all the above route configurations including multiple depots, partial charging and vehicle-depot constraint. Our BP algorithm is able to solve problems involving multiple depots, partial charging and vehicle-depot constraint. To compare the performance of the BP algorithm with other existing algorithms, we tested 10 batches of benchmark instances with different scales according to the method used in [15]. The number of depots involved ranges

TABLE VI
COMPUTATIONAL RESULTS OF BP ALGORITHM AND ALNS HEURISTIC ON BENCHMARK INSTANCES

Batch	ID	# V		Obj		Run Time (s)		Gap (%)
		BP	ALNS	BP	ALNS	BP	ALNS	
1-300	1	51	51	54957.1	54969.1	689	308	0.02
	2	54	54	57833.2	57833.2	296	272	0.00
	3	48	49	52580.1	53496.7	590	363	1.71
1-400	1	72	72	77109.0	77152.6	2618	941	0.06
	2	64	66	70733.1	71559.3	2059	1033	1.15
	3	70	72	75166.0	76859.4	1019	965	2.20
2-300	1	49	49	53502.0	53578.5	753	397	0.14
	2	54	54	58004.1	58233.2	678	365	0.39
	3	49	50	53269.4	53888.0	1008	341	1.15
2-400	1	62	62	68931.5	69102.4	4173	1021	0.25
	2	63	64	69224.6	69941.3	4366	1096	1.02
	3	71	71	75968.4	76452.5	3195	1103	0.63
3-300	1	49	50	53647.3	53841.8	1067	392	0.36
	2	56	56	59576.9	59654.4	776	437	0.13
	3	50	50	54173.1	54178.1	1008	399	0.01
3-400	1	66	66	72408.4	72820.8	4060	1253	0.57
	2	70	70	75085.5	75263.6	4089	1212	0.24
	3	67	67	72813.4	73087.1	4546	1246	0.37
4-300	1	52	52	55875.3	56053.0	1003	503	0.32
	2	52	52	55274.3	56553.5	1170	487	2.26
	3	48	48	52633.8	52805.4	1181	462	0.32
4-400	1	67	67	73047.7	73539.4	3335	1295	0.67
	2	66	67	74171.6	74330.5	3850	1304	0.21
	3	73	73	77791.0	77894.0	2810	1278	0.13
5-300	1	55	56	59171.5	60408.0	1222	489	2.05
	2	57	57	60585.5	61157.6	914	524	0.94
	3	59	59	62144.6	62627.8	809	531	0.77
5-400	1	67	68	73521.8	73741.9	3080	1406	0.30
	2	73	73	77778.1	78822.1	2887	1338	1.32
	3	68	69	73873.7	74614.9	4295	1368	0.99

from 1 to 5, and the number of trips ranges from 100 to 400. In order to create random instances, the start time of each trip is an integer number randomly generated from interval [operation start time, operation end time]. The travel distance of each trip is randomly generated from set {20, 25, 30, 35, 40, 45}. The travel time of each trip was then calculated according to an average travel speed of 18 km/h. The energy consumption rate and charging rate are set to be 0.45 kWh/min and 1.6 kWh/min, respectively.

We compared our BP algorithm with the ALNS algorithm introduced in [10] which is able to solve e-bus scheduling instances involving multiple depots, partial charging and vehicle-depot constraint. Table VI shows the computational results of the BP algorithm and ALNS heuristic on the benchmark instances. Each instance batch is named in format "the number of depots – the number of trips". In each batch, three instances were randomly generated and identified by

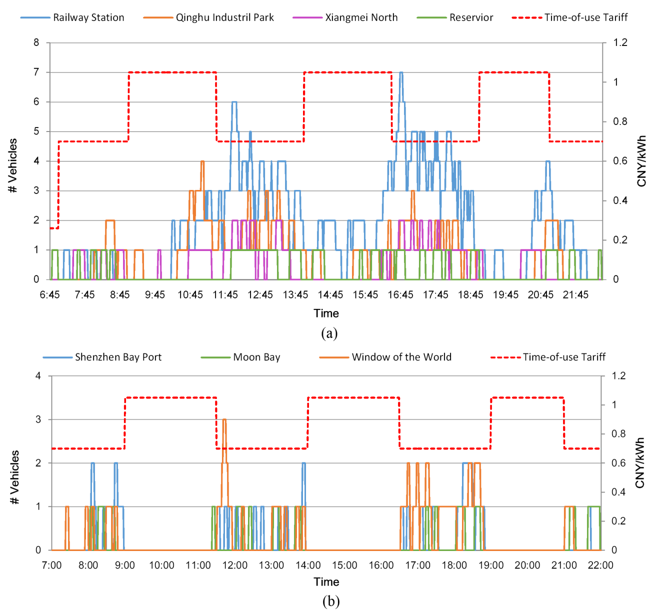


Fig. 13. The number of e-buses charging at the depots during the operation time in (a) case I; (b) case II.

the ID number. The number of e-buses used (#V), objective (Obj) and computational time (Run Time) of the BP algorithm and ALNS heuristic, and the solution gap between the BP algorithm and ALNS heuristic are reported (Gap). The gap is calculated as $Gap = (Z_{ALNS} - Z_{BP})/Z_{ALNS} \cdot 100$ where Z_{BP} and Z_{ALNS} are the objectives obtained by the BP algorithm and ALNS heuristic, respectively.

The BP algorithm is able to generate instances with up to 400 trips within a computational time limit of 3 hours. We can observe that the objective values obtained by the BP are generally superior to those obtained by the ALNS heuristic with a gap of up to 2.26%. However, ALNS heuristic has a shorter computational time due to the nature of heuristics. The computational time grows fast with the increase of the number of trips; while with more depots involved, the computational time of the two algorithms both increase but with a lower speed. In conclusion, BP algorithm can generated solutions of higher quality compared with the ALNS heuristic but requires a longer computational time. Compared with the number of depots, the number of trips has a greater impact on the computational time of both algorithms. For problem instances with less than 400 trips, BP algorithm is computationally efficient and is recommended since it can obtain better solution. When solving the problem instances of a larger scale, heuristic methods such as the ALNS heuristic are more practical to be applied.

VI. Conclusion

In this paper, we addressed an MD-EVSP problem considering the vehicle-depot constraint. An MIP model is formulated with the aim to minimize the total operational cost. An efficient BP algorithm is developed to solve the problem. In the BP algorithm, we devised a heuristic method to generate good initial solutions and adopted heuristic decisions in the label setting algorithm to solve the pricing problem. In this way, the efficiency of the BP algorithm is improved and the large-sized problem instances can be solved. We conducted numerical experiments based on the fixed-route and multi-route operation cases in Shenzhen. The results verified the model and showed that the BP algorithm is able to generate high-quality solutions for problem instances with around 400 trips within a computational time of 3 hours. Sensitivity analysis indicated that increasing the battery capacity and charging rate can reduce the operational cost. The impact of battery capacity and charging rate on the operational cost is different on different energy consumption rate and should be analyzed according to specific e-bus operation cases. The operational cost is more sensitive to the changes of battery capacity and charging rate under a higher energy consumption rate. The lack of charging facilities in the depots can result in an increase in the total operational cost. We demonstrated the benefits of multi-route operation mode in reducing the fleet size and thus the operational cost. Based on the above results, the transit agencies are suggested to equip the bus terminals and depots with chargers to maintain operational efficiency. The number of charging piles reserved for e-bus day-time charging can be determined based on the optimal charging schedule. The proper charging power and battery capacity should be determined according to the operational characteristics including the route length, operation time and energy consumption rate etc.

In this study, the largest solvable instance within a 3-hour computational time limit is around 400 trips according to the results of the numerical experiments. However, in real-world practices, e-bus scheduling cases with larger number of timetabled trips exist which need be addressed by algorithms with higher computational efficiency such as heuristics. Besides, we assumed a linear charging rate and energy consumption rate to reduce the complexity of the model and algorithm which enables that the problem instances of relatively large-scale is solvable. In future studies, e-bus scheduling algorithms should be developed to solve the problem instances of larger scale considering more operational characteristics such as nonlinear charging, more realistic energy consumption model, the number of chargers at the depots, and the interaction between the charging operations and the grid. Regarding the charging technology, fast opportunity charging at bus stops and wireless in-motion charging are the emerging technologies that should be considered in the e-bus scheduling problem.

References

[1] (2020). Bloomberg New Energy Finance (BNEF). *Electric Vehicle Outlook*. [Online]. Available: https://www.sustainable-bus.com/news/electric-vehicle-outlook-2020-bnef-electric-buses/

[2] G. Daily. (2017). *Shenzhen has Achieved an Overall Electrification of its City Bus Fleets*. Accessed: Aug. 20, 2021. [Online]. Available: http://shenzhen.sina.com.cn/news/s/2017-12-28/detail-ifyqcwaq5128608.shtml

[3] J.-Q. Li, "Battery-electric transit bus developments and operations: A review," *Int. J. Sustain. Transp.*, vol. 10, no. 3, pp. 157–169, Jan. 2016.

[4] (Mar. 20, 2020). Editorial. *Electric Bus Range, Focus on Electricity Consumption: A Sum-up*. Sustainable Bus. Accessed: Nov. 21, 2021. [Online]. Available: https://www.sustainable-bus.com/news/electric-bus-range-focus-on-electricity-consumption-a-sum-up/

[5] S. Elkosantini and S. Darmoul, "Intelligent public transportation systems: A review of architectures and enabling technologies," in *Proc. Int. Conf. Adv. Logistics Transp.*, May 2013, pp. 233–238.

[6] A. A. Bertossi, P. Carraresi, and G. Gallo, "On some matching problems arising in vehicle scheduling models," *Networks*, vol. 17, no. 3, pp. 271–281, 1989.
[7] S. Bunte and N. Kliewer, "An overview on vehicle scheduling models," *Public Transp.*, vol. 1, no. 4, pp. 299–317, 2009.
[8] A.-S. Pepin, G. Desaulniers, A. Hertz, and D. Huisman, "A comparison of five heuristics for the multiple depot vehicle scheduling problem," *J. Scheduling*, vol. 12, no. 1, pp. 17–30, 2009.
[9] Z. Chao and C. Xiaohong, "Optimizing battery electric bus transit vehicle scheduling with battery exchanging: Model and case study," *Proc. Social Behav. Sci.*, vol. 96, pp. 2725–2736, Nov. 2013.
[10] Y. Yang, M. Guan, and J. Ma, "Battery electric transit bus scheduling problem based on column generation approach," *J. Transp. Syst. Eng. Inf. Technol.*, vol. 16, no. 5, pp. 198–204, 2016.
[11] J.-Q. Li, "Transit bus scheduling with limited energy," *Transp. Sci.*, vol. 48, no. 4, pp. 521–539, Nov. 2014.
[12] M. Wen, E. Linde, S. Ropke, P. Mirchandani, and A. Larsen, "An adaptive large neighborhood search heuristic for the electric vehicle scheduling problem," *Comput. Oper. Res.*, vol. 76, pp. 73–83, Dec. 2016.
[13] M. E. van Kooten Niekerk, J. M. van den Akker, and J. A. Hoogeveen, "Scheduling electric vehicles," *Public Transp.*, vol. 9, nos. 1–2, pp. 155–176, 2017.
[14] G.-J. Zhou, D.-F. Xie, X.-M. Zhao, and C. Lu, "Collaborative optimization of vehicle and charging scheduling for a bus fleet mixed with electric and traditional buses," *IEEE Access*, vol. 8, pp. 8056–8072, 2020.
[15] Ş. Yıldırım and B. Yıldız, "Electric bus fleet composition and scheduling," *Transp. Res. C, Emerg. Technol.*, vol. 129, Aug. 2021, Art. no. 103197.
[16] L. Zhang, S. Wang, and X. Qu, "Optimal electric bus fleet scheduling considering battery degradation and non-linear charging profile," *Transp. Res. E, Logistics Transp. Rev.*, vol. 154, Oct. 2021, Art. no. 102445.
[17] M. Janovec and M. Koháni, "Exact approach to the electric bus fleet scheduling," *Transp. Res. Proc.*, vol. 40, pp. 1380–1387, Jan. 2019.
[18] T. Liu and A. A. Ceder, "Battery-electric transit vehicle scheduling with optimal number of stationary chargers," *Transp. Res. C, Emerg. Technol.*, vol. 114, pp. 118–139, May 2020.
[19] M. Rinaldi, E. Picarelli, A. D'Ariano, and F. Viti, "Mixed-fleet single-terminal bus scheduling problem: Modelling, solution scheme and potential applications," *Omega*, vol. 96, Oct. 2020, Art. no. 102070.
[20] L. Li, H. K. Lo, and F. Xiao, "Mixed bus fleet scheduling under range and refueling constraints," *Transp. Res. C, Emerg. Technol.*, vol. 104, pp. 443–462, Jul. 2019.
[21] E. Yao, T. Liu, T. Lu, and Y. Yang, "Optimization of electric vehicle scheduling with multiple vehicle types in public transport," *Sustain. Cities Soc.*, vol. 52, Jan. 2020, Art. no. 101862.
[22] T. Lu, E. Yao, Y. Zhang, and Y. Yang, "Joint optimal scheduling for a mixed bus fleet under micro driving conditions," *IEEE Trans. Intell Transp*, vol. 22, no. 4, pp. 1–12, Apr. 2021.
[23] S. S. G. Perumal, T. Dollevoet, D. Huisman, R. M. Lusby, J. Larsen, and M. Riis, "Solution approaches for integrated vehicle and crew scheduling with electric buses," *Comput. Oper. Res.*, vol. 132, Aug. 2021, Art. no. 105268.
[24] Y. Shen, J. Xu, and J. Li, "A probabilistic model for vehicle scheduling based on stochastic trip times," *Transp. Res. B, Methodol.*, vol. 85, pp. 19–31, Mar. 2016.
[25] F. He, J. Yang, and M. Li, "Vehicle scheduling under stochastic trip times: An approximate dynamic programming approach," *Transp. Res. C, Emerg. Technol.*, vol. 96, pp. 144–159, Nov. 2018.
[26] X. Tang, X. Lin, and F. He, "Robust scheduling strategies of electric buses under stochastic traffic conditions," *Transp. Res. C, Emerg. Technol.*, vol. 105, pp. 163–182, Aug. 2019.
[27] S. Hulagu and H. B. Celikoglu, "An electric vehicle routing problem with intermediate nodes for shuttle fleets," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 2, pp. 1–13, Feb. 2020.
[28] S. Hulagu and H. B. Celikoglu, "Electric vehicle location routing problem with vehicle motion dynamics-based energy consumption and recovery," *IEEE Trans. Intell. Transp. Syst.*, early access, Jul. 8, 2021, doi: 10.1109/TITS.2021.3089675.
[29] A. De, J. Wang, and M. K. Tiwari, "Hybridizing basic variable neighborhood search with particle swarm optimization for solving sustainable ship routing and bunker management problem," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 3, pp. 986–997, Mar. 2020.
[30] B. Ji, X. Yuan, Y. Yuan, X. Lei, and H. H. C. Iu, "An adaptive large neighborhood search for solving generalized lock scheduling problem: Comparative study with exact methods," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 8, pp. 3344–3356, Aug. 2020.
[31] H. Zhang, C. J. R. Sheppard, T. E. Lipman, and S. J. Moura, "Joint fleet sizing and charging system planning for autonomous electric vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 11, pp. 4725–4738, Nov. 2020.
[32] S. Zhang and K. Leung, "Joint optimal power flow routing and vehicle-to-grid scheduling: Theory and algorithms," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 1, pp. 1–14, Jan. 2020.
[33] S. Irnich and G. Desaulniers, "Shortest path problems with resource constraints," in *Column Generation*, G. Desaulniers, J. Desrosiers, and M. M. Solomon, Eds. Boston, MA, USA: Springer, 2005, pp. 33–65.
[34] G. Ozbaygin, O. E. Karasan, M. Savelsbergh, and H. Yaman, "A branch-and-price algorithm for the vehicle routing problem with roaming delivery locations," *Transp. Res. B, Methodol.*, vol. 100, pp. 115–137, Jun. 2017.
[35] M. Jiang, Y. Zhang, and Y. Zhang, "Multi-depot electric bus scheduling considering operational constraint and partial charging: A case study in Shenzhen, China," *Sustainability*, vol. 14, no. 1, p. 255, Dec. 2021.
[36] T. H. J. Muller and P. G. Furth, "Integrating bus service planning with analysis, operational control, and performance monitoring," in *Proc. ITS Amer. Annu. Meeting*, Washington, DC, USA, 2000, pp. 1–20.

Mengyan Jiang is currently pursuing the Ph.D. degree with the Intelligent Transportation and Logistics Systems Laboratory, Tsinghua-Berkeley Shenzhen Institute, Tsinghua University, Shenzhen, China. Her research interests include planning and operation of intelligent transportation and logistics systems.

Yi Zhang (John) (Member, IEEE) is currently a Professor with the Department of Automation, Tsinghua University, Beijing, China. His research interests include intelligent vehicle-infrastructure cooperative systems, analysis of urban transportation systems, urban road networks management, traffic data fusion and dissemination, and urban traffic control and management.

Yi Zhang (Edwina) is currently an Assistant Professor with the Shenzhen International Graduate School, Institute of Future Human Habitat, Tsinghua University, Shenzhen, China. Her research focuses on application planning of renewable energy at city scale with city information modeling, intelligent building energy systems, big data analysis on energy consumption behavior, electric vehicles operation, DC power, and distributed electricity storage in buildings.