ARIMA - GARCH

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Ref: Ruppert (2011).

1 Literacies

2 Branchies

Bayessian GARCH.

3 Remarks

ARMA is used to model the Mean, while GARCH model is used to model the votality(Variance).

ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant.

ARCH is an acronym meaning Auto-Regressive Conditional Heteroskedasticity. In ARCH models the conditional variance has a structure very similar to the structure of the conditional expectation in an AR model.

GARCH (Generalized ARCH) models, which model conditional variances much as the conditional expectation is modeled by an ARMA model.

As we have seen, an AR(1) process has a nonconstant conditional mean but a constant conditional variance, while an ARCH(1) process is just the opposite. If both the conditional mean and variance of the data depend on the past, then we can combine the two models. In fact, we can combine any ARMA model with any of the GARCH models.

4 ARIMA Processes

4.1 AR(p) model

autoregressive processes.

Section 12.6 AR(p) Models.

$$y_t = \mu + \underline{\lambda_1 y_{t-1} + ... + \lambda_p y_{t-p}} + \epsilon_t \tag{1}$$

Where $\{\epsilon_t\} \sim i.i.d\ N(0, \sigma_\epsilon^2)$.

One problem with AR models is that they often need a rather large value of p to fit a data set.

4.2 MA(q) Model

A process y_t is a *moving average process* if y_t can be expressed as a weighted average (moving average) of the past values of the white noise process $\{\epsilon_t\}$.

EQ.12.24

$$y_t = \mu + \underline{\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + ... + \theta_q\epsilon_{t-q}} \tag{2}$$

4.3 ARMA Processes

Section 12.8

Stationary time series with complex autocorrelation behavior often are more parsimoniously modeled by mixed autoregressive and moving average (ARMA) processes than by either a pure AR or pure MA process.

ARMA(p,q):

$$y_t = \mu + \underline{\lambda_1 y_{t-1} + ... + \lambda_p y_{t-p}} + \underline{\epsilon_t + \theta_1\epsilon_{t-1} + ... + \theta_q\epsilon_{t-q}} \tag{3}$$

4.4 ARIMA

A time series Y_t is said to be an $ARIMA(p, d, q)$ process if its d-order difference $\Delta^d Y_t$ is $ARMA(p, q)$ process. For example, A random walk is an ARIMA(0, 1, 0) model, and white noise is an ARIMA(0, 0, 0) model.

5 GARCH Process

5.1 An Example

Conciding a regression model with k variables in (EQ.1):

$$y_t = \mu + \underline{\lambda_1 y_{t-1} + ... + \lambda_p y_{t-p}} + \epsilon_t$$

This is called the mean value equation, and the turbulence part is:

$$\epsilon_t = \sigma_t z_t \tag{4}$$

Where σ_t is the standard deviation of ϵ_t, and z_t is the random part sampled from the standard normal distribution $N(0, 1)$. The variance of ϵ_t is:

$$\sigma_t^2 = Var(\epsilon_t|\epsilon_{t-1}, ...) \tag{5}$$

5.2 ARCH(p) Model

The idea of this model is that variance σ_t^2 in (EQ.5) is an autoregressive process, which is determined by the historical data of disturbance ϵ_t: (Just like $MA(q)$ process)

$$\sigma_t^2 = \kappa + \underline{\alpha_1 \epsilon_{t-1}^2 + ... + \alpha_p \epsilon_{t-p}^2} \tag{6}$$

Then we call ϵ_t in (EQ.4) a ARCH(p) process.

5.3 GARCH(p,q)

Assume the variance σ_t^2 is an ARMA(q, **p**) process. EQ. (14.8) in Ruppert (2011)

$$\sigma_t^2 = \kappa + \underline{\gamma_1 \sigma_{t-1}^2 + ... + \gamma_q \sigma_{t-q}^2} + \underline{\alpha_1 \epsilon_{t-1}^2 + ... + \alpha_p \epsilon_{t-\mathbf{p}}^2} \tag{7}$$

However, in Matlab‘s garch function, GARCH(p,q) means ARMA(p,q). p is the order of AR, and q is the order of MA.

5.4 GJR-GARCH

Both the GJR and the GARCH-specifications are used quite often in the finance literature. The GARCH is defined as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

and the GJR-GARCH reads as follows (GLOSTEN et al. (1993))

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \mathbb{I}_{t-1} \varepsilon_{t-1}^2 \tag{8}$$

where $\mathbb{I}_{t-1}$ is the indicator function:
$\mathbb{I}_{t-1}(\varepsilon_{t-1}) = \varepsilon_{t-1}$ for $\epsilon_{t-1} > 0$ and
$\mathbb{I}_{t-1}(\varepsilon_{t-1}) = 0$ otherwise.

According to research (Laurent et al. and Brownlees et al.) the GJR models generally perform better than the GARCH specification. Thus, including a leverage effect leads to enhanced forecasting performance.

6 ACF PCF

before doing ARMA-GARCH regression, we need to determine the order (lag numbers) for the model, through ACF and PCF analysis.

References

GLOSTEN, L. R., JAGANNATHAN, R., and RUNKLE, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48(5):1779–1801.

Ruppert, D. (2011). Statistics and data analysis for financial engineering, 2nd edition, volume 13. Springer.

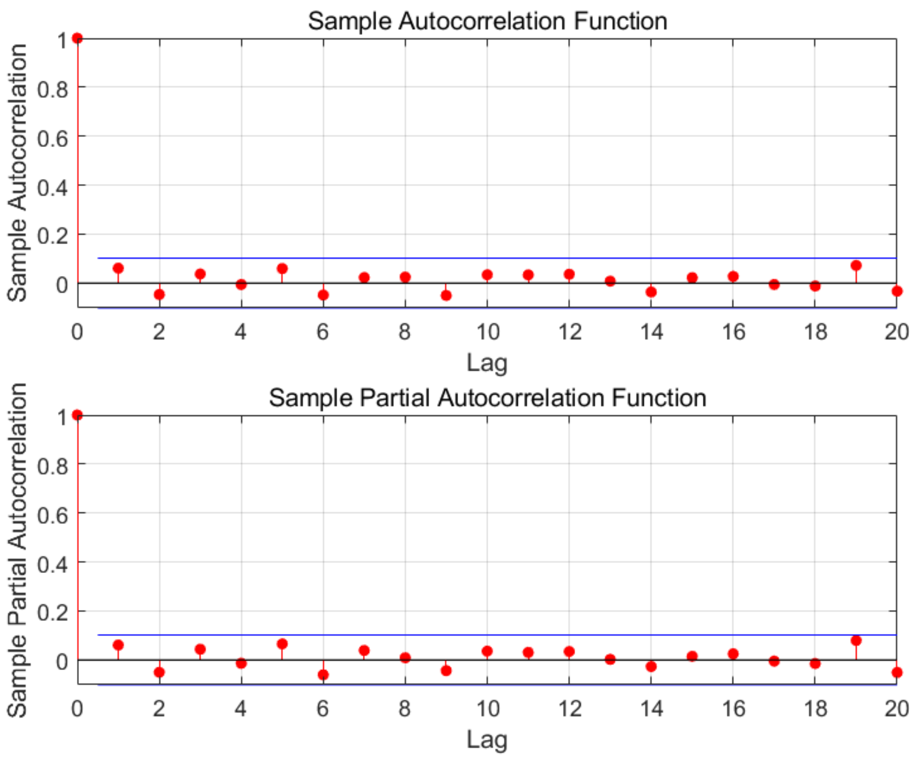

Figure 1: ACF PDF