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Optimal service zone and headways for flexible-route bus services for multiple periods

Myungseob (Edward) Kim and Austin Roche

Department of Civil and Environmental Engineering, Western New England University, Springfield, MA, USA

ABSTRACT
Where private ridesharing companies such as Uber and Lyft are transforming the transit sector by making flexible transit cheaper and more readily available than ever before, it may be beneficial for public transportation agencies to adapt to the changing marketplace and provide flexible bus services in low to mid-demand density areas such as suburban or rural areas. This paper proposes an optimization model in which the service area for the flexible bus operations is jointly optimized with headways with time-dependent demand densities. The closed-form solution that minimizes the average costs of user and operator is obtained with analytic optimization. The model presented in this paper may be a helpful planning resource to determine the feasibility of flexible bus services where fixed-route transit operation is not desirable due to the lack of ridership, especially in suburban and rural regions. Numerical case studies and sensitivity analyses for critical operation factors are presented.

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Public transportation; flexible bus services; headways; service area; analytic solution

Introduction

Bus transit systems may be categorized into two branches: (1) fixed-route bus systems, which can also be referred to as conventional bus services, and (2) unconventional bus services such as dial-a-ride services, ridesharing (e.g. Uber and Lyft), or flexible bus services. Fixed-route bus operations have characteristics that the bus routes are predetermined and its operating schedules are preset. Therefore, conventional bus operations are preferred to transport high ridership demand with high service frequencies. Conventional services are typically assigned to busy corridors to transport passengers with low average cost per person. Kocur and Hendrickson (1982) investigated local bus service design with conventional bus operations.

Flexible-route bus services, however, have flexibility in routing and schedules. Flexible-route bus services are typically considered for many-to-one or one-to-many demand patterns such as delivering passengers from the terminal to the residential areas. Wilson and Hendrickson (1980) proposed models to analyze system performances of flexibly routed transportation services. Stein (1978) proposed the tour length approximation model for dial-a-ride services, and Daganzo (1984) found that the coefficient for the tour length

CONTACT Myungseob (Edward) Kim myungseob.kim@wne.edu Department of Civil and Environmental Engineering, Western New England University, Springfield, MA 01119, USA

approximation are 0.90 for Euclidean space and 1.15 for rectilinear space, respectively. Chang and Schonfeld (1991) compared fixed- and flexible-route bus services with the objective of minimizing the total cost that includes bus operator's cost, user's in-vehicle cost, and user's waiting cost. The user's access cost is considered for fixed-route bus formulations and door-to-door pick-up or drop-off is assumed for flexible-route bus services (i.e. no access cost). They confirmed that there exists the demand threshold between conventional and flexible bus services when the average cost per person is compared. Daganzo and Ouyang (2019) recently explored demand-responsive transportation services, namely taxi, ridesharing and dial-a-ride options. In that paper, several perspectives namely level of service, resource reduction, and passenger capture among three service options are investigated.

Kim and Schonfeld (2012) extended Chang and Schonfeld (1991) to compare conventional, flexible and variable-type bus services in which the variable-type bus operation is that with the same size of the vehicle, the type of bus operation is switched between conventional and flexible services based on the demand density. For instance, high demand periods are served with conventional bus operations and flexible bus operation covers passengers with low demand periods. As the joint use (i.e. integration) of conventional and flexible bus operations based on the demand thresholds may be beneficial to save the total system cost, further studies are proposed such as mixed fleet operations for conventional and flexible services (Kim and Schonfeld 2013), passenger transfers between conventional and flexible bus operations (Kim and Schonfeld 2014), the analysis of maximum system welfare for conventional and flexible services (Kim and Schonfeld 2015). Chien and Schonfeld (1998) used conventional bus services as a bus feeder for a rail transit line.

The passengers, under flexible-route bus operations, do not have to transport themselves from their home to a pre-determined bus stop. This door-to-door functionality is particularly favorable to elderly and disabled people as they face more challenges than able-bodied people. In terms of the average cost per passenger, flexible bus services are particularly favorable in areas with a lower demand density, in comparison to conventional bus operations. In suburban areas where the ridership density is low in comparison to highly populated regions such as NYC or Boston, it provides a reasonable transportation option to those who need it. As an extension from Kim et al. (2019) which is to jointly optimize the service zone and headway for flexible-route services, this paper presents an optimization model that minimizes the average cost per passenger with the decision variables of the service area and headways for flexible services with multiple demand periods. The analytic solution is obtained for the optimal service area, and the headway is obtained from the relation of the maximum allowable headway based on the service area, vehicle size, and the demand density. The model presented in this paper may be of interest for transit planners to evaluate the feasibility of flexible-route bus services where frequent conventional services are not justified due to the low demand in suburban or rural regions.

Materials and methods

Bus operation description and cost formulation

We seek to design the model in which many-to-one demand patterns are served. For instance, passengers in rural or suburban areas are transported to the terminal (or

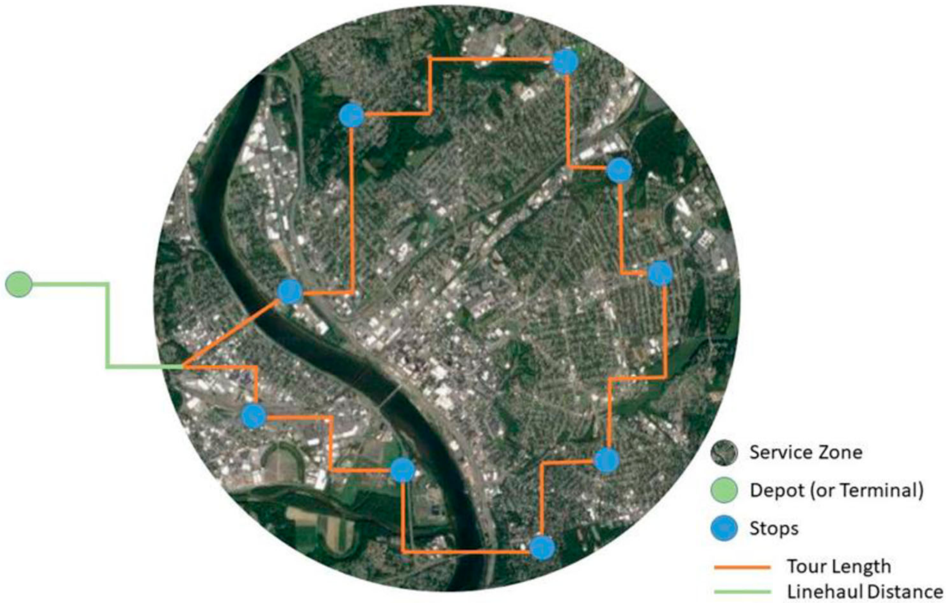

Figure 1. Description of flexible bus tour.

Central Business District) in the downtown or vice versa. It is assumed that the route of the bus operations and its schedules are flexible. Stein's (1978) approximation is used to estimate the length of the vehicle tour within the zone. The zone is assumed to be fairly compact and fairly convex. In a larger system perspective, a region can be subdivided into multiple service zones while each zone is connected to the main terminal for passenger transfers so that many-to-many travel demand patterns within the region are covered with multiple flexible-route bus services. The travel demand is uniformly distributed over time and space within the zone, and the minimum number of stops within a tour is at least five. A singular zone and the line haul to the terminal are shown in Figure 1. The parameters and variables that are used in the formulations are presented in Table 1.

Flexible bus formulation

In the flexible bus service formulations, three cost terms are considered, namely supplier's operating cost, user's in-vehicle cost and user's waiting cost. Kim, Levy, and Schonfeld (2019) formulated the flexible-route bus service for one-time period so their formulation is adapted in this study and are extended to cover demand variations during multiple periods while the service zone is designed to be consistent.

The supplier's vehicle operating cost is formulated with the fleet size N and the unit operating cost of the vehicle c as shown in Equation (1). The unit cost of operation for a vehicle is based on a fixed cost component and a variable cost component dependent to the size of the vehicle (i.e. the number of seats) as in Table 1.

$$C_S = N \times c \tag{1}$$

Table 1. Notation and baseline values for inputs.

Symbol	Variable	Units	Base value
A	Zone size (area)	Square miles	–
a	Parameter for bus operating cost	$ per hour	30
ASC	Average system cost	$ per passenger	–
b	Parameter for bus operating cost	$ per seat hour	0.3
C_A	Average total cost	$ per passenger	–
C_T	Total cost	$ per hour	–
C_S	Supplier cost	$ per hour	–
C_V	In-vehicle cost	$ per hour	–
C_W	Waiting cost	$ per hour	–
c	Unit bus operating cost (= a + b * S)	$ per bus hour	–
D_C	Tour length within zone	Miles	–
h_i	Headway for time period *i*	Hours	–
i	Number of time periods		
J	Line Haul distance	Miles	10
l	Load factor	Dimensionless	1.0
N	Fleet size	Buses	–
n	Number of stops	Number	–
Ø	Stein's constant	Dimensionless	1.15
Q_i	Demand density for time period *i*	Passengers per square mile per hour	–
q	Demand for time period *i*	Passengers per hour	–
R	Round trip time	Hours	–
S	Bus capacity	Seats per bus	45
TSC	Total system cost	$	–
u	Number of passengers per stop	Number of passengers	1
V_X	Line haul speed	Miles per hour	30
V_L	Average local speed	Miles per hour	–
v_v	Value of in-vehicle time	$ per passenger hour	12
v_w	Value of waiting time	$ per passenger hour	15
w	Waiting time	Hours	–
y	Ratio of local speed to express speed	Dimensionless	0.9

The length of the tour distance D_c within the zone is approximated by Stein (1978)'s formula in Equation (2). The tour distance is the function of the number of stops in the tour n and the size of the zone A. The coefficient Ø is 1.15 for rectilinear space and 0.9 for Euclidean space (Daganzo 1984). This paper assumes the rectilinear space within the zone:

$$D_c = \text{Ø}\sqrt{nA} \tag{2}$$

The number of stops during the tour n is found from the relation of demand density Q, zone size, A, headway h, and the number of passengers (boarding or alighting) per stop u:

$$n = \frac{QAh}{u} \tag{3}$$

As the actual hourly demand per zone q (in passengers/hour) is product of the demand density Q and the zone size A, the demand q is expressed as:

$$q = QA \tag{4}$$

Then, the tour length D_c in Equation (2) is rearranged as:

$$D_c = \text{Ø}\sqrt{\frac{qAh}{u}} \tag{5}$$

The round trip time, R, is the sum of 2-directional trip time for line haul (i.e. express segment) and trip time in the local zone. The line haul distance J, the express speed V_X, and local speed V_L are used to compute the vehicle round trip time R:

$$R = \frac{2J}{V_X} + \frac{D_c}{V_L} \tag{6}$$

We assume the local speed is a function of express speed V_X. We denote the ratio of local speed V_L, to express speed V_X, as y. Then the round travel time R is expressed in Equation (8) as a function of express speed V_X, but the local speed V_L can also be treated as an independent input parameter.

$$V_L = y\, V_X \tag{7}$$

$$R = \frac{2J}{V_X} + \frac{D_c}{y\, V_X} \tag{8}$$

The required fleet size N is then obtained by dividing the round travel time R by the headway for the service h.

$$N = \frac{R}{h} \tag{9}$$

The supplier's operating cost C_S is then formulated in Equation (10).

$$C_S = \frac{2Jc}{hV_X} + \frac{Øc}{y\, V_X}\sqrt{\frac{qA}{uh}} \tag{10}$$

The user's in-vehicle cost C_V is formulated by multiplying the average value of passenger's in-vehicle time v_v, the actual demand q, and average travel time for passengers, which is the half of the round travel time R, as expressed in Equation (11).

$$C_V = v_v q\, \frac{R}{2} \tag{11}$$

Equation (10) is re-written as:

$$C_V = \frac{v_v qJ}{V_x} + \frac{Øv_v}{2y\, V_x}\sqrt{\frac{q^3 hA}{u}} \tag{12}$$

The average waiting time for passengers is assumed as the half of the headway. This assumption may be valid as we develop a planning model with aggregated demand. Thus, the user's waiting cost C_W is formulated with the average waiting time $h/2$, the value of the waiting time v_w, and the actual demand q.

$$C_W = v_w q \frac{h}{2} \tag{13}$$

The total cost for the flexible service is the sum of operating cost, in-vehicle cost and waiting cost as shown in Equation (14) and detailed as in Equation (15):

$$C_T = C_S + C_V + C_W \tag{14}$$

$$C_T = \frac{2Jc}{hV_X} + \frac{\text{Ø}c}{y\,V_X}\sqrt{\frac{qA}{uh}} + \frac{v_v qJ}{V_x} + \frac{\text{Ø}v_v}{2y\,V_x}\sqrt{\frac{q^3 hA}{u}} + v_w q\frac{h}{2} \tag{15}$$

The average cost per passenger C_A can be found by dividing the total cost in Equation (15) by the passenger flow q:

$$C_A = \frac{C_T}{QA} = \frac{C_T}{q} \tag{16}$$

Thus, the average cost for the service is:

$$\begin{aligned} C_A = {} & \frac{2Jc}{hV_X}\frac{1}{QA} + \frac{\text{Ø}c}{y\,V_X QA}\sqrt{\frac{QA \cdot A}{uh}} + \frac{1}{QA}\frac{v_v QA \cdot J}{V_x} \\ & + \frac{\text{Ø}v_v}{2yV_X}\frac{1}{QA}\sqrt{\frac{(QA)^3 hA}{u}} + \frac{v_w QAh}{2QA} \end{aligned} \tag{17}$$

Equation (17) is rewritten as:

$$C_A = \frac{2Jc}{V_X QAh} + \frac{\text{Ø}c}{y\,V_X}\sqrt{\frac{1}{Qhu}} + \frac{v_v J}{V_X} + \frac{\text{Ø}v_v A}{2yV_X}\sqrt{\frac{Qh}{u}} + \frac{v_w h}{2} \tag{18}$$

Equation (18) is can be optimized for the service area A and the headway h for one time period (or one time demand). As we assume the maximum allowable headway policy $h_{\max} = \frac{Sl}{QA}$ is used, Equation (18) can be reduced for one decision variable (i.e. service area A) when we extend the model for multiple time periods. The average cost for the flexible-route bus service with the maximum allowable headway policy is formulated as Equation (19).

$$C_A = \frac{2Jc}{V_X Sl} + \frac{\text{Ø}c}{y\,V_X}\sqrt{\frac{A}{uSl}} + \frac{v_v J}{V_X} + \frac{\text{Ø}v_v}{2yV_X}\sqrt{\frac{ASl}{u}} + \frac{v_w Sl}{2QA} \tag{19}$$

The total cost is product of the average cost per passenger C_A and the demand q, as expressed in Equation (20) and is rearranged in Equation (21).

$$C_T = C_A \times q = C_A \times Q \times A \tag{20}$$

$$C_T = \frac{2JcQA}{V_X Sl} + \frac{\text{Ø}cQ}{y\,V_X}\sqrt{\frac{A^3}{uSl}} + \frac{v_v JQA}{V_X} + \frac{\text{Ø}v_{vQ}}{2yV_X}\sqrt{\frac{A^3 Sl}{u}} + \frac{v_w Sl}{2} \tag{21}$$

The total cost for one time period in Equation (21) is then used to formulate the total system-wide cost *TSC* that covers multiple time periods, as denoted *i* in Equation (22).

$$TSC = \sum_T^C = \sum \left\{ \frac{2JcQ_iA}{V_X Sl} + \frac{\text{Ø}cQ_i}{y\ V_X}\sqrt{\frac{A^3}{uSl}} + \frac{v_v JQ_iA}{V_X} + \frac{\text{Ø}v_{vQ_i}}{2yV_X}\sqrt{\frac{A^3 Sl}{u}} + \frac{v_w Sl}{2} \right\} \tag{22}$$

The average system cost *ASC* is obtained by the total system cost in Equation (22) by the total number of passengers, and is expressed in Equation (23)

$$ASC = \frac{\sum_T^C}{\sum^q} = \frac{\sum_T^C}{A\sum_i^Q} \tag{23}$$

The average system cost *ASC* is rearranged using Equations (22) and (23).

$$ASC = \frac{1}{A\sum_i^Q} \sum \left\{ \frac{2JcQ_iA}{V_X Sl} + \frac{\text{Ø}cQ_i}{y\ V_X}\sqrt{\frac{A^3}{uSl}} + \frac{v_v JQ_iA}{V_X} + \frac{\text{Ø}v_{vQ_i}}{2yV_X}\sqrt{\frac{A^3 Sl}{u}} + \frac{v_w Sl}{2} \right\} \tag{24}$$

Equation (24) is simplified as Equation (25), and the average system cost *ASC* has one unknown variable, which is the service area *A*. One the system-wide decision value of *A* is determined, the headway for each time period can be found by the maximum allowable headway policy $h_{\max} = \frac{Sl}{QA}$.

$$ASC = \frac{2Jc}{V_X Sl} + \frac{\text{Ø}c}{y\ V_X}\sqrt{\frac{A}{uSl}} + \frac{v_v J}{V_X} + \frac{\text{Ø}v_v}{2yV_X}\sqrt{\frac{ASl}{u}} + \frac{v_w Sl}{2A(\sum^Q)} \tag{25}$$

Results

Analytical solution

In this paper, we seek to find the analytic solution as it can provide the closed form solutions and the insight for relations among design parameters. As we develop a macro level planning model, it also makes sense to have an analytic solution for aggregated ridership demand.

To find the optimal value of service area *A*, the first derivative of the average system cost in Equation (25) with respect to service area *A* is set to equal to zero. The parameter *n* is denoted as the total number of demand variations (or number of time periods).

$$\frac{\partial ASC}{\partial A} = \frac{\text{Ø}c}{2y\ V_X\sqrt{uSlA}} + \frac{\text{Ø}v_v}{4yV_X}\sqrt{\frac{Sl}{uA}} + \frac{v_w S\ln}{2A^2(\sum^Q)} \tag{26}$$

By solving Equation (26) we find the optimal zone size A^* for the flexible-route bus operations, as expressed in Equation (27).

$$A^* = \left\{ \frac{2nv_w y\ V_X Sl\sqrt{u}}{\emptyset(\sum^Q)(2c + v_v Sl)} \right\}^{2/3} \tag{27}$$

It is essential to check whether the optimal solution of service area A^* yields the globally minimum for the objective function in Equation (25). The second-order derivative for the service area A is shown in Equation(28) and it should be positive for all values of A to guarantee the global minimum.

$$\frac{\partial^2 ASC}{\partial A^2} = -\frac{\emptyset c}{4y\ V_X\sqrt{uSlA^3}} - \frac{\emptyset v_v}{8yV_X}\sqrt{\frac{Sl}{uA^3}} + \frac{v_w S\ln}{A^3(\sum^Q)} > 0 \tag{28}$$

By rearranging Equation (28), we obtain the necessary condition for the service area A and is the maximum possible value for A. Thus, we denote it as $A_{\max}$:

$$A^* < A_{\max} = \left\{ \frac{8nv_w yV_X Sl\sqrt{u}}{\emptyset(\sum^Q)(2c\ +\ v_v Sl)} \right\}^{2/3} \tag{29}$$

To ensure the optimal service area A^* is always less than its maximum value as shown in Equation (29), we divide Equation (29) by Equation (27) and the relation is expressed in Equation (30):

$$\frac{A_{\max}}{A^*} = \frac{\left\{ \frac{8nv_w yV_X Sl\sqrt{u}}{\emptyset(\sum^Q)(2c\ +\ v_v Sl)} \right\}^{2/3}}{\left\{ \frac{2nv_w y\ V_X Sl\sqrt{u}}{\emptyset(\sum^Q)(2c\ +\ v_v Sl)} \right\}^{2/3}} \tag{30}$$

We find that Equation (30) is simplified as $\sqrt[3]{16} = 2.52$. Therefore, it is confirmed that the analytical solution obtained in Equation (27) yields the globally minimum solution.

Baseline numerical analysis

Input values

As a baseline analysis, we consider multiple time periods for the analysis. It is assumed that the demand for one day of the service is categorized into 10 demand densities, and is sorted with the decreasing order, as shown in Table 2. Other input parameters and values applied are presented in Table 1.

Table 2. Demand inputs for multiple time periods.

Time period	Demand density (passengers/sq. mile)	Time period	Demand density (passengers/sq. mile)
1	20	6	10
2	18	7	8
3	16	8	6
4	14	9	4
5	12	10	2

Table 3. Baseline case results.

Time period	Demand density (passengers/sq. mile)	Service area (sq. mile)	Headway (h)	Average cost ($/passenger)	Average system cost ($/passenger)
1	20	9.83	0.23	12.60	14.01
2	18		0.25	12.79	
3	16		0.29	13.03	
4	14		0.33	13.34	
5	12		0.38	13.75	
6	10		0.46	14.32	
7	8		0.57	15.18	
8	6		0.76	16.61	
9	4		1.14	19.47	
10	2		2.29	28.05	

Baseline results

For a time period with a demand density of two passengers per square mile, the average cost is the highest among the average costs shown in Table 3. As the demand density increases, which means increasing ridership, the average system cost per passenger drastically decreases. When the demand density increases from two to four passengers per sq. mile, the average cost per passenger decreases by 30.1% from $28.05 to $19.47. As the demand density increases further, the cost per passenger continues to decrease yet the rate slows down. Demand density increase from four to eight and eight to 16 passengers/sq. mile yield 22.0% and 14.2% decreases in the average cost per passenger, respectively. The data are shown graphically in Figure 2 and are summarized in Table 3.

The headways for each time period are found based on the maximum allowable headway policy. As the demand density increase, the required fleet size N increases as well, and as the result, the vehicles operate more frequently, which means short headways. The exponential decay pattern for headways is displayed in Figure 3.

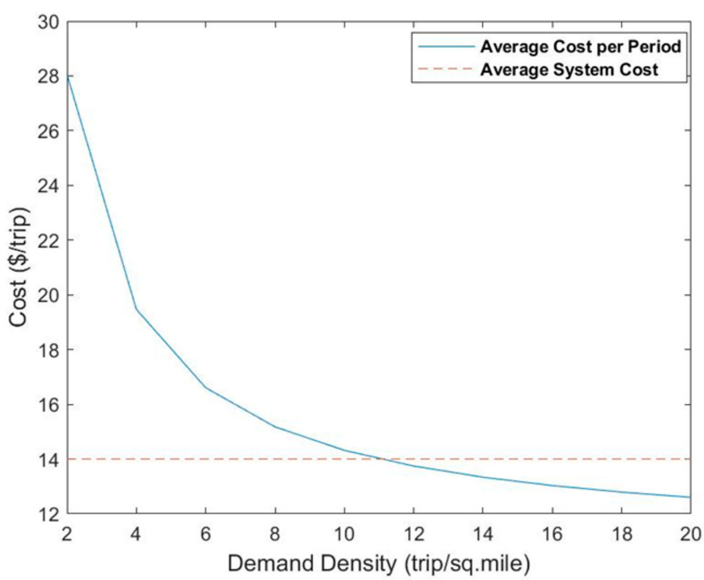

Figure 2. System-wide average cost and average cost of trip for demand density variations.

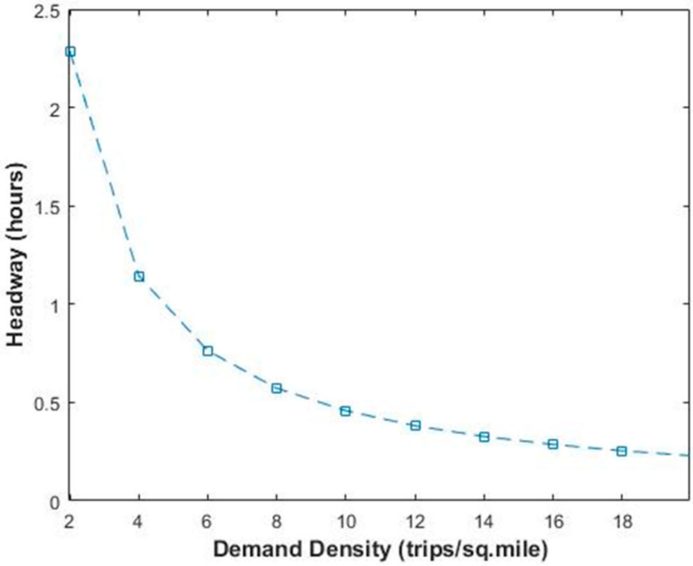

Figure 3. Headway variations over demand densities.

With the closed form solution is shown in Equation (27), the optimal service area A^* is found as 9.83 sq. miles, and headway decisions are found for each time period, respectively, as shown in Table 3. The headway shows variations between 0.23 and 2.29 h. As we evaluate the headway decisions for various demand density variations, it is noted that when the demand density is very low such as two passengers per sq. mile, the headway is determined as 2.29 h. It is possible that the optimized headway is longer than the round travel time R. If the transit agency plans to provide at least a vehicle operating within the time of the round travel, it is necessary to adjust the headway to be less than or equal to the round travel time R. However, it is noted the numerical example presented in this paper shows how demand variations including very low demand density affect the headway policy decisions. In practical operations, the transit agencies typically set the maximum headway policy, which as an example can be less than an hour (i.e. means that the bus service is provided at least every hour). For low demand densities namely two to four passengers per sq. mile, the resulting headway is over an hour so it is assumed that the demand is uniformly generated within the zone while the vehicle is being held at the terminal due to the low demand.

Sensitivity analysis

In the sensitivity analysis, we explore how the variation of critical variables affects the system cost of the flexible bus services. The input values for demand densities and other parameters are kept the same as the baseline case study.

Vehicle size

We change the input values of the vehicle size S, ranging from 5 seats per bus to 45 seats per vehicle. As the vehicle size increases, the area of coverage and cost per

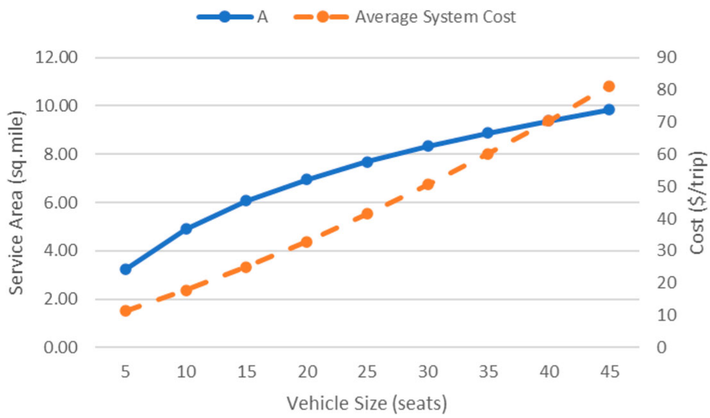

Figure 4. Sensitivity of service area and cost with respect to vehicle size.

passenger also increase. This is due to the capacity of the vehicle to carry more passengers on a given trip. It is noted that as vehicle size increases, the optimal value of service area A and the total system cost increase. The sensitivity of system costs with respect to the size of the vehicle is shown visually in Figure 4 and are summarized in Table 4.

Number of people per stop

Increasing the number of people per stop also increases the efficiency of a flexible-route bus operations. In this sensitivity analysis, we vary the value of u (number of people per stop), ranging from 1.0 to 2.0. When the value of u increases, the number of stops n decreases and it results in decreasing tour distance. This causes the average cost per passenger to decrease significantly. As a result, this increase in efficiency causes a slight increase in service area for one vehicle. This increase is shown in the analytical solution. The analytical solution shows that the service area is proportional to $\sqrt[3]{u}$. The data are shown visually in Figure 5 and is summarized in Table 5.

As the service area A increases with the number of people per stop, this inevitably increase the total system cost. This is because, assuming uniform demand density, an increase in service zone results in more ridership generated and, therefore, it increases the total system cost. Although the total cost is rising in this scenario, it is important to remember that an increase in number of riders per stop causes a decrease in cost

Table 4. Sensitivity of system cost over vehicle size.

Vehicle size (seats/bus)	Service area (sq. mile)	Average system cost ($/passenger)	Total system cost ($/day)
5	3.24	$11.36	$4,043.57
10	4.91	$17.85	$9,643.52
15	6.07	$25.04	$16,710.57
20	6.96	$32.91	$25,190.90
25	7.69	$41.41	$35,047.89
30	8.33	$50.51	$46,254.72
35	8.88	$60.18	$58,790.73
40	9.38	$70.41	$72,639.46
45	9.83	$81.17	$87,787.38

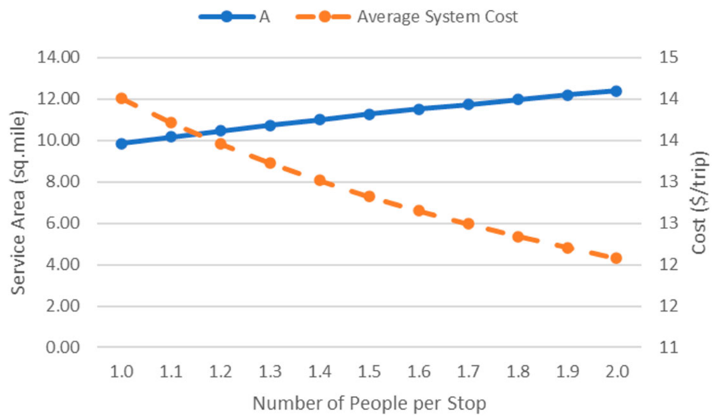

Figure 5. Sensitivity of service area and cost with respect to number of people per stop.

Table 5. Sensitivity of system cost over the number of people per stop.

Number of people per stop	Service area (sq. mile)	Average system cost ($/passenger)	Total system cost ($/day)
1.00	9.83	$14.01	$15,147.93
1.10	10.15	$13.71	$15,310.07
1.20	10.45	$13.45	$15,462.66
1.30	10.73	$13.22	$15,606.99
1.40	11.00	$13.01	$15,744.09
1.50	11.25	$12.82	$15,874.82
1.60	11.50	$12.65	$15,999.85
1.70	11.73	$12.49	$16,119.78
1.80	11.96	$12.34	$16,235.09
1.90	12.18	$12.20	$16,346.21
2.00	12.39	$12.08	$16,453.49

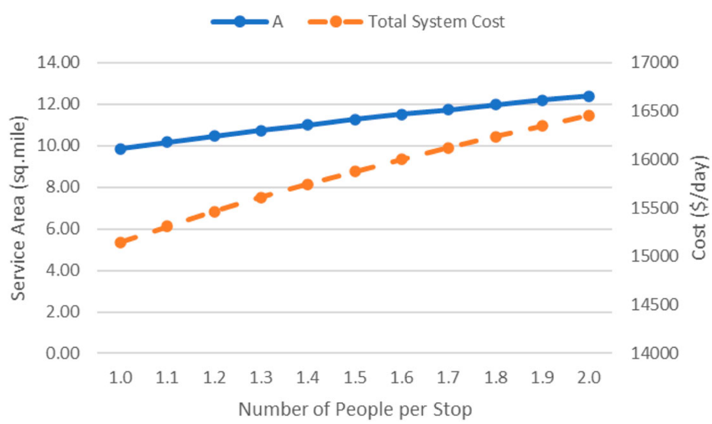

Figure 6. Sensitivity of service area and total cost with respect to number of people per stop.

per passenger. Doubling the number of passengers per stop from one to two yields an 11.6% decrease in average cost per passenger. The data are presented in Figure 6 and Table 5.

Conclusions

This paper has formulated a solution for flexible-route bus operations with demand variations. The previous study by Kim, Levy, and Schonfeld (2019) has been extended to consider multiple time periods. The optimal solution for the service area was obtained analytically with the assumption of the maximum allowable headway policy. With the optimal service area, which is constant across the multiple time periods, the optimal headway was obtained for each time period. The results of the numerical analyses showed that a lower demand requires a system with a high average cost per passenger. We noted the drastic fluctuations in average cost per passenger from low-demand to high-demand time periods. As expected, for the high demand time period, the optimal service area decreased and the headway also decreased. Sensitivity analyses found that increasing the vehicle size or increasing number of people per stop increased the optimal service zone area. As the number of people per stop increased, we noted the total system cost was increasing mainly due to the increased service area generating more ridership. However, this increase resulted in the decreased number of stops so that the average cost per passenger decreased.

The analytic closed-form solution provides the insights between the design parameters and decision variables. For instance, the value of waiting time positively affects the optimal service area. The increase of the vehicle operating speed also increases the service coverage of the vehicle as it decreases the travel time for both passengers and vehicle operators. We also found that the sum of demand density negatively affected the optimal service area. Regarding the global optimality of the solution, we confirmed that the optimal value of service areas (shown in Equation (27)) is always less than its maximum value (presented in Equation (29)). The analytical solution and cost formulations for flexible-route bus services may be used as a useful guideline to examine an alternative transit operation where the demand is not sufficiently high to justify frequent services using fixed-route bus operations.

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ORCID

Myungseob (Edward) Kim http://orcid.org/0000-0002-8433-4950

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