Cross-layers Evolution of Opinions: Viral Marketing in Multiplex Social Networks

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ABSTRACT

The inner dynamics of the multiple channels for information diffusion - i.e., online social networks and face-to-face communication, - plays a fundamental role in the individual opinion of an innovation in viral marketing. In the namely multiplex social networks, the agent's opinion is evolved across multiple channels because of the cross-layers propagation of related information and the diverse diffusion efficiencies of the conjoint layers. However, much emphasis has been put on the effect of the structural feature of a simplex network and the impact of opinion evolution in multiple channels has largely been ignored. Here, we propose a novel agent-based diffusion model to capture the cross-layers evolution of opinions in viral marketing. By theoretical studies and extensive simulations, we reveal the coupling relationship between the diffusion of an innovation and the consensus formation of different distributions of initial opinions. It is found that the consensus formation indicates the termination of the diffusion dynamics in multiplex networks and determines the final rate of the adoption. In conclusion, we think that the concept of cross-layers evolution of opinions may provide new insights into further study of viral marketing in multiplex networks.

Categories and Subject Descriptors

I.2 [**Artificial Intelligence**]: Multiagent Systems; J.4 [**Computer Applications**]: Social and Behavioral Sciences

General Terms

Human Factors, Experimentation.

Keywords

Opinion Evolution, Viral Marketing, Diffusion Dynamics, Multiplex Networks, Multiagent.

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1. INTRODUCTION

Viral marketing is in the exploitation of existing social networks in maximizing the adoption rate of an innovation product with the aid of the word of mouth effects between friends [1, 2]. The word of mouth effects demonstrate the individual opinion of an innovation, and the positive opinions can facilitate the adoption behavior of an innovation [3-5].

Currently, the individual opinion of an innovation highly relies on the diffusion processes in the multiplex social networks, where individuals contact through different linking types [6-8]. For instance, people are linked together in the physical network [8] through the conventional face-to-face communication. Meanwhile, the online social networks (OSN) have become a new formal way of life for people to gain the information of intercourse, news and sales promotion [1, 9, 10]. In the multiplex social networks, the opinion held by the social agent is cross-layers evolved [7], which indicates two fundamental paradigms: the cross-layers propagation of information [7, 11] and the diverse diffusion efficiencies of the multiple layers [12, 13]. First, the cross-layers propagation indicates that people can post the experience of using an innovation to OSN and talk about online rumors in daily life [8]. A vivid and recent example of the cross-layers propagation is the photos of "bent iphone 6" spreading in the online social networks. Second, information disseminates quickly in OSN, which can help to increase the awareness of an innovation; but the communication in the physical network is more persuasive, especially with the people who have adopted the innovation [1]. Therefore, the purchase intention (opinion) evoked by the online promotion may be reinforced or reduced by the oral consultation.

In most previous works on viral marketing, the diffusion of an innovation is modeled as the approximated process of epidemic or information diffusion in a simplex network [9]. The hot issue is the influence maximizing problem which aims at finding the optimal nodes to maximize the scope of influence (adoption) according to the degree distribution [14-16] or the shortest path [17] of a single social network. Recently, some studies have realized the role of human characteristics in the diffusion of innovations, such as the checking time [18], the novelty decay of rumors [19] and the relationships between individuals [20]. These approximated processes simplify the human behavior in viral marketing and cannot capture how the decision of adopting an innovation is affected by the word of mouth effects. To the best of our knowledge, there is scant literature that made the systematic analyses on the cross-layers evolution of agents' opinions and the effect on the diffusion dynamics of an innovation in multiplex networks.

In this paper, we propose a novel diffusion model of viral marketing with the aid of agent-based method. Each agent has 3-tuples states, which indicate the awareness of an innovation, the opinion and the adoption state, respectively. The viral marketing consists of the diffusion of the awareness, the opinion evolution and the diffusion of the adoption. The opinion of an agent is evolved based on the conformity bias rule [21-23]. Meanwhile, each agent changes the adoption state (makes buying decision) according to its opinion of an innovation. To manifest the cross-layers propagation, each agent is assumed to sequentially assimilate the opinions held by interacted agents in the multiple layers. To capture the diverse diffusion efficiencies of multiple layers, the proposed model also implements the spatial-temporal synchronization process [18, 24, 25], which means that the agent checks all of the unread messages in OSN periodically and meets one or several friends in the physical network occasionally.

We mainly analyze the cross-layers evolution of uniform, normal and polarized distributions of initial opinions, which represent three typical public opinions of an innovation. It is found that the consensus formation is closely related to the adoption rate and diffusion velocity of an innovation. First, the formation of consensus indicates the termination of the diffusion processes in multiplex networks. Second, the high rate of the adoption rate relies on the high point of the consensus, instead of the wide scope of the awareness of an innovation. Meanwhile, the time difference between the diffusion of the awareness and the consensus formation causes an interesting double-peak pattern of adopting an innovation. Because different distributions of initial opinions lead to the diverse rates of the adoption, we also discuss the applicability of the viral marketing. It is suggested that the viral marketing can be implemented only if there are enough agents hold positive opinions of an innovation. We hope that the introducing of cross-layers evolution of opinions can provide new insights into the study of viral marketing in multiplex networks.

The rest of the paper is organized as follows. We outline the details of the proposed model in Section 2. In Section 3, we analyze the diffusion processes in multiplex networks. Simulation results and analyses are presented in Section 4. In Section 5, we conclude our findings and point out the future outlook of our research.

2. MODEL OUTLINE

2.1 Topology, Agent State and Initial Opinion

The multiplex networks consist of an online social network and a physical network which are denoted by L_1 and L_2, respectively [8]. The notion a_i denotes the agent i according to the identification of an agent. As shown in Figure 1, each agent is represented by two conjoint nodes in the duplex layers of the multiplex network. Let N_i^1 and N_i^2 denote the sets of neighbors of a_i in L_1 and L_2.

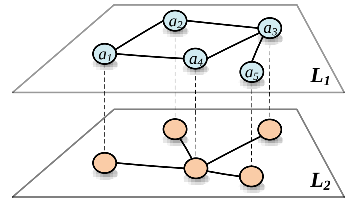

Figure 1 Illustration of Multiplex Networks

Different from the binary state of the agent in previous works on viral marketing [9, 18, 19], a 3-tuple factor $X_i(t) = \{w_i(t), o_i(t), s_i(t)\}$ is used to represent the state of a_i at discrete time t. If a_i is not aware of the innovation, the element $w_i(t)$ is equal to zero. Otherwise, $w_i(t)$ is equal to one. Generally, a_i perceives the innovation through two channels: checking messages sent by agents in N_i^1 and contacting with agents in N_i^2. If a_i has bought the product, $s_i(t) = 1$. Otherwise, $s_i(t)$ is equal to zero. The element $o_i(t) \in [-1, 1]$ denotes the opinion of the innovation held by a_i. Then, $o_i(t) \geq 0$ means that a_i holds the positive attitude to the product and $o_i(t) < 0$ indicates the negative opinion. Based on the classical SIR model in epidemics [26], we derive the states of a_i as follows.

$$X_i(t) = \begin{cases} Sus., \quad Susceptible \begin{cases} w_i(t) = 0 \\ s_i(t) = 0 \end{cases} \\ Inf., \quad Infected \begin{cases} w_i(t) = 1 \\ s_i(t) = 0 \end{cases} \\ Rec., \quad Recovered \begin{cases} w_i(t) = 1 \\ s_i(t) = 1 \end{cases} \end{cases} \tag{1}$$

Each agent is initially set as susceptible state and the opinion $o_i(t)$ is set as a random value according to a given distribution. If a_i has perceived the innovation and become the infected state, a_i will change the opinion and participate in the diffusion process of the innovation. The viral marketing in multiplex networks consists of two types of diffusion processes: the awareness and the adoption of the innovation. The recovered state means the agent has adopted the innovation. In our model, $o_i(t)$ also represents the attitude of a_i to the innovation when a_i spreads the rumor in L_1 and discusses the innovation with others agents in L_2.

The evolution of the opinion is the core part and closely related to the distributions of initial opinions. In this paper, the initial opinions are assumed to follow uniform, normal and polarized distributions. The uniform distribution is $U[-1, 1]$. The normal distribution means that most agents hold the neutral point of view. According to reference [27], the probability density function of initial opinion is given by

$$f_{nor} = \frac{5}{\sqrt{2\pi}} e^{-12.5x^2}, \ x \in [-1, 1]. \tag{2}$$

The polarized distribution indicates that agents are generally distributed in two camps, and the probability density function [27] is given by

$$f_{por} = \begin{cases} \dfrac{5}{\sqrt{2\pi}} e^{-12.5(x+1)^2}, \; x < 0 \\ \qquad\qquad\qquad\qquad\qquad , \; x \in [\text{-}1,\ 1] \\ \dfrac{5}{\sqrt{2\pi}} e^{-12.5(x-1)^2}, \; x \geq 0 \end{cases} \tag{3}$$

We mainly analyze the effects of the cross-layers evolution of different initial opinions on the final adoption rate and the velocity of the diffusion. The diffusion process of the innovation is triggered by a recovered agent that is randomly selected and set as the constant positive opinion. This random selection is different from the optimal selection in many previous works [9, 18, 19]. The great complexity of our model is the main reason, which will be discussed in Section 3.2.

2.2 Cross-Layers Evolution

Depending on the opinions held by interacted agents in multiple layers, each agent's opinion is cross-layers evolved.

2.2.1 Sequences of Cross-Layers Evolution

As suggested in [7], the cross-layers propagation of information is directional. In L_1, each agent can check the unread messages and forward information. In L_2, each agent can contact with linking neighbors. In real life, the above three behaviors are intertwined. To simplify the sequences of the cross-layers evolution, it is assumed that each agent checks and forwards messages after the contacting in L_2. Then, the agent that has not adopted the innovation will make buying decision according to oneself opinion. After the adoption of the innovation, the subsequent evolution of individual opinion mostly depends on the expectation of individual and the quality or availability of the innovation [4]. This part needs to establish the complex model of the relationship between agents and the product, which is beyond the scope of this paper. Therefore, it is also assumed that $o_i(t)$ becomes constant if $s_i(t)$ = 1. Flow diagrams of the cross-layers evolution are shown in Figure 2.

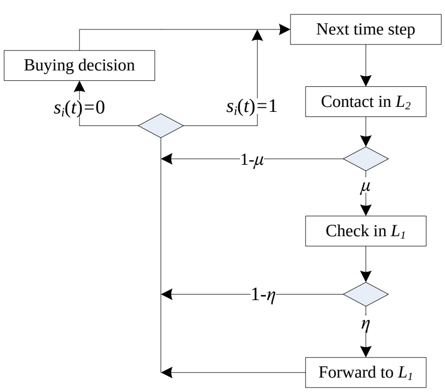

Figure 2 Flow Diagrams of Cross-Layers Evolution

2.2.2 Spatial-Temporal Synchronization Process

The diffusion efficiencies of the conjoint layers are diverse because of the two main reasons. First, the temporal dynamics in the multiple layers are different. In the real world, individual always logins to the online social network periodically and check all the unread information in one time [18, 24, 25]. On the contrary, the contacts between agents in the physical network are sparser because a person occasionally meets his/her friends or acquaintances. The phenomenon is generally named as the spatial-temporal synchronization process [25], which has to be captured in the cross-layers evolution of opinions. Second, the face-to-face communication is more persuasive. Therefore, the recovered agents in L_2 show larger effects on the opinion evolution

The opinion of an agent evolves based on the conformity bias rule [21-23]. Each agent can iteratively update its opinion by partly assimilating other agents' opinions which are within a confidence region. The confidence region is denoted by ε [22], and the factor α represents the biased weight of assimilation [23]. The confidence region and the weight of assimilation are both identical for all agents. In the following, the spatial-temporal synchronization process is introduced according to the flow diagrams of cross-layers evolution as shown in Figure 2.

In the physical network L_2, two linking agents communicate with a given probability λ at each discrete time t. The contact in physical network is synchronous for two agents. Therefore, the contact probability λ can be simplified as the probability that the edge linking two agents in L_2 is available at each time interval and is assumed as identical for all agents. The agent that has adopted the innovation shows larger influence on the opinion evolution. Thus, the opinion of a recovered agent will affect a_i even if the opinion is not within the confidence region. Let $E_i(t)$ denote the set of agents, that will influence a_i at time t

$$E_i(t) = \{a_j \in N_i^2 \mid s_j(t) = 1 \; or \; \; |o_j(t\text{-}1) - o_i(t\text{-}1)| \leq \varepsilon\}\,. \tag{3}$$

If $E_i(t)$ is empty, $o_i'(t)$ = $o_i(t$ - 1). Otherwise, $o_i'(t)$ is calculated by

$$o_i'(t) = (1\text{-}\alpha) o_i(t\text{-}1) + \alpha \frac{\sum_{a_j \in E_i(t)} o_j(t\text{-}1)}{|E_i(t)|}\,. \tag{4}$$

At each discrete time, a_i will check the unread messages in L_1 with the probability μ_i and forward messages with the probability η_i. Similarly, the checking probability and forwarding probability are assumed to be identical for all agents. If a_i checks unread messages at time t, $E_i'(t)$ denote the set of agents, that will affect a_i at time t

$$E_i'(t) = \{a_j \in N_i^1 \mid \; |o_j(t') - o_i'(t)| \leq \varepsilon\}\,. \tag{5}$$

The parameter $o_j(t')$ indicates the opinion held by agent j forwarded at time t' which is between the time t and the last checking time of a_i. Then, $o_i(t)$ is updated by

$$o_i(t) = (1\text{-}\alpha) o_i'(t) + \alpha \frac{\sum_{a_j \in E_i'(t)} o_j(t')}{|E_i'(t)|}\,. \tag{6}$$

If $E_i'(t)$ is empty, $o_i(t)$ = $o_i'(t)$. Then, agent i will forward $o_i(t)$ in OSN with the probability η.

For convenience, the probabilities of checking and forwarding messages in L_1 and the contacting probability in L_2 are denoted as the probability set {μ, η, λ} of the spatial-temporal synchronization process. By setting λ = 0 or μ = η = 0 (other elements are non-zero), our model can simulate the diffusion process in the single OSN or the physical network.

2.2.3 Buying Decision

Each agent will decide to change the adoption state after the evolution of the opinion in multiple layers. We generalize the linear threshold model [7, 9, 15] to simulate the adoption of the innovation. Agent i is assigned with a threshold $\theta_i \in [0, 1]$. Then, a_i makes the buying decision if

$$o_i(t) \geq \theta_i. \tag{7}$$

Positive opinion indicates the positive intention, attitude or emotion and negative opinion cannot induce the buying behavior. The final ratio of adopting the innovation highly relies on the threshold distribution in social networks. However, some empirical studies have suggested that the reliable formulas of threshold distributions according to different types of collective human behaviors still remain unknown [28]. In this paper, θ_i is initialized to follow the uniform distribution [7].

3. ANALYSIS

In this section, we first analyze the relationship between different distributions of initial opinions and the diffusion process of the innovation. Then, we present the complexity of the proposed model by proving the non-monotonicity and non-submodularity. Meanwhile, the relationship between the scope of awareness and the adoption rate is discussed.

3.1 Different Distributions of Initial Opinions

The final adoption rate and the velocity are two main parameters associated with the diffusion dynamics in social networks [7]. The fraction of recovered agents in the multiplex networks indicates the adoption rate of the innovation. Generally, the time to reach a certain rate of the adoption is utilized to represent the velocity of the adoption. Let $\delta(t)$ and $\delta'(t)$ denote the adoption rate with any two distributions of opinions $f_t(x)$ and $g_t(x)$ at time t, respectively.

Lemma 1 *$\delta(t) > \delta'(t)$, if $\int_0^1 xf_t(x)dx > \int_0^1 xg_t(x)dx$.*

Proof: The fraction of recovered agents indicates the probability of a randomly selected agent that remains the recovered state. With the opinion distribution $f_t(x)$, the probability of an agent with the recovered state at time t is $p_t(x > \theta)$, which is the joint probability distributions of $f_t(x)$ and $\Theta(\theta)$. The factor $\Theta(\theta) = 1$ which indicates the uniform distribution of thresholds. Then,

$$p_t(x > \theta) = \frac{\int_0^1 f_t(x)dx \int_0^x \Theta(\theta)d\theta}{\int_{-1}^1 f_t(x)dx \int_0^1 \Theta(\theta)d\theta} = \frac{\int_0^1 xf_t(x)dx}{\int_{-1}^1 f_t(x)dx} \tag{8}$$

According to the known condition, $\int_{-1}^1 f_t(x)dx = 1$. Then, $p_t(x > \theta) = \int_0^1 xf_t(x)dx$. Likewise, $p_t'(x > \theta) = \int_0^1 xg_t(x)dx$. Therefore, $p_t(x > \theta) > p_t'(x > \theta)$. Because the probability of being in recovered state with $f_t(x)$ is larger than the one with $g_t(x)$, $\delta(t) > \delta'(t)$. □

The $f_t(x)$ and $g_t(x)$ means the distributions of evolved opinions at time t. Let $f_0(x)$ and $g_0(x)$ denote the two distributions of initial opinions at time $t = 0$.

Theorem 2 *For any $f_0(x)$ and $g_0(x)$, $\delta(t) > \delta'(t)$ at any time t, if $\int_0^1 xf_0(x)dx > \int_0^1 xg_0(x)dx$ and $f_0(x)$ and $g_0(x)$ are both axially symmetric.*

Proof: Because $\int_0^1 xf_0(x)dx > \int_0^1 xg_0(x)dx$, the opinions of agents with the distribution $f_0(x)$ approach the positive extreme side. Meanwhile, based on the Lemma 1, it can be derived that more agents with the distribution $f_0(x)$ become the recovered state at time 0. Let $h_t(x)$ denote the distribution of opinions of recovered agents with the distribution $f_0(x)$ at time t. Thus, $\int_{-1}^0 xh_t(x)dx = 0$. The factor n is the number of agents in multiplex networks. Let $b(t)$ denote the numbers of recovered agents at time t. The opinion $o'(1)$ of an agent that has not become recovered state at time 0 firstly evolves in L_2. Based on the mean-field method which means the homogeneous contacts with infected agents and recovered agents [26], $o'(t)$ can be calculated by

$$o'(1) = (1-\alpha)o(0) + \alpha \frac{n\int_{o(0)-\varepsilon}^{o(0)+\varepsilon} xf_0(x)dx - b(t)\int_{o(0)-\varepsilon}^{o(0)+\varepsilon} xh_0(x)dx + b(t)\int_0^1 xh_0(x)dx}{n\int_{o(0)-\varepsilon}^{o(0)+\varepsilon} f_0(x)dx - b(t)\int_{o(0)-\varepsilon}^{o(0)+\varepsilon} h_0(x)dx + b(t)\int_0^1 h_0(x)dx}. \tag{9}$$

Because the opinions of agents with the distribution $f_0(x)$ approach the positive extreme side, the assimilated opinions of interacted agents is larger than the one with the distribution $g_0(x)$. With the distribution $f_0(x)$, the increment of opinion $(o'(1) - o(0))$ is larger. Then, $o'(1)$ evolves in L_1 and the opinions assimilated by the agent in L_1 is also larger than the one with the distribution $g_0(x)$

$$o(1) = (1-\alpha)o'(1) + \alpha \frac{n\int_{o'(1)-\varepsilon}^{o'(1)+\varepsilon} xf_0(x)dx}{n\int_{o'(1)-\varepsilon}^{o'(1)+\varepsilon} f_0(x)dx} \tag{10}$$

Thus, the increment of opinion $(o(1) - o(0))$ with the distribution $f_0(x)$ is larger and it can still be satisfied that $\int_0^1 xf_1(x)dx > \int_0^1 xg_1(x)dx$ at time 1 and $\int_0^1 xf_t(x)dx > \int_0^1 xg_t(x)dx$ by the same reasoning. Based on Lemma 1, it can be derived that $\delta(t) > \delta'(t)$. □

Based on Theorem 2, it can be derived that the final adoption rate with the polarized distribution of initial opinions is the largest of the three typical distributions. Meanwhile, the velocity with the polarized distribution is also the largest.

3.2 Complexity of the Model

With the aid of two simple case studies, we show the complicated diffusion processes triggered by different agents and prove the non-monotonicity and non-submodularity of the proposed model. The final adoption rate of the innovation in viral marketing also highly relies on the properties of the selected agents that trigger the diffusion process. The selected agents are usually named as seeds [9].

As shown in Figure 3, the 3-tuple states of agents are marked near the corresponding nodes in L_1. The confidence region and threshold are both 0.3 and identical for all agents. The biased weight of assimilation is 0.8. The red arrow indicates the opinion evolution between agents if the agent in the arrow side assimilates the opinion of the agent in the initial point. In order to facilitate the analyses of the diffusion processes, the three elements of the spatial-temporal synchronization process (probability set) are all set as one.

The first case study shows the non-monotonicity of the proposed model and discusses the relationship between the scope of the awareness and the rate of the adoption. The monotonicity means that the final adoption rate increases if more agents are selected to trigger the diffusion process [9]. Mathematically, the function of the adoption rate σ is monotone if the sets of seeds $S' \subset$ S and $\sigma(S') \leq \sigma(S)$. The factor $\sigma(S)$ means the final rate of the adoption triggered by the agents in the set S. In Figure 3(a), only a_1 is selected and $S' = \{a_1\}$. It can be found that all agents in the multiplex networks successively become recovered and the diffusion of the adoption coincides with the diffusion of the awareness. The opinion held by a_2 evolves in L_1, and a_5 decides to adopt the innovation because of the contact with a_4 in L_2. The opinions held by a_3 and a_4 are cross-layers evolved.

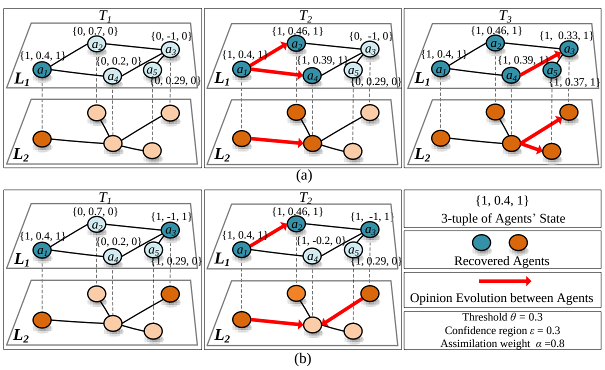

Figure 3 Illustrations of Non-monotonicity: (a) Seeding a_1; (b) Seeding a_1 and a_3.

In Figure 3(b), the set of seeds S is $\{a_1, a_3\}$. Because a_3 initially holds the extreme and negative opinion, a_4 is affected and holds the negative opinion. Meanwhile, the sum of agents' opinions in L_2 is less than zero, and the positive opinions held by a_1 in L_1 and a_5 in L_2 are not within the confidence region. Thus, the opinion of a_4 remains negative after the cross-layers evolution of many time intervals. Then, a_4 and a_5 will not adopt the innovation. Compared with the diffusion process in Figure 3(a), one more agent is selected as seeds but the final adoption rate decreases. This phenomenon violates the rule of the monotonicity, and the negative opinion of the innovation is the main reason. Meanwhile, the awareness of the innovation spreads more quickly than the one in Figure 3(a).

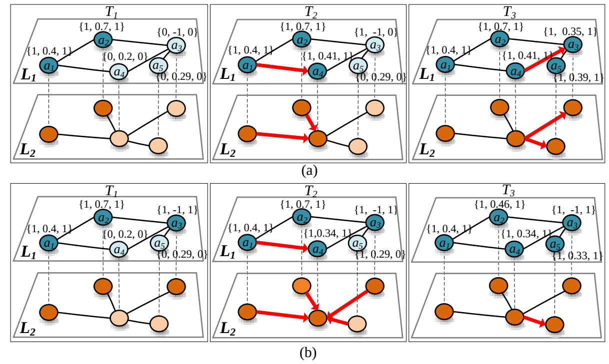

Figure 4 Illustrations of Non-submodularity: (a) Seeding a_1 and a_2; (b) Seeding a_1, a_2 and a_3.

The second case study shows the non-submodularity. If for all sets $S' \subset S \subset V$ and any $A \subset V$ satisfy $\sigma(S' \cup A) - \sigma(S') \geq \sigma(S \cup A) - \sigma(S)$, the function of the adoption rate σ is submodular. The submodularity indicates that the increment of the adoption rate decreases if more and more agents are selected as seeds [9]. Compared with the diffusion processes in Figure 3, the set A only consists of the additional seed a_2. It can be found that $\sigma(\{a_1, a_2\}) - \sigma(\{a_1\}) \leq \sigma(\{a_1, a_2, a_3\}) - \sigma(\{a_1, a_3\})$, which does not meet the rule of the submodular. Compared with the diffusion process in Figure 3(a), the additional seed a_2 only improves the positive opinions of the innovation instead of the adoption rate and velocity. The reason of the non-submodularity is complicated, probably because of the cumulative effects of the seeds on some crucial agents.

The complexity of the model indicates the great difficulty of the seeds selection because the monotonicity and submodularity mean that the approximately optimal seeds (lead to the largest adoption rate) can be found by greedy algorithms [9, 18]. Therefore, in order to reduce the influences of the different seeds on the final adoption rate, we first randomly select an agent and then set the opinion as a constant value. The first case study shows that the wide scope of awareness about the innovation does not mean the high rate of the adoption. This relationship will be validated by extensive simulations.

4. SIMULATION

The viral marketing in multiplex networks has been simulated on a computer. In this field, there are still no real data of the conjoint online social network and the physical network. Many existing studies perform the simulations on the synthetic multiplex networks which are generated by random or small-world network models [7, 8]. In order to improve the accuracy of the simulation, L_1 is the New Orleans regional network in Facebook with an average node degree of 25.3 [29]. As most users in the regional network live in a city, L_2 is assumed to be a small-world network. In the small-world layer, the degree of each node is 20 and the probability of interpolating between regular lattices is 0.1. There are 63,731 agents in the simulated multiplex network. The confidence region is 0.3. The thresholds of agents follow uniform distribution $U(0, 1)$.

The average fraction of recovered agents ($s_i(t) = 1$) and the average fraction of agents with $w_i(t) = 1$ in stationary states indicate the final adoption rate and the scope of the awareness, respectively. The velocity of diffusion process is evaluated by comparing the time when half of agents have known or adopted the innovation. Each trial is performed with 200 replications. Each replication takes 1000 time intervals.

4.1 Systematic Effect of Cross-Layers Evolution

To present the systematic effect of cross-layers evolution of opinions on the diffusion processes of the awareness and the adoption, we conduct experiments with different parameter settings. This section consists of two parts of experiments. First, to clarify the impacts of the multiple layers on diffusion processes, the probability set $\{\mu, \eta, \lambda\}$ is initialized as {0.5, 0.5, 0}, {0, 0, 0.1} and {0.5, 0.5, 0.1}, respectively. Then, the diffusion processes in the OSN, the physical network and the multiplex networks can be compared with each other. The assimilation weight α is set as 0.2 and 0.8. An agent is randomly initialized as the recovered state and the opinion is set as 0.5 to trigger the diffusion process. The results are shown in Figure 5 and Figure 6.

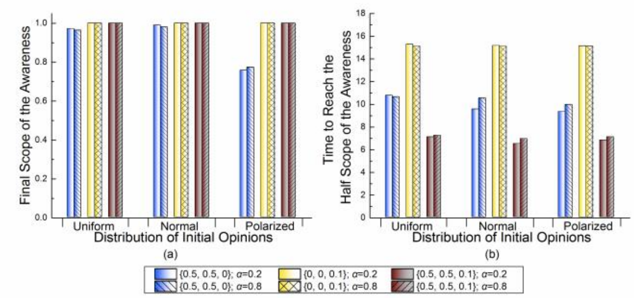

Figure 5 Diffusion of the Awareness: (a) Final Scope of the Awareness; (b) Time to Reach the Half Scope of Awareness

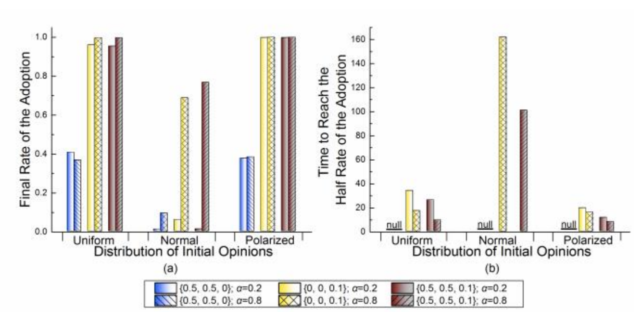

Figure 6 Diffusion of the Adoption: (a) Final Rate of the Adoption; (b) Time to Reach the Half Scope of

From Figure 5, it can be found that the distributions of initial opinions have no obvious influence on the diffusion processes of the awareness of the innovation. Nearly all agents in social networks have known the innovation except that information spreads on in L_1 (OSN) and the distribution of initial opinions is polarized. The probable reason is that the great differences between agents' opinion and many nodes with very small degrees inhibit the diffusion process. As shown in Figure 5(b), information spreads quickly in OSN, compared with the one in the physical network. In the multiplex networks, the velocity of diffusion process increases by conjoining the OSN and the physical network together. Meanwhile, the weight of assimilating others'

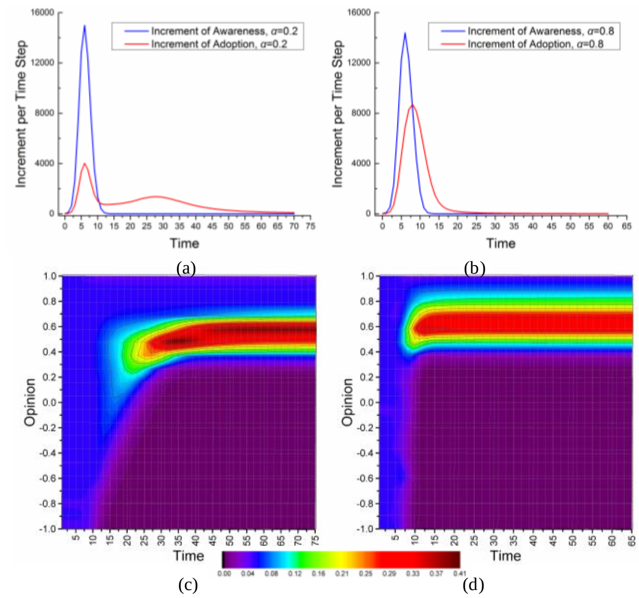

Figure 7 Uniform Distribution of Initial Opinions

opinions has little influence on the diffusion processes of the awareness.

However, the global diffusion of the awareness cannot always lead to the high rate of the adoption as shown in Figure 6. First, with probability set {0.5, 0.5, 0}, the rate of adoption in single L_1 is the lowest compared with other probability sets. The multiplex networks with probability set {0.5, 0.5, 0.1} can lead to the highest rate of adoption within the shortest time. Second, the adoption rate is the lowest if the initial opinions follow the normal distribution. It is because that the consensus between agents is near zero and it is lack of enough agents with positive opinions (near one) if the initial opinions follow normal distribution. Meanwhile, the final adoption rate and the velocity increase with a larger value of the assimilating weight. It is because that each agent adjusts the opinion by a big margin and it is easy to reach a consensus for a group of agents if the assimilating weights are large.

In the second part of experiments, we further discuss the close relationship between the cross-layers evolution of opinions and the diffusion process. Only the probability set (0.5, 0.5, 0.1) is utilized. The assimilation weight α is set as 0.2 and 0.8. The line graphs indicate the diffusion processes of the innovation which mean the incremental numbers of agents that have known and adopted the innovation per time step. The processes of opinion evolution are presented with the aid of the heat maps. The change in color of the heat map indicates the change in the fraction of agents that hold different opinions. The abscissa of the line graph is corresponding to the one of the heat map with the same distribution of initial opinions and assimilation weight.

It can be found that a double-peak pattern of the increment of the adoption emerges if the weight of assimilation is 0.2 in Figure 7 and Figure 8. The first peak of the increment of the adoption emerges after the peak of the diffusion process of the awareness. If the initial opinions follow the uniform or polarized distribution, many agents with positive opinions adopt the innovation quickly. Most recovered agents in the period of the first peak are “impulsive” shoppers with more positive opinions or smaller thresholds.

According to the assumption of our model, agents that have adopted the innovation hold constant and positive opinions. Therefore, agents with negative opinions can be generally persuaded and positive consensus on the innovation can emerge. This progressive formation of positive consensus on the innovation is the main reason of the second peak. The trend of the second peak is smoother than the one of the first peak, and it emerges when the diffusion of the awareness is terminated. By comparing the abscissas of the line graph and the heat map in the (a) and (c) sub-graphs of Figure 7 and Figure 8, agents gradually make consensus on the innovation in the duration of the second peak. In this period, most agents are “demanding” shoppers with negative opinions or large thresholds. On the contrary, the second peak of the increment disappears if the weight of assimilation is 0.8 in Figure 7 and Figure 8. It means that the positive consensus on the innovation emerges quickly and nearly follows the diffusion of the awareness. However, it can still be found that the opinions of agents gradually become constant after the peak in the (d) sub-graphs of Figure 7 and Figure 8.

The relationship between evolved opinions and diffusion process of the innovation is more complex with the normal distribution of initial opinions. As shown in Figure 9, the double-peak pattern emerges if the assimilation weight is 0.8. Before the diffusion process of the adoption is terminated, agents have made consensus at many points of opinions. Meanwhile, the point of the final consensus is much lower than the one with uniform and polarized distributions. The normal distribution of initial opinions means that agents have no preference for the innovation and certainly leads to the very low rate and velocity of the adoption. By comparing the consensus points in heat maps with the adoption rates in Figure 6(a), it can be found that low point of consensus leads to the low rate of the adoption.

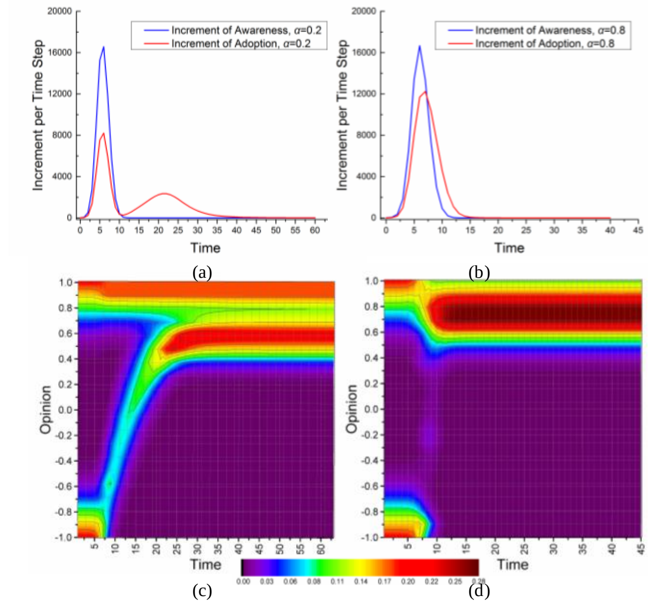

Figure 8 Polarized Distribution of Initial Opinions

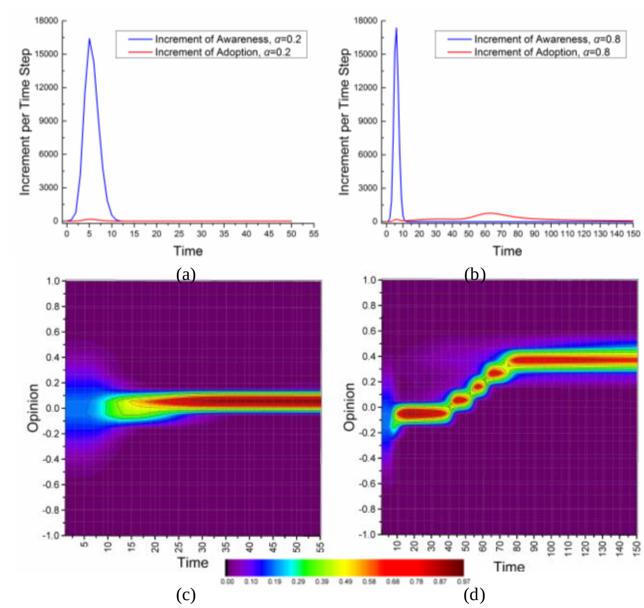

Figure 9 Normal Distribution of Initial Opinions

In conclusion, we show that the high scope of awareness and the high rate of the adoption are not invariably linked. The consensus formation of opinions is closely related to the diffusion process of the innovation. First, consensus formation indicates the termination of the diffusion process. Second, the point of consensus reflects the rate of the adoption.

In fact, people generally have similar and positive comments on the best-selling products. The double-peak pattern indicates an interesting phenomenon in the case of no competition. In the real world, the agents that hold negative opinions on the innovation may appreciate similar products of other companies. The second peak of the increment of the adoption may disappear because people can buy the alternative product such as the Android camp and the Apple camp of smart phone.

4.2 Is Viral Marketing the Best Means?

The main object of this section is to further analyze the differences between diverse distributions of initial opinions and discuss the applicability of the viral marketing. At first, an agent is randomly initialized as infected state and the opinion is set as 0.5 to trigger the diffusion process of the awareness. The sale of the innovation begins after the diffusion of the awareness in order to highlight the effect of the evolved opinions. Until the time of open sale (t_{op}), agents cannot buy the innovation (become recovered states). The probability set is {0.5, 0.5, 0.1} and the weight of assimilation is set as 0.2 and 0.8. The results are shown in Figure 10 and Figure 11.

From Figure 10, it can be found that the adoption rate decreases with the delay of the time of open sale if the initial opinions follow the uniform distribution. With normal distribution of initial opinions, the rate and velocity of the adoption are both the lowest. However, the t_{op} has no obvious influence on the adoption rate with polarized distribution of opinions but greatly improves the velocity of the adoption as shown in Figure 11. Meanwhile, the velocity of the adoption with uniform distribution of initial opinions is the fastest if the t_{op} is 10 and decreases if the t_{op} is larger than 10. It means that a certain time of the diffusion of the awareness can facilitate the adoption of the innovation.

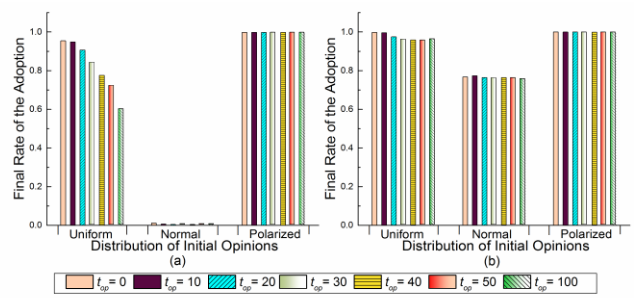

Figure 10 Final Rate of the Adoption: (a) α = 0.2; (b) α = 0.8

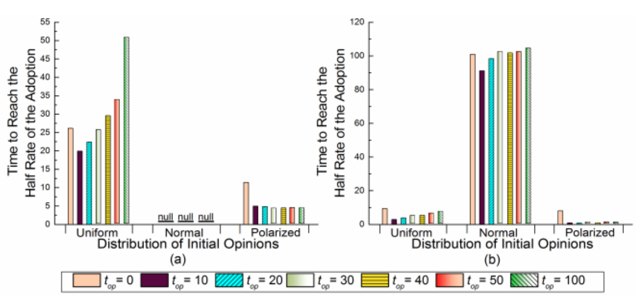

Figure 11 Time to Reach the Half Rate of the Adoption: (a) α = 0.2; (b) α = 0.8

In conclusion, the diffusion of the innovation can be facilitated if the initial opinions follow the polarized distribution. This conclusion accords with the Theorem 2. Because many agents hold positive and extreme opinions, the adoption rate is still high and stabile even if the opinions are evolved. However, with uniform and normal distributions, the number of agents hold extreme opinions decreases as the opinions gradually evolves.

As Watts and coworkers declare, the viral marketing is hard to implement [30]. The unknown distribution of initial opinions and the unexpected process of opinion evolution may be the underlying reasons. It is suggested that the viral marketing is a suitable and economic advertising method if there are many supporters (fans) in the social networks or the innovation can evoke great interest of most people. In some cases, the viral marketing is only a complementary method of advertising to the traditional mass marketing [30]. For the normal distribution of initial opinions, the mass marketing in TV or the advertising column of online social networks can help to introduce the innovation and induce more positive opinions.

5. DISCUSSION AND FUTURE WORK

In this paper, we propose a novel agent-based model to capture the cross-layers evolution of opinions and simulate the diffusion of an innovation in multiplex networks. We analyze the evolution process of different distributions of initial opinions and find that the formation of the consensus is closely related to the diffusion of an innovation. The formation of the consensus indicates the termination of the diffusion process. The high point of the consensus is equivalent to the high rate of the adoption. Moreover, we discuss the applicability of the viral marketing according to different distributions of initial opinions.

A restriction of our work is that we assume the identical model of opinion evolution in the online social network and the physical network. This may not be appropriate for some types of diffusion processes. For instance, a person can examine the comments of a web link or a rumor spreading in the OSN. The consensus or the major point of these comments can greatly influence the person, instead of the opinions of linking neighbors. Extending our model to capture the different features of opinion dynamics in OSN and physical network is possible in principle. It may require a diffusion process of a specific entity, which is supported by real field example.

Another restriction, which we have already discussed, is the simplification of the relationship between agents and the innovation. This relationship leads to the more complicated diffusion process, and the conjoint layers will be a double-edges sword. In the multiplex networks, the online social network provides the quick spreading of the information about the innovation, and the contact in the physical network is essential to persuade the following adopters. However, the unexpected experiences of a few users or some gossip also spread quickly. Future work aimed at capturing this relationship should carefully consider the effect of the negative information [24] on the cross-layers evolution of opinions and the adoption of the innovation.

It is hoped that our work can provide new insights into the study of viral marketing in multiplex networks. For instance, besides the widest scope of the awareness about the innovation, the selected seeds should induce the most positive consensus, compared with other sets of agents. For two competing brands, it is very important to promote competitive product before the second peak of adopting an innovation emerges. Moreover, how to allocate the limited budget [14] into the multiple channels is also a new challenge to the issue of viral marketing in multiplex social networks, because the costs of handing out free samples in the physical network and promoting methods in the online social networks are different.

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