Programación de Modelos en Ecuaciones Diferenciales

Ecuaciones Lineales

Ecuación diferencial de primer orden

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Forma estándar:

$$\frac{1}{a_1(x)}\left(a_1(x)\frac{dy}{dx} + a_0(x)y\right) = \frac{1}{a_1(x)}g(x)$$
$$\frac{dy}{dx} + P(x)y = R(x)$$

Factor Integrante

Imagina una función $u(x)$ tal que $\frac{d}{dx}\left(u(x)y\right) = u(x)R(x)$.

Multiplicando la forma estándar por $u(x)$, se obtiene:

$$\boxed{u(x)\frac{dy}{dx} + u(x)P(x)y} = u(x)R(x)$$

Expandiendo la derivada de $u(x)y$, se obtiene:

$$\frac{d}{dx}\left(u(x)y\right) = u(x)\frac{dy}{dx} + \frac{du}{dx}y = \boxed{u(x)\frac{dy}{dx} + u(x)P(x)y}$$

$\therefore \frac{du}{dx}y = u(x)P(x)y$ & $\frac{du}{dx} = u(x)P(x)$.

$$\Rightarrow \frac{1}{u}\,du = P(x)\,dx$$
$$\int \frac{1}{u}\,du = \int P(x)\,dx$$
$$\ln(u) = \int P(x)\,dx + C$$
$$u = e^{\int P(x)\,dx + C}$$
$$u = e^{\int P(x)\,dx}e^{C}$$
$$u = e^{\int P(x)\,dx}C$$
$$C = 1 \rightarrow u = e^{\int P(x)\,dx}$$

$$\therefore \frac{d}{dx}\left[e^{\int P(x)\,dx}y\right] = e^{P(x)\,dx}R(x)$$