SDSC6015 (Semester A, 2025)

Stochastic Optimization for Machine Learning

Assignment 1

Please submit both your code (e.g. ".py" file if you are using Python) and your results when programming is required. *You can submit either a .py or a .txt file. If you are using other formats, you can copy and paste your code to a .txt file for submission.*

Question 1. [1 point] Consider the perspective function $\boldsymbol{p} : \mathbb{R}^n \times \mathbb{R}_{++} \to \mathbb{R}^n$,

$$\boldsymbol{p}(\boldsymbol{z}, t) = \frac{1}{t}\,\boldsymbol{z}.$$

Suppose $C \subseteq \mathbf{dom}(\boldsymbol{p})$ is a convex set, prove that its image

$$\boldsymbol{p}(C) = \{\boldsymbol{p}(\boldsymbol{x}) \mid \boldsymbol{x} \in C\} \tag{1}$$

is convex.

Question 2. [1 point] Consider a function $p_{\boldsymbol{x}}(t) = \sum_{k=1}^{n} x_k \cdot \cos(k \cdot t)$, where $\boldsymbol{x} \in \mathbb{R}^n$. Prove that the set

$$S = \{\boldsymbol{x} \in \mathbb{R}^n \mid |p(t)| \leq 1 \;\text{ for }\; |t| \leq \pi/3\}$$

is convex.

Question 3. [1 point] Let K be a nonempty cone. Prove that the set

$$K^{\star} = \{\boldsymbol{y} \mid \boldsymbol{x}^{\top}\boldsymbol{y} \geq 0, \;\; \forall \boldsymbol{x} \in K\}$$

is a convex cone.

Question 4. [2 point] Show that the function $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(\boldsymbol{x}) = \log(e^{x_1} + \cdots + e^{x_n})$$

is convex by using second order condition.