Algorithm analysis

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- What if you just bought a faster computer?
- (And, honestly, it's too difficult.)

Usually, we don't care about exact timings (real-time systems excepted).

Primitive operations

Instead of estimating time in seconds, we estimate the number of primitive operations:

- variable assignments
- arithmetic operations
- essentially, CPU instructions

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- essentially, CPU instructions

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But what if you buy a computer with a different CPU instruction set?

It doesn't matter, because we'll abstract away from this level of detail.

Scalability

We want to know how algorithms' performance scales as the input gets large:

- Express running times as functions of input size, n.
- e.g., "Inserting into a sorted list takes time proportional to n."
- e.g., "... into a priority queue takes time proportional to $\log_2(n)$."

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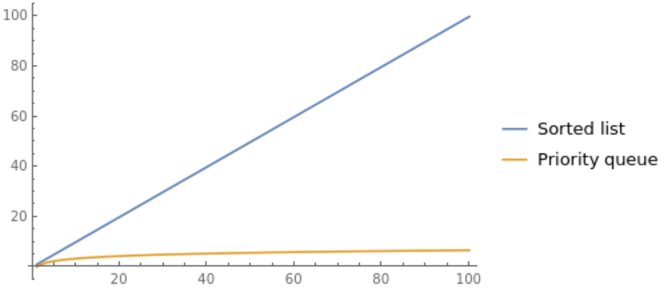

Asymptotic behaviour

We're interested in **asymptotic** performance: large inputs, as $n \to \infty$.

> Suppress constant factors and lower-order terms.
>
> — Tim Roughgarden, *Algorithms Illuminated, Part I*.

Ignoring constant factors

"The number of steps to insert an item into a sorted list is proportional to the list's length" – What's the constant of proportionality?

- Depends on hardware, compiler, etc.
- Exactly what we're trying to abstract away.
- It's probably a small number (like 10, not a million) so it doesn't matter much.

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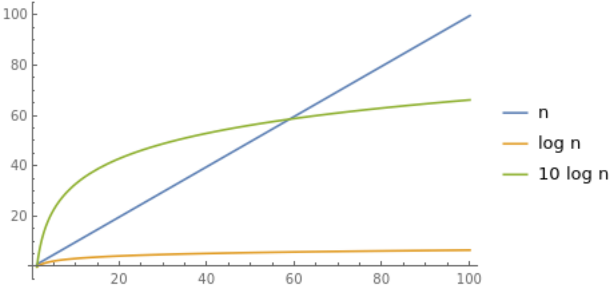

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- e.g., implementing stacks as arrays vs lists.

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When Choosing between similar approaches:

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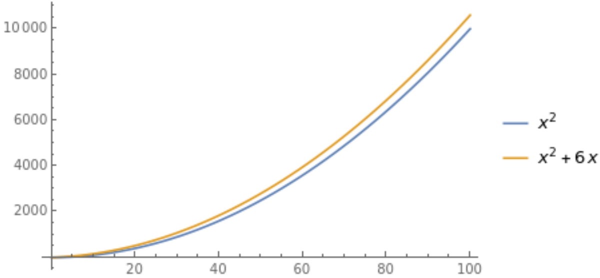
- e.g., implementing stacks as arrays vs lists.

When writing code: twice as fast is twice as fast.

Ignoring lower-order terms

Lower-order terms are ones that become insignificant as $n \to \infty$.



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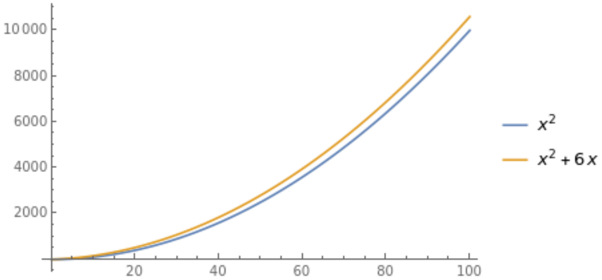

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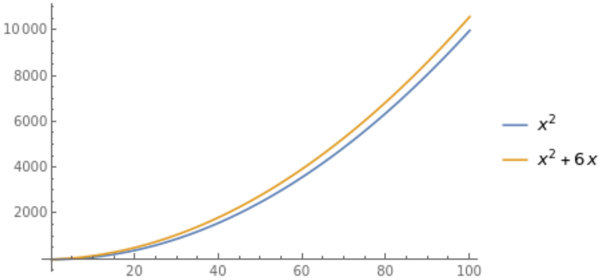

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- For $x = 10$, $6x$ is 40% of $x^2 + 6x$.
- For $x = 100$, it's 6%; for $x = 1000$, it's 0.6%.
- For $x > 6$, $x^2 + 6x < 2x^2$ and we already agreed to ignore constant factors.

Can we really ignore lower-order terms?

Usually, yes.

- For small inputs, any algorithm will do.
- It's large inputs that need us to be smart.
- Lower-order terms are negligible for large inputs.

Asymptotic notation

We want a system for comparing functions while ignoring constant factors and lower-order terms.

Definition

Let $f(n)$ and $g(n)$ be functions $\mathbb{N}_{>0} \to \mathbb{N}_{>0}$.
We write $f(n) = O(g(n))$ if there are constants n_0 and c such that, for all $n \geq n_0$, $f(n) \leq c\, g(n)$.

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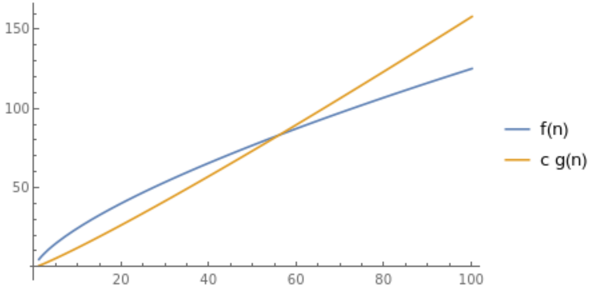
We say "f is big-O of g".

"Constant" means that n_0 and c cannot depend on n in any way.

Big-O, visually



Think of $f = O(g)$ as "f is sort-of less than g."

Big-O examples (1)

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Big-O examples (3)

For any constants k, and $a_0, \ldots, a_k$,
let $p(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$.

E.g., $p(n) = 5n^4 - 16n^3 + 0n^2 + 3n - 1$.

Claim: $p(n) = O(n^k)$.

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Summary. Any polynomial is big-O of its leading term.

Big-O is ignoring constant factors and lower-order terms.

Big-O examples (4)

$n^{k+1} \neq O(n^k)$.

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For all c and all $n \geq c$, $n^{k+1} \geq c\, n^k$.

So we cannot find c and n_0 such that $n^{k+1} \leq c\, n^k$ for all $n \geq n_0$.

Properties of big-O

Big-O behaves a lot like $\leq$ on numbers:

- $f(n) = O(f(n))$ for all functions f.
- $f(n) + g(n) = O(\max\{f(n), g(n)\})$.
- If $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = O(h(n))$.

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- $\log n = O(n^k)$ for any $k > 0$, even non-integer k.
- E.g., $\log n = O(\sqrt{n})$, since $\sqrt{n} = n^{1/2}$.

Big-O and constants

Let $f(n) = k$ be any constant function.

For all $n \geq 1$, $f(n) \leq k \cdot 1$, so $f(n) = O(1)$.

$f(n) = O(1)$ if, and only if, there is a constant c such that $f(n) \leq c$ for all $n \geq 1$.

Big-O cheat sheet

Common functions ordered by big-O ("sort-of less than"):

$$k, \ldots, \log(\log n), \log n, \sqrt{n} = n^{1/2}, n,$$
$$n, n \log n, n^{1.0001}, n^2, n^3, \ldots, 2^n, 2^{n^2}, \ldots$$

Big-O's friends (1)

Definition

Let $f(n)$ and $g(n)$ be functions $\mathbb{N}_{>0} \to \mathbb{N}_{>0}$.
We write $f(n) = \Omega(g(n))$ if there are constants n_0 and c such that, for all $n \geq n_0$, $f(n) \geq c\, g(n)$.

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So big-Ω is "kind-of greater than".

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We write $f(n) = \Theta(g(n))$ if there are constants n_0, c and d such that, for all $n \geq n_0$, $c\,g(n) \leq f(n) \leq d\,g(n)$.

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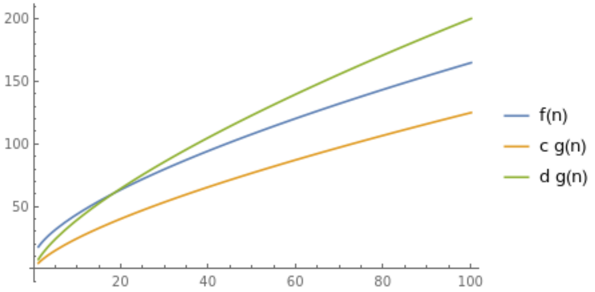
Equivalently, $f(n) = \Theta(g(n))$ if, and only if, $f(n) = O(g(n))$ and $g(n) = O(f(n))$.

So big-Θ is "approximately proportional to".

Big-Θ, visually

Here, $f(n) = \Theta(g(n))$.



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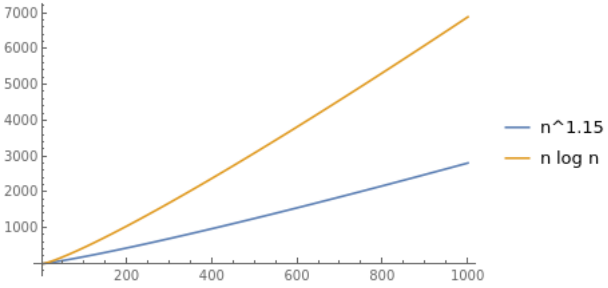
Works for most functions you'll come across in this course, but be careful!



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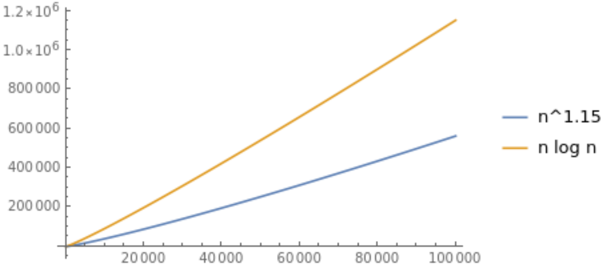

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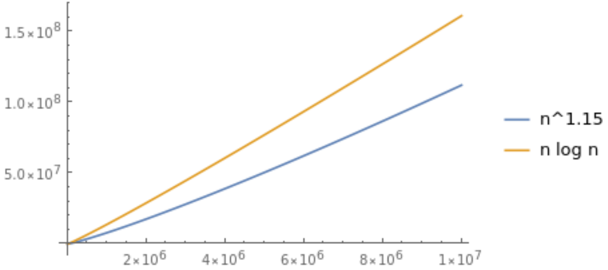

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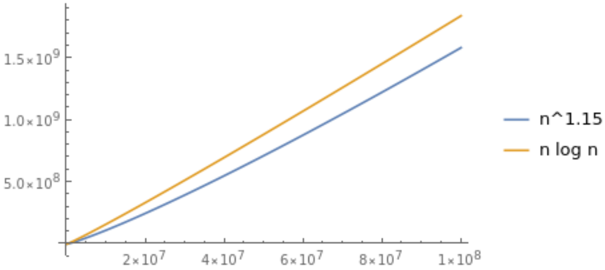

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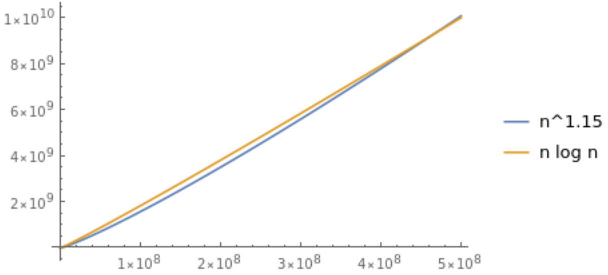

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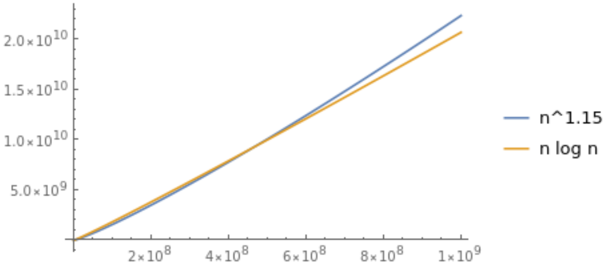

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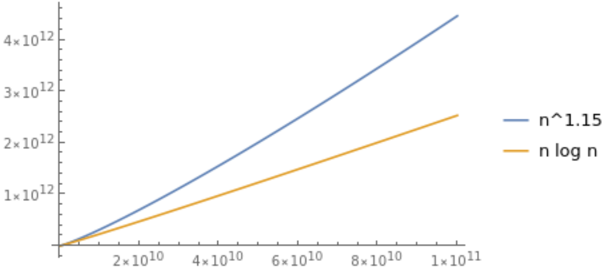

Tight bounds

Asymptotic notation allows us to bound one function in terms of another.

A bound is **tight** if it can't be simplified or made more precise.

Examples:

- $\pi < 4$ and $\pi > 3$ are tight integer bounds on pi.
- $\pi \leq 4$ and $\pi > 0$ are not tight – but true and may be useful!
- $n^2 + 3n + 4 = O(n^2)$ is tight.
- $n^2 + 3n + 4 = O(n^2 + n)$ and $= O(n^3)$ are not tight.

Pet peeve

There is no such thing as "the big-O of a function", as in "What is the big-O of $n \log n + 3n$?"

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There are infinitely many correct answers:

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- $n \log n + 3n = O(n \log n),\ O(n^2),\ O\big(2^{2^n}\big)$, ...

Instead, ask "What is a tight big-O bound for $n \log n + 3n$?"

Not my pet peeve

Some people object vehemently to writing, e.g., "$3n + 4 = O(n)$" and insist on "$3n + 4 \in O(n)$".

Their point:

- $3n + 4$ is a function
- formally, $O(n)$ is a set of functions
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This is true but most people write $=$.

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- f is at most approximately proportional to g;
- For all large n, $f(n) \leq c\,g(n)$.

Asymptotic notation – summary

"Suppress constant factors and lower-order terms."

$f(n) = \Theta(g(n))$

- f is approximately proportional to g;
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$f(n) = \Omega(g(n))$

- f is at least approximately proportional to g;
- For all large n, $f(n) \geq c\,g(n)$.

Practical program analysis

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If your program's input has length n and it runs in time...

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- $\Theta(n \log n)$ then doubling n slightly more than doubles the time.
- $\Theta(2^n)$ then adding 1 to n doubles the time. (Eek!)

Analysis of Java programs (1)

Simple statements take time $O(1)$

- assignments
- evaluating expressions without method calls
- jumps (e.g., `break`, skipping past an `if` when the condition is false)

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Method calls:

- Analyze the method to find it runs in time $O(f(n))$.
- Jumping to the method and back takes time $O(1)$.
- Total is $O(1 + f(n)) = O(f(n))$;

Analysis of Java programs (2)

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if ([condition]) {
    [block 1]
} else {
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if ([condition]) {
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} else {
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- Analyze the condition and two blocks to find they run in time $O(f_c)$, $O(g_1)$ and $O(g_2)$.
- Total is $O(1 + f_c + \max\{g_1, g_2\}) = O(\max\{f_c, g_1, g_2\})$.

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- Analyze the loop to find it runs $O(t(n))$ times.

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}
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- Analyze `[condition]` and `[block]` to find they run in time $O(f_{\mathrm{c}})$, $O(g)$.
- Analyze the loop to find it runs $O(t(n))$ times.
- Total is $O(1 + (1 + g(n))t(n)) = O(g(n)t(n))$.

Analysis of Java programs (4)

```
for ([initializer]; [condition]; [increment]) {
    [block]
}
```

is equivalent to

```
[initializer]
while ([condition]) {
    [block]
    [increment]
}
```

Analysis of Java programs (5)

A specific, common case of `for` loops:

```
for (int i = 0; i < n; i++) {
    [block]
}
```

- We know the loop runs n times.
- Total cost is n times the cost of executing `[block]`.

Example analysis

```
1  for (int i = 0; i < n; i++)
2      for (int j = 0; j < n; j++)
3          if (A[i] == B[j])
4              duplicates++;
```

- Line 4 runs in time $O(1)$ (simple statement).

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- `for (j...)` runs lines 3–4 n times: takes time $O(n \times 1) = O(n)$.
- `for (i...)` runs lines 2–4 n times: takes time $O(n \times n) = O(n^2)$.