



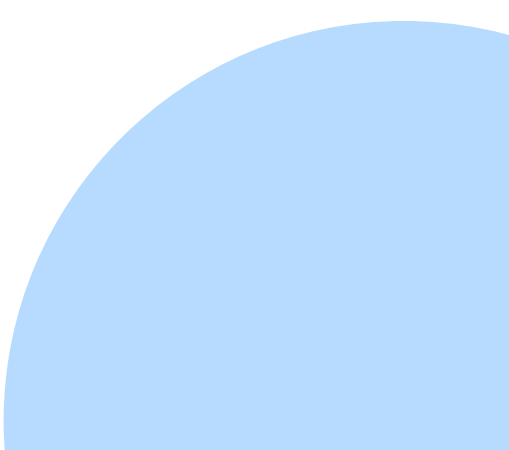
Note of Math Statistics

Course Note

2024-11-04

Lawrence

2024 Year
Fall Season Class



1 Introduction

1.1 Definition

We set a population F and X_1, X_2, \dots, X_n is a random sample of size n from the population F . If X_1, X_2, \dots, X_n are independent and identically distributed random variables, (i.i.d) then their common probability distributed function is:

$$F(x_1)F(x_2)\dots F(x_n) = \prod_{i=1}^n F(x_i) \quad (1.1.1)$$

Their common probability density function is:

$$f(x_1)f(x_2)\dots f(x_n) = \prod_{i=1}^n f(x_i) \quad (1.1.2)$$

Statistic Given a random sample X_1, X_2, \dots, X_n from a population F , a statistic is any real function $g(X_1, X_2, \dots, X_n)$ of the sample values X_1, X_2, \dots, X_n .

Sample mean The sample mean is the statistic defined by:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \quad (1.1.3)$$

Sample variance The sample variance is the statistic defined by:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \quad (1.1.4)$$

Sample standard deviation The sample standard deviation is the statistic defined by:

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2} \quad (1.1.5)$$

They have the following properties:

$$1. \quad \sum_{i=1}^n (X_i - \bar{X}) = 0 \quad (1.1.6)$$

2. We apply a transformation to sample $Y_i = aX_i + b$. Then the sample mean and sample variance of Y_i are:

$$\bar{Y} = a\bar{X} + b \quad (1.1.7)$$

$$S_Y^2 = a^2 S_X^2 \quad (1.1.8)$$

1. To any constant c , we have:

$$\sum_{i=1}^n (X_i - c)^2 \geq \sum_{i=1}^n (X_i - \bar{X})^2 \quad (1.1.9)$$

Sample moment The k -th sample origin moment is the statistic defined by:

$$a_{n,k} = \frac{1}{n} \sum_{i=1}^n X_i^k \quad (1.1.10)$$

The k -th sample central moment is the statistic defined by:

$$m_{n,k} = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^k \quad (1.1.11)$$

Order statistics Given a random sample X_1, X_2, \dots, X_n from a population F , we arrange the sample values in increasing order: $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$. We call $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ the order statistics of the sample.

Sample median The sample median is the statistic defined by:

