Left-Orderable Groups

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Overview

1. What is a Left-Orderable Group?
2. The Positive Cone
3. An Example: $\mathbb{Z}^2$
4. Free Groups
5. Why do we care?

What is a Left-Orderable Group?

Strict Orderings

Let G be a group. $<$ is a *strict, total ordering* on G if it is:
i) Transitive ($g < h, g < h \Rightarrow g < h$)
ii) $\forall g, h \in G$, exactly one of $g < h$, $h < g$ or $g = h$ holds.

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Invariant Orderings

G is *left-ordered* if $<$ is *left-invariant*, i.e.

$$\forall g, h, f \in G, g < h \Rightarrow fg < fh$$

The Positive Cone

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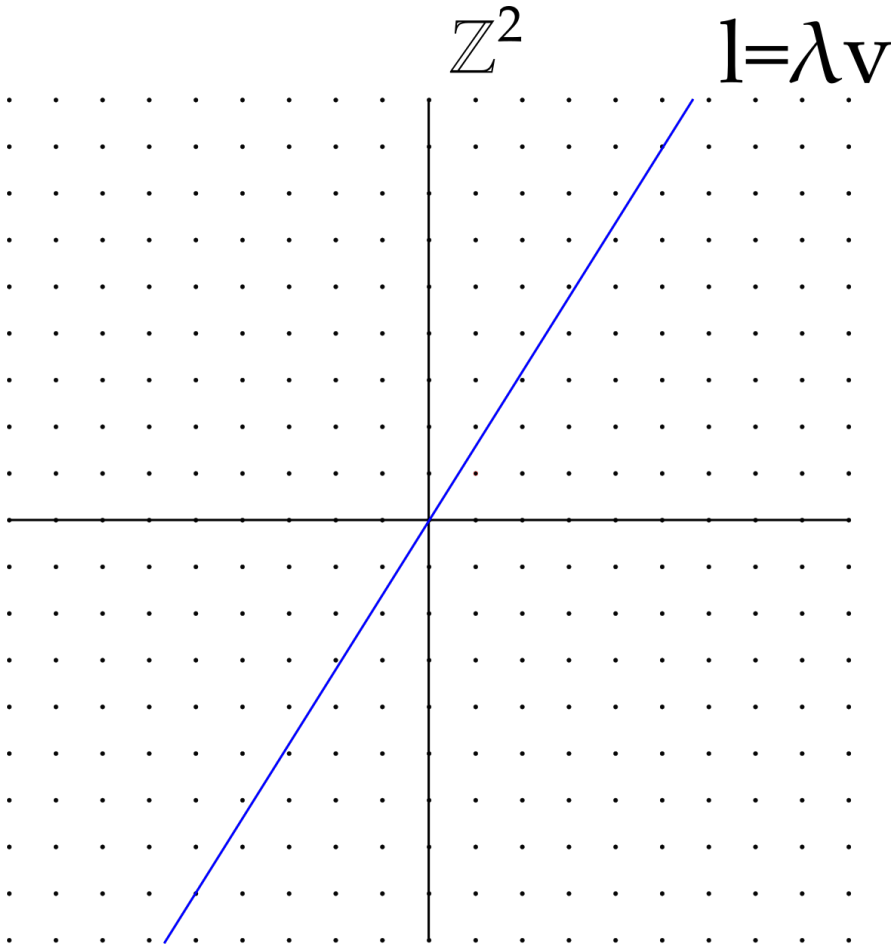
- P must satisfy:
  i) P is a subsemigroup (i.e. $g, h \in P \Rightarrow gh \in P$)
  ii) $P \cup P^{-1} = P \backslash \{1\}$
  iii) $P \cap P^{-1} = \emptyset$

An Example: $\mathbb{Z}^2$

- v a vector in $\mathbb{R}^2$ with irrational gradient.
- $\underline{x} < \underline{y} \iff \underline{x} \cdot v < \underline{y} \cdot v$



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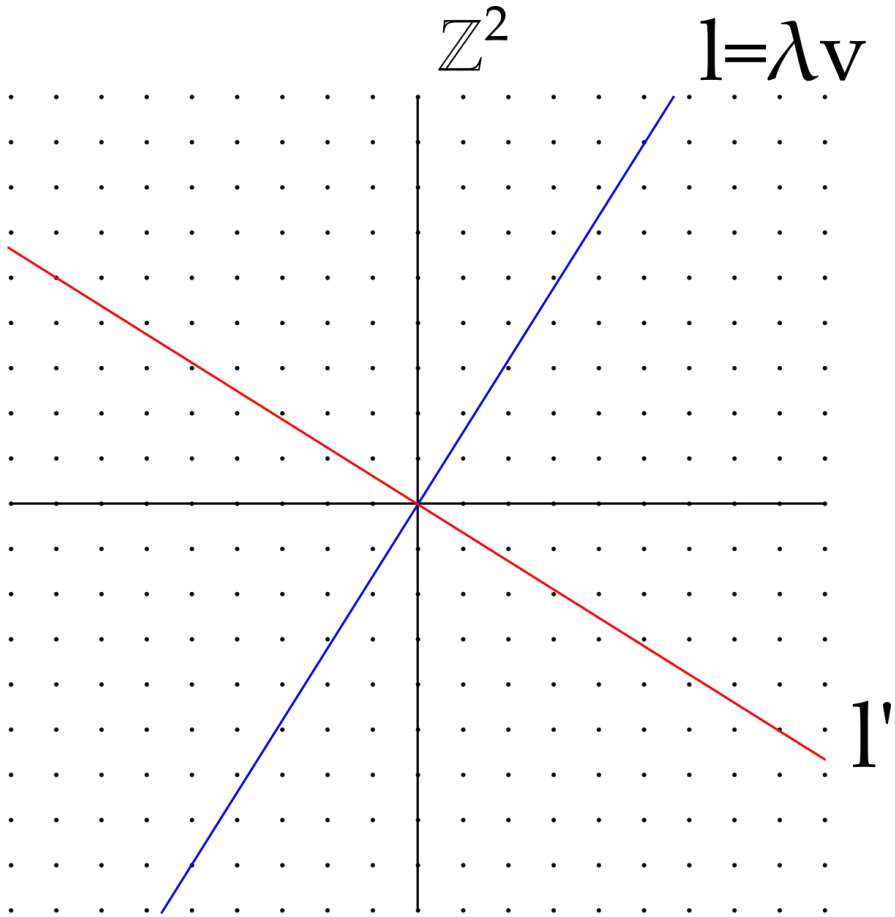
- v a vector in $\mathbb{R}^2$ with irrational gradient.
- $\underline{x} < \underline{y} \iff \underline{x} \cdot v < \underline{y} \cdot v$
- P is one side of $l' \perp l$.



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No Slide Numbering

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Typesetting and Math

The packages `inputenc` and `FiraSans`[1,2] are used to properly set the main fonts.

This theme provides styling commands to typeset *emphasized*, alerted, **bold**, example text, ...

`FiraSans` also provides support for mathematical symbols:

$$e^{i\pi} + 1 = 0.$$

[1] `https://fonts.google.com/specimen/Fira+Sans`
[2] `http://mozilla.github.io/Fira/`

Section 2

Blocks

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Block

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Example block

Example text.

Columns

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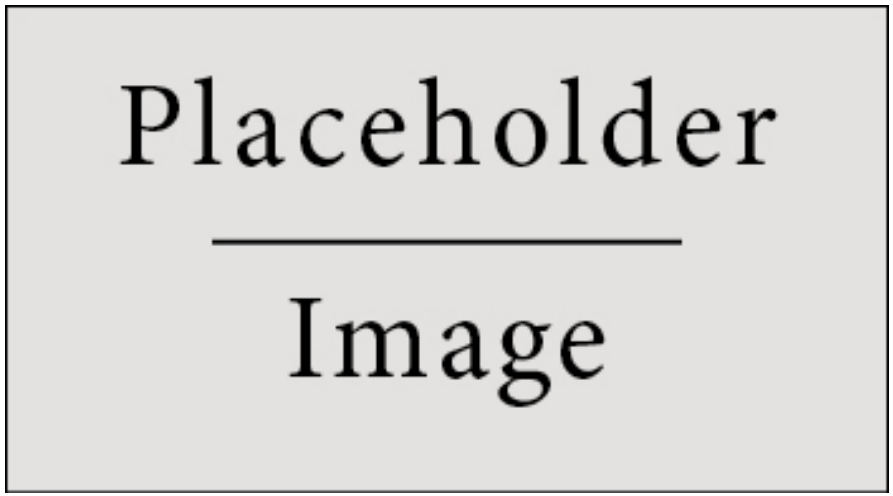

Lists

Items:

- Item 1
  - Subitem 1.1
  - Subitem 1.2
- Item 2
- Item 3

Enumerations:

1. First
2. Second
   2.1 Sub-first
   2.2 Sub-second
3. Third

Descriptions:

First Yes.

Second No.

Table

Discipline	Avg. Salary
Engineering	**$66,521**
Computer Sciences	$60,005
Mathematics and Sciences	$61,867
Business	$56,720
Humanities & Social Sciences	$56,669
Agriculture and Natural Resources	$53,565
Communications	$51,448
Average for All Disciplines	**$58,114**

Table: Table caption

Thanks for using **Focus**!

References

Backup Slide

This is a backup slide, useful to include additional materials to answer questions from the audience.

The package `appendixnumberbeamer` is used to refrain from numbering appendix slides.