

Single Depot Heterogeneous Capacitated Vehicle Routing Problem with Simultaneous Delivery and PickUp for Disaster Management Systems

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Abstract - We address the challenge of cost management for pre-disaster emergency funds, with ample warning time available for completing emergency operations. We formulate a mathematical model for a complex vehicle routing problem involving a single depot, a fleet of heterogeneous vehicles with limited capacities, and simultaneous delivery and pickup tasks. Each vehicle type is assigned a specific road network based on vehicle-road compatibility. We develop heuristic approaches to generate high-quality solutions for this problem and compare them with a state-of-the-art commercial solver. Our findings reveal that our heuristics perform exceptionally well for very large problem instances, while the commercial solver outperforms them for smaller instances. Moreover, our algorithms can handle scenarios where customers have either delivery or pickup demands, as well as cases where both operations are required. We evaluate the performance of our exact formulations extending existing data, as well as generate new data sets demonstrating the effectiveness of our bioinspired heuristic methods in achieving satisfactory outcomes.

Keywords - OR in Disaster Relief, Vehicle Routing Problem, Simultaneous Delivery and PickUp, Heterogeneous Fleet, Vehicle-Road Compatibility

I. INTRODUCTION

Delays in intelligent integration of infrastructure and pre-planning in developing nations have led to calamities being made of natural disasters especially enhanced by climate change. Prediction and pre-disaster evacuation are essential to reduce potential damages, and much research is being done in this regard to upgrade and install new resilient infrastructure. It is therefore pertinent to gauge the existing financial constraints of disaster relief funds to strategically plan for pre-disaster evacuation operations performed during the disaster-warning period. We develop a more compact exact formulation for the vehicle routing problem [1] with simultaneous pickup and delivery [2] with a heterogeneous fleet of vehicles [3, 4], compared to those discussed in [5] and [6] while also considering vehicle compatibility with road networks. We develop Machine Tuning (MT) and Jelly Fishing (JF) Heuristics to find good solutions. We use a novel dataset¹ to highlight the necessary differences in distance matrices for each Vehicle-Type layer, due to different shortest paths between the same Nodes for different Vehicle Types due to Vehicle-Road compatibility.

¹ Dataset: <https://github.com/SanTanBan/DataSet-IEEM2023>

II. PROBLEM DESCRIPTION AND FORMULATION

We consider the multiple heterogeneous fixed-fleet vehicle routing problem with simultaneous delivery and pickup. We modify the formulation from [5] where they use different vehicle layers (i.e., each vehicle is referred to by an extra subscript of their decision variables) and redundant constraints (Eq. 9 and Eq. 10 in [5]). We reduce this to layers specific to vehicle types and further reduce the number of continuous (load flow) variables used. We further modify the cost-minimization objective function in [5], incorporating vehicle type to road compatibility by introducing different distance matrices for each vehicle type.

TABLE 1: NOTATIONS OF SETS AND PARAMETERS USED

N	Set of all Nodes/Relief Points, where $ N = n$
N_0	Set of all Relief Points and the Depot (Depot being the 0 th Node), where $ N_0 = n+1$
p_i	PickUp quantity associated with Node i
d_i	Delivery quantity associated with Node i
C_{ijk}	Shortest distance required to travel from node i to node j , ($i \neq j$) for vehicles of type k . The vehicle type-specific network layers are generated for each vehicle type; according to road compatibility with that vehicle type.
T	Set of all Vehicle Types, generally the k subscript is used to refer to this
Q_k	Capacity of a vehicle of type k
H_k	Number of vehicles of type k available at Depot
F_k	Fixed cost related to dispatching a vehicle of type k
V_k	Variable cost per unit length of route related to dispatching a vehicle of type k

TABLE 2: NOTATIONS OF DECISION VARIABLES USED IN THE EXACT MILP FORMULATION

x_{ijk}	Binary decision variable; refers to a k^{th} type of vehicle journeying from node i to node j , ($i \neq j$) on layer k if the variable takes value of 1
y_{ij}	Continuous variable; refers to the amount of pickup (humans, rescued cattle, etc.) being carried by a vehicle from node i to j , ($i \neq j$)
z_{ij}	Continuous variable referring to the amount of delivery (Relief kits including Food, Water, Medicine, Waterproof-sheets, Sanitary Items, etc.) being carried from node i to j , ($i \neq j$)

The mixed integer linear programming (MILP) formulation is developed, using notations in TABLE 1 and TABLE 2, as below (equations 1 to 11):

$$\text{Min. } \sum_{k \in T} \sum_{i \in N_0} \sum_{\substack{j \in N_0 \\ (i \neq j)}} c_{ijk} x_{ijk} V_k + \sum_{j \in N} \sum_{k \in T} x_{0jk} F_k \quad (1)$$

subject to the following constraints:

$$\sum_{\substack{j \in N_0 \\ (i \neq j)}} x_{ijk} \leq 1, \quad \forall i \in N, \quad (2)$$

$$\sum_{\substack{j \in N_0 \\ (i \neq j)}} (x_{ijk} - x_{jik}) = 0, \quad \forall k \in T, \forall i \in N_0 \quad (3)$$

$$\sum_{j \in N} x_{0jk} \leq H_k, \quad \forall k \in T, \quad (4)$$

$$y_{0j} = 0, \quad \forall j \in N, \quad (5)$$

$$z_{i0} = 0, \quad \forall i \in N, \quad (6)$$

$$\sum_{\substack{j \in N_0 \\ (i \neq j)}} (y_{ij} - y_{ji}) = p_i, \quad \forall i \in N, \quad (7)$$

$$\sum_{\substack{j \in N_0 \\ (i \neq j)}} (z_{ji} - z_{ij}) = d_i, \quad \forall i \in N, \quad (8)$$

$$y_{ij} + z_{ij} \leq \sum_{k \in T} x_{ijk} Q_k, \quad \forall i, j \in N_0, (i \neq j), \quad (9)$$

$$y_{ij}, z_{ij} \geq 0, \quad \forall i, j \in N_0, (i \neq j), \quad (10)$$

$$x_{ijk} \in \{0,1\}, \quad \forall i, j \in N_0, (i \neq j), \quad \forall k \in T. \quad (11)$$

We further propose heuristics for obtaining good solutions and develop a novel dataset for a computational study.

III. HEURISTIC ALGORITHM

The heuristics in [5] allow unlimited vehicles to be used while the heuristic in [6] may not be compared easily (due to better than optimal results reported in Table 6 in [6]). Upon placing the exact-algorithm from [6], within the adaptive hybrid local search [5] (replacing their Dijkstra algorithm), poor solutions were obtained (even after introducing new neighbourhood structures). Since no good heuristic exists for our considered problem, we therefore adapt the Adaptive Threshold Search (ATS) in [5] developing MT and JF heuristics, naming our heuristic versions as ATSA-MT and ATSA-MT-JF Heuristics; the latter A in ATSA referring to Algorithm implementation of ATS (as the Flowchart logic in Fig. 2 in [5] is slightly different). MT progressively tunes the mean values of our Edge Selection Criteria which are essentially dynamic logics. JF refers to the continued approach of expanding

and contracting the range of values for individual parameters involved in the selection criteria of Edges (edges refer to encoded routes as in [5]). The heuristic components are described below:

A. Node Sequence (NoSe)

A NoSe is a linear array of the Nodes in any order, always starting with the Depot (0th Node). The initial NoSe is the ascending sequence of Node Numbers.

B. Neighbourhood Structures (NeSt)

We generate new NoSe extending on the original 4 NeSt of [5], always ensuring the first Node remains the Depot:

1. Shuffle Random: Randomly shuffling existing NoSe
2. Reversal: Reversing an arbitrary length (we kept it >3 to distinguish it from Adjacent Swap) of the NoSe
3. Single Insertion: Taking one Node from the NoSe and placing it anywhere else within the NoSe
4. General Swap: Swapping any two Nodes of the NoSe
5. Adjacent Swap: Swapping adjacent Nodes of NoSe
6. SubArray Transplant allowing intermediate Operations (inspired from Transposons in Biology):
 - a) Generate the SubArray randomly by either:
 - i) Taking a random length of consecutive elements from the NoSe
 - ii) Taking a random number of elements from arbitrary locations within the NoSe

The probability of using the first method was kept double than that of using the second method.

- b) Remove the SubArray from the NoSe creating a new shorter Trimmed NoSe (i.e. without the SubArray elements).
- c) Perform either of the below operations on the SubArray:
 - i) Reverse the SubArray
 - ii) Shuffle the SubArray elements randomly

The above operations were allowed to happen with separate one-third probabilities; thereby allowing the SubArray to remain intact as well.
- d) Perform following operations on the Trimmed NoSe (with separate one-third probabilities):
 - i) Reverse the Trimmed NoSe
 - ii) Shuffle the Trimmed NoSe randomly
- e) Insertion of the SubArray within the Trimmed NoSe is done using either of the following ways:
 - i) Single Bulk Insertion of the SubArray, in a reversed manner, at a random location within the Trimmed NoSe
 - ii) Insert each element of the SubArray at random locations in the Trimmed NoSe

In our case, the first insertion method had double the probability of being used than the second.
7. Linear Cutting and Accumulation: Each of the previous 6 NeSt are performed independently on some limited portion of the NoSe with separate one-third probabilities of using any NeSt, if at all. The selected portions are allowed to overlap amongst themselves.

8. Linear Cutting once without Overlap: A random portion of the NoSe is chosen. On this chosen portion, each of the first 6 NeSt is performed with independent one-third probabilities.

C. Decoding Mechanism (DeMe)

The basic representation of feasible Route into Edge in the NoSe is taken from [5] combined with below components:

- 1) Edges: A directed connection between two Nodes of the NoSe containing a Starting Node, Stopping Node, Vehicle Type used; and Cost of the Edge which is calculated by a tour starting from the Depot sequentially covering all Nodes in the NoSe from after the Starting Node till and including the Stopping Node and then returning back to the Depot.
- 2) Mother Graph: Each Mother Graph consists of some Edges on the Nose. The first of these starts from the Depot to a Node in the NoSe (say Node L). The next Edge starts from this Node L and ends further down the NoSe array. The Mother Graph is termed Traversed if connected Edges are present such that the Stopping Node of the last Edge is also the end of NoSe.
- 3) Daughter Edge: A non-Traversed Mother Graph generates multiple Daughter Edges using leftover vehicles of each type [FIGURE 1], which start from the Stopping Node of its last Edge. One among these feasible Daughter Edges is selected and included in the Mother Graph for the next set of Daughter Edge generation-and-selection till the Mother Graph is Traversed.

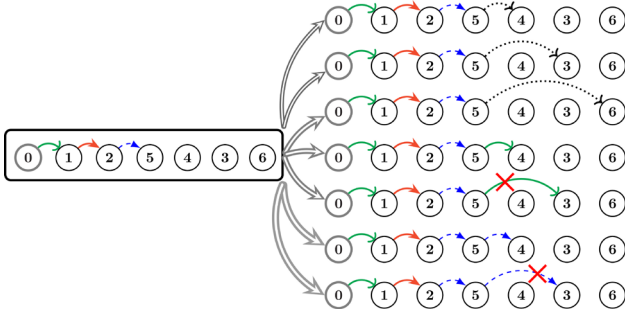


FIGURE 1: DAUGHTER EDGE GENERATION FROM A MOTHER GRAPH (RED CROSSED-OUT EDGES ARE INFEASIBLE, PREVENTING SELECTION OF THESE VEHICLE-TYPE-EDGES)

- 4) Edge Selection Strategy: 3 expressions are developed using following 3 parameters of each Daughter Edge:
 - a) Length (Γ): Equal to the number of PickUp-&-Delivery Nodes the Edge caters to.
 - b) Fixed Cost (F) of route represented by the Edge
 - c) Variable Cost (Λ) of route represented by Edge
 Edge selection criterias (equations 12, 13 or 14) are used independently forming different versions of the ATSA-MT and the respective Heuristics are named according to the number of parameters in these expressions (6, 10 or 19 respectively). The Daughter Edge which gives the minimum value for the used Edge Selection Expression is chosen to extend the Mother Graph and continue the next set of Daughter Edge generation and selection, till the

Mother Graph is Traversed. The parameters take values from a dynamic normal distribution. Their independent starting values of Mean (μ) and SD (σ) were obtained from TABLE 3.

$$\text{Expression 1: } h\Lambda^i\Gamma^j + k\Gamma^lF^m \quad (12)$$

$$\text{Expression 2: } aF^b\Lambda^c\Gamma^d + h\Lambda^i\Gamma^j + k\Gamma^lF^m \quad (13)$$

$$\text{Expression 3: } aF^b\Lambda^c\Gamma^d + eF^f\Lambda^g + h\Lambda^i\Gamma^j + k\Gamma^lF^m + nF^o + p\Lambda^q + r\Gamma^s \quad (14)$$

TABLE 3: RANGE OF UNIFORM DISTRIBUTION AS USED FOR OBTAINING STARTING VALUES OF THE EXPRESSONAL PARAMETERS

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
μ	0,1	0,1	0,1	$-5/4, 1$	$-1/4, 1/4$	$-1/4, 1/4$	$-1/4, 1/4$
σ	$5/4$	1	1	1	$3/4$	$3/4$	$3/4$
	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	
μ	0,2	0,1	-2,1	0,2	-2,1	0,1	
σ	$1, 5/4$	$1, 5/4$	$1, 3/2$	$5/4$	$1, 3/2$	1	
	<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	
μ	$-1/5, 1/5$	$-1/4, 1/4$	$-1/5, 1/5$	$-1/4, 1/4$	$-1/2, 1/2$	-2,2	
σ	$3/4$	$3/4$	$3/4$	$3/4$	1	1	

A sample expression for updating μ and σ of the first parameter (*a*) is shown in equations 15 and 16:

$$\mu_a^{new} = \mu_a + (a - \mu_a) \frac{F}{F_{itr}} \quad (15)$$

$$\sigma_a^{new} = \sigma_a \left(1 - \frac{F}{F_{itr}}\right) \quad (16)$$

$$F_{itr} = 2000 \left(1 - \left(\frac{|N|}{1500}\right)^{0.1} + \left(1 - \frac{|N|}{1000}\right)^5\right) \quad (17)$$

$$F_{itr} = 3250 \left(1 - \left(\frac{|N|}{2750}\right)^{0.1} + \left(1 - \frac{|N|}{750}\right)^5\right) \quad (18)$$

F_{itr} is the maximum number of iterations without any solution improvement after which the ATSA-MT Heuristic terminates.

The following two DeMe are constructed:

- Original Decoding Mechanism (ODM): Parameter values are kept unchanged throughout each iteration to obtain a single Traversed Mother Graph. If the final objective value is found better than best solution obtained yet, then each parameter's Mean and S.D. are independently updated as per equations 15 and 16.
 - Randomized Decoding Mechanism (RDM): This generates new parameter values during each set of Daughter Edge selection. No parameter value updation can be done when RDM is used.
- 5) Continuing the DeMe: The acceptance of a solution within a threshold range has been taken from [5], where ii is number of solutions obtained within threshold and C is number of best solutions obtained. The threshold t is further detailed in FIGURE 2.

IV. RESULTS AND DISCUSSION

The formulation and developed Heuristics were tested on Gurobi 9.5.2 in Python 3.10. The data for instances with Nodes 5 upto 35 is taken from Tables 1 and 2 in [5] by sequentially increasing the number of Nodes and using respective p -values from TABLE 5 for distance calculations.

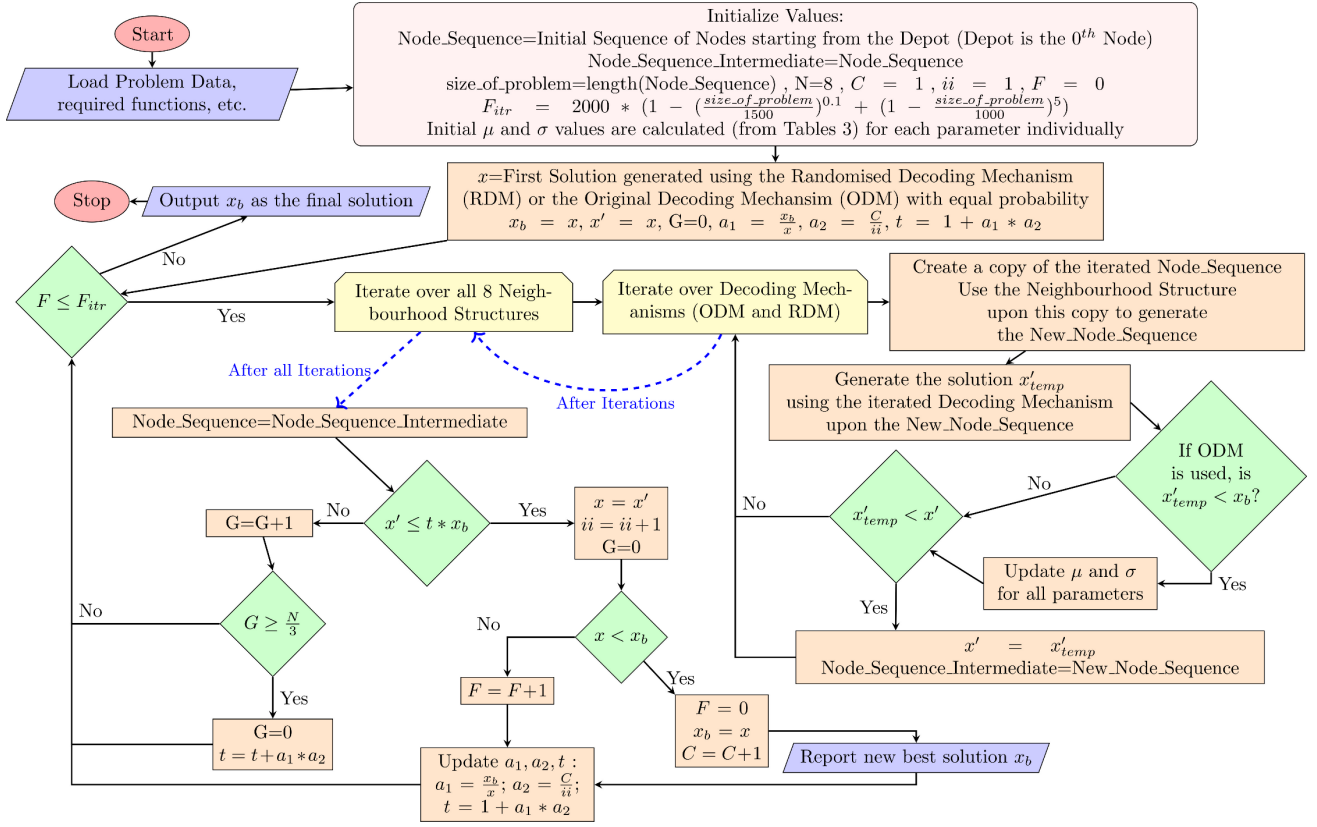


FIGURE 2: FLOWCHART REPRESENTING ATSA-MT LOGIC

TABLE 4: COMPUTATIONAL RESULTS (COMPUTER SPECIFICATIONS: INTEL® CORE™, I5-10500 CPU @ 3.1 GHZ AND INTEL® XEON® W-2133 CPU @ 3.60GHZ, X64-BASED PROCESSOR WITH INSTALLED RAM OF 16 GB HAVING 64-BIT WINDOWS 10 PRO AND 11 OPERATING SYSTEMS)

No. of Nodes	Vehicle Types Used	Available Vehicles of respective Types	Exact MILP Formulation			ATSA-MT-6 (30 runs for each problem instance)			ATSA-MT-19-JF (30 runs for each problem instance)		
			Obj. Value (A)	Soln. Time (sec)	MIP Gap % or Lower Bound	Min. Obj. Value (B)	Avg. Soln. Time	Relative Gap % §	Min. Obj. Value (B)	Avg. Soln. Time	Relative Gap % §
5	1,2,3,4	1,1,1,1	341.00	1.25	0%	341.00	7.84	0%	341.00	10.76	0%
10	1,2,3,4	2,2,2,2	508.55	2.17	0%	508.55	23.36	0%	508.55	26.24	0%
15	1,2,3,4	3,3,3,3	606.33	29.36	0%	606.33	43.51	0%	606.33	38.11	0%
20	1,2,3,4	4,4,4,4	700.73	623.33	0%	734.67	56.68	4.62%	720.34	53.09	2.72%
25	1,2,3,4	3,3,3,3	808.03	1982.06	0%	808.03	81.36	0%	841.75	65.88	4.01%
30	1,2,3,4	2,2,2,2	1016.79	3600	12.83%	952.39	93.75	-6.76%	989.63	104.66	-2.74%
35	1,2,3,4	1,1,1,1	1108.18	3600	12.40%	1106.81	63.60	-0.12%	1094.38	52.17	-1.26%
225	1,2,4	23,26,13	#	86400	6264.8	9903.23	968.64	-	9356.55	2096.64	-
275	2,3	32,38	#	86400	9168.3	12570.27	844.96	-	12499.63	1814.50	-
325	1,2,3,4	15,20,20,15	#	86400	8107.8	13203.62	2490.94	-	13270.76	3066.31	-
375	1,4	22,33	#	86400	12291.3	18112.37	863.00	-	18012.02	1531.38	-
425	2,3,4	19,14,30	#	86400	11331.5	18151.18	1179.52	-	18937.51	1338.07	-
475	1,2,3	31,28,25	#	86400	15027.5	22590.61	901.80	-	23606.26	1161.83	-
512	1,2,4	25,27,29	#	86400	15425.1	24148.51	887.72	-	25964.67	1717.59	-
555	1,3,4	23,27,25	#	86400	14159.8	23697.73	1009.31	-	23791.36	2410.61	-

No feasible solution was found within 24 hours of the MILP run

§ For the respective Heuristics of ATSA-MT-6 or ATSA-MT-19-JF, the Relative Gap % = (B - A) * 100 / B

For detailed comparative results, please refer to the **Supplementary Comparison Table** available at: <https://github.com/SanTanBan/DataSet-IEEM2023>

The p -values in TABLE 5 are used for calculating the Minkowski Distance² (i.e., L^p Norm) between Nodes, for each Vehicle Type (specifications provided in Table 1 in [5]) and as used throughout our computational studies.

TABLE 5: EXTENSION OF TABLE 1 IN AVCI TOPALOGLU 2016 [5]

Vehicle Type	1	2	3	4
p -value	1.8	1.6	1.4	1.2

Varying p -values approximate practical deviations in the actual road distance, due to large vehicles not being able to ply on the smallest of roads and therefore having to detour via compatible broader roads. The Latitude, Longitude, PickUp, and Delivery values are generated randomly from a uniform distribution within the range of [0, 125] for the larger instances with Nodes 225 and more. For these larger instances, we use Eq.18 (instead of Eq.17) to calculate F_{itr} .

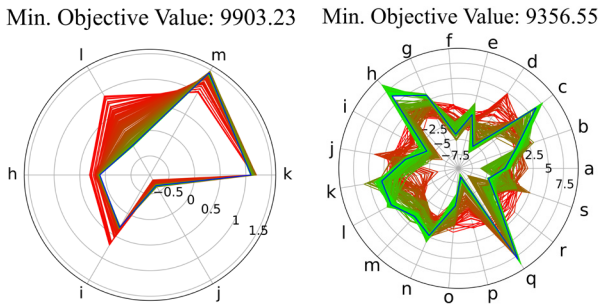


FIGURE 3: PROGRESS OF MEAN VALUE OF PARAMETERS (RED TO BLUE THROUGH GREEN IN THE RADAR CHART) FOR THE BEST RUN OF ATSA-MT-6 AND ATSA-MT-19-JF IN THE 225 NODES INSTANCE

A JF extension of the ATSA-MT is developed since the parameter S.D. values get exceedingly small as the search progresses. Inspired from the motion³ of the bell of a jellyfish's hood, we allow the S.D. to increase once it breaches a lower bound and again decrease upon breaching an upper bound, continuing until the stopping criteria is satisfied. This extensive (Relax Phase) and intensive (Squeeze Phase) search greatly reduces the average values of solutions, without major improvements in the minimum objective function values. We introduce independent Search Directions (D) for each parameter, taking value 1 when the corresponding parameter's S.D. is less than 10^{-9} , and -1 when the corresponding parameter's S.D. is more than 10^0 .

$$\mu_a^{new} = \mu_a + (a - \mu_a) \left(\frac{1 - 100^{\frac{-3F}{F_{itr}}} + \left(\frac{F}{F_{itr}} \right)^{\frac{1}{15}}}{2} \right)^{\frac{1}{2}} \quad (19)$$

$$\sigma_a^{new} = \sigma_a \left(\frac{1 + D_a \frac{F - R(0,1)}{F_{itr} + R(0,1)}}{1 - D_a \frac{F - R(0,1)}{F_{itr} + R(0,1)}} \right) \quad (20)$$

We use equations 19 and 20 to update μ and σ for each parameter, where $R(\alpha, \beta)$ refers to a random number in the range $[\alpha, \beta]$. We did computational studies for all combinations of the expressions (creating heuristic versions ATSA-MT 6, ATSA-MT-10, ATSA-MT-19) as

well as later incorporating the JF search separately. We highlight the changes of parameter means in FIGURE 3 for one instance; and report experimental results in TABLE 4 for the best performing heuristic variants.

V. CONCLUSION

Our exact MILP formulation produced faster results compared to [5] and [6], possibly being the optimal formulation for this VRP variant. JF search, which produced good average-objective-function values, may be incorporated within meta-heuristics like simulated annealing by varying the temperature within a range. Extensions towards a rich-VRP should encompass different challenges of the humanitarian logistics as detailed in [7, 8, 9, 10] by considering vehicle-load compatibility, introducing functional-vertices alongwith hierarchical multi-modal transshipment-ports, allowing multiple vehicular trips over traffic-congested time-varying transportation networks, combining split and simultaneous delivery-pickup nodes within a single formulation, to ultimately develop an International Disaster Resource Planner.

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² https://en.wikipedia.org/wiki/Minkowski_distance

³ <https://youngzine.org/news/changing-ecosystems/secret-jellyfishs-motion>