Software reliability: probability

- Calculating probability of orthogonal events
- Elementary probability
- Calculating the Probability of Failure on Demand (POFOD)

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What is probability?

- A real number in the range 0..1
- Represents the chance that a certain event will occur
- Examples:
  - The probability of "heads" in a coin toss: 0.5
    - Represents a 50%-50% chance that it be "heads"
  - The probability for rain tomorrow: 0.14
    - Based on statistical analysis, meter/map readings…
  - The probability for a student to drop out before graduation is 0.4
    - Based on numbers of students who dropped out before graduation

How is probability calculated?

- Difficulty: Prediction under uncertainty
  - How can we tell if it will rain tomorrow?
  - How can we tell if the coin toss will result in “heads”?
  - How can we tell if ‘ A Smith’ will drop-out before graduation?
- We cannot!
- All we can do is: “predict” chances based on history

Calculating probability of equiprobable discrete events

- When all events have equal probability:

$$\frac{\#\text{suitable events}}{\#\text{possible events}}$$

Prob. of discrete (equiprobable) events: example

- Probability for "head" in one coin toss:

$$\frac{\text{H}}{\text{H/T}} = \frac{1}{2} = 0.5$$

- Probability for all "heads" in two coin tosses:
  - Possible events:
    - head, head
    - Head, tail
    - Tail, head
    - Tail, tail

$$\frac{\text{HH}}{\text{HH/HT/TH/TT}} = \frac{1}{4} = 0.25$$

Context: Probability of Failure on Demand

- **POFOD (probability of failure on demand)**
- This is the probability that the system will respond correctly when a request is made for service at a given point in time. This metric is used for systems where demands for service are intermittent and relatively infrequent over the lifetime of the system.
- Example - The likelihood that the system will fail when a user requests service. A biometric authentication device that fails to correctly identify or reject users on an average of **once** out of a **hundred** times has a POFOD of 1%.

Probability of Failure on Demand

- **POFOD (probability of failure on demand)**
  - Measured using the history
  - Number of failed tests divided by the total number of tests

$$\frac{\#\text{failed tests}}{\#\text{tests}} = \frac{2}{20} = 0.1$$

Probability of complementary events

- If p_e is the probability that en event e occurs, what is the probability that e will NOT occur?

$$1 - p_e$$

NOT probability: examples

- Probability for "head" NOT to appear in one coin toss:

$$1 - p_{head} = 1 - \frac{1}{2} = 0.5$$

- Probability for anything OTHER THAN two "heads" in two coin tosses:

$$1 - p_{head,head} = 1 - \frac{1}{4} = 0.75$$

- Probability for no failure during execution:

$$1 - p_{failure} = 1 - \frac{2}{20} = 0.9$$

Probability: lottery example

- No. of lottery tickets: 2,000
- No. of prizes: 20
- Probability of winning the lottery:

$$\frac{20}{2,000} = 0.01$$

- Probability of NOT winning in the lottery:
  - Calculation 1:

$$1 - p_{win} = 0.99$$

  - Calculation 2:

$$\frac{1,980}{2,000} = \frac{99}{100} = 0.99$$

AND Probability of orthogonal (independent) events

- *Probability of event P1 AND P2 happening*
- P_1
  - probability of event #1
- P_2
  - probability of event #2
- $P_1 \times P_2$
  - probability of event #1 AND event #2

AND probability: examples

- Probability of getting the number 6 in tossing a die: $\frac{1}{6}$
- Probability of getting the numbers 6, 6 in tossing two dice:
- Probability of NOT winning the lottery: 0.99 $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
- Probability of NOT winning the lottery twice: 0.99x0.99

Example: software failure

- A PRINT operation from MS Outlook will succeed if both software components do not fail:
  - MS Outlook (failure probability: 0.1)
  - Printer driver (failure probability: 0.2)
- What is the probability that PRINT succeeds?

$$p_{success} = p_{succOutlook} \times p_{succPDF} = \\ = (1-0.1) \times (1-0.2) = 0.72$$

- What is the probability that PRINT fails?

$$p_{fail} = 1 - p_{success} = 0.28$$

OR probability of mutually exclusive events

- *Probability of event P1 OR event P2*
- P_1
  - probability of event #1
- P_2
  - probability of event #2
- Events are mutually exclusive
- $P_1 + P_2$
  - probability of event #1 OR event #2

OR probability: examples

- Probability for "head" OR tail to appear in one coin toss:
  - 0.5 + 0.5 = 1

- Probability for anything OTHER THAN two "heads" in two coin tosses:
  - $P_{TT} + P_{TH} + P_{HT} = 0.25 + 0.25 + 0.25 = 0.75$

- Also can be calculated using the NOT rule:
  - Same as the probability of NOT getting two "heads"
  - $1 - P_{HH} = 1 - 0.25 = 0.75$

Summary

- Probability of (equiprobable, discrete) event:

$$\frac{\#\text{suitable events}}{\#\text{possible events}}$$

- NOT probability (complement):

$$1 - p_e$$

- AND probability (orthogonal events):

$$p_1 \times p_2$$

- OR probability (mutually exclusive events):

$$p_1 + p_2$$

- POFOD

$$\frac{\#\text{failed tests}}{\#\text{tests}} = \frac{2}{20} = 0.1$$

  - likelihood that the system will fail when a user requests service

Exercise

- The probability of an error for each cell (mobile phone network unit) is 0.1. Your mobile phone needs only one cell to connect to the network, and it is situated within the range of three cells. You start your phone and check connection (which takes about a second). What is the probability that this operation fails?