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ORIGINAL ARTICLE

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A two-phase heuristic method for the multi-depot location routing problem taking inventory control decisions into consideration

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Abstract The multi-depot location-routing problem (MDLRP) combines depot location and vehicle routing decisions, in order to determine the locations of depots and find the optimal set of vehicle schedules and routes. Inventory control decisions are interrelated with vehicle routing and depot location. However, the inventory control decisions are always ignored in MDLRP. In this paper, a mathematical model for the single-product multi-depot location-routing problem taking inventory control decisions into consideration is proposed. Since finding the optimal solution(s) for this model is an NP (non-polynomial) problem, we propose a two-phase heuristic method to find solutions for this problem. In phase 1, the initial solution using a route-first, location-allocation second approach based on the minimal system cost (including location, transportation, and inventory costs) is determined. In phase 2, an improvement heuristic search for a better solution based on the initial solution in phase 1 is developed. One sample description is presented for demonstration purposes. At last, the proposed heuristic method is tested and evaluated via simulation. The results show the proposed heuristic method is better than those existing methods without taking inventory control decisions into consideration.

Keywords Heuristic methods · Multi-depot location-routing problem (MDLRP) · Inventory control · NP problem

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1 Introduction

The multi-depot location-routing problem (MDLRP) is to determine locations of depots from several candidates and find the optimal set of vehicle schedules and routes based on the shortest travelling distance criterion [1,2,3]. MDLRP consists of two sub-problems: a location allocation problem (LAP) and a vehicle routing problem (VRP) [3]. Location allocation problems are to choose the best locations for depots from a given set of candidate sites to minimize the sum of the depot-opening cost and the cost of assigning customers to the depots. Vehicle routing problems are to determine the optimal delivery routes from a given depot to some geographically dispersed customers. For example, there are eight customers, $C_1 \sim C_8$, and four candidate depots, $D_1 \sim D_4$, in Fig. 1. Based on the objective of minimizing travelling distance, D_2 and D_4 are open and three routes, $\{D_2, C_1, C_2, C_3\}$, $\{D_2, C_4, C_5\}$, $\{D_4, C_6, C_7, C_8\}$, are determined. However, inventory control decisions seem ignored in most MDLRP research [1,2,3]. Since inventory control decisions, such as order quantity (or shipment quantity), order frequency, etc., affect both the inventory cost and the transportation cost, they should be considered in the MDLRP to achieve the objective of minimal inventory and transportation cost [4,5]. For example, shipment in smaller quantities and higher frequency leads to reductions in inventory cost but requires additional transportation cost. In addition, the depot location decision is affected by both inventory control decisions and vehicle routing decisions. The depot location allocation should be determined based on minimising the sum of the inventory cost and the transportation cost (The minimal travelling distance cost is not the only criterion.). Since the inventory control decision is interrelated to vehicle routing and depot location, the total system cost (including location, transportation, and inventory) may increase when inventory control decisions are not taken into consideration [6,7,8]. Hence, how to determine depot

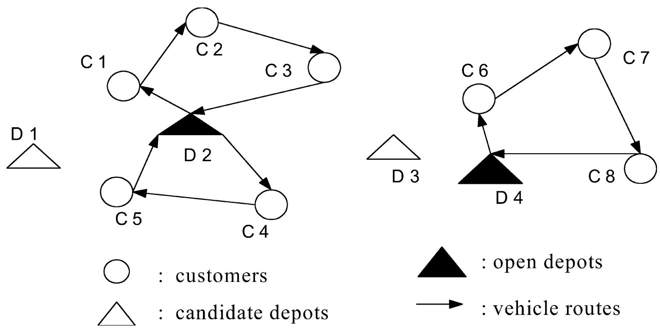

Fig. 1 Multi-depot location routing problem

locations, vehicle routing, and inventory policy become important issues in distribution systems [9].

According to a survey by Min et al. [10], there are two types of solution methods used for solving MDLRP: (1) Exact algorithm and (2) Heuristic. Both sub-problems (LAP and VRP) of MDLRP have been shown to be NP-hard; thus, MDLRP also belongs to the class of NP-hard problems [2, 11]. Since finding the optimal solution(s) for this model is an NP (non-polynomial) problem, the exact algorithm seems infeasible to solve the problem. Hence, it is worthwhile to develop some heuristic methods. There are four classified approaches for MDLRP in the heuristic methods [10]: (1) location-allocation first, route-second, (2) route-first, location-allocation second, (3) savings/insertion, and (4) tour improvement/exchange. Among these four approaches, the tour improvement/exchange method is used more widely than the others [1,2,3,10,11,12]. The tour improvement/exchange approach creates a solution in such a way that a feasible solution is altered continuously to yield another feasible solution with a reduced cost until no additional cost reductions are possible. Perl and Daskin [11] proposed a new solution procedure. The initial solution is found based on the route-first, location-allocation second approach and then an improvement procedure is used separately for vehicle routing and depot locations. Srivastava [1] found an initial solution based on the route-first, location-allocation second approach. Then an improvement approach for local optimality is used for the final solution. Renaud et al. [12] proposed a tabu search algorithm for the multi-depot vehicle routing problem with capacity and route length restrictions based on the exchange improvement approach. Tuzun and Burke [2] also presented a two-phase tabu search architecture for the solution of the location-routing problem. The route-first, location-allocation second approach is used first. Then the improvement search is used based on the tabu method. Wu et al. [3] proposed a decomposition-based method for solving the location-routing problem. The problem is divided into two sub-problems: the location-allocation problem and the vehicle routing problem. Each sub-problem is then solved in a sequential, iterative and improved manner by the simulated annealing algorithm.

In this paper, a mathematical model for the multi-depot location-routing problem taking inventory control decision into considerations is proposed. Since finding the optimal solution(s) for this model is an NP problem, a two-phase heuristic method that consists of finding the initial solution(s) and improving the initial solution(s) is also proposed. Phase 1 is to find a better initial solution based on route-first, location-allocation second approach. Phase 2 is to improve the initial solution. According to the above literature review, any one of these two phases can improve the final solution. Hence, these two approaches (phases) are used together.

2 Model formulation for the multi-depot location routing problem

2.1 Assumptions and notations

2.1.1 Assumptions

1. We are dealing with the single-product multi-depot location-routing problem.
2. Each customer is served by exactly one vehicle.
3. Each route is served by one vehicle.
4. The total demand on each route is less than or equal to the vehicle service capacity.
5. Each route begins and ends at the same depot.
6. Fleet type is homogeneous (vehicle capacities are the same).
7. We know the following:
   a. number of candidate depots;
   b. number of customers;
   c. demand of each customer, which is stochastic;
   d. vehicle capacity;
   e. vehicle service capacity;
   f. ordering cost;
   g. depot establishing cost;
   h. shortage cost;
   i. holding cost;
   j. probability density function for customers' demand of each route during lead time.

2.1.2 Notations

M: number of candidate depots
N: number of customers
K: number of vehicles (or routes)
b: vehicle capacity
$MaxSup$: vehicle service capacity
NOD: number of open depots
FC_j: cost of establishing depot j
c: cost of dispatching vehicles
cm: traveling cost / unit distance
h^+: holding cost / unit time/ unit product

hs: shortage cost/unit product
A: ordering cost/each order
h: index of depots or customers ($1 \le h \le N+M$)
g: index of depots or customers ($1 \le g \le N+M$)
i: index of customers ($1 \le i \le N$)
j: index of depots ($N+1 \le i \le N+M$)
k: index of vehicles or routes ($1 \le k \le K$)
V_k: set for route k with an open depot ($1 \le k \le K$)
Dis_{kgh}: total distance for route k.
Q_{kgh} : number of units produced for route k during each production run.
UL_{kgh}: average demand for route k during lead time.
D_{kgh}: total demand for route k.
R_{kgh}: order-up-to level for replenishment of route k.
$B(R_{kgh})$: expected shortage number for route k during each production run.
$f_L(x)$: probability density function for customers' demand of each route during lead time L, x is the random demand during lead time L.
Y_{ij}: 1, if customer j is allocated to depot i; 0 otherwise.
Z_j: 1, if depot j is established; 0 otherwise.
X_{kgh}: 1, if point g immediately proceeds point h on route k; 0 otherwise.

2.2 Model formulation

Before the mathematic model is developed, three relevant costs: depot establishing, transportation and inventory, used in this paper are introduced first. The depot establishing cost depends on whether depot j is open or not. The expression is as follows: $FC_j \times Z_j$. The transportation cost includes the cost of dispatching vehicles, c, plus the travelling cost among customers, $cm \times Dis_{kgh}$, multiplying the average transportation times in a period, D_{kgh}/Q_{kgh}. The expression of transportation cost is as follows: $(c+cm \times Dis_{kgh}) \times D_{kgh}/Q_{kgh}$. As for inventory, since the model for continuous review systems is widely used [13,14], this paper will adopt the model as inventory policy. There are three costs considered in this inventory model: (1) holding cost, (2) ordering cost, and (3) shortage cost (Since the demand for each customer is stochastic, some orders may not be satisfied and shortage occurs.). The detailed computation for these three costs is as follows: (1) holding cost $= (Q_{kgh}/2+R_{kgh}-UL_{kgh}) \times h^+$. (2) ordering cost $= (D_{kgh}/Q_{kgh}) \times A$. (3) shortage cost $= hs \times B(R_{kgh}) \times \frac{D_{kgh}}{Q_{kgh}}$ As for the computation of $B(R_{kgh})$, please refer to Eq. 1 in Appendix 1. After these relevant costs are discussed, the model formulation is as follows:

$$\text{Minimize} \sum_{j=N+1}^{N+M} FC_j \times Z_j + \sum_{k=1}^{K} \sum_{g=1}^{N+M} \sum_{h=1}^{N+M} \left(\left(c + cm \times Dis_{kgh}\right) \times \frac{D_{kgh}}{Q_{kgh}} + \left(\frac{Q_{kgh}}{2} + R_{kgh} - UL_{kgh}\right) \times h^+ + \frac{D_{kgh}}{Q_{kgh}} \times A + hs \times B\left(R_{kgh}\right) \times \frac{D_{kgh}}{Q_{kgh}} \right) \times X_{kgh}$$

s.t.

$$Q_{kgh} \leqslant b \tag{1}$$

$$D_{kgh} \leqslant MaxSup \tag{2}$$

$$\sum_{k=1}^{K} \sum_{h=1}^{N+M} X_{ikh} = 1, i = 1, ..., N \tag{3}$$

$$\sum_{g \in v} \sum_{h \in v} \sum_{K=1}^{k} X_{ghk} \geqslant 1, \forall (v, \bar{v}) \tag{4}$$

$$\sum_{g=1}^{N+M} X_{hgk} - \sum_{g=1}^{N+M} X_{ghk} = 0, k = 1, ..., K, h = 1, ..., N + M \tag{5}$$

$$\sum_{j=N+1}^{N+M} \sum_{i=1}^{N} X_{ijk} \leqslant 1, k = 1, ..., K \tag{6}$$

$$\sum_{h=1}^{N+M} X_{ihk} + \sum_{h=1}^{N+M} X_{jhk} - Y_{ij} \leqslant 1, i = 1, ..., N, \quad j = N + 1, ..., N + M, K = 1, ..., K \tag{7}$$

$$X_{kgh} = 0.1\ g = 1, ..., N + M, h = 1, ..., N + M, k = 1, ...k\ K \tag{8}$$

$$Z_j = 0, 1\ j = N + 1, ..., N + M \tag{9}$$

$$Y_{ij} = 0, 1\ i = 1, ..., N,\ j = N + 1, ..., N + M \tag{10}$$

In the above formulation, the objective function is to minimize the sum of depot establishing cost, transportation cost, and inventory cost. Constraint (1) states the amount of each delivery to customers must be less than or equal to vehicle capacity. Constraint (2) insures the total demand for route k is less than or equal to vehicle service capacity. Constraint (3) states each customer appears in only one route. Constraint (4) insures each route begins and ends at the same depot. Constraint (5) insures that every point entered by the vehicle should be the same point the vehicle leaves. Constraint (6) insures a route cannot be served by multiple depots. Constraint (7) states a customer can be allocated to a depot only if there is a route passing by that customer. Constraints (8)–(10) insure the integrality of decision variables.

3 The proposed heuristic method

For obtaining a solution for MDLRP taking inventory control decisions into consideration, a two-phase heuristic method is proposed in this paper. Phase 1 finds the initial solution using route-first, location-allocation second approach based on minimal system cost. Phase 2 develops an improvement heuristic search for a better solution based on the initial solution in phase 1. The detailed procedure for this method is as follows (also see Fig. 2):

Phase 1: finding the initial solution

Step 1. (1) Set $k=1$, $r=1$, $MaxSup$ = vehicle service capacity, $max\_swap=0$. (2) Put all customers into a set F. (3) Put all depots into a set E.

Step 2. (1) Randomly select a customer from F. (2) Put this customer into the set V_k. (3) Delete the customer from F.

Step 3. Select a customer, W, from F with the minimal marginal cost C_s as the next candidate customer (For computing C_s, the number of units produced Q_{kgh} and the order-up-to level R_{kgh} should be calculated first. After these two values are available, $C_s=S(V_k+\{W\})-S(V_k)$. $S(V_k)$ or $S(V_k+\{W\})$ can be computed as follows:

$$\left(c+cm\times Dis_{kgh}\right)\times\frac{D_{kgh}}{Q_{kgh}}+\left(\frac{Q_{kgh}}{2}+R_{kgh}-UL_{kgh}\right)\times h^{+}+\frac{D_{kgh}}{Q_{kgh}}\times A+hs\times B\left(R_{kgh}\right)\times\frac{D_{kgh}}{Q_{kgh}}.$$

Please refer to Appendix 1 for the detailed procedure of computing Q_{kgh} and R_{kgh}).

Step 4. Is the total demand of customers in V_k and the candidate customer less than or equal to $MaxSup$? If yes, (1) put the candidate customer into V_k, (2) delete the candidate customer from F, (3) go to step 5. Otherwise, (1) set $k=k+1$, (2) put the candidate customer into V_k, (3) delete the candidate customer from F, (4) go to step 5.

Step 5. Is F empty? If yes, go to step 6. Otherwise, go to step 3.

Step 6. Compute the centroid of V_t for $1 \le t \le k$.

$$X_t=\frac{\sum_{i=1}^{q\_cm_t} x_i\times d_i}{\sum_{i=1}^{q\_cm_t} d_i},\quad Y_t\frac{\sum_{i=1}^{q\_cm_t} y_i\times d_i}{\sum_{i=1}^{q\_cm_t} d_i}$$

(X_t, Y_t) is the coordinate of the centroid for V_t, q_cm_t is the number of customers in V_t, d_i is the demand of customer i in V_t and (x_i, y_i) is the coordinate of customer i for $i=1$ to q_cm_t).

Step 7. (1) Select a depot from E with the shortest path to the centroid of V_r. (2) Put the depot into V_r. (3) Delete the depot from E. (4) Set $r=r+1$.

Step 8. Is r greater than k? If yes, go to step 9. Otherwise, go to step 7.

Step 9. (1) Compute the total system cost SC (It can be computed based on the objective function mentioned in 2.2). (2) Set the initial solution SC and V_t as the temporary best solution (Let $SC'=SC$ and $U_t=V_t$ for $1 \le t \le k$).

Phase 2: improving the initial solution

Step 10. (1) Close a depot with the minimal system cost SC. (2) Set $NOD=NOD$—1 (The initial value of NOD is equal to the number of routes k.).

Step 11. Is SC less than SC'? If yes, set $SC'=SC$, $U_t=V_t$ for $1 \le t \le k$ and go to step 12. Otherwise, go to step 12.

Step 12. Is NOD equal to 1? If yes, go to step 13. Otherwise, go to step 10.

Step 13. (1) Set $V_t=U_t$ for $1 \le t \le k$. (2) Randomly select a closed depot and an open depot from the best solution and exchange each other (the selected open depot is substituted by the selected closed depot in V_t). (3) Compute the total system cost SC.

Step 14. Is SC less than SC'? If yes, set $SC'=SC$, $U_t=V_t$ for $1 \le t \le k$ and go to step 15. Otherwise, set $max\_swap=max\_swap+1$ and go to step 15.

Step 15. Is max_swap greater than a default value (usually the default value is set equal to half of number of candidate depots [2])? If yes, go to step 16. Otherwise, go to step 13.

Step 16. The best solution (SC' and U_t for $1 \le t \le k$) is obtained and stop.

4 An illustrative example

An example is used for illustrating the proposed heuristic method. We assume that there are three depots, D_1, D_2, and D_3 and four customers, C_1, C_2, C_3, and C_4. The vehicle dispatching cost is 25 for each time. The distance cost is 1/unit distance. The holding cost is 0.5/unit/year. The ordering cost is 20 for each order. The shortage cost is 2/unit. The vehicle capacity is 150 units. The vehicle service capacity is 1000 units. This is detailed in the following tables:

Depots	Location (coordinate)	Depot establishing cost
D_1	(38, 133)	209
D_2	(22, 76)	467
D_3	(174, 193)	143

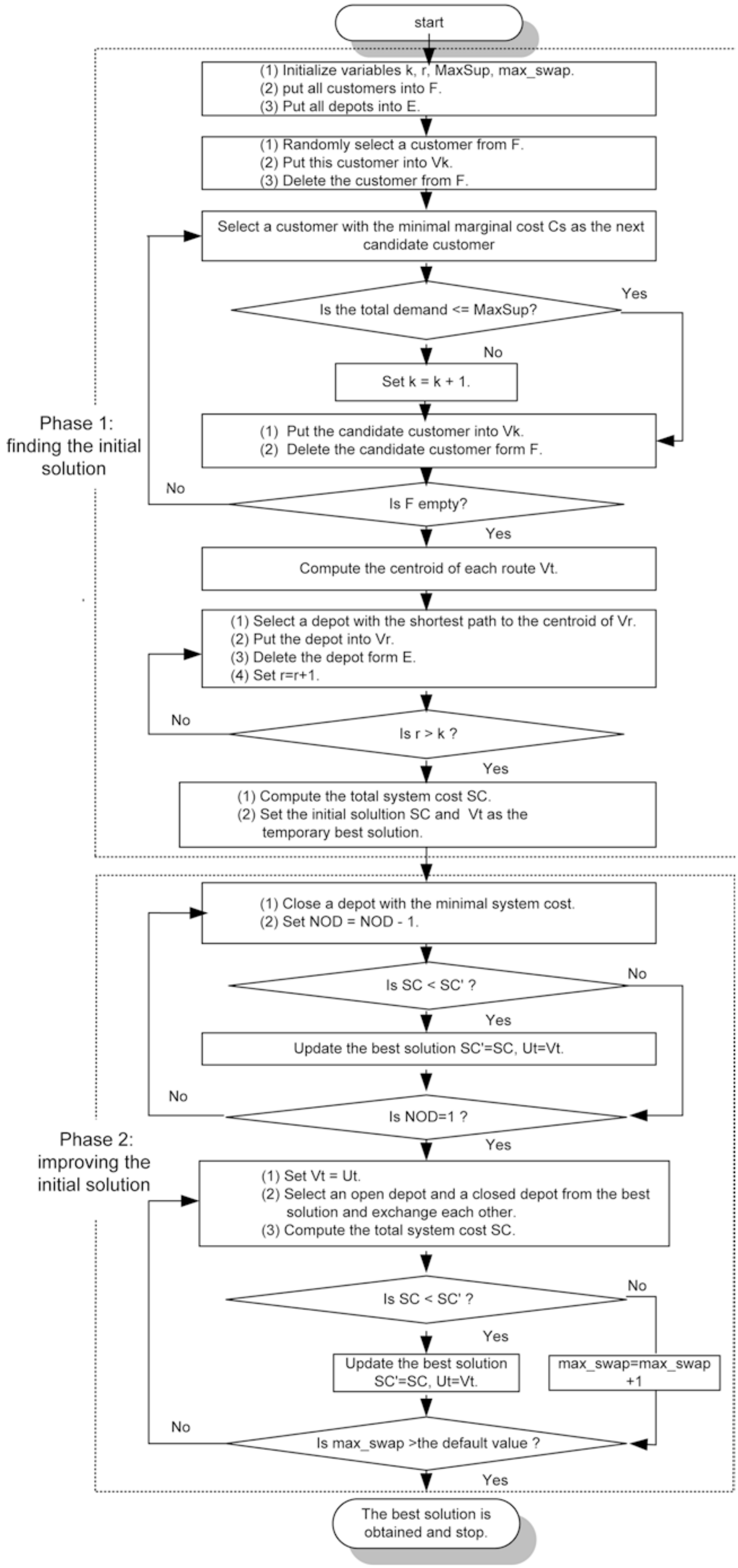

Fig. 2 Flowchart for the proposed heuristic method

Customer	Location (coordinate)	Demand (year)	Demand during lead time (uniform distribution)
C_1	(155, 98)	474	$U[0, 3]$
C_2	(178, 191)	365	$U[0, 8]$
C_3	(78, 88)	522	$U[0, 6]$
C_4	(50, 65)	200	$U[0, 7]$

The procedure for the heuristic method is as follows:

Step 1. (1) Set $k=1$, $r=1$, $MaxSup=1000$, $max\_swap=0$. (2) Put C_1, C_2, C_3, and C_4 into F. (3) Put D_1, D_2, and D_3 into E.

Step 2. (1) Randomly select C_2 from F. (2) Put C_2 into V_1. (3) Delete C_2 from F.

Step 3. Select C_1 from F with the minimal marginal cost C_s as the next candidate customer. $(Q, R)=(150, 10.5)$ and $B(R)=0.01$ for $S(V_1+\{C_1\})$, and $(Q, R)=(150, 2.76)$ and $B(R)=0.01$ for $S(V_1)$ are deduced based on the procedure in Appendix 1. $C_s(C_1)=S(V_1+\{C_1\})-S(V_1)=1363.49–785.84=577.65$. $C_s(C_3)=810.11$ and $C_s(C_4)=985.68$.

Step 4. The total demand $(474+365=839)$ of the customers in V_1 $(=\{C_2\})$ and the candidate customer C_1 is less than $MaxSup$. (1) Put C_1 into V_1. (2) Delete C_1 from F (3) Go to step 5.

Step 5. F is not empty and go to step 3.

Step 3. Select C_3 from F with the minimal marginal cost C_s as the next candidate customer.

Step 4. The total demand $(474+365+522=1361)$ of the customers in V_1 $(=\{C_2, C_1\})$ and the candidate customer C_3 is greater than $MaxSup$. (1) Set $k=k+1=2$. (2) Put C_3 into V_2. (3) Delete C_3 from F. (4) Go to step 5.

Step 5. F is not empty and go to step 3.

Step 3. Select C_4 from F with the minimal marginal cost C_s as the next candidate customer.

Step 4. The total demand $(522+200=722)$ of the customers in V_2 $(=\{C_3\})$ and the candidate customer C_4 is less than $MaxSup$. (1) Put C_4 into V_2. (2) Delete C_4 from F. (3) Go to step 5.

Step 5. F is empty and go to step 6.

Step 6. Compute the centroid of V_1 and V_2. $X_1=((155\times474)+(178\times365))/(474+365)=165$, $Y_1=((98\times474)+(191\times365))/(474+365)=138.46$, $X_2=((78\times522)+(50\times200))/(522+200)=70.24$, $Y_2=((88\times522)+(65\times200))/(522+200)=81.63$.

Step 7. (1) Select D_3 with the shortest path to the centroid of V_1 $(=55.28)$. (2) Put D_3 into V_1. (3) Delete D_3 from E. (4) Set $r=r+1=2$.

Step 8. r is equal to k and go to step 7.

Step 7. (1) Select D_2 with the shortest path to the centroid of V_2 $(=48.57)$. (2) Put D_2 into V_2. (3) Delete D_2 from E. (4) Set $r=r+1=3$.

Step 8. r is greater than k and go to step 9.

Step 9. (1) Compute the total system cost SC.

1. In route 1 $(V_1=\{C_2, C_1, D_3\})$, $(Q, R)=(150, 10.5)$ and $B(R)=0.01$ are deduced based on the procedure in Appendix 1. The total demand D is 839 $(474+365)$. The total distance (Dis) is 197.16 and the average total demand during lead time is 5.5 $((3+8)/2)$. Depot establishing cost for D_3 is 143.
   Transportation cost: $(c+cm\times Dis)\times(D/Q)$
   $=(25+1\times197.16)\times(839/150)=1242.61$.
   Inventory cost:
   $(Q/2+R-UL)\times h^+ +(D/Q)\times A+hs\times B(R)\times(D/Q)=$
   $(150/2+10.5-5.5)\times0.5+(839/150)\times20$
   $+2\times0.01\times(839/150)=151.98$
2. In route 2 $(V_2=\{C_3, C_4, D_2\})$, $(Q, R)=(150, 12.3)$ and $B(R)=0.02$ are deduced based on the procedure in Appendix 1. The total demand D is 722 $(522+200)$. The total distance (Dis) is 123.59 and the average total demand during lead time is 6.5 $((6+7)/2)$. Depot establishing cost for D_2 is 467.
   Transportation cost: $(c+cm\times Dis)\times(D/Q)$
   $=(25+1\times123.59)\times(722/150)=715.21$.
   Inventory cost:
   $(Q/2+R-UL)\times h^+ +(D/Q)\times A+hs\times B(R)\times(D/Q)$
   $=(150/2+12.3-6.5)\times0.5+(722/150)\times 20$
   $+2\times0.02\times(722/150)=136.86$
   Hence, the total system cost $SC=2856.66$ (the sum of depot establishing, transportation, and inventory costs in V_1 and V_2).(2) Set $SC'=SC$, $U_t=V_t$ for $1\le t\le k$.

Phase 2: improving the initial solution

Step 10. (1) Close D_2 with the minimal system cost SC $(=3511.77)$. Thus, the first route V_1 and the second route V_2 are delivered by D_3. Set $NOD=NOD-1=1$ (The initial value of NOD is equal to the number of routes which is equal to 2.).

Step 11. SC is greater than SC'. Go to step 12.

Step 12. NOD is equal to 1. Go to step 13.

Step 13. (1) Set $V_t=U_t$ for $1\le t\le k$. (2) Randomly select the only closed depot D_1 and an open depot D_2 from $\{D_2, D_3\}$, and the open depot D_2 is substituted by D_1 in V_t. (3) Compute the total system cost SC.

1. In route 1 $(V_1=\{C_2, C_1, D_3\})$, $(Q, R)=(150, 10.5)$ and $B(R)=0.01$ are deduced based on the procedure in Appendix 1. The total demand D is 839 $(474+365)$. The total distance (Dis) is 197.16 and the average total demand during lead time is 5.5 $((3+8)/2)$.
   Depot establishing cost for D_3 is 143.
   Transportation cost: $(c+cm\times Dis)\times(D/Q)$
   $=(25+1\times197.16)\times(839/150)=1242.61$.
   Inventory cost:
   $(Q/2+R-UL)\times h^+ +(D/Q)\times A+hs\times B(R)\times(D/Q)$
   $=(150/2+10.5-5.5)\times0.5+(839/150)\times20$
   $+2\times0.01\times(839/150)=151.98$

2. The second route V_2 ($=\{C_3, C_4, D_1\}$) is delivered by D_1. $(Q, R)=(150, 12.3)$ and $B(R)=0.02$ are deduced based on the procedure in Appendix 1. The total demand D is $522+200=722$. The total distance (Dis) is 165.49 and the average total demand during lead time is $(6+7)/2=6.5$.
   Depot establishing cost for D_1 is 209.
   Transportation cost:
   $(c+cm\times Dis)\times(D/Q)=(25+1\times165.49)\times(722/150)$
   $=916.89$.
   Inventory cost:
   $(Q/2+R-UL)\times h^{+}+(D/Q)\times A+hs\times B(R)\times(D/Q)$
   $=(150/2+12.3-6.5)\times0.5+(722/150)\times20$
   $+2\times0.02\ (722/150)=136.86$

The total system cost SC is 2800.34.

Step 14: SC is less than SC', set SC' $=SC$ and $U_t=V_t$ for $1\le t\le k$. Go to step 15.
Step 15: *max_swap* ($=0$) is less than the default value (= number of candidate depots/2 = 3/2). Thus, go to step 13.
Step 13. (1) Set $V_t=U_t$ for $1\le t\le k$. (2) Randomly select the only closed depot D_2 and an open depot D_3 from $\{D_1, D_3\}$, and then the open depot D_3 is substituted by D_2 in V_t. (3) Compute the total system cost SC ($=3621$).
Step 14. SC is greater than SC' ($=2800.34$). Set *max_swap* = *max_swap* + 1 = 1. Go to step 15.
Step 15: *max_swap* ($=1$) is less than the default value. Thus, go to step 13.
Step 13. (1) Set $V_t=U_t$ for $1\le t\le k$. (2) Randomly select the only closed depot D_2 and an open depot D_1 from $\{D_1, D_3\}$, and then the open depot D_1 is substituted by D_2 in V_t. (3) Compute the total system cost SC ($=2856.66$).
Step 14. SC is greater than SC' ($=2800.34$). Set *max_swap* = *max_swap* + 1 = 2. Go to step 15.
Step 15: *max_swap* ($=2$) is greater than the default value. Thus, go to step 16.
Step 16: The best solution SC' is 2800.34, $U_1=\{C_2, C_1, D_3\}$, $U_2=\{C_3, C_4, D_1\}$ and stop.

The solution for the proposed heuristic method is 2800.34. The optimal solution is 2800.34 based on the LP model in 2.2. It is found the optimal solution can be obtained by the proposed heuristic method for this small-sized problem.

5 Comparative evaluations

Numerical experiments were conducted to examine the computational effectiveness and efficiency of the proposed heuristic method (HM-1) by comparing it with two existing heuristic methods: HM-2 (SAV1 [1], please refer to Appendix 2 for the details) and HM-3 (a tabu search proposed by [2], please refer to Appendix 3 for the details). The heuristic methods are coded using the Visual C++ programming language and the tests are carried out on a PC Pentium 1.4G.

Considering all possible combinations of problem parameters and algorithmic parameters (The detailed values are listed in Table 1), 144 different problem instances ($3^2\times2^4$) were designed to evaluate the performance of the heuristic solutions. Each problem instance contained 5 tests. The demand for each customer in any test is randomly selected from a uniform distribution $U[450, 600]$ for each month. The demand during lead time for each customer is randomly selected from a uniform distribution U [0, 10]. The location (x, y) of each customer and candidate depot is randomly selected from a uniform distribution $U[0, 100]$. The other relevant cost values are listed in Table 2 [15].

Table 3 shows the average solutions and average CPU times for this proposed heuristic method and two other heuristic methods described earlier. It is found HM-1 is better than HM-2 and HM-3 in any problems. When the number of candidate depots increases, the total system costs for all the three heuristic solutions decrease. When the number of customers increases, the total system costs for all the three heuristic solutions increase. When vehicle service capacity increases, the total system costs for all the three heuristic solutions increase. When vehicle capacity increases, the total system costs for all the three heuristic solutions decrease. When distance cost and depot establishing cost increase (the ratio of distance cost and depot establishing cost to inventory cost becomes high) in the cost structure, the total system costs for all the three heuristic solutions increase. In addition, HM-1 and HM-2 are better than HM-3 in terms of average CPU time. However, these three methods are efficient (all average CPU times in Table 3 are less than 60 seconds). The CPU time of HM-

Table 1 The parameters and their values in different problem instances

Parameters	Values
Method used	HM-1, HM-2, HM-3
Number of depots	20, 10
Number of customers	200, 150, 100
Vehicle service capacity	4000, 3000
Vehicle capacity	300, 150
Cost structure (depot establishing cost: distance cost: shortage cost)	1200:2:2(High), 600:1:2(Low)

Table 2 The other relevant cost values used in different problem instances

Parameters	Values
Ordering cost	20
Vehicle dispatching cost	25
Shortage cost	2
Holding cost	0.5

Table 3 Results for different problem instances

number of depots	number of customers	vehicle service capacity	vehicle capacity	cost structure	HM-1		HM-2		HM-3	
					cost[1]	cpu[2]	cost[1]	cpu[2]	cost[1]	cpu[2]
10	100	3000	150	Low	68014	9	87166	6	84628	11
10	100	3000	300	Low	37721	7	45577	5	44055	9
10	100	4000	150	Low	78826	10	99938	8	96639	16
10	100	4000	300	Low	41870	9	51102	8	49710	14
10	150	3000	150	Low	94155	11	114052	9	110828	17
10	150	3000	300	Low	51586	10	59750	9	57961	22
10	150	4000	150	Low	107008	15	136468	11	132494	25
10	150	4000	300	Low	56312	14	70515	11	68170	28
10	200	3000	150	Low	118349	14	141786	13	136686	25
10	200	3000	300	Low	64464	16	75113	12	71663	20
10	200	4000	150	Low	132250	14	162103	14	157479	28
10	200	4000	300	Low	70419	16	83108	14	81270	27
10	100	3000	150	High	78638	7	102565	5	100548	9
10	100	3000	300	High	45346	7	54239	5	52853	9
10	100	4000	150	High	87832	10	112847	8	109463	18
10	100	4000	300	High	49025	10	58746	7	56841	13
10	150	3000	150	High	105950	11	148950	9	143835	20
10	150	3000	300	High	60686	11	77115	8	74966	18
10	150	4000	150	High	117510	14	163012	11	158362	25
10	150	4000	300	High	64844	15	83238	10	81784	22
10	200	3000	150	High	131333	17	188798	13	185241	26
10	200	3000	300	High	75036	14	97134	12	96246	23
10	200	4000	150	High	145126	17	206353	14	200343	28
10	200	4000	300	High	81087	17	103302	14	103206	27
20	100	3000	150	Low	64270	19	85526	15	84005	45
20	100	3000	300	Low	35470	16	44011	13	43700	30
20	100	4000	150	Low	73623	25	95225	20	93422	40
20	100	4000	300	Low	40032	23	48500	20	48058	44
20	150	3000	150	Low	86563	25	110661	22	109380	46
20	150	3000	300	Low	47778	24	57843	21	57129	41
20	150	4000	150	Low	99022	25	130391	22	128535	43
20	150	4000	300	Low	53643	23	68047	20	66162	39
20	200	3000	150	Low	106869	27	138759	23	135689	45
20	200	3000	300	Low	59419	25	72117	22	70987	51
20	200	4000	150	Low	122727	28	160585	25	155907	51
20	200	4000	300	Low	65981	26	81829	23	80416	50
20	100	3000	150	High	123399	15	179670	12	174437	23
20	100	3000	300	High	70108	15	91357	11	89667	26
20	100	4000	150	High	140025	17	196472	14	190749	30
20	100	4000	300	High	77938	20	100191	16	97564	32
20	150	3000	150	High	161200	20	245044	16	240819	31
20	150	3000	300	High	91553	17	125861	14	124137	27
20	150	4000	150	High	184460	24	283812	21	276516	41
20	150	4000	300	High	103190	25	143094	22	139897	47
20	200	3000	150	High	197001	26	294904	23	288256	45
20	200	3000	300	High	113102	23	151949	20	148495	42
20	200	4000	150	High	226471	26	341949	22	333931	48
20	200	4000	300	High	125074	26	172552	22	170438	43

cost[1]: The average system cost of 5 test problems per instance
cpu[2]: The average CPU time of 5 test problems per instance (unit: second)

3 increases when the problem size (number of candidate depots and number of customers) increases.

6 Conclusions

The inventory control decision seems ignored in most MDLRP research. In this paper, we have developed an effective and efficient heuristic method for the single-product multi-depot location-routing problem taking inventory control decisions into consideration, which still remains computationally intractable. This proposed heuristic method overcomes the problem of not taking inventory into consideration. The solution of the proposed two-phase heuristic method is better than those not taking inventory into consideration in terms of system cost and CPU time.

Two related research directions are as follows: (1) develop a model and methods for the multi-product multi-depot location-routing problem, and (2) develop an optimal solution for the single-product multi-depot location-routing problem.

7 Appendix 1

The procedure proposed by [13] is used to deduce Q_{kgh} and R_{kgh} as follows:

Step 1. Let $p=1$ and $(R_{kgh})_p=0$.

Step 2. Compute the shortage number $B((R_{kgh})_p)$ in each route.

$$B\left(\left(R_{kgh}\right)_p\right) = \int_{R_{kgh}}^{\infty} \left(x - \left(R_{kgh}\right)_p\right) f_L(x)dx \tag{A1}$$

Step 3. Put $B((R_{kgh})_p)$ into Eq. 2 and $\left(Q_{kgh}\right)_p^{'}$ can be deduced.

$$\left(Q_{kgh}\right)_p^{'} = \sqrt{\frac{2D_{kgh}\left(cm \times Dis_{kgh} + c + hs \times B\left(\left(R_{kgh}\right)_p\right)\right)}{h^+}} \tag{A2}$$

Step 4. Put $\left(Q_{kgh}\right)_p^{'}$ into Eq. 3 and compute the new value of $\left(R_{kgh}\right)_p^{'}$.

$$F_L\left(\left(R_{kgh}\right)_p^{'}\right) = 1 - \frac{h^+ \times \left(Q_{kgh}\right)_p^{'}}{hs \times D_{kgh}} \tag{A3}$$

where $F_L\left(\left(R_{kgh}\right)_p^{'}\right)$ is a cumulative distribution of $f_L(x)$ at $x = \left(R_{kgh}\right)_p^{'}$.

Step 5. If $\left(Q_{kgh}\right)_p^{'} \approx \left(Q_{kgh}\right)_{p-1}^{'}$ or $\left(R_{kgh}\right)_p \approx \left(R_{kgh}\right)_p^{'}$, the optimal solution $\left(Q_{kgh}, R_{kgh}\right) = \left(\left(Q_{kgh}\right)_p^{'}, \left(R_{kgh}\right)_p^{'}\right)$ is found and stop. Otherwise, let $\left(R_{kgh}\right)_{p+1} = \left(R_{kgh}\right)_p^{'}$, $p=p+1$, and go to step 2.

8 Appendix 2

The detailed procedure for SAV1 method used in [1] is as follows:

Step 1. Initialize all depots open.
Step 2. Calculate modified savings for all open depots and all customers.
Step 3. Assign customers to open depots based on maximum frequency of positive savings.
Step 4. Create vehicle routes for each open depot.
Step 5. Compute total system cost for the current number of open depots.
Step 6. Compute opportunity penalty of not using best depot for each customer link *i-j*.
Step 7. Compute maximum opportunity penalty for each customer with every open depot.
Step 8. Compute the sum of the maximum opportunity penalty of all customers for each open depot.
Step 9. Close the depot with the smallest penalty sum.
Step 10. Is the number of open depots equal to required number or 1? If yes, go to step 11. Otherwise, go back to step 2.
Step 11. Select the system configuration with system cost (or with required number of depots).
Step 12. Stop.

9 Appendix 3

The detailed procedure for the tabu-search method used in [2] is as follows:

Step 1. Open one depot randomly, route customers using savings algorithm.
Step 2. Is the number of non-improving routing insert moves less than *max_route*? If yes, go to step 3. Otherwise, go to step 4.
Step 3. Perform best non-tabu *routing_insert* move, m^*, declare m^* tabu and go back to step 2.
Step 4. Is the number of non-improving routing swap moves less than *max_route*? If yes, go to step 5. Otherwise, go to step 6.
Step 5. Perform best non-tabu *routing_swap* move, m^*, declare m^* tabu and go back to step 4.
Step 6. Update routing with the best one found in routing phase.
Step 7. Is the number of non-improving location swap moves less than *max_swap*? If yes, go to step 8. Otherwise, go to step 9.
Step 8. Perform best non-tabu *location_swap* move, m^*, declare m^* tabu and go back to step 2.
Step 9. Update solution with the best one found in location swap phase.
Step 10. Is the number of non-improving add location moves less than *max_swap*? If yes, go to step 11. Otherwise, go to step 12.
Step 11. Perform best non-tabu *location_add* move, m^*, declare m^* tabu and go back to step 2.
Step 12. Update solution with the best one found in location add phase.
Step 13. Stop.

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