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A heuristic for bi-objective vehicle routing with time window constraints

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Abstract

This paper is concerned with the bi-objective vehicle routing problem with time window constraints (BVRPTW). The BVRPTW is to determine the most favorable vehicle routes that minimize the total vehicle travel time and the total customer wait time which are, more often than not, conflicting. We construct a linear goal programming (GP) model for the BVRPTW and propose a heuristic algorithm to relieve a computational burden inherent to the application of the GP model. The heuristic algorithm consists of a parallel insertion method for clustering and a sequential linear goal programming procedure for routing. The results of computational experiments showed that the proposed algorithm finds successfully the more favorable solutions in most cases than the Potvin an Rousseau's method that is known as a very good heuristic for the VRPs with time window constraints. The proposed algorithm was capable of generating a numerous nondominated solutions through the change of the target values of the two objectives and the decision maker's preference on the objectives expressed as α. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Bi-objective VRP; Time windows; Goal programming; Heuristic

1. Introduction

The main concern of distribution management is to design the optimal vehicle routes for customer service. The classic vehicle routing problem (VRP) is to determine a set of routes that minimizes total travel time or distance, originating and terminating at a central depot, for a fleet of vehicles which services a set of customers with known demands or supplies. Each customer is serviced exactly once and, furthermore, all the customers must be assigned to vehicles such that the restrictions on the capacity of vehicles and the duration of a route are met.

Recently, the customer service to meet the required service time has been considered as an important issue in VRPs. The time window constraint is based upon the fact that a customer may require the earliest and latest service times by vehicle. The earliest and latest service times mean the lower and upper bounds of time window, each. The time window constraints exist in many areas such as retail distribution, school bus routing, industrial refuse collection, and after-sales service.

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The VRPs with time window constraints include temporal aspects as well as spatial aspects of vehicle movements, and need new solution techniques different from the existing solutions for the classic VRP. During the last 10 years, a considerable amount of research has been done on vehicle routing and scheduling problems with time windows. Excellent surveys may be found in Desrochers et al. [1] and Solomon and Desrosiers [2].

Badeau et al. [3] classified the methodologies for solving the problems as

(a) exact algorithms – Desrochers et al. [4], Kohl and Madsen [5], Fisher et al. [6];
(b) route construction heuristics – Potvin and Rousseau [7], Russell [8];
(c) route improvement heuristics – Potvin and Rousseau [9], Solomon et al. [10];
(d) composite heuristics that include both route construction and route improvement procedures – Kontoravdis and Bard [11], Russell [8];
(e) meta heuristics – tabu search [3,12,13], simulated annealing [14] and genetic algorithms [15].

In the VRPs with time window constraints, the service delay may be considered as a customer wait time when a vehicle arrives at a customer after its earliest service time. This is because, in some cases, the earliest service time can be considered as the most desirable time for customer to be served. The customer satisfaction will be improved by minimizing the customer wait time as an objective function. When a vehicle arrives at a customer before its earliest service time, the vehicle must wait to start a service until the earliest service time. The vehicle wait time is explicitly treated in the constraint for the vehicle return time to depot. The vehicle should arrive at customers before their latest service times. So far, we were not able to find out any paper in literature survey that treats explicitly the minimization of the customer wait time as an objective function in VRPs with time window constraints.

In this paper, we deal with the bi-objective vehicle routing problem with time window constraints (BVRPTW). Two conflicting objectives are the minimization of total vehicle travel time and the minimization of total customer wait time. First, we construct a linear goal programming (GP) model and solve a simple example by applying it. Next, we propose a two-stage heuristic algorithm for the BVRPTW. In the first clustering stage, a parallel insertion procedure clusters a set of customers into various groups. In the second stage, a sequential linear GP (SLGP) procedure [16] is applied to each group to determine the most favorable vehicle routes. Finally, we perform the computational experiments to evaluate the proposed heuristic algorithm using the 21 test problems.

2. Goal programming model for BVRPTW

A linear GP model for BVRPTW is constructed on the basis of the existing mathematical formulation of VRP [17]. In the GP model, NV virtual nodes ($N+1, \ldots, N+\mathrm{NV}$), whose locations are same as the starting depot node, augmented. These nodes correspond to returning depots for each of NV vehicles, respectively.

The notation used in the GP model is summarized below:

Constants

N	number of locations including depot (depot node number = 1)
NV	number of vehicles
Q	capacity of vehicle
R	permissible vehicle return time to depot
c_i	quantity to be collected at location i ($c_i = 0$ for $i = 1, N+1, \ldots, N+\mathrm{NV}$)
s_i	service time at location i ($s_i = 0$ for $i = 1, N+1, \ldots, N+\mathrm{NV}$)
t_{ij}	travel time from location i to j
E_i	earliest service time at location i ($E_1 = 0$)
L_i	latest service time at location i ($L_1 = 0$)
TT	target value of total vehicle travel time
B_1, B_2	a very large number

Decision variables

$$x_{ij} \quad \begin{cases} 1 & \text{if the link } (i,j) \text{ is included in a vehicle route} \\ 0 & \text{otherwise} \end{cases}$$

a_i vehicle arrival time at location i ($a_1 = 0$)
q_i quantity carried by the vehicle leaving location i ($q_1 = 0$)
$n_{(l)}$ negative deviations of constraint l ($l = 2,\dots,11$)
$p_{(l)}$ positive deviations of constraint l ($l = 2,\dots,11$)

With this notation the BVRPTW for collection service may be formulated as the following GP model:

$$\begin{aligned}\text{Min } \bar{A} &= \Bigg\{ \Bigg[\sum_{j=2}^{N+\mathrm{NV}} (n_{(2)j} + p_{(2)j}) + \sum_{i=2}^{N} (n_{(3)i} + p_{(3)i}) \\ &\quad + (n_{(4)} + p_{(4)}) + \sum_{i=1}^{N} \sum_{\substack{j=2 \\ i \neq j}}^{N+\mathrm{NV}} n_{(5)ij} \\ &\quad + \sum_{i=1}^{N} p_{(6)i} + \sum_{i=N+1}^{N+\mathrm{NV}} p_{(7)i} + \sum_{i=1}^{N} \sum_{\substack{j=2 \\ i \neq j}}^{N+\mathrm{NV}} n_{(8)ij} \\ &\quad + \sum_{i=N+1}^{N+\mathrm{NV}} p_{(9)i} \Bigg], p_{(10)}, \sum_{i=1}^{N} p_{(11)i} \Bigg\}\end{aligned} \tag{1}$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{N} x_{ij} + n_{(2)j} - p_{(2)j} = 1, \quad j = 2,\dots,N+\mathrm{NV}, \tag{2}$$

$$\sum_{\substack{j=2 \\ i \neq j}}^{N+\mathrm{NV}} x_{ij} + n_{(3)i} - p_{(3)i} = 1, \quad i = 2,\dots,N, \tag{3}$$

$$\sum_{j=2}^{N} x_{1j} + n_{(4)} - p_{(4)} = \mathrm{NV}, \tag{4}$$

$$\begin{aligned} &a_j - a_i - n_{(11)i} - B_1 x_{ij} + n_{(5)ij} - p_{(5)ij} \\ &\quad = t_{ij} + s_i - B_1, \quad i = 1,\dots,N, \\ &\quad j = 2,\dots,N+\mathrm{NV};\ i \neq j, \end{aligned} \tag{5}$$

$$a_i + n_{(6)i} - p_{(6)i} = L_i, \quad i = 1,\dots,N, \tag{6}$$

$$a_i + n_{(7)i} - p_{(7)i} = R, \quad i = N+1,\dots,N+\mathrm{NV}, \tag{7}$$

$$\begin{aligned} &q_j - q_i - B_2 x_{ij} + n_{(8)ij} - p_{(8)ij} = c_j - B_2, \\ &\quad i = 1,\dots,N, \quad j = 2,\dots,N+\mathrm{NV}, i \neq j, \end{aligned} \tag{8}$$

$$q_i + n_{(9)i} - p_{(9)i} = Q, \quad i = N+1,\dots,N+\mathrm{NV}, \tag{9}$$

$$\sum_{i=1}^{N} \sum_{j=2}^{N+\mathrm{NV}} t_{ij} x_{ij} + n_{(10)} - p_{(10)} = \mathrm{TT}, \tag{10}$$

$$a_i + n_{(11)i} - p_{(11)i} = E_i, \quad i = 1,\dots,N, \tag{11}$$

$$a_i, q_i \geqslant 0, \quad i = 1,\dots,N+\mathrm{NV}. \tag{12}$$

In the GP model, Eqs. (2)–(9) are the system constraints for defining the BVRPTW and Eqs. (10) and (11) are the goal constraints. The achievement vector (1) includes minimizing deviations, either negative or positive, or both, from a set of constraints, with certain pre-emptive priority weights assigned by the decision maker. A primal priority should be given to the first nine system constraints. The second goal priority is given to the minimization of total vehicle travel time and the third goal priority is given to the minimization of total customer wait time. The pre-emptive priority of two goal constraints may be changed by the decision maker's preference.

Constraints (2)–(4) assure that each customer is visited exactly once and exactly NV vehicles are used. Constraints (5) compute the arrival time at location j taking into account the vehicle wait time and service time at the previous location i, and travel time from i to j. $p_{(5)ij}$ are required for the case of $x_{ij} = 0$. Constraints (6) ensure that a vehicle arrives at location j before its latest service time. Constraints (6) and (11) impose the time windows that are defined in terms of the feasible service start times at locations. Constraints (7) imply that a vehicle must return to depot by the specified time after completing its collection service. Constraints (8) compute the quantity carried by the vehicle leaving location j. Constraints (9) ensure that the capacity of each vehicle is not surpassed. Both constraints (5) and (8) operate as subtour elimination constraints, that are a generalization of the subtour elimination constraints for the traveling salesman problem [17]. Constraint (10) represents that the total travel time must be kept within a reasonable bound TT, or target value. Constraints (11) compute the vehicle wait time at location i, $n_{(11)i}$ and the customer wait time at location i, $p_{(11)i}$. If we want to minimize the total vehicle travel time including the vehicle wait times at locations, $\sum_{i=1}^{N}\sum_{j=2}^{N+\mathrm{NV}} t_{ij}x_{ij}$ in Eq. (10) is substituted by $\sum_{i=N+1}^{N+\mathrm{NV}} a_i$.

We attempted to solve a small-size example problem with one vehicle by applying the GP model. Table 1 summarizes the example problem.

Table 1
Input data of example problem

Number of customers	Vehicle capacity		Vehicle return time	Customer service time	Avg. length of time windows
8	200		230	10	55.4
Customer no. (i)	Coordinates		Supply (c_i)	Earliest service time (E_i)	Latest service time (L_i)
	x_i	y_i			
1 (depot)	35	35	—	—	—
2	35	17	7	22	87
3	45	20	11	37	96
4	42	7	5	72	131
5	23	3	7	111	162
6	32	12	7	77	134
7	44	17	9	56	109
8	28	18	26	79	116
9	18	18	17	123	195

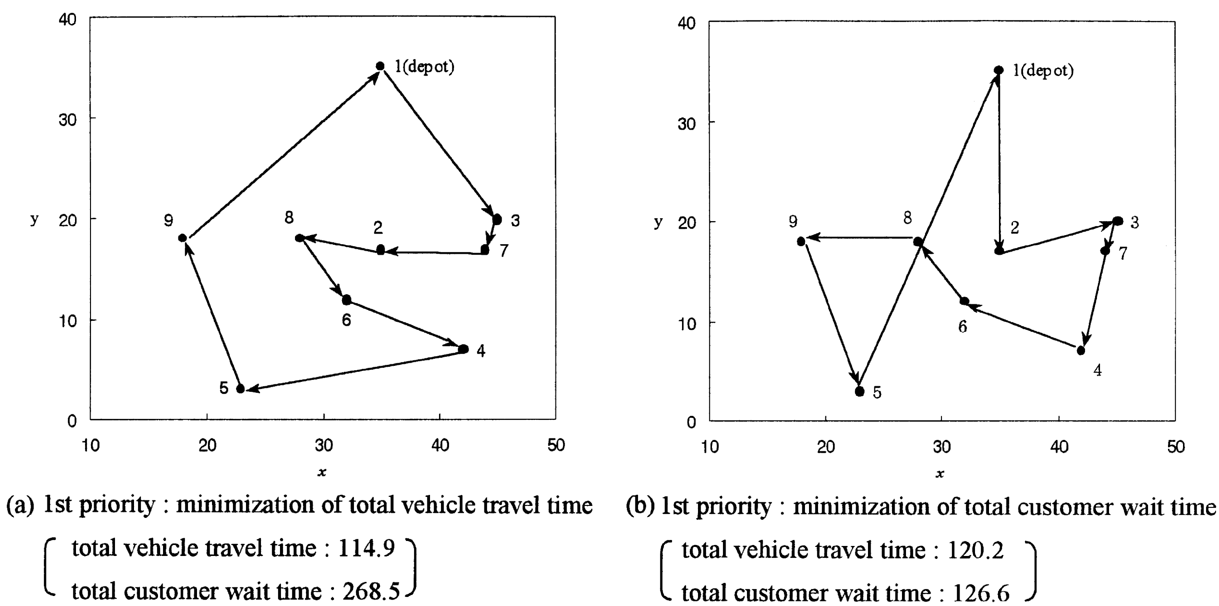

Fig. 1. Two solutions of example problem.

The travel time between customers is assumed to be equal to the Euclidean distance between them. The GP model constructed was executed with LINGO [18] on an IBM compatible MMX 200 MHz.

Fig. 1 shows the most favorable solutions obtained for two different goal priority structures. Comparing the solution of (b) with one of (a), the total customer wait time is reduced to less than half owing to the slight increase of the total vehicle travel time by 5.3 time units. We confirmed that the numerous nondominated solutions can be easily generated through the change of target values of the two goal constraints and the decision makers goal priority structure.

3. Heuristic algorithm for BVRPTW

The GP formulation to determine the most favorable solution as it stands has a serious computational difficulty in its application. To overcome this problem, we developed a heuristic algorithm. The heuristic algorithm consists of two stages. In the first clustering stage, a parallel insertion procedure clusters a set of customers into various groups considering the given constraints and the decision makers preference on the two objectives expressed as α. In the second routing stage, a sequential linear GP (SLGP) procedure [8] is applied to each group to determine the most favorable routes. The number of required vehicles is equal to the number of groups obtained in the clustering stage.

3.1. Clustering stage

It is known that, in general, the parallel routing approaches perform better than the sequential routing approaches in the VRPs with time windows [7]. The proposed clustering stage classifies the customers into various groups using the parallel routing approach. The procedural steps of the clustering method are described as follows:

Step 1 (*Initialization*): A temporary route of each group is constructed by connecting its seed location with a central depot. The number of seed locations becomes the number of initial groups. The seed locations for each group are determined as below.

(1) Select the furthest location from a central depot as the first seed location and put it in the seed set.
(2) Search for a location among the remaining locations which cannot be visited directly after any location in the seed set due to the time and vehicle constraints, and add it to the seed set if exists and repeat (2). Stop otherwise.

Step 2: Eqs. (13) and (14) are calculated for all the unassigned locations x that can be inserted between the adjacent locations i and j in the temporary route of group r. The feasibility test is performed with respect to the constraints for vehicle capacity, vehicle return time, and latest service time at customer.

$$c_{1r}(i,x,j) = t_{ix} + t_{xj} - t_{ij}, \quad \forall (i,j), \forall x, \forall r, \tag{13}$$

$$c_{2r}(i,x,j) = \mathrm{cw}_x + \sum_{k \in S} \mathrm{PF}_k, \quad \forall (i,j), \forall x, \forall r, \tag{14}$$

$$\mathrm{PF}_k = \begin{cases} b'_k - b_k, & k = j, \\ \mathrm{Max}[(\mathrm{PF}_{k-1} - vw_k), 0], & k \in S, \end{cases}$$

where (i,j) is the link between adjacent locations i and j in the current temporary route, t_{ij} the vehicle travel time from location i to j, cw_x the customer wait time at location x after inserting location x between i and j in the current temporary route, vw_k the vehicle wait time at location k before inserting location x between i and j in the current temporary route, b'_k the service start time at location k after inserting location x between i and j in the current temporary route, b_k the service start time at location k before inserting location x between i and j in the current temporary route, S is a set of locations following after location j in the current temporary route.

Eqs. (13) and (14) mean the increases of total vehicle travel time and total customer wait time, respectively, resulted from inserting location x between i and j in the current temporary route r. In Eq. (14), PF_k means the increased amount of customer wait time at location k by inserting location x between i and j in the current temporary route.

Step 3: The best insertion place of location x is determined by minimizing Eq. (15), that is, $r^*_{(i,j)} = \mathrm{Min}_r \mathrm{Min}_{(i,j)} c_r(i,x,j)$. In Eq. (15), α is a weight for the objective of travel time minimization that is determined on the basis of the decision makers preference for the two objectives. $c_r(i,x,j)$ may be considered as a penalty function expected by inserting location x between i and j in the current temporary route.

$$c_r(i,x,j) = \alpha \cdot c_{1r}(i,x,j) + (1-\alpha) \cdot c_{2r}(i,x,j), \quad 0 \leqslant \alpha \leqslant 1 \tag{15}$$

Step 4: Location x^* with the largest value of $A(x)$ is selected and inserted into its best place. $A(x)$ is computed by Eq. (16). If there does not exist the next best place to insert, then $p'(x) = 0$. Hence, a location with a large difference of penalty function between the best and next best insertion places and a small number of groups possible to be inserted into would be considered first for clustering.

$$A(x) = [p'(x) - p(x)]/v(x), \tag{16}$$

where $p(x) = c_r(i, x, j)$ for the best insertion place of x, $p'(x) = c_r(i, x, j)$ for the next best insertion place of x, and $v(x)$ the number of groups which location x can be inserted into.

If there exist locations impossible to be inserted into any of the current groups because of the constraints violation, then new groups are formed with them as the seed locations.

Step 5: The procedure is moved back to Step 2 and repeated until all customers are clustered into groups.

Step 6: The groups are reformed by applying both Or-opt and 2-opt improvement procedures [12,13] that may cause the move of customers among groups and thus result in the reduced number of groups. And stop.

3.2. Routing stage

Once a set of customers is clustered into groups in the clustering stage, the GP models for routing the customers in each group are constructed. The GP formulation for each group becomes much simpler than the complete GP model for the BVRPTW in Section 2. Each GP model is solved by applying the SLGP procedure.

The SLGP decomposes the GP formulation into two single objective LP subproblems according to the value of α. When α is greater than 0.5, the total vehicle travel time is minimized first. This results in the establishment of a single objective LP model with an objective function of Min $\sum_{i=1}^{N}\sum_{j=2}^{N+1} t_{ij}x_{ij}$ subject to the system constraints. Once the optimal solution for the LP model is obtained, then the total customer wait time is minimized next with an objective function of Min $\sum_{i=1}^{N} p_{(11)i}$ subject to $\sum_{i=1}^{N}\sum_{j=2}^{N+1} t_{ij}x_{ij} \leqslant \mathrm{TT}$ and the system constraints. TT is determined by $[1 + (1-\alpha)] \times$ (optimal value of total vehicle travel time obtained in the first LP model).

When α is less than 0.5, the first LP model is constructed to minimize the total customer wait time with an objective function of Min $\sum_{i=1}^{N} p_{(11)i}$ and the system constraints. Then the total vehicle travel time is minimized next with an objective function of Min $\sum_{i=1}^{N}\sum_{j=2}^{N+1} t_{ij}x_{ij}$ subjective to $\sum_{i=1}^{N} p_{(11)i} \leqslant \mathrm{TW}$ and the system constraints. TW is determined by $(1+\alpha) \times$ (optimal value of total customer wait time obtained in the first LP model). When α is equal to 0.5, one of two objectives is arbitrarily selected as an objective function for the first LP model.

The solution(s) to the second LP model is considered as the most favorable vehicle routes for a group based on the decision makers preference on the objectives expressed as α. Many different routes can be obtained through the change of the target values and the decision maker's preference on the objectives.

A flowchart of the heuristic algorithm for the BVRPTW is shown in Fig. 2.

4. Computational results

We evaluated the performance of the proposed heuristic algorithm using the R1- and C1-type set of Solomon's [19] test problems. The geographical data of the test problems which are all 100-customer Euclidean problems were randomly generated using a uniform distribution (R1 type) and were clustered (C1 type). Table 2 summarizes the two types of test problems. The proposed algorithm was programmed in Visual Basic 5.0, batch-executed with LINGO [18], and implemented on an MMX 200MHz.

In order to evaluate the performance of the proposed heuristic algorithm for the BVRPTW, we compared it with the Potvin and Rousseau [7] method (P&R method) that is known as a very good route construction heuristic algorithm for VRPs with time window constraints, because we were not able to find the solution techniques right for the BVRPTW in literature. In the P&R method, $c_1(i,u,j) = \alpha \times c_{11}(i,u,j) + (1-\alpha) \times c_{12}(i,u,j)$ is used for computing the feasible customer insertion cost. The first component in the equation, $c_{11}(i,u,j)$, means the increase of vehicle travel time and the

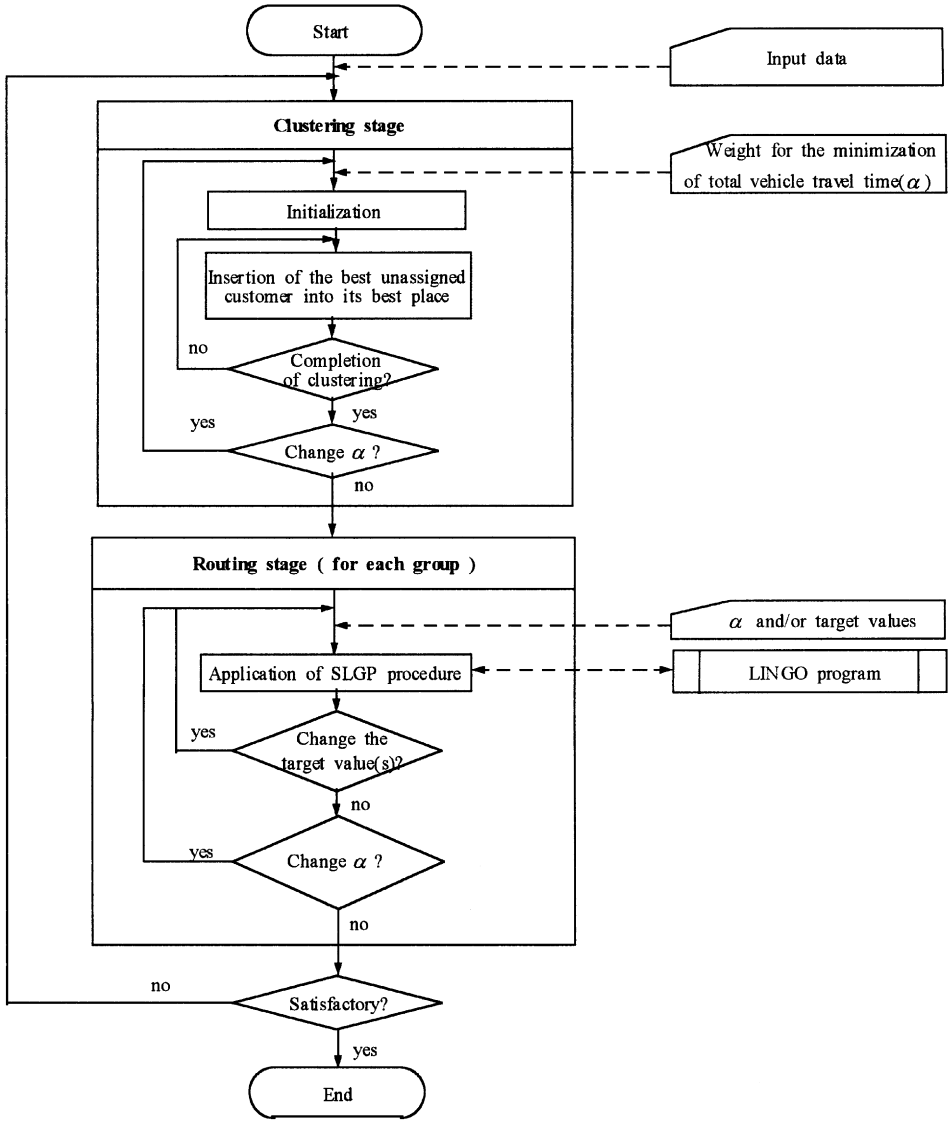

Fig. 2. Flowchart of proposed heuristic for BVRPTW.

second component, $c_{12}(i, u, j)$, means the increase of customer wait time at customer j due to the insertion of customer u between customer i and j. The parameters α and $(1 - \alpha)$ are used to weight two components of the summing equation.

We performed the computational experiments on the 21 test problems while changing the weight of the minimization of total vehicle travel time, α, from 0.0 to 1.0 by 0.25. The proposed algorithm was fairly successful on all cases with different values of α.

Table 3 shows the computational results obtained by applying both the proposed algorithm and the P&R method to all the test problems with $\alpha = 0.75$. The computational results are so impressive. The proposed algorithm shows a better performance for both objectives than the P&R method on 13 out of 21 test problems. The P&R method shows a better performance for both objectives than the proposed algorithm on only 2 out of 21 test problems. On the remaining 6 test problems, neither method generates the dominant solutions for both objectives. Especially, it is shown that the proposed algorithm generates better solutions for the objective of total customer wait time minimization on 17 out of 21 test problems. This outcome implies that the proposed algorithm performs very well, particularly with respect to the minimization of total customer wait time.

It is also an important outcome in Table 3 that the proposed algorithm requires 1–3 vehicles less than or at least same number of vehicles as the P&R method on all the test problems except problem R101. However, the proposed algorithm shows a significant drawback in computation time (CPU time) on some problems such as R104, R107, R108, R110, R111, R112, C102, C103, and C104. The proposed algorithm may consume much computation time in the routing stage that is affected mainly by the number of customers in a group and the lengths of time windows.

Table 4 shows the comparison of the solutions obtained by applying the proposed algorithm to all the test problems with $\alpha = 1.0$ and their best-known solutions. The numerical values inside parentheses represent the number of vehicles required for the solution. To compare the solutions by the proposed algorithm with the best known solutions, the error rate is computed by $100 \times$(solution by proposed algorithm − best-known solution)/best-known solution.

It is observed from Table 4 that the proposed algorithm generates the solutions pretty close to the best-known solutions with an average error rate

Table 2
Summary of test problems

Problem type	Number of customers	Vehicle capacity	Vehicle return time	Service time
R1 (12 problems)	100	200	230	10
C1 (9 problems)	100	200	1236	90

Table 3
Comparison of proposed algorithm and Potvin and Rousseau's method ($\alpha = 0.75$)

Problem	Total vehicle travel time		Total customer wait time		Number of vehicles		CPU time (s)	
	Proposed alg.	P&R	Proposed alg.	P&R	Proposed alg.	P&R	Proposed alg.	P&R
R101	1846.1	1744.1	123.57	429.3	22	21	116	166
R102	1710.6	1589.9	912.8	3403.8	19	19	68	187
R103	1493.0	1382.9	4751.0	6560.8	13	14	625	187
R104	1121.2	1201.5	6951.3	8643.0	11	11	5437	112
R105	1607.9	1505.4	897.8	1205.5	15	16	88	123
R106	1508.2	1401.5	2623.4	5110.8	13	14	189	125
R107	1298.6	1337.9	5548.1	7348.0	11	12	2928	117
R108	1160.1	1131.6	7929.7	8088.0	10	10	2055	115
R109	1411.4	1389.9	1906.2	2861.1	13	14	109	107
R110	1320.3	1305.3	3275.8	4028.8	11	13	3862	144
R111	1348.3	1342.8	4003.0	5267.2	11	13	1888	194
R112	1201.5	1095.3	5794.8	5132.3	10	11	2404	115
C101	858.8	1208.1	2729.7	2628.1	10	12	29	50
C102	912.4	1204.4	15865.7	17320.0	10	11	717	65
C103	1314.2	1144.1	27174.4	26264.0	10	11	960	83
C104	1545.6	1281.4	28731.0	42414.0	10	10	5274	74
C105	1189.1	1189.1	2785.9	2785.9	12	12	77	73
C106	1030.8	1229.2	4725.9	6536.6	11	13	62	105
C107	1217.9	1314.4	4273.4	8252.0	12	14	94	134
C108	1084.1	1257.9	7269.5	12220.0	11	12	82	136
C109	1066.2	1210.7	16685.2	12268.0	10	13	143	148

Table 4
Comparison of proposed algorithm ($\alpha = 1.0$) and best known solutions

Problem	Total vehicle travel time			CPU time (s)		Reference
	Proposed alg.	Best known	Error rate (%)	Proposed alg.	Best known	
R101	1806.1 (21)	1607.7 (18)	12.3	167	1064.2	[4]
R102	1531.6 (18)	1434.0 (17)	6.8	70	756.9	[4]
R103	1382.4 (13)	1207.0 (13)	14.5	165	1500	[20]
R104	1094.5 (10)	982.0 (10)	11.5	3346	2700	[21]
R105	1465.6 (16)	1377.1 (14)	6.4	115	2700	[21]
R106	1442.8 (13)	1252.0 (12)	15.2	329	2700	[21]
R107	1271.9 (11)	1159.9 (10)	9.7	534	13774	[21]
R108	1099.2 (10)	981.0 (9)	12.1	4573	13774	[13]
R109	1297.9 (13)	1235.7 (11)	5.0	73	13774	[13]
R110	1271.4 (12)	1080.4 (11)	17.7	213	2700	[21]
R111	1141.2 (11)	1129.9 (10)	1.0	743	13774	[13]
R112	1063.6 (11)	953.6 (10)	11.5	5141	2700	[21]
C101	828.9 (10)	827.3 (10)	0.2	22	434.5	[4]
C102	829.7 (10)	827.3 (10)	0.3	2636	1990.8	[4]
C103	845.1 (10)	828.1 (10)	2.1	3755	3200	[21]
C104	877.0 (10)	824.8 (10)	6.3	5274	3200	[21]
C105	828.9 (10)	828.9 (10)	0.0	34	14630	[13]
C106	831.5 (10)	827.3 (10)	0.5	36	724.8	[4]
C107	828.9 (10)	827.3 (10)	0.2	58	1010.4	[4]
C108	956.3 (11)	827.3 (10)	15.6	81	1613.6	[4]
C109	857.2 (10)	828.9 (10)	3.4	143	14630	[13]
Average			7.25			

[4]: Sun SPARC 1.
[21]: Silicon Graphics Indigo (100 MHz).
[17]: Sun SPARC 10/50.
[20]: NEXT 68040 (25 MHz).
Proposed alg.: MMX 200 MHz.

of 7.25% on all the test problems and particularly works well with an average error rate of 3.2% on C1-type test problems when the total vehicle travel time is minimized as a single objective ($\alpha = 1.0$). The computation times are not comparable because the computers used are different for the test problems. However, when the kinds of the computers are taken into account, it may be implicitly acceptable that the proposed algorithm requires generally less computation times than the other methods for the best known solutions.

We did not perform the detailed experiments to examine the effect of problem types on the performance of the proposed algorithm. Nevertheless, it was not found in all cases with different values of α as in a case with $\alpha = 0.75$ shown in Table 3 that the geographical aspect of customer locations has any significant impact on its performance. Through the application of the proposed heuristic algorithm, we ascertained that the two objectives are, more often than not, conflicting. Furthermore, we confirmed that the proposed algorithm generates a numerous nondominated solutions by changing the target values of two objectives and also the increase in number of vehicles (groups) reduces significantly the total customer wait time in most cases.

5. Conclusion

In this paper, we dealt with the bi-objective vehicle routing problem with time window constraints

(BVRPTW). The two conflicting objectives are the minimization of total vehicle travel time and the minimization of total customer wait time. We first constructed a linear GP model and then proposed a heuristic algorithm for the BVRPTW. The heuristic algorithm consists of the clustering and routing stages. In the first clustering stage, a parallel insertion procedure clusters a set of customers into various groups considering the given constraints and the decision maker's preference on two objectives expressed as α. In the second routing stage, a SLGP procedure is applied to each group to determine the most favorable vehicle routes.

The results of computational experiments on 21 test problems showed that the proposed algorithm finds successfully the more favorable solutions than the Potvin and Rousseau method in most cases. The proposed algorithm required 1–3 vehicles less than or at least same number of vehicles as the P&R method on all the test problems except one problem. However, the proposed algorithm showed a significant drawback in CPU time on some problems. The proposed algorithm generated the solutions pretty close to the best known solutions and particularly worked well on C1 type test problems when the total vehicle travel time is minimized as a single objective ($\alpha = 1.0$). We confirmed through the computational experiments that the proposed algorithm is capable of generating a numerous nondominated solutions by changing the target values of two objectives and the decision makers preference on the objectives expressed as α.

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