CE213 Artificial Intelligence – Lecture 13

Neural Networks: Part 1

What Is a Neural Network?
Why Do We Need Neural Networks?
How to Design or Train a Neural Network?

McCulloch-Pitts Neuron and Neural Networks (NN Part 1)

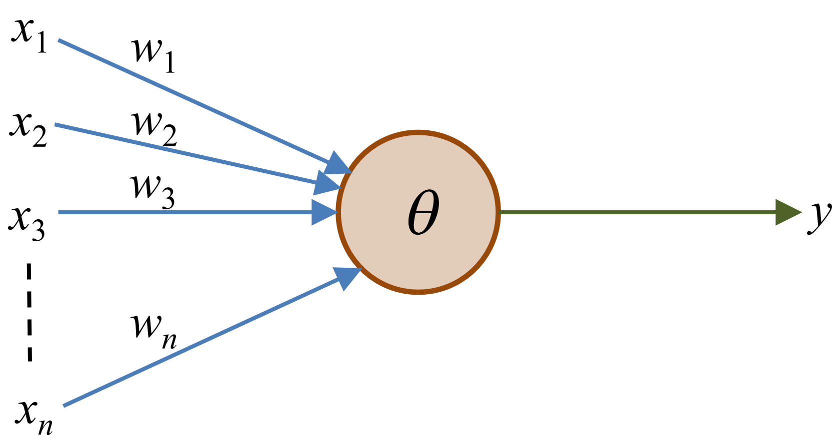
Learning Using The Delta Rule (NN Part 2)

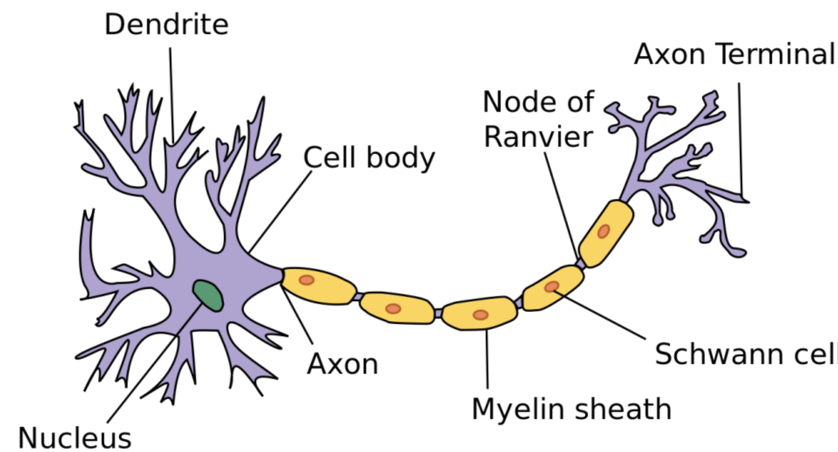
The Generalised Delta Rule & Back-Propagation Networks (NN Part 3)

McCulloch-Pitts Neuron Model

It is called an MP neuron (figure on the left) for simulating a typical biological neuron (figure on the right).

The relationship between the input and output of an MP neuron can be described by the following diagram:





where

x_i is the i^{th} input

w_i is the weight of the i^{th} input

θ is a threshold

y is the output

A typical biological neuron (Approximately 100 billion neurons in the human brain)

McCulloch-Pitts Neuron Model (2)

The output of an MP neuron is computed from the input as follows:

1) Calculate the weighted summation of inputs:

$$\sum_{i=1}^{n} w_i x_i = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$$

2) Compare the weighted sum of inputs with the threshold to generate output:

$$\sum_{i=1}^{n} w_i x_i \geq \theta \rightarrow y = 1$$

$$\sum_{i=1}^{n} w_i x_i < \theta \rightarrow y = 0$$

The output of an MP neuron is binary. Its input takes continuous values in general, but binary values for logic.

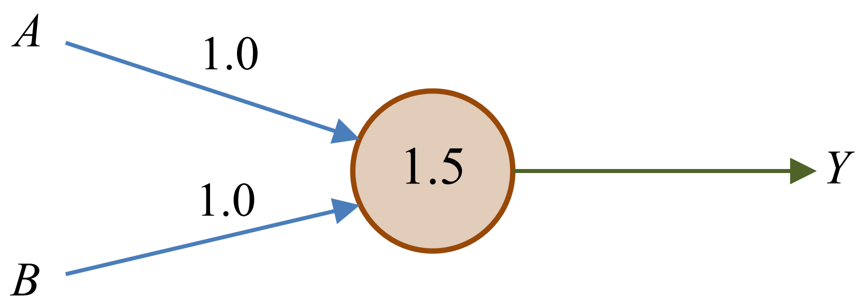
Basic information processing in an MP neuron: **weighted sum** and **thresholding**, which could be quite powerful (e.g., fault diagnosis).

This is more like a logic gate than a biological neuron.

What Can an MP Neuron Compute?

Assume inputs are binary (i.e., 1 or 0)

Implementation of AND



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

If A + B $\geq$ 1.5, Y = 1.

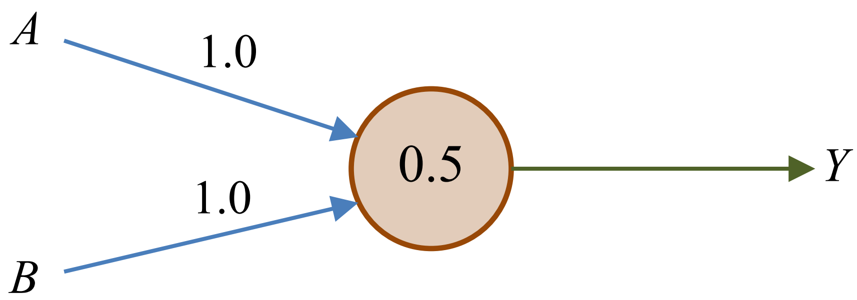
Otherwise, Y = 0.

Of course, there could be many other correct designs, e.g., changing 1.0 to 1.1 or 1.2 would do.

(*A* and *B* correspond to input x_1 and x_2 in the MP neuron model presented in the previous slides. The weighted sum is $w_A \cdot A + w_B \cdot B$, where $w_A = w_1 = 1.0$ and $w_B = w_2 = 1.0$.)

What Can an MP Neuron Compute? (2)

Implementation of OR



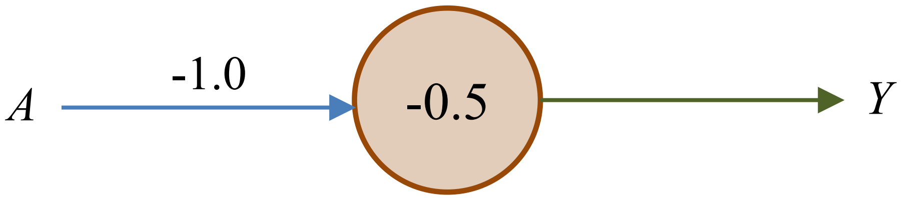
If A + B $\geq$ 0.5, Y = 1 .

Otherwise, Y = 0.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

What Can an MP Neuron Compute? (3)

Implementation of NOT



If -A < -0.5 or A > 0.5, Y = 0.

Otherwise, Y = 1.

A	Y
0	1
1	0

What Can an MP Neuron Compute? (4)

Are there any Boolean functions that an MP neuron cannot compute?

How about implementation of XOR (exclusive OR)?

A	B	A xor B
1	1	0
1	0	1
0	1	1
0	0	0

(Truth table of XOR)

What Can an MP Neuron Compute? (5)

Suppose the weights are w_A and w_B, and weighted sum of inputs to the MP neuron is $w_A \cdot A + w_B \cdot B$.

Then the following equations should be satisfied for computing XOR (check the equations on slide 3):

$w_A + w_B < \theta$	(1)	(for A=1, B=1, Y=0)
$w_A \geq \theta$	(2)	(for A=1, B=0, Y=1)
$w_B \geq \theta$	(3)	(for A=0, B=1, Y=1)
$0 < \theta$	(4)	(for A=0, B=0, Y=0)

According to (2), (3) and (4), θ, w_A and w_B must be positive. If both (2) and (3) are true, then (1) must be false,

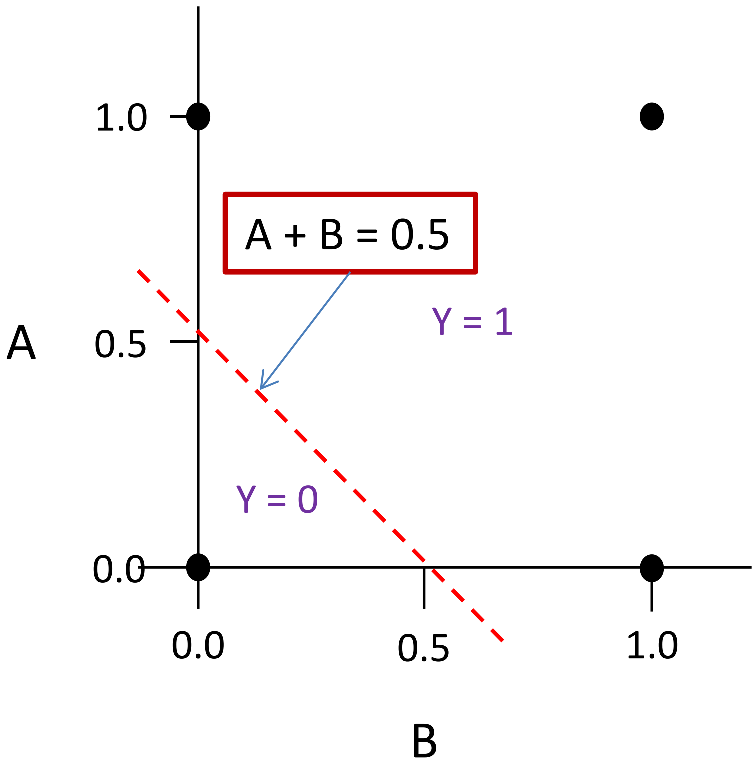
Therefore, there is no combination of values for weights and threshold that can create an MP neuron for computing the XOR function.

An MP neuron cannot compute XOR!

What Can an MP Neuron Compute? (6)

A Graphical Interpretation

Let's consider the implementation of **OR:**
the values of weights and threshold
of the MP neuron are:
$w_A = w_B = 1;\ \theta = 0.5$



What values of A and B will make Y = 1?

Y=1 at all points where $w_A \cdot A + w_B \cdot B \geq \theta$

i.e., whenever A + B $\geq$ 0.5

Therefore, from the figure on the right we can see that

All points above or on the line described by **A + B = 0.5** lead to Y =1

All points below the line lead to Y = 0

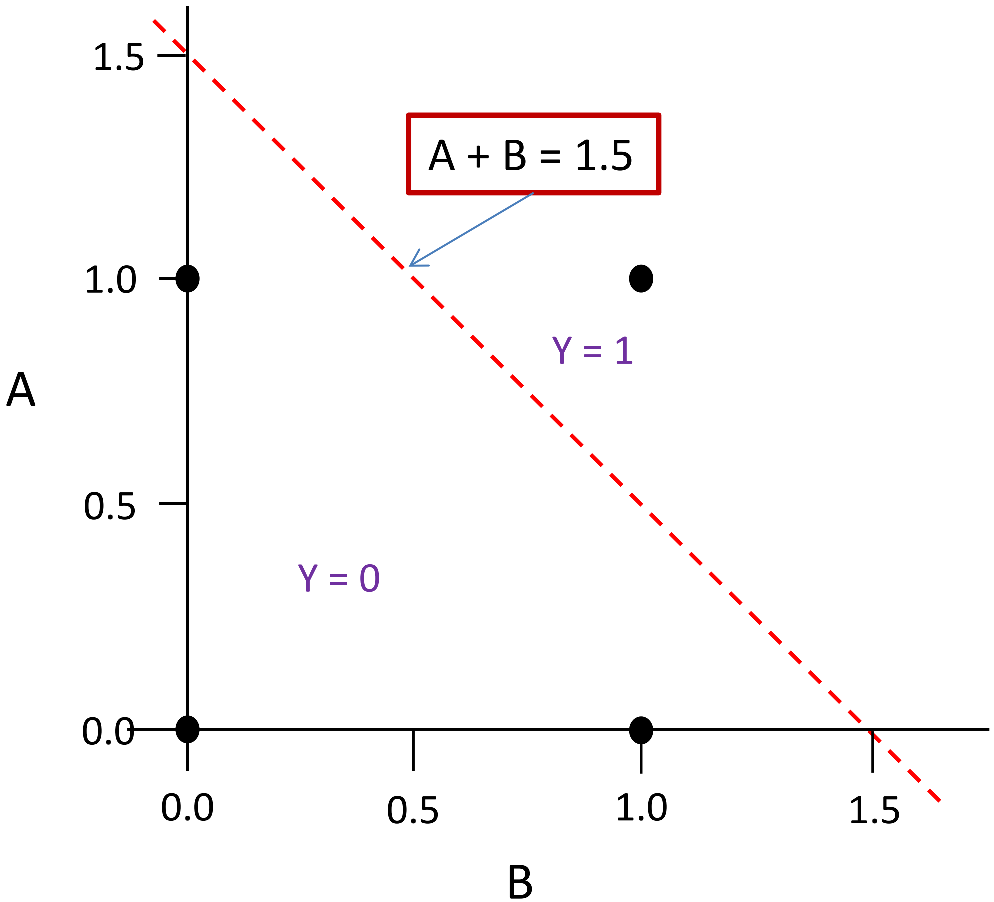
What Can an MP Neuron Compute? (7)

Similarly, for the implementation of **AND**, the values of weights and threshold of the MP neuron are:

$w_A = w_B = 1$ and $\theta = 1.5$,

which define a line in the figure on the right:

$w_A \cdot A + w_B \cdot B = \theta$, or A+B = 1.5.



Therefore,

All points above or on the line described by **A + B = 1.5** lead to Y =1

All points below the line lead to Y = 0.

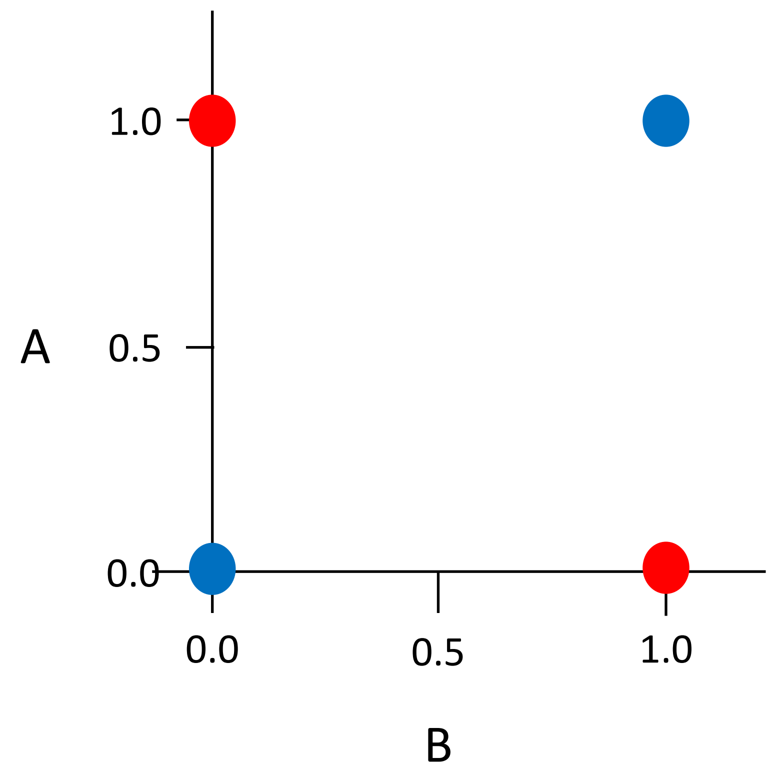
What Can an MP Neuron Compute? (8)

So, what about implementation of XOR?

Y should be 1 at red spots.
Y should be 0 at blue spots.

There is no straight line that can separate the blue spots (Y=0) from the red spots (Y=1).

Therefore, it is impossible for an MP neuron to implement logic XOR !



What Can an MP Neuron Compute? (9)

When an MP neuron has more than 2 Inputs (one neuron only)

The function implemented by such an MP neuron depends on

$$\sum_{i=1}^{n} w_i x_i = \theta$$

This is the equation of a **hyperplane** in *n*-dimensional space.

So, an MP neuron divides the space into two regions separated by a hyperplane. On one side of this hyperplane, Y=0, and on the other side, Y=1.

The values of the weights and threshold of the MP neuron determine the position and orientation of this hyperplane. – This may solve complicated classification problems in high-dimensional space! (This had led to the first golden age of neural network research in 1960s)

Linear Separability

The functions that an MP neuron with *n* inputs can implement are those defined by a linear surface:

when $n = 2$ it is a straight line

when $n = 3$ it is a plane

when $n > 3$ it is a hyperplane

Such functions are said to be ***linearly separable***.

For this reason, MP neurons are sometimes called:

***Linear Separation Units* (LSUs)**

(They are not linear units, but linear separation units)

McCulloch-Pitts Neural Networks - More Than One MP Neuron

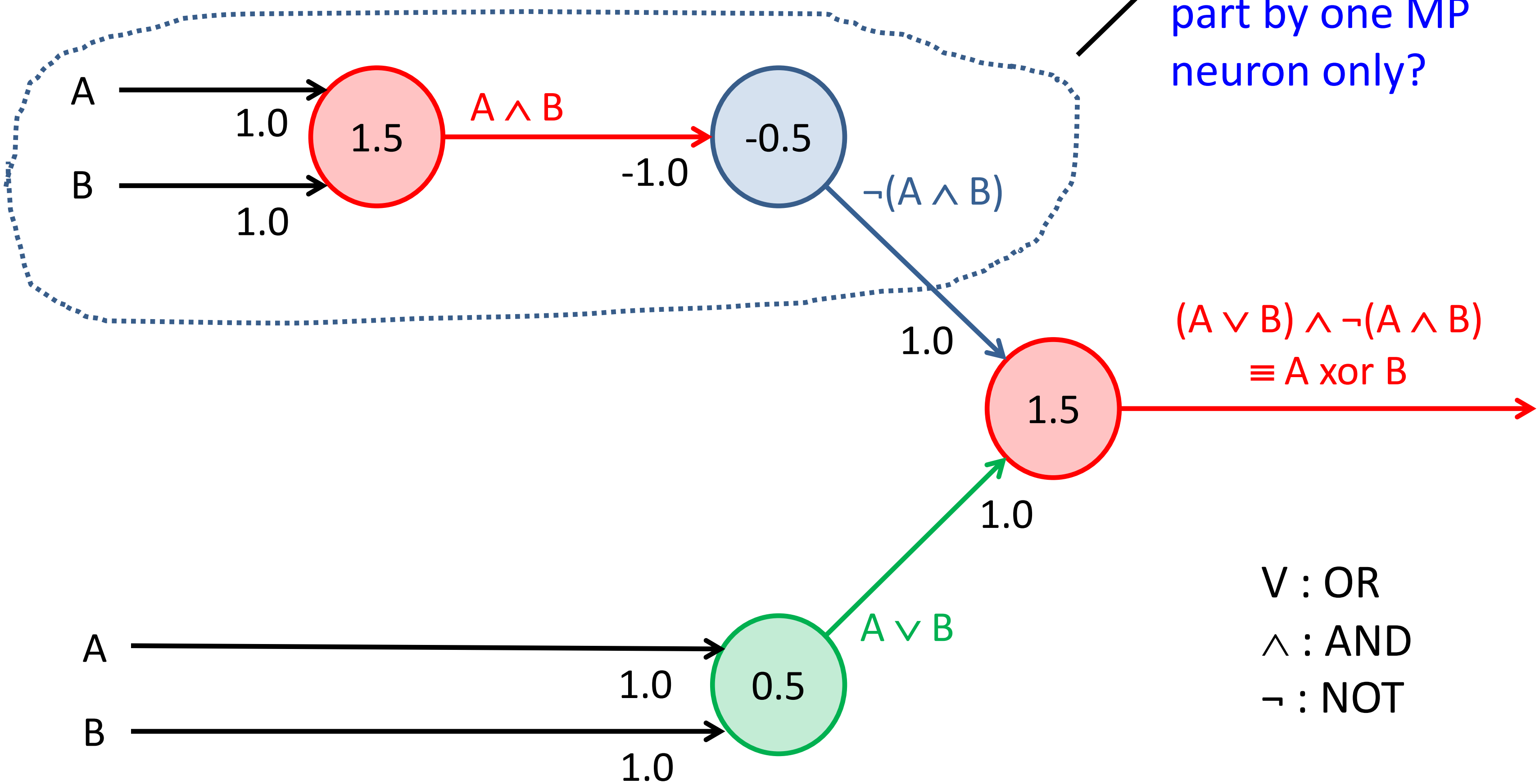
We could obviously use the output of one MP neuron as an input to another MP neuron.

In such a way we could build an ***artificial neural network***.

Could such a neural network (more than one neuron) compute functions that a single neuron could not? - Yes

McCulloch-Pitts Neural Networks (2)

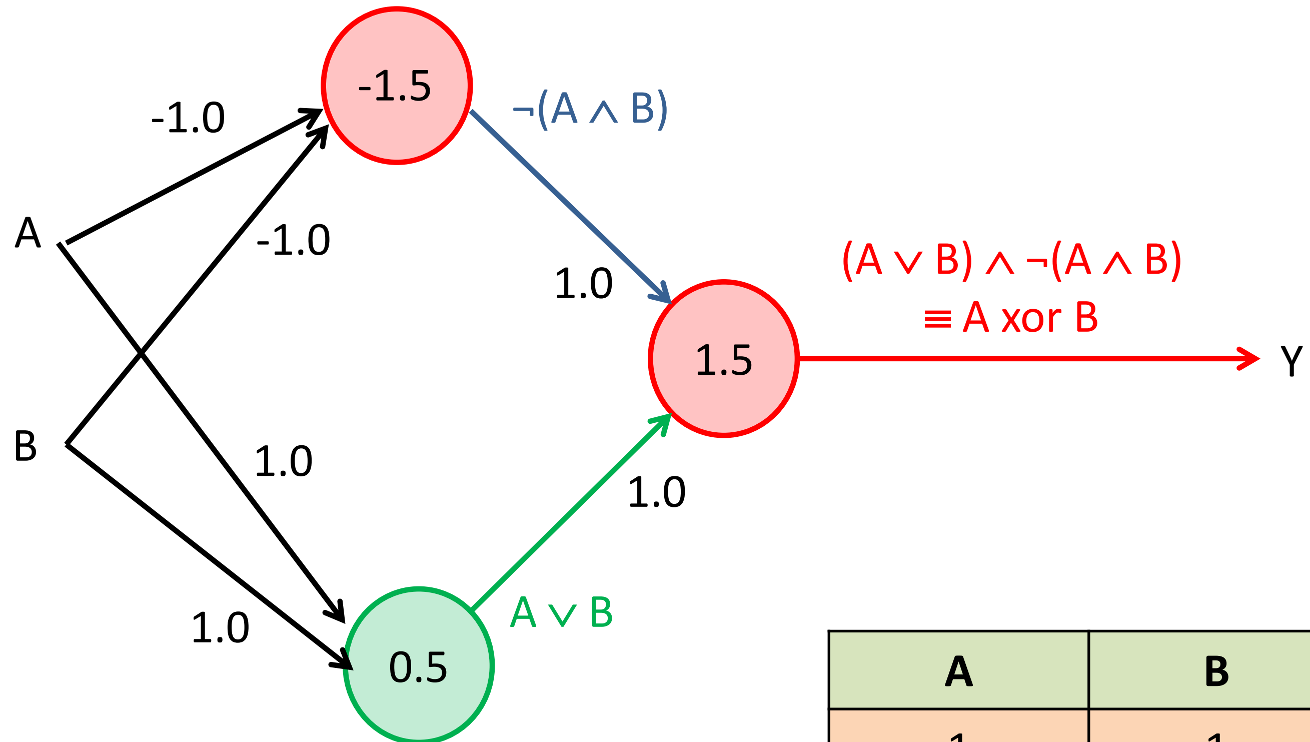
Implementation of XOR with 4 MP neurons



Is it possible to implement this part by one MP neuron only?

V : OR
∧ : AND
¬ : NOT

A Neural Network with 3 MP Neurons for Computing XOR:



Minimum number of MP neurons required for computing XOR?

Calculate the network's output Y with different values of A and B to prove it solves the XOR problem.

A	B	Y
1	1	?
1	0	?
0	1	?
0	0	?

Summary

McCulloch-Pitts Neuron and Neural Networks

MP Neuron (described by a diagram or equations)

What an MP Neuron Can Compute

Linear Separability – One Neuron with More Than 2 Inputs

Neural Network - More Than One Neuron

Designing an MP neural network like the one for computing XOR seems difficult, especially for high-dimensional input problems.

Solution – Learning from data/experience (topic for next lecture)