Proof By Induction Of My GCD Function

Code:

```
def gcd(x: int, y: int ):
    if y == 0:
        return a
    else:
        return gcd(y, x%y)
```

Theorem:

Given any two non-negative integers, that are not both zero, my function gcd, which is listed above in the code section, will produce the greatest common divisor between the two non-negative integers given to the function

Proof by Induction:

In this proof, I will use x and y to represent the two non-negative integers given to my gcd function that are not both zero, f_{xy} will represent the result of the function, z will be any non-negative number less than f_{xy} and x' and y' will represent my x and y variable through multiple recursions.

Let me first start by stating the definition of the greatest common divisor. If f_{xy} is the greatest common divisor of the two non-negative integers given to my function then the following statements are true:

```
x % fxy = 0
y % fxy = 0
```

And given that z <= f_{xy} and that z is a non-negative number then we can also state:

```
x % z = 0
y % z = 0
fxy % z = 0
```

Now let me state my two cases in my gcd function. We will have two possible inputs that I need to handle. The first being when y = 0 and the second is when y > 0.

Case 1:

When y = 0 we can see in my code that we will first hit my base case, if b = 0, then terminate the gcd function and return the value in x.

Let's now check this with my definition of the greatest common factor. So, first, we must check does x % x = 0. The answer to this will always be yes because anything divide by itself will have no remainder. Second, since y = 0 anything moded with zero will produce zero. So, y % x = 0 stands true in the case. It also stands to reason then that any non-negative number less than x, z which is also a divisor of x, will fit the second part of my greatest common divisor definition.

Case 2:

When y > 0 we can see in my code that we will now go to the second case. Which will call my gcd method again, but this time we will pass y in as the first variable and x % y for the second. We will continue to repeat this process unit the second variable given to my gcd function is equal to zero. At this point, we will hit my base case and return the value in the variable x.

First, let me clarify something important in this process. In the second case, we are passing the second variable in as the result of x%y, as known as y'. So, we can assume that:

```
y' <= x,
y' < y
```

With this stated, we will continue to get numbers smaller and smaller than x' until we produce an x' and y' when moded together give us zero. At this point, we will hit my base case and walk through the same logic as stated in my case 1.

Conclusion:

With all that is stated above, I have shown that for any non-negative x and y values, that are also not both zero, provided to my gcd function you will get a f_{xy} that is the true greatest common divisor of that x and y.