Evaluation of on-demand line-based bus services

Arne Schulz*[1,2] and Tobias Vlćek[3]

[1]Helmut Schmidt University, Institute of Quantitative Logistics, Holstenhofweg 85, 22043 Hamburg, Germany, *arne.schulz@hsu-hh.de*
[2]Universität Hamburg, Institute of Operations Management, Moorweidenstraße 18, 20148 Hamburg, Germany, *arne.schulz@uni-hamburg.de*
[3]Universität Hamburg, Institute of Logistics, Transport and Production, Moorweidenstraße 18, 20148 Hamburg, Germany, *tobias.vlcek@uni-hamburg.de*

July 8, 2024

Abstract

***Keywords*: mobility service, on-demand, linear programming, network flow formulation**

1 Introduction

Modern societies need to solve the trade-off between individual mobility and the necessary reduction of CO_2 emissions. As a consequence several alternative technologies like electric-driven vehicles or cars with hydrogen propulsion as well as mobility services like sharing economy (car sharing, bike sharing, e-scooter) or ridepooling services have been evaluated. Currently it is an open question which of them should be used in which context and how they are ideally combined with traditional line-based public transport.

In this paper, we evaluate the potential of a different solution which can be classified between a traditional line-based bus service and a door-to-door ridepooling service. The first has the advantage of significant pooling, as all customers need to adapt themselves to the time schedule and the bus route. A bus is used best if the bus schedule addresses the mobility demand well for as many people as possible. However, this is not the case in off-peak hours or rural areas. In these cases, an on-demand ridepooling service has advantages, as the service is offered only in case of demand. Thus, no empty or oversized

*Corresponding author

buses drive. Moreover, a ridepooling service has the advantage of door-to-door transportation which increases the comfort and the access to the mobility service especially for people with reduced mobility.

Our setting is between a traditional line-based bus service and a ridepooling service. We consider a line-based bus service, but the bus does not drive the entire line as scheduled but only on-demand. This means that customers need to announce their interest in the service upfront via smartphone application or telephone call. In practice, this is done with a lead time of two hours. Then, the bus drives only if it is required. Thus, we know the demand upfront such that resources can be used well and we avoid empty or oversized buses driving. This setting is currently used in rural areas in the north of Germany. In rural areas, villages are rather small and concentrated around a centre. Hence, transportation requests mainly occur between villages or between a village and the next city, which means that even a line-based bus is close to a door-to-door transportation.

Due to the line-based scheduled service, we know all possible requests upfront. We will see in the paper that we can model this setting by an acyclic graph which allows us to model the problem as a network flow problem which can be solved efficiently by linear programming. Thus, the contribution of the paper is as follows:

- We present a network flow formulation for an on-demand line-based bus service.
- We extend the service to include heterogeneous buses and driver breaks.
- We use the models to evaluate the potential of on-demand line-based bus services as a mobility service especially in rural areas.

By the last point we also address an issue named in the survey paper by Vansteenwegen et al. (2022) to evaluate the potential of bus services and how and where they are used best.

The paper is constructed as follows: First, we classify our setting into the literature (Section 2). Then, we give a formal problem description (Section 3) and present our model formulations (Section 4). Afterwards, the models are used to evaluate the potential of an on-demand line-based bus service in a comprehensive computational study (Section 5). Finally, the paper closes with a conclusion in Section 6.

2 Literature review

Our setting is, as already mentioned, a mixture between a traditional line-based bus service and a ridepooling service. We only need to serve bus tours with a positive demand. Therefore, the literature review is separated into the two research areas public bus services and ridepooling. Moreover, we shortly refer to literature regarding the shift scheduling of drivers.

2.1 Public bus services

Public bus services are a backbone of the public transport system. Especially in rural areas, a main focus of public bus services is on the transportation of students to and from schools which leads to two demand peaks direct before and after school times while there is only a moderate demand in between. In line with the practical demand, a significant amount of papers focuses on school bus routing. However, during these peak times the demand can be predicted very well, as it is known how many students need to be transported between a stop and a school (students often have subsidized tickets for exactly their demand). Since our setting focuses on the low demand off-peak and weekend hours, we shortly refer to the survey on the school bus routing problem by Park and Kim (2010) for further details on school bus routing. Nevertheless, note that also these papers often use mixed-integer programming formulations for their solution approaches (Bektaş and Elmastaş (2007), Fügenschuh (2009), Bögl et al. (2015)). Beside line-based public transport there are also other applications for bus services like event tours, e.g. in the tourism sector (Brandinu and Trautmann, 2014).

Our approach to serve bus tours only (partly) if there is a positive known demand can be classified as a flexible bus service. The survey paper by Errico et al. (2013) classifies a bus service as semi-flexible if there is some buffer time in the bus tour which can be used to deviate from the fixed bus route and serve nearby on-demand requests. As one setting the authors describe marked stops on the bus tour where users can ask for service by waving their hand (so called flag requests). In fact, our setting is very similar. We also have marked stops, as all bus stops are on-demand marked stops. However, the customers cannot simply wave their hand when the bus passes the stop because the bus only serves the stop if there is a pre-known demand. In this point, we deviate by the fact that users need to announce their request upfront by a smartphone app or via telephone. As we know upfront whether there is a demand at a stop, we only need to visit stops with demand and, hence, can optimize the travelled bus routes visiting only these stops.

Another approach allowing vehicles to deviate from the fixed line-based path to serve customers within a surrounding service area is Mobility Allowance Shuttle Transit (MAST) investigated amongst others by Quadrifoglio et al. (2007). A different variation of flexibility is presented by Qiu et al. (2014). They integrated the pickup and delivery points of accepted curb-to-curb customers as temporary stops in their system which can then be used by other customers.

The system investigated in Pei et al. (2019) is semi-flexible in the sense that the buses, serving the same line in both directions with a U-turn at the end of the line, can shorten the line at the end if there is no demand. This means that the buses can perform their U-turn before the last stop and thereby shorten their tours at the end as well as the beginning of the next tour serving the line backwards. In the paper at hand, we somehow extend this setting to several lines such that buses can interrupt the service of a line if the bus is empty and switch to another line. This can be at the end of a line, i.e. the rest of the line is skipped or somewhere in between if another bus serves the rest of the previous line with positive demand. A recent review paper on on-demand bus services is given by Vansteenwegen et al. (2022).

Kim and Schonfeld (2014) coordinate conventional bus services with flexible doorstep services allowing transits. The authors use probabilistic optimization models to reduce the customers transfer times. The problem of integrating ridepooling requests into the public transport network, i.e. serving customers with a ridepooling vehicle and public transport while allowing transfers between both, is also known as the integrated dial-a-ride problem (Häll et al., 2009).

In practical applications, bus schedules underlie further restrictions like multiple depots such that each bus needs to return to its own depot in the evening and different bus types (e.g. with different capacities). Gintner et al. (2005) present a two-phase method to solve such multiple depots and vehicle types bus scheduling problems close to optimality even for instances with thousands of scheduled bus tours. In the paper on hand, we restrict ourselves to a single depot. However, our second and third setting also allow to include multiple depots by defining a vehicle type k once for every depot for the price of a significantly increased network size. By this, we could also ensure that the vehicle returns to its own depot. Moreover, depots' capacities can be included (Kliewer et al., 2006). Besides, we do not consider any recharging or refueling operations. For a survey on electric bus planning and scheduling see Perumal et al. (2022).

From the methodological point of view, as already mentioned, many papers use mathematical programming formulations to solve bus scheduling problems as we do in this paper. However, there are also papers combining mathematical programming with further methods like heuristics (Gintner et al., 2005) or constraint programming (De Silva, 2001). A survey paper on modelling approaches for vehicle scheduling models is given by Bunte and Kliewer (2009).

2.2 Ridepooling

A ridepooling service is a door-to-door on-demand service where customers can request a ride between a pickup and a delivery location usually via a smartphone app. Thereby, the pickup and the delivery location are almost fully flexible and the request time is either immediately or within a given future time frame (Schulz and Pfeiffer, 2024). We defined semi-flexible bus lines as bus lines where the bus can leave the line in between to serve some on-demand customers nearby. A ridepooling service is in this sense a fully flexible bus service where no predefined time table or bus line is given (Vansteenwegen et al., 2022).

It seems to be apparent that a line-based bus service is used best if there is a high transportation demand from similar pickup to similar delivery locations while a ridepooling service is more attractive if there are very heterogeneous transportation requests of a reasonable density within a service area. Then, the ridepooling provider can pool them such that still all customers are served between their pickup and delivery locations but not necessarily directly. Prior studies show that significant pooling rates are possible while still maintain acceptable detours (Pfeiffer and Schulz, 2022). In comparison to a ridepooling service, our line-based service leads already to some kind of clustered requests. Customers cannot ask for a request from any location to any other but only between the line-based stops, i.e. the customers themselves cluster their requests to the bus line. Therefore, it is likely to reach a reasonable pooling rate. Of course, this is also the case in any conventional line-based bus service. However, as only those tours with positive demand need to be served, more bus lines can be offered without increasing the costs. How many more lines a bus service can offer with the on-demand line-based setting is exactly the question we want to answer in this paper.

The underlying tour scheduling problem of a ridepooling provider is the Dial-a-Ride Problem (DARP) which has been investigated for decades (Psaraftis, 1980). The DARP was originally introduced to serve people with reduced mobility where it is also used in practice. Borndörfer et al. (1999) developed a vehicle scheduling approach called Berlin's Telebus. Telebus is a bus service for handicapped people who are not able to use the public transport system. For the DARP different solution approaches were developed: the three-index formulation (Cordeau, 2006), the two-index formulation (Ropke et al., 2007), the restricted fragments based formulation (Rist and Forbes, 2021) or the event-based formulation (Gaul et al., 2022, 2023).

Ridepooling services have also been investigated in cities (Pfeiffer and Schulz, 2022) as well as in rural areas with interrelated trips (Johnsen and Meisel, 2022).

2.3 Shift scheduling

As our paper does not mainly focus on shift scheduling but only addresses the effect of driver breaks on the planning solution, we only shortly refer to the survey on tour scheduling by Alfares (2004) as well as to the shift scheduling paper for ridepooling services by Berthold et al. (2024). However, note that we use the simplifying assumptions that breaks are already scheduled. In fact, there is some degree of freedom in the break scheduling due to labour regulations (Boyer et al., 2018). Moreover, we assume that there is a predefined assignment of one driver to a bus whenever the bus is outside the depot. Thus, driver changes are only allowed when the bus visits the depot between tours. Perumal et al. (2019) allow driver changes also outside the depot.

3 Problem description

We investigate a line-based bus service. The bus service needs to serve several bus lines at different times according to a given time schedule. We consider every combination of a bus line and a starting time as one line $l = 1, ..., n$. The line consists of m_l stops $s_1^l, ..., s_{m_l}^l$. Every stop is associated with a time $\bar{t}_{s_j^l}$, $l = 1, ..., n$, $j = 1, ..., m_l$, at which the bus stops at the stop. We assume that the time for boarding and deboarding is in comparison to the driving time negligible such that the bus stops and departs at the same time at each bus stop. This means that $\bar{t}_{s_{m_l}^l}$ indicates the end of bus tour l. Furthermore, travel times between any pair of stops are given by $t_{s_j^l, s_{j'}^{l'}} > 0$, $l, l' = 1, ..., n$, $j = 1, ..., m_l$, $j' = 1, ..., m_{l'}$. Travel times between the single depot D and the beginning of tour l as well as between the end of tour l and the depot are given by $t_{D, s_1^l} > 0$ and $t_{s_{m_l}^l, D} > 0$, $l = 1, ..., n$, respectively. We assume the triangle inequality to hold for all travel times. The bus company uses K buses $k = 1, ..., K$ with capacity Q_k to serve the bus lines.

In our first setting, we assume all buses to be homogeneous, i.e. $Q_k = Q_{k'}$ for all pairs $k, k' = 1, ..., K$. Moreover, we assume them to be autonomous in this setting. This means that drivers do not need to be considered.

In the second setting, buses are heterogeneous, i.e. $Q_k \neq Q_{k'}$ might hold for some $k, k' = 1, ..., K$. In this setting, we can also include customer demand $d_{s_j, s_{j'}}$ for every line $l = 1, ..., n$ and stops $j, j' = 1, ..., m_l$ with $j' > j$. Note here that we need at most $K = n$ buses, one for every tour.

In the third setting, we additionally include drivers. Thus, one driver is assigned to every bus and a time a_k at which the bus is available (the driver's shift starts), a time b_k at which the associated bus driver starts a break of a given length p, and a time c_k at which the driver's shift ends are given. Thus, bus k can only serve tours such that the bus starts not before a_k at the depot, stops at the depot between time b_k and $b_k + p$ and ends the last tour not later than c_k at the depot. Hence, the difference between settings two and three is that in setting three the availability of the bus is reduced to the time intervals $[a_k, b_k]$ and $[b_k + p, c_k]$. Thereby, we assume that driver shifts are given by the time intervals and that drivers are assigned to buses according to their capabilities if necessary.

The first setting can be interpreted as the setting with homogeneous autonomous buses such that no differentiation between the vehicles is necessary. The second setting can be interpreted as the setting with heterogeneous autonomous buses. However, it is sufficient to define bus types, i.e. one type for every bus type with different capacity. The third setting can be interpreted as the current situation with driver-driven buses. Thus, the three settings allow us to evaluate the potential of on-demand line-based bus services in today's situation with drivers as well as in a future situation with autonomous buses. Moreover, the different capacities allow us to also answer the question of ideal bus capacities. Vansteenwegen et al. (2022) asked to answer this question in the investigation of bus services.

Our aim in this paper is to evaluate the effect if only those parts of a bus tour are served for which a positive demand is given in comparison to the service of all bus tours independent of the demand. Thus, we minimize the number of buses, and thereby drivers, required to serve the bus tours. We do this analysis for all three settings.

Of course, we also use some simplifying assumptions in this paper which still allow us to obtain realistic results in our analysis. These are:

- We consider one common depot for all buses.
- We consider driver shift schedules fulfilling the labour regulations to be given.
- We assume fixed driver breaks fulfilling labour regulations to be given and assume that there is only one break in a shift.
- We assume that there is a predefined assignment of one driver to each bus whenever the bus is on tour. Thus, we assume that there is always a driver who has the capabilities necessary to drive the bus. Otherwise, the bus can simply be excluded from the analysis. This also includes that no driver changes are possible outside the depot.
- We do not consider any refueling or recharging operations but assume the buses to be refueled/recharged when starting in the depot and that the tank/battery is sufficient to serve all tours until the depot is reached again.
- We do not differentiate between bus configurations beside the capacity.

The last point can be integrated by excluding that a certain bus is allowed to serve a certain tour if the configuration is not sufficient. However, we cluster buses with the same characteristics in the computational study to reduce the model's size. If there would be too many different configurations, the model's size would become intractably large.

4 Model formulations

We start with the modelling of the first setting.

4.1 Network flow formulation for the first setting

Let us first consider the case where all tours need to be served.

4.1.1 Service of all tours

The setting can be represented by a graph $G = (V, A)$ with

$$V = \{D, s_1^1, ..., s_1^n, s_{m_1}^1, ..., s_{m_n}^n\}$$

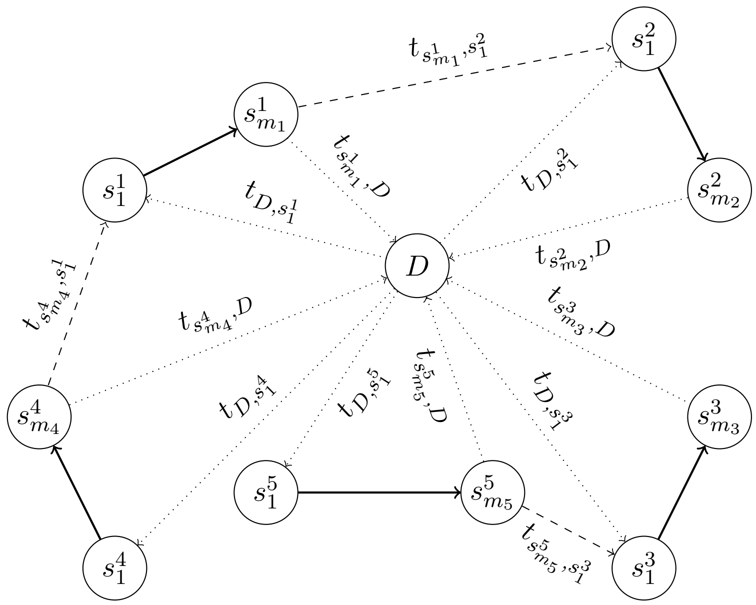


Figure 1: Graph for setting 1

as in Figure 1. In the graph, a bus schedule with five lines is pictured. Moreover, the travel times t_{D,s_1^l} between the depot and the start of the line as well as between the end of the line and the depot $t_{s_{m_l}^l,D}$ are given by dotted lines. These relations must always be possible, as otherwise a line could not be served (at least if triangle inequality holds). Moreover, dashed lines represent possible tour connections with travel times $t_{s_{m_l}^l,s_1^{l'}}$. A connection of two tours is possible if $\bar{t}_{s_{m_l}^l} + t_{s_{m_l}^l,s_1^{l'}} \leq \bar{t}_{s_1^{l'}}$ holds. Thus,

$$
\begin{aligned}
A =& \{(D, s_1^1), ..., (D, s_1^n), (s_{m_1}^1, D), ..., (s_{m_n}^n, D), (s_1^1, s_{m_1}^1), ..., (s_1^n, s_{m_n}^n)\} \\
& \cup \{(s_{m_l}^l, s_1^{l'}) : \bar{t}_{s_{m_l}^l} + t_{s_{m_l}^l,s_1^{l'}} \leq \bar{t}_{s_1^{l'}}\}.
\end{aligned}
$$

In the concrete setting, two tours are possible serving bus tours 4, 1, as well as 2 and 5 as well as 3, respectively, in this sequence. As all travel times are positive, $\bar{t}_{s_{m_l}^l} + t_{s_{m_l}^l,s_1^{l'}} \leq \bar{t}_{s_1^{l'}}$ implies that the graph is acyclic.

In the following, we present a network flow formulation for problem setting 1 using the variables x_{ij} which are 1 if a tour uses the arc $(i, j) \in A$ and 0 otherwise.

$$
\min \sum_{j:(D,j)\in A} x_{Dj} \tag{1}
$$

with the constraints

$$
\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = 0 \qquad \forall i \in V \tag{2}
$$

$$x_{s_1^l, s_{m_l}^l} = 1 \qquad \forall l = 1, ..., n \qquad (3)$$

$$x_{ij} \geq 0 \qquad \forall (i,j) \in A \qquad (4)$$

The objective function (1) minimizes all outgoing flows of the depot, i.e. all starting bus tours. Constraints (2) are the flow conservation constraints which ensure that every node which is reached is left again. Constraints (3) ensure that all tours are served. As all nodes beside the depot node D have only one outgoing (s_1^l) or only one ingoing ($s_{m_l}^l$) arc, Constraints (2) and (3) also ensure that there is exactly one ingoing and outgoing flow for every node beside the depot node D. Constraints (4) are the non-negativity constraints for the flow variables.

The formulation can be interpreted as a minimum cost flow model with a sufficient number of buses, i.e. flow starting in the source (the depot), whereat a flow from the depot to the depot is possible and all arcs are cost-neutral beside those between the depot and the bus line starts which have costs of 1. Additionally, Constraints (3) ensure that every bus line is served (solid lines in Figure 1. We can replace the arc $(s_1^l, s_{m_l}^l)$ by a sink $sink_l$ with inflow 1 and arc $(s_1^l, sink_l)$ as well as a source $source_l$ with outflow 1 and arc $(source_l, s_{m_l}^l)$. This is equivalent to $x_{s_1^l, s_{m_l}^l} = 1$. Note that adding additional sources and sinks does not violate the network flow property, as we can add a supersource requiring the sum of all required flows and arcs between the supersource and the sources with the capacity of the source. We can proceed analogously with the sinks (Cormen et al., 2009). Thus, (1)–(4) is equivalent to a minimum cost flow model formulation.

4.1.2 Service of tours with positive demand only

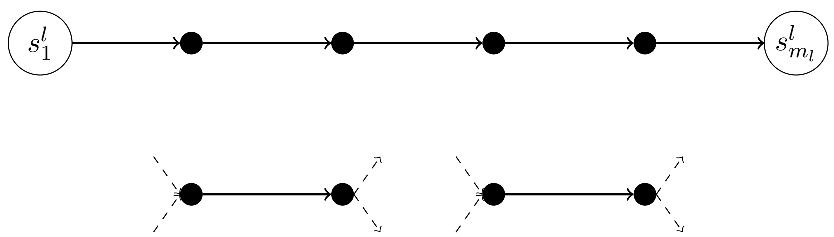
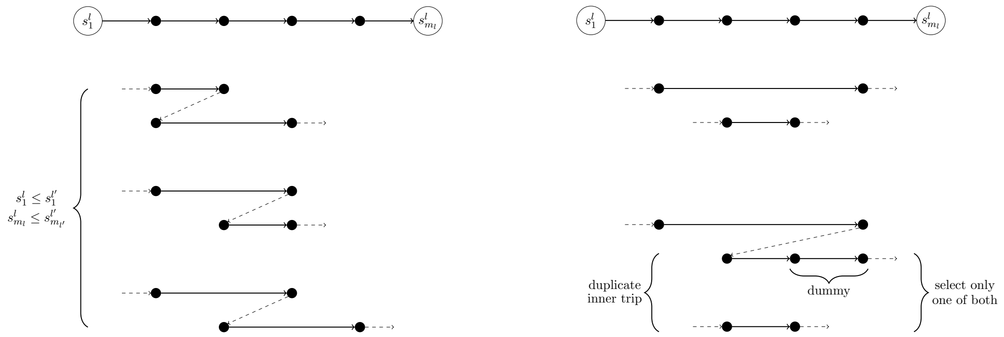


Figure 2: Graph for setting 1 (tours only served if demand positive)

In this part of setting 1, we only serve those parts of a tour which have a positive demand. Figure 2 shows the situation. Instead, of serving the entire tour between s_1^l and $s_{m_l}^l$, the bus needs only to serve the parts between the second and third as well as between the fourth and fifth stop of the tour. However, it is not necessary that the same bus serves both of them and if the same bus does, it is allowed that another customer trip is served in between. Generally, the approach in Section 4.1.1 can directly be applied by simply defining each customer trip $l = 1, ..., n$ as one tour s_1^l, ..., $s_{m_l}^l$. As each customer has one pickup and one delivery location without any (by this customer) required stop in between, $m_l = 2$ for all $l = 1, ..., n$. However, now several customer trips can share parts



(a) partly overlapping customer trips (b) one customer trip comprises the other

Figure 3: Possible overlapping situations

of one original bus line tour. If their trips belong to the same bus line tour but do not overlap, this is no problem. However, if the trips overlap, we need to ensure that they can be served together by one bus. Figure 3 shows the two relevant cases. In the left Subfigure 3a, two customer trips overlap, but one of them joins and leaves the vehicle not later than the other, i.e. $s_1^l \leq s_1^{l'}$ and $s_{m_l}^l \leq s_{m_{l'}}^{l'}$. Then, we can simply allow to serve trip l' after trip l, i.e. add arc $(s_{m_l}^l, s_1^{l'})$ to A, although trip l is not finished before trip l' begins. However, both trips share the same geographical path, so, they can be served together.

The situation is more challenging if one trip fully comprises the other (Subfigure 3b). In this case, the first trip l starts before and ends after the second trip l'. Thus, the bus would be free earlier if we add the second trip after the first. Instead, we duplicate the inner trip l'. One of the duplicates is then extended to the end of the first trip, i.e. $s_{m_{l'}}^{l'_1} = s_{m_l}^l$ (the 1 represents the first duplicate). Then, we can add again the second trip after the first and are back in the setting of Subfigure 3a. The second duplicate stays unmodified. It represents the case that the second trip is not combined with the first one and is not set into relation to the first trip.

We have to ensure now that only one of the two duplicates l'_1 and l'_2 is served. For it, we add a constraint

$$\sum_{i=1}^{2} \sum_{l'':(l'_i, l'') \in A} x_{l'_i l''} = 1. \tag{5}$$

It remains to show that the model modification by adding Constraint (5) does not violate the network flow property. Constraint (5) means that one of the duplicates has no successor in A while the other has. We can add additional sources and sinks to integrate this into our network flow model. First, we proceed as in Section 4.1 to add $x_{s_1^{l'_1}, s_{m_{l'}}^{l'_1}} = 1$

and $x_{s_1^{l'_2}, s_{m_{l'}}^{l'_2}} = 1$ to the model. Moreover, we add an additional source $source'_l$ with outflow 1 and arcs $(source'_l, s_1^{l'_1})$ and $(source'_l, s_1^{l'_2})$ as well as an additional sink $sink'_l$ with inflow 1 and arcs $(s_{m_{l'}}^{l'_1}, sink'_l)$ and $(s_{m_{l'}}^{l'_2}, sink'_l)$. Then, a flow of 1 needs to flow from $source'_l$ via one of the duplicates $s_1^{l'_1}$ and $s_{m_{l'}}^{l'_1}$ or $s_1^{l'_1}$ and $s_{m_{l'}}^{l'_1}$ to $sink'_l$. Because of $x_{s_1^{l'_1}, s_{m_{l'}}^{l'_1}} = 1$ and $x_{s_1^{l'_2}, s_{m_{l'}}^{l'_2}} = 1$ there must also be a flow through the other duplicate which is equivalent to (5).

Noch ein Beispiel und ggf. ne Prozedur für komplexere Verschatelungen einfügen. Man wird dann ein Duplikat für jeden einzelnen Kunden der Verschatelung haben und eins für jedes Tuple von Kunden.

4.2 Integer programming formulation for the second setting

In the second setting, buses are heterogeneous. Thus, we need to include an index for the buses $k = 1, ..., K$. Let x_{ijk} be 1 if tour k uses the arc $(i, j) \in E$ and 0 otherwise. If the bus capacity is not sufficient to serve tour i or j, we set $x_{ijk} = 0$. In the case, where only trips with a positive demand are served, we need to consider the capacity when generating the duplicates for joint delivery. If a set of customers cannot be served together by a bus, the duplicate is not generated. The same is true in the partly overlapping case. If trips cannot be served together, the corresponding x variable is set to zero. The model formulation is as follows:

$$\min \sum_{k=1}^{K} \sum_{j:(D,j)\in A} x_{Djk} \tag{6}$$

with the constraints

$$\sum_{j:(i,j)\in A} x_{ijk} - \sum_{j:(j,i)\in A} x_{jik} = 0 \qquad \forall i \in V, k = 1, ..., K \tag{7}$$

$$\sum_{k=1}^{K} x_{s_1^l, s_{m_l}^l, k} = 1 \qquad \forall l = 1, ..., n \tag{8}$$

$$\sum_{j:(D,j)\in A} x_{Djk} \leq 1 \qquad \forall k = 1, ..., K \tag{9}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall (i, j) \in A, k = 1, ..., K \tag{10}$$

Constraints (7) are again the flow conservation constraints. Constraints (8) ensure that every request is served by a bus. Constraints (9) take care that every bus is used at most once. This is ensured by the fact that every bus leaves the depot at most once. Constraints (10) are the binary constraints. Note that the different buses can be interpreted as different commodities and the multi commodity integral flow problem is known to be NP-hard for only two commodities (Even et al., 1976). Constraint (5) can analogously be adapted to

$$\sum_{k=1}^{K}\sum_{i=1}^{2}\sum_{l'':(l',l'')\in A} x_{l'_i l'' k} = 1 \tag{11}$$

such that only one duplicate is served by one bus tour.

The presented formulation has the disadvantage that the number of variables strongly increases by every additional bus. In fact, the entire tour network is modelled once for every bus. To reduce the network's size, we merge buses of the same type, i.e. with the same capacity Q_k. Thus, index k represents buses of the same type. Then, index k comprises a number of O_k buses and we need to adapt Constraints (9) to

$$\sum_{j:(D,j)\in A} x_{Djk} \leq O_k \qquad \forall k = 1, ..., K \tag{12}$$

As a bus company will have only a small subset of different bus types in practice, the model's size can be restricted to a treatable size in practical applications.

4.3 Adaptation for the third setting

The third setting is a very straightforward extension, as the information which bus serves which trip is already included in the model formulation (x_{ijk} variables). We simply need to set all variables x_{ijk} to zero if the bus would need to leave the depot outside the time windows $[a_k, b_k]$ and $[b_k + p, c_k]$. This can directly be done in the preprocessing given the t_{D,s_1^l}, $t_{s_{m_l}^l,D}$, $\bar{t}_{s_1^l}$, and $\bar{t}_{s_{m_l}^l}$ parameters. Variable x_{ijk} is set to zero if either

(i) $\bar{t}_{s_1^l} - t_{D,s_1^l} < a_k$

(ii) $\bar{t}_{s_{m_l}^l} + t_{s_{m_l}^l,D} > b_k$ and $\bar{t}_{s_1^l} - t_{D,s_1^l} < b_k + p$ or

(iii) $\bar{t}_{s_1^l} - t_{D,s_1^l} > c_k$

is fulfilled. (i) ensures that no tour can be served by bus k if it needs to depart at the depot before a_k. Due to (ii) no tour can be served by the bus if the tour starts before the end of the break ($\bar{t}_{s_1^l} - t_{D,s_1^l} < b_k + p$) and ends later than the first part of the shift ($\bar{t}_{s_{m_l}^l} + t_{s_{m_l}^l,D} > b_k$). This includes the two cases that the tour starts in the first interval $[a_k, b_k]$ but ends later than b_k and that the tour starts in the break (and therefore ends after b_k). Finally, no tour served by bus k is allowed to end later than c_k (case (iii)). In fact, this allows the bus drivers to drive back to the depot after the last tour before their break, spend the break in the depot, and leave the depot not before the end of the break again. Again, we can reduce the model's size by merging bus-driver-assignments to one type if the buses' capacity and the drivers' break coincide.

5 Computational evaluation of on-demand line-based bus services

Setting 1a: Wie viele Busse sind nötig, um alle Linien zu bedienen?
Setting 1b: Wie viele Busse sind nötig, um alle Linien zu bedienen, auf denen tatsächlich Demand ist (ganze Linien)?
Setting 1c: Wie viele Busse sind nötig, um nur den tatsächlichen Demand zu bedienen?

Settings 2a, 2b, 2c: Selbe Analyse mit kapazitätsrestringierten Bussen (z.B. Kleinbusse). Tradeoff zwischen Anzahl Fahrten und Busgröße.

Settings 3a, 3b, 3c: Selbe Analyse mit Fahrerpausen. Welchen Einfluss haben die Pausen?

Evaluation unterschiedlicher Fahrzeuggrößen. Evaluation dynamischer Anfragen (rolling horizon). Rejections erlauben, wie ändern sich die Ergebnisse, wenn ein Servicelevel von 90, 95, 99 oder 100% erreicht werden muss?

Wenn Anzahl Touren minimiert ist, nach der Lösung suchen, wo am meisten Puffer ist, um Kunden auch direkt am Zielort statt an Haltestellen rauszulassen (Survey Errico et al.) => Robustheit gegen stochastische Fahrtzeiten

Unterschiedliche Szenarien (geringe, mittlere, hohe Nachfrage) für den konkreten Fahrplan aus MeckPomm rechnen

6 Conclusion

References

Alfares, H. K. (2004). Survey, categorization, and comparison of recent tour scheduling literature. *Annals of Operations Research*, 127:145–175.

Bektaş, T. and Elmastaş, S. (2007). Solving school bus routing problems through integer programming. *Journal of the Operational Research Society*, 58(12):1599–1604.

Berthold, L., Fliedner, M., and Schulz, A. (2024). A shift scheduling model for ridepooling services.

Bögl, M., Doerner, K. F., and Parragh, S. N. (2015). The school bus routing and scheduling problem with transfers. *Networks*, 65(2):180–203.

Borndörfer, R., Grötschel, M., Klostermeier, F., and Küttner, C. (1999). *Telebus Berlin: Vehicle scheduling in a dial-a-ride system*. Springer.

Boyer, V., Ibarra-Rojas, O. J., and Ríos-Solís, Y. Á. (2018). Vehicle and crew scheduling for flexible bus transportation systems. *Transportation Research Part B: Methodological*, 112:216–229.

Brandinu, G. and Trautmann, N. (2014). A mixed-integer linear programming approach to the optimization of event-bus schedules: a scheduling application in the tourism sector. *Journal of Scheduling*, 17:621–629.

Bunte, S. and Kliewer, N. (2009). An overview on vehicle scheduling models. *Public Transport*, 1(4):299–317.

Cordeau, J.-F. (2006). A branch-and-cut algorithm for the dial-a-ride problem. *Operations research*, 54(3):573–586.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., and Stein, C. (2009). *Introduction to algorithms*. MIT press, Cambridge, Massachusetts, 3 edition.

De Silva, A. (2001). Combining constraint programming and linear programming on an example of bus driver scheduling. *Annals of Operations Research*, 108(1):277–291.

Errico, F., Crainic, T. G., Malucelli, F., and Nonato, M. (2013). A survey on planning semi-flexible transit systems: Methodological issues and a unifying framework. *Transportation Research Part C: Emerging Technologies*, 36:324–338.

Even, S., Itai, A., and Shamir, A. (1976). On the complexity of timetable and multicommodity flow problems. *SIAM Journal on Computing*, 5(4):691–703.

Fügenschuh, A. (2009). Solving a school bus scheduling problem with integer programming. *European Journal of Operational Research*, 193(3):867–884.

Gaul, D., Klamroth, K., Pfeiffer, C., Schulz, A., and Stiglmayr, M. (2023). A tight formulation for the dial-a-ride problem. *arXiv preprint arXiv:2308.11285*.

Gaul, D., Klamroth, K., and Stiglmayr, M. (2022). Event-based milp models for ridepooling applications. *European Journal of Operational Research*, 301(3):1048–1063.

Gintner, V., Kliewer, N., and Suhl, L. (2005). Solving large multiple-depot multiple-vehicle-type bus scheduling problems in practice. *OR Spectrum*, 27:507–523.

Häll, C. H., Andersson, H., Lundgren, J. T., and Värbrand, P. (2009). The integrated dial-a-ride problem. *Public Transport*, 1:39–54.

Johnsen, L. C. and Meisel, F. (2022). Interrelated trips in the rural dial-a-ride problem with autonomous vehicles. *European Journal of Operational Research*, 303(1):201–219.

Kim, M. E. and Schonfeld, P. (2014). Integration of conventional and flexible bus services with timed transfers. *Transportation Research Part B: Methodological*, 68:76–97.

Kliewer, N., Mellouli, T., and Suhl, L. (2006). A time–space network based exact optimization model for multi-depot bus scheduling. *European Journal of Operational Research*, 175(3):1616–1627.

Park, J. and Kim, B.-I. (2010). The school bus routing problem: A review. *European Journal of Operational Research*, 202(2):311–319.

Pei, M., Lin, P., and Ou, J. (2019). Real-time optimal scheduling model for transit system with flexible bus line length. *Transportation Research Record*, 2673(4):800–810.

Perumal, S. S., Larsen, J., Lusby, R. M., Riis, M., and Sørensen, K. S. (2019). A matheuristic for the driver scheduling problem with staff cars. *European Journal of Operational Research*, 275(1):280–294.

Perumal, S. S., Lusby, R. M., and Larsen, J. (2022). Electric bus planning & scheduling: A review of related problems and methodologies. *European Journal of Operational Research*, 301(2):395–413.

Pfeiffer, C. and Schulz, A. (2022). An alns algorithm for the static dial-a-ride problem with ride and waiting time minimization. *Or Spectrum*, 44(1):87–119.

Psaraftis, H. N. (1980). A dynamic programming solution to the single vehicle many-to-many immediate request dial-a-ride problem. *Transportation Science*, 14(2):130–154.

Qiu, F., Li, W., and Zhang, J. (2014). A dynamic station strategy to improve the performance of flex-route transit services. *Transportation Research Part C: Emerging Technologies*, 48:229–240.

Quadrifoglio, L., Dessouky, M. M., and Palmer, K. (2007). An insertion heuristic for scheduling mobility allowance shuttle transit (mast) services. *Journal of Scheduling*, 10:25–40.

Rist, Y. and Forbes, M. A. (2021). A new formulation for the dial-a-ride problem. *Transportation Science*, 55(5):1113–1135.

Ropke, S., Cordeau, J.-F., and Laporte, G. (2007). Models and branch-and-cut algorithms for pickup and delivery problems with time windows. *Networks: An International Journal*, 49(4):258–272.

Schulz, A. and Pfeiffer, C. (2024). Using fixed paths to improve branch-and-cut algorithms for precedence-constrained routing problems. *European Journal of Operational Research*, 312(2):456–472.

Vansteenwegen, P., Melis, L., Aktaş, D., Montenegro, B. D. G., Vieira, F. S., and Sörensen, K. (2022). A survey on demand-responsive public bus systems. *Transportation Research Part C: Emerging Technologies*, 137:103573.