

Applying a Capacitated Heterogeneous Fleet Vehicle Routing Problem with Multiple Depots Model to Optimize a Retail Chain Distribution Network

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Abstract - Planning and operating retail chain distribution processes is becoming more challenging, due to the increasing demand and urban congestion. This research applied a Capacitated Heterogeneous Fleet Vehicle Routing Problem with Multiple Depots (CHFVRPMD) model to optimize a retail chain distribution network with a real-world road network. The model attempts to obtain a distribution plan that minimizes the total distribution cost. A hybrid solution algorithm was proposed to solve the CHFVRPMD model. The solution algorithm was developed by combining the heuristic and metaheuristic methods. A case study was carried out to collect actual data to test the proposed optimization model and the solution algorithm. All the input parameters were estimated using a real-world industry application, to minimize the gap between the theoretical model and the real-world applicability. Computation experiments demonstrate the proposed hybrid solution algorithm's effectiveness in optimizing the CHFVRPMD model in a feasible computational time. The overall findings reveal that the CHFVRPMD model has achieved about 10.7% savings in daily distribution cost, in optimizing the selected retail distribution network. The proposed model and solution algorithm assist industry practitioners to mitigate transport inefficiencies in retail distribution networks.

Keywords - Heterogeneous fleet, heuristic, metaheuristic, multiple depots, retail chains, vehicle routing

I. INTRODUCTION

The demand for commodities has been increasing exponentially due to the growth of the global population [1], [2]. This growth has caused a significant increase in urban freight flows in retail chains [1]. The number of delivery vehicles is forecast to increase by 36% over the next decade [3]. These trends cause numerous challenges in managing urban traffic, thereby creating significant problems for the maintenance of a smooth flow of urban distribution systems [3]. Although most urban logistics studies apply mathematical optimizations, most fail to analyze the nature of different logistics systems [3]. Hence, it is necessary to focus on the individual operating practices and characteristics of different distribution systems [4].

Vehicle Routing Problem (VRP) based optimization packages have achieved significant cost savings (5% to 20%) in global transportation [5]. Hence, this research intends to apply an extension of VRP to optimize a distribution network in retail chains with multiple

distribution centers. A Capacitated Heterogeneous Fleet Vehicle Routing Problem with Multiple Depots (CHFVRPMD) model was applied to optimize the retail chain distribution network with the application's real-world attributes, including real-world driving distances [3].

This research employed a hybrid solution method, so that Initial Basic Feasible Solutions (IBFS) were obtained using a heuristic method, and, subsequently, solutions were improved using a metaheuristic method. The majority of VRP research is aimed at pure investigation and not applications. This research is significant, since both the CHFVRPMD model and the suggested hybrid solution method are tested using a real-world industry application.

The remainder of the paper is organized as follows. Section II conducts a review of the existing literature. Section III provides the details about the applied optimization model, while details about the suggested solution method are presented in Section IV. Section V presents the results of the numerical experiments. Finally, in Section VI, the research findings and future research directions are highlighted.

II. LITERATURE REVIEW

VRP is one of the most extensively studied areas in the domain of Operations Research [6]. Dantzig and Ramser [7] formulated a model to solve a "truck dispatching problem" as their first attempt at VRP. Later VRP problems were generalized as a linear optimization problem [8]. Mathematically, VRP is presented as a directed graph $G(V, A)$, where $V = \{0, 1, 2, n\}$ set of nodes, and $A = \{(i, j): i, j \in N\}$ represents a set of arcs connecting any two nodes [9]. Nodes represent distribution centers and geographically dispersed outlets [9]. Previous researchers applied various extensions of VRPs to optimize retail chains in different contexts [10].

Many practically motivated VRPs match with a directed graph with asymmetric costs [11]. Specifically, routings within city limits are asymmetric, due to the one-way roads and traffic congestion [11]. Most real-world vehicle fleets are heterogeneous, and real-world distances are frequently asymmetric, especially in urban transportation [12]. Existing research rarely considered characteristics such as heterogeneous fleets, multiple depots, and real-world distances, simultaneously [11]. The research attempts to fill this gap by incorporating the above-mentioned real-world characteristics simultaneously.

Most existing research employed more problem-tailored solution approaches to solve the different extensions of VRP, which might not be applied to other problem types [9]. More research is required to compare the performance of different metaheuristic methods [13]. Therefore, it is vital to compare the performance of different metaheuristic methods in solving different extensions of VRP; thereby, more effective solution methods can be determined.

III. MODEL FORMULATION

The CHFVRPMD model's mathematical formulation is shown below. The CHFVRPMD model used in this study was inspired by the optimization model presented in [14]. The following mathematical formulation presents the CHFVRPMD model's assumptions, notations, objective function, and constraints.

A. Assumptions

The proposed optimization model was developed, using the assumptions listed below. Assumptions are based on the operating practices of the selected retail chain distribution network.

- Each truck starts the deliveries from a central distribution center and returns to the origin point after providing services to allocated retail outlets
- Demand at each retail outlet is known before dispatching trucks
- Split delivery is not allowed, and each outlet is served by exactly one truck
- Trucks have deterministic heterogeneous capacities

B. Notations

K	Set of trucks
R	Set of retail outlets
D	Set of distribution centers
N	N=RUD; Set of nodes
A	Set of arcs
d_{ij}	Real-world driving distance of arc $(i, j) \in A$
t_{ij}^k	Driving time of k^{th} truck for an arc $(i, j) \in A$
q_j	Total quantity delivered to retail outlet j (No. of crates)
Q_k	The capacity of truck k
U_j^k	Cumulative quantity delivered by k^{th} truck at retail outlet j
U_i^k	Cumulative quantity delivered by k^{th} truck at retail outlet i
ST_i	Average service time at retail outlet i
t_j^k	Time k^{th} truck visits retail outlet j
t_i^k	Time k^{th} truck visits retail outlet i
F_k	Fixed cost of dispatching the trucks
c_k	Fuel cost of k^{th} truck per km
φ	Large number

$$x_{ij}^k = \begin{cases} 1, & \text{If truck } k \text{ is used for arc } (i, j) \\ 0, & \text{Otherwise} \end{cases}$$

C. Objective Function

$$\text{Minimize } \sum_{k \in K} F_k + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_k x_{ij}^k d_{ij} \quad (1)$$

The objective function attempts to minimize the total distribution cost incurred during retail chain distribution. The objective function's first term represents the fixed cost of dispatching the trucks, including the labor and maintenance costs, so that the model attempts to minimize the number of trucks associated with the distribution process and therefore to maximize the truck capacity utilization. The second term of objective (1) is related to the fuel cost. It attempts to minimize the fuel consumption associated with retail chain distribution. Therefore, the proposed model finds a more economical route plan.

D. Constraints

$$\sum_{k \in K} \sum_{i \in N} x_{ij}^k = 1 \text{ for } \forall j \in R \quad (2)$$

Constraint (2) ensures that each retail outlet should be served by exactly one truck. This is to avoid split deliveries. In split deliveries, the total quantity can be delivered by several trucks.

$$\sum_{k \in K} \sum_{i \in D} x_{ij}^k = \sum_{k \in K} \sum_{i \in D} x_{ji}^k \text{ for } \forall j \in R \quad (3)$$

Constraint (3) ensures that each truck should start from a distribution center. Further, all trucks should return to their distribution center after providing services to designated retail outlets.

$$U_i^k + q_j = U_j^k \text{ for } \forall x_{ij}^k = 1 \text{ and } i, j \in R \text{ and } k \in K \quad (4)$$

Equation (4) ensures that the total supply quantity up to the j^{th} outlet is the aggregate quantity supplied up to the previous retail outlet and the quantity supplied to the j^{th} outlet

$$U_j^k \leq Q^k \text{ } \forall j \in R \text{ and } k \in K \quad (5)$$

Constraint (5) ensures that the total quantity supplied using the k^{th} truck shall not exceed the capacity of that truck. Since the proposed model incorporates a truck fleet with heterogeneous capacities, this constraint considers each truck capacity separately.

$$t_j^k = t_i^k + t_{ij}^k + ST_i \text{ for } \forall x_{ij}^k = 1 \text{ and } i, j \in R \quad (6)$$

$$t_j^k \geq t_i^k - \varphi(1 - x_{ji}^k) \text{ for } \forall x_{ij}^k = 1 \text{ \& } i, j \in R \quad (7)$$

Constraints (6)-(7) eliminate sub-tours in the routing plan for the retail distribution network.

IV. HYBRID SOLUTION METHOD

Researchers used several categories of solution approaches to solve the different extensions of MDVRP (Multi-Depot VRP) and HFVRP (Heterogenous Fleet VRP) models. These could be highlighted as exact, heuristic, metaheuristic, and hybrid methods [14]. Fig. 1 presents a comparison, carried out in this research, of different solution methods.

Exact Method	Heuristic Method
Useful to find global optimal solutions Not applicable for large-size problems with real-world complexities due to unrealistic search time	Useful to find solutions in feasible computational time Highly problem-tailored and difficult to customize for different types of problems
Metaheuristic Method	Hybrid Method
Helpful to find near optimal solutions in feasible computational time Possible to customize for different types of problems Limitations in improving the computation time	Helpful to find near optimal solutions in feasible computational time Possible to customize for different types of problems Useful to further improve the search algorithms in terms of the quality of the solutions and the computation time

Fig. 1. Comparison of different categories of solution methods.

The comparison highlights the advantage of hybrid methods in solving the various VRP extensions. Furthermore, as shown in [15], hybrid methods are the least used solution category. Therefore, this research attempted to develop a hybrid solution method. In hybrid methods, two solution methods are combined to avoid limitations such as low solution quality, impracticable computation time, and optimal solutions trapped in local minima [16]. To achieve better results, researchers used combined exact-metaheuristic, heuristic-metaheuristic, and metaheuristic-metaheuristic methods [16]. This research combined heuristic and metaheuristic methods.

A. Initial Solution Method

A heuristic approach was applied to obtain Initial Basic Feasible Solutions (IBFS) for the proposed CHFVRPMD model. Using heuristic approaches to obtain IBFS is important [17]. Thereby, the number of iterations can be reduced in the improvement process of IBFS [17]. This paper used the Clarke and Wright (CW) algorithm proposed in scheduling vehicle routing [8]. The CW algorithm has two versions, which can be defined as sequential and parallel [18]. In practice, the parallel version

of CW performed better [18]. Therefore, the parallel version of CW was selected to obtain the IBFS.

B. Iterative Improvement of IBFS

Metaheuristics are widely used by researchers, since these methods are helpful for obtaining better solutions in less computation time [13]. This research used Guided Local Search (GLS) to improve the IBFS obtained using the CW heuristic. The metaheuristic approaches improve the IBFS, using a fundamentally iterative process called “neighborhood search” [9]. Neighborhood search methods examine the candidate solutions by continuously applying changes to the previous solution [19]. Neighborhood structures for improving routing plans can be used within a route or between routes [19]. The research applied one neighborhood structure operating within the route (two-opt procedure) and three neighborhood structures operating between the routes (relocate, exchange, and cross procedures) [19]. The algorithm terminates once it reaches the optimal solution or when the defined stopping condition has been met [9]. This research used the number of accepted neighbors as the stopping criteria.

GLS is a memory-based meta-heuristic that applies a penalty-based augmented cost function, shown below, to proceed with the local search [19].

$$g(x) = f(x) + \lambda \sum_i (I_i(x) * p_i) \quad (8)$$

Here, $g(x)$ indicates the augmented cost function, and $f(x)$ indicates the original objective function of the proposed optimization model. The fundamental logic of this method is to penalize the features of the previously found local minimum. Thereby, the method searches for a better solution than before. In the above formula, $I_i(x)$ indicates a decision variable that can be either zero or one. The value of $I_i(x)$ becomes one when the current solution has similarities to previously found solutions. Further, p_i represents the penalty associated with those similarities. The penalty factor (λ) tunes the search procedure in this method. For example, for a large value of λ , this method conducts a more diverse search, while, for a small value, it conducts a more intensive search. However, the entire penalty term repeatedly updates until we get a near-optimal solution [19]. For the current research, we tuned the parameters and used the value 0.1 for the λ .

V. COMPUTATION EXPERIMENT

A. Data for the Computation Experiment

All the computational experiments were carried out based on the data collected from one of the largest retail chains in Sri Lanka. The selected distribution network is related to delivering fresh vegetables to its retail outlets. Data including location data, demand data, fleet data, and

operation data were collected to test the proposed CHFVRPMD model and the hybrid solution method.

Location data contain 247 retail outlets and 2 distribution centers. Based on those data, the distance matrix was created. Distances were measured using the driving distances, since the research focused on real-world applications. It is highlighted that the VRP model fails to capture the optimal route plan when Euclidean distances (straight-line distances) are used to solve real-world applications [20]. Therefore, the research employed OSRM (Open-Source Routing Machine) API to measure the real driving distances [21].

Demand data were collected as the aggregate demand for the multiple product types for each retail outlet. Further, demand was measured using vegetable crates as the unit of measure. Fleet data contain details about the trucks and their capacities. Finally, operation practices and the distribution network cost were collected, using unstructured interviews.

B. Details of the Experiment

All the computation experiments were carried out using a computer with a Core i5, 5200U processor, running at 2.40 GHz - 2.42 GHz, using 8 GB RAM in Windows 10 Home 64 bit. Algorithms were developed using OR-Tools version 7.2 [22] with Python version 3.9.6 in Visual Studio Code version 1.60.

The computation experiments of this research consist of two parts. In the first one, the performance of the hybrid solution method was tested. After that, the CHFVRPMD model was tested in solving the real-world application. This research applied the real driving distances, to develop the distance matrix. Thus, this real-world application is an extension of Asymmetric Costs VRP (ACVRP) [23]. The literature posits that there is a lack of published best-known solutions and benchmark instances for the ACVRP [23]. Hence, this research compared the suggested hybrid solution method with a metaheuristic method (GLS) and a heuristic (CW heuristic) method. This comparison used daily distribution cost and computation time as performance criteria.

C. Experiment Results

We adhered to the following process: retrieving data, applying the CHFVRPMD model, and hybrid solution method, to obtain the near-optimal distribution plan. In computation experiments, the above process was repeatedly applied, to obtain the experimental results below.

Test results of the first computation experiment are highlighted in Fig. 2 and Fig. 3. Problem instances were defined based on the size of the problem (i.e., number of outlets| number of trucks). According to Fig. 2 and Fig. 3, the proposed hybrid solution method has outperformed the GLS and CW heuristic in terms of distribution cost and the computation time, in solving the employed industry

application. Fig. 5 was obtained using the hybrid solution method for further evaluations.

Fig. 4 indicates how the cost gets reduced in the iterative improvement process. Further, it indicates computation time is increasing non-linearly with the number of iterations. Therefore, one needs to determine an optimal balance between the quality of the solution and the computation time, when applying the model to solve real-world applications.

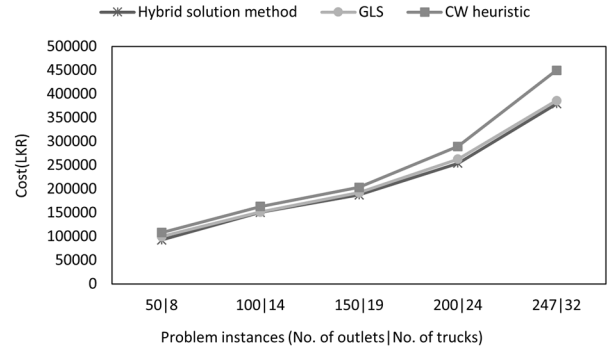


Fig. 2. Distribution cost comparison of solution methods.

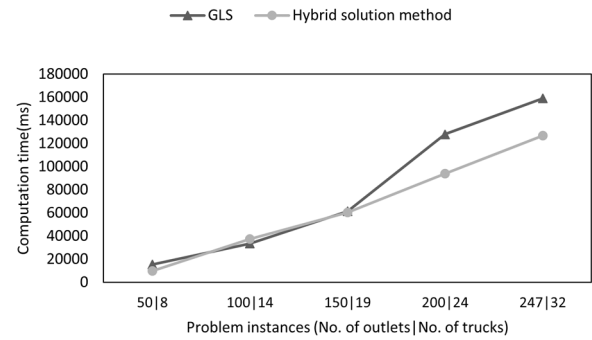


Fig. 3. Computation time comparison of solution methods.

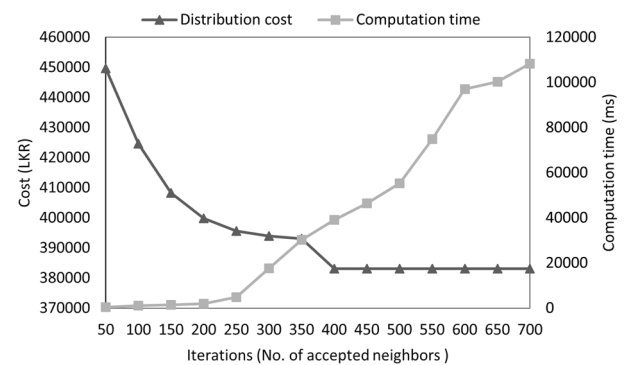


Fig. 4. Iteration results for hybrid solution method.

This computation experiment intends to measure the effectiveness of the CHFVRPMD model in optimizing multiple depot distribution networks. Therefore, instance-1 and instance-2 were compared, to investigate the effectiveness of the CHFVRPMD model.

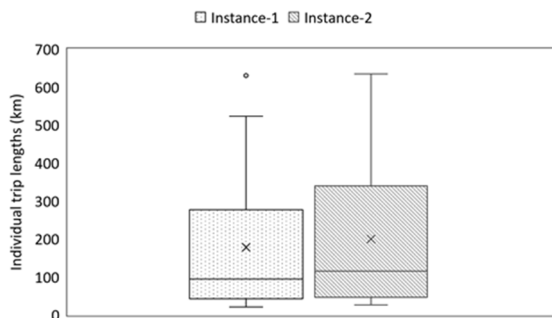


Fig. 5. Effectiveness of the proposed CHFVRPMD model.

- Instance-1: The CHFVRPMD model was tested, to optimize the selected retail chain distribution network with multiple depots.
- Instance-2: The CHFVRP (Capacitated Heterogeneous Fleet Vehicle Routing Problem) model was applied, to optimize the distribution network.

In Fig. 5, the two instances were compared, using the individual trip lengths. It shows that instance-1 has shorter trip lengths, compared to instance-2. This is an indication of the effectiveness of the proposed CHFVRPMD model. Further, evidence suggests that instance-1 has achieved 10.7% of cost-saving over instance-2, when considering daily distribution cost. Additionally, computation time was compared for the two instances. Results highlight that the CHFVRPMD model is capable of optimizing this entire distribution network within 528311 milliseconds (8.805minutes). For instance-2, the CHFVRP model consumed 423312 milliseconds (7.0552 minutes). The experiment showed that the CHFVRPMD model is feasible for real-world application, due to its significant savings of daily operation cost (10.7%) at one minute of additional computation time.

VI. CONCLUSION

The proposed CHFVRPMD model efficiently assigns distribution centers to retail outlets, thereby significantly reducing daily operating costs. Further, the model assists in optimizing truck fleets with heterogenous capacities. The proposed hybrid solution method helps to solve the optimization model in reasonable computation time that allows it to be employed for large-scale real-world applications. Future research should explore an integrated VRP model with order scheduling to optimize the retail chain distribution processes, to advance our understanding. Further, we suggest using different combinations of heuristic and metaheuristic approaches to test hybrid solution methods, to conduct a comprehensive comparison to determine the best combinations.

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