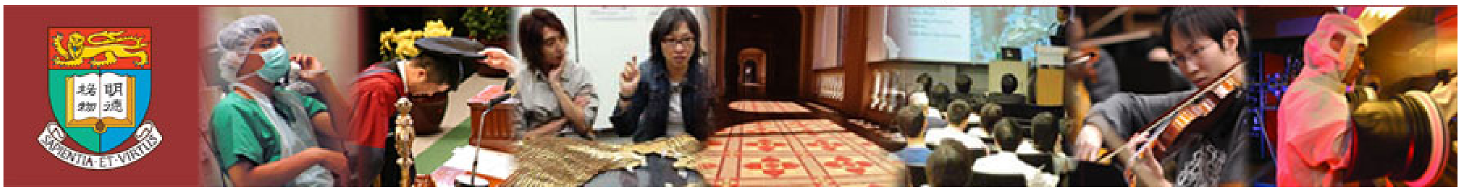

An Artificial Bee Colony Algorithm for the Capacitated Vehicle Routing Problem

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Abstract

This paper introduces an artificial bee colony heuristic for solving the capacitated vehicle routing problem. The artificial bee colony heuristic is a swarm-based heuristic, which mimics the foraging behavior of a honey bee swarm. An enhanced version of the artificial bee colony heuristic is also proposed to improve the solution quality of the original version. The performance of the enhanced heuristic is evaluated on two sets of standard benchmark instances, and compared with the original artificial bee colony heuristic. The computational results show that the enhanced heuristic outperforms the original one, and can produce good solutions when compared with the existing heuristics. These results seem to indicate that the enhanced heuristic is an alternative to solve the capacitated vehicle routing problem.

Keywords: Routing, Artificial Bee Colony, Metaheuristic

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1. Introduction

The *Capacitated Vehicle Routing Problem* (CVRP) (i.e., the classical vehicle routing problem) is defined on a complete undirected graph $G=(V,E)$, where $V=\{0,1,\dots,n\}$ is the vertex set and $E=\{(i,j):i,j\in V,i<j\}$ is the edge set. Vertices $1,\dots,n$ represent customers; each customer i is associated with a nonnegative demand d_i and a nonnegative service time s_i. Vertex 0 represents the depot at which a fleet of m homogeneous vehicles of capacity Q is based. The fleet size is treated as a decision variable. Each edge (i,j) is associated a nonnegative traveling cost or travel time c_{ij}. The CVRP is to determine m vehicle routes such that (a) every route starts and ends at the depot; (b) every customer is visited exactly once; (c) the total demand of any vehicle route does not exceed Q; and (d) the total cost of all vehicle routes is minimized. In some cases, the CVRP also imposes duration constraints where the duration of any vehicle route must not exceed a given bound L. Mathematical formulations of the CVRP can be found in Toth and Vigo (2002).

As the CVRP is a NP-hard problem, only instances of small sizes can be solved to optimality using exact solution methods (e.g., Toth and Vigo, 2002; Baldacci et al., 2010), and this might not even be possible if it is required to use limited amount of computing time. As a result of this, heuristic methods are used to find good, but not necessarily guaranteed optimal solutions using reasonable amount of computing time. During the past two decades, an increasing number of publications on heuristic approaches have been developed to tackle the CVRP. The work can be categorized into evolutionary algorithms (Baker and Ayechew, 2003; Berger and Barkaoui, 2003; Prins, 2004; Mester and Bräysy, 2007; Prins, 2009; Nagata and Bräysy, 2009), ant colony optimization (Bullnheimer et al., 1999; Reimann et al., 2004; Yu et al., 2009), simulated annealing (Osman, 1993; Lin et al., 2009), tabu search (Taillard, 1993; Gendreau et al., 1994; Rego and Roucairol, 1996; Rego, 1998; Cordeau et al., 2001; Toth and Vigo, 2003; Derigs and Kaiser, 2007), path-relinking (Ho and Gendreau, 2006), adaptive memory procedures (Rochat and Taillard, 1995; Tarantilis and Kiranoudis, 2002; Tarantilis, 2005), large neighborhood search (Ergun et al., 2006; Pisinger and Ropke, 2007), variable neighborhood search (Kytöjoki et al., 2007; Chen et al., 2010), deterministic annealing (Golden et al., 1998; Li et al., 2005), honey-bees mating optimization (Marinakis et al., 2010), particle swarm optimization (Ai and Kachitvichyanukul, 2009) and hybrid Clarke and Wright's savings heuristic (Juan et al., 2010).

Some of the above CVRP heuristics are based on previously developed heuristics. In the following we will mention some of the relationships. Rochat and Taillard's (1995) adaptive memory procedure used the idea of genetic algorithms of combining solutions to construct new solutions and employed the tabu search of Taillard (1993) as an improvement procedure. Tarantilis (2005) had a similar framework as Tarantilis and Kiranoudis (2002) (which is a variant of the idea proposed by Rochat and Taillard), but is different in that it does not rely on probability as in Tarantilis and Kiranoudis (2002) and also uses a more sophisticated improvement procedure. The *Unified tabu search* (Cordeau et al., 2001) and the tabu search used by Ho and Gendreau (2006) contain some of the features found in *Taburoute* (Gendreau et al., 1994); infeasible solutions are considered by extending the objective function with a penalty function and the use of continuous diversification. *Granular tabu search* (Toth and Vigo, 2003) restricts the neighborhood size by removing edges from the graph that are unlikely to appear in an optimal solution. Later, other researchers have also applied this idea on their CVRP heuristics (e.g., Li et al., 2005; Mester and Bräysy, 2007; Chen et al., 2010). For extensive surveys on VRP metaheuristics, the reader is referred to Gendreau et al. (2002), Cordeau et al. (2005) and Gendreau et al. (2008).

While the above heuristics have been widely used and successfully applied to solve the CVRP for several years, Artificial Bee Colony (ABC) is a fairly new approach introduced just a few years ago by Karaboga (2005), but has not yet been applied to solve the CVRP. However, ABC has been applied to solve other problems with great success (Baykasoğly et al., 2007; Kang et al., 2009; Karaboga, 2009; Karaboga and Basturk, 2007, 2008; Karaboga and Ozturk, 2009; Singh, 2009). It is worthwhile to evaluate the performance of the ABC algorithm for solving the CVRP.

In this paper, we evaluate the performance of the ABC heuristic using classical benchmark instances. An enhanced version of ABC heuristic is also proposed to improve the performance of the basic ABC heuristic for solving the CVRP. Computational results show that the enhanced ABC heuristic can produce much better solutions than the basic one, and the computation time of the enhanced ABC has slightly increased. The results obtained by the enhanced ABC heuristic are also competitive to those obtained from other metaheuristics. The paper is organized as follows. In Section 2, we give a review of the basic ABC algorithm. Refinements of the basic ABC algorithm and how they are applied to the CVRP are presented in Section 3. Experimental results showing the improvements in the performance of the enhanced ABC algorithm are presented in Section 4. Finally, Section 5 concludes the paper.

2. Artificial bee colony algorithm

The ABC algorithm belongs to a class of evolutionary algorithms that are inspired by the intelligent behavior of the honey bees in finding nectar sources around their hives. This class of metaheuristics has only started to receive attention recently. Different variations of bee algorithms under various names have been proposed to solve combinatorial problems. But in all of them, some common search strategies are applied; that is, complete or partial solutions are considered as food sources and the groups of bees try to exploit the food sources in the hope of finding good quality nectar (i.e., high quality solutions) for the hive. In addition, bees communicate between themselves about the search space and the food sources by performing a waggle dance.

In the ABC algorithm, the bees are divided into three types; employed bees, onlookers and scouts. Employed bees are responsible for exploiting available food sources and gathering required information. They also share the information with the onlookers, and the onlookers select existing food sources to be further explored. When the food source is abandoned by its employed bee, the employed bee becomes a scout and starts to search for a new food source in the vicinity of the hive. The abandonment happens when the quality of the food source is not improved after performing a maximum allowable number of iterations.

The ABC algorithm is an iterative algorithm, and it starts by generating random solutions as the food sources and assigning each employed bee to a food source. Then during each iteration, each employed bee finds a new food source near its originally assigned (or old) food source (using a neighborhood operator). The nectar amount (fitness) of the new food source is then evaluated. If the new food source has more nectar than the old one, then the old one is replaced by the new one. After all employed bees have finished with the above exploitation process, they share the nectar information of the food sources with the onlookers. Then, each onlooker selects a food source according to the traditional roulette wheel selection method. After that, each onlooker finds a food source near its selected food source (using a neighborhood operator) and calculates the nectar amount of the neighbor food source. Then, for each old food source, the best food source among all the food sources near the old food source is determined. The employed bee associated with the old food source is assigned to the best food source and abandons the old one if the best food source is better than the old food source. A food source is also abandoned by an employed bee if the quality of the food source has not been improved for *limit* (a predetermined number) successive iterations. That employed bee then becomes a scout, and searches for new food source randomly. After the scout finds a new food source, the scout becomes an employed bee again. After each employed bee is assigned to a food source, another iteration of the ABC algorithm begins. The whole process is repeated until a stopping condition is met.

The steps of the ABC algorithm are summarized as follows:

```
1. Randomly generate a set of solutions as initial food sources $x_i$, $i = 1, \ldots, \tau$. Assign each
   employed bee to a food source.
2. Evaluate the fitness $f(x_i)$ of each of the food sources $x_i$, $i = 1, \ldots, \tau$.
3. Set $v = 0$ and $l_1 = l_2 = \ldots = l_\tau = 0$.
4. Repeat
      a. For each food source $x_i$
            i. Apply a neighborhood operator on $x_i \rightarrow \tilde{x}$.
            ii. If $f(\tilde{x}) > f(x_i)$, then replace $x_i$ with $\tilde{x}$ and $l_i = 0$, else $l_i = l_i + 1$.
      b. Set $G_i = \emptyset$, $i = 1, \ldots, \tau$, where $G_i$ is the set of neighbor solutions of food source $i$.
      c. For each onlooker
            i. Select a food source $x_i$ using the fitness-based roulette wheel selection
               method.
            ii. Apply a neighborhood operator on $x_i \rightarrow \tilde{x}$.
            iii. $G_i = G_i \cup \{\tilde{x}\}$
      d. For each food source $x_i$ and $G_i \neq \emptyset$
            i. Set $\hat{x} \in \arg\max_{\sigma \in G_i} f(\sigma)$.
            ii. If $f(x_i) < f(\hat{x})$, then replace $x_i$ with $\hat{x}$ and $l_i = 0$, else $l_i = l_i + 1$.
      e. For each food source $x_i$
            i. If $l_i = limit$, then replace $x_i$ with a randomly generated solution.
      f. $v = v + 1$
5. Until $(v = MaxIterations)$
```

3. Application to the CVRP

In this section, different components of the algorithm are first explained, and then how they are woven together and applied to the CVRP is described.

3.1 Solution representation

In order to maintain the simplicity of the ABC algorithm, a rather straightforward solution representation scheme is adopted. Suppose n customers are visited by m vehicle routes. The representation has the form of a vector of length $(n+m)$. In the vector, there are n integers between 1 and n inclusively representing customers' identity. There are also m 0s in the vector representing the start of each vehicle route from the depot. The sequence between two 0s is the sequence of customers to be visited by a vehicle. Figure 1 illustrates a representation of a CVRP instance with $n = 7$ and $m = 3$. As shown in Figure 1, customers 4 and 1 are assigned to the same vehicle route and the vehicle visits customer 4 before customer 1.

0	4	1	0	2	7	5	0	3	6

Figure 1. Solution representation

3.2 Search space and cost functions

The search may become restrictive if only the feasible part of the solution space is explored. Hence, we also allow the search to be conducted in the infeasible part of the solution space, so that the search can oscillate between feasible and infeasible parts of the solution space X. Each solution $x \in X$ consists of m vehicle routes where each one of them starts and ends at the depot, and every customer is visited exactly once. Thus, x may be infeasible with regard to the capacity and/or duration constraints. For a solution x, let $c(x)$ denote its travel cost, and let $q(x)$ and $t(x)$ denote the total violation of the capacity and duration constraints respectively. The routing cost of a vehicle k corresponds to the sum of the costs c_{ij} associated with the edges (i, j) traversed by this vehicle. The total violation of the capacity and duration constraints is computed on a route basis with respect to Q and L. Each solution x found in the search is evaluated by a cost function $z(x) = c(x) + \alpha q(x) + \beta t(x)$ that includes the considerations mentioned above. The coefficients α and β are self-adjusting positive parameters that are modified at every iteration. The parameter α is adjusted as follows: If the number of solutions with no violation of the capacity constraints is greater than $\tau / 2$, the value of α is divided by $1+\delta$, otherwise it is multiplied by $1+\delta$. The same rule also applies to β with respect to the duration constraints.

3.3 Initial solution

An initial solution is constructed by assigning one customer at a time to one of the m vehicle routes. The selection of the customer is randomly made. The customer is then assigned to the location that minimizes the cost of assigning this customer over the current set of vehicle routes. The above procedure is repeated until all customers are routed.

A total of τ initial solutions are generated by the above procedure.

3.4 Neighborhood operators

A neighborhood operator is used to obtain a new solution $\tilde{x}$ from the current solution x in Step 4 of the ABC heuristic. A number of neighborhood operators (among those listed below) are chosen in advance. Any combinations of the listed operators are possible, even one. Then, whenever a new solution $\tilde{x}$ is needed, a neighborhood operator is randomly chosen from the set of pre-selected neighborhood operators and applied *once* (i.e., no local search is performed) to the solution x. The set of pre-selected operators is determined by experimental testing discussed in Section 4.1. The possible operators include:

1. Random swaps

   This operator randomly selects positions (in the solution vector) i and j with $i \neq j$ and swaps the customers located in positions i and j. See Figure 2, where $i = 3$ and $j = 7$.

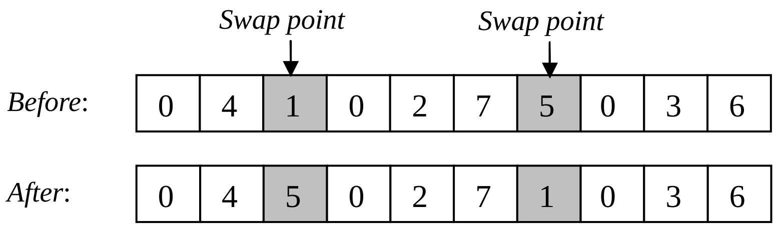


Figure 2. Random swaps

2. Random swaps of subsequences

   This operator is an extension of the previous one, where two subsequences of customers and depot of random lengths are selected and swapped. An example is shown in Figure 3.

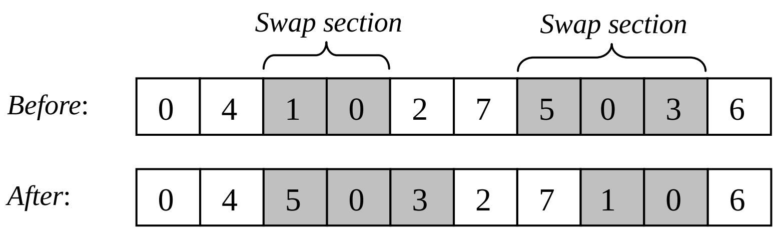


Figure 3. Random swaps of subsequences

3. Random insertions

   This operator consists of randomly selecting positions i and j with $i \neq j$ and relocating the customer from position i to position j. See Figure 4, where customer 5 is relocated from position 7 to position 3.

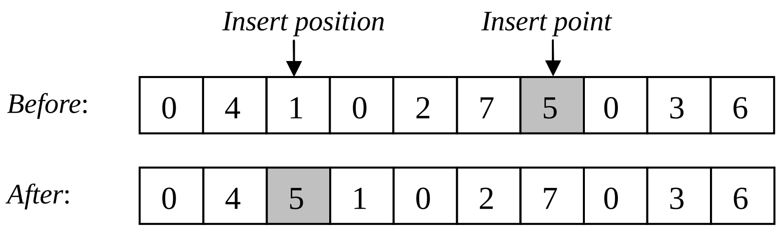


Figure 4. Random insertions

4. Random insertions of subsequences

   This operator is an extension of the operator of random insertions where a subsequence of customers and depot of random length starting from position i is relocated to position j. Positions i and j are randomly selected. Figure 5 gives an example of this operator.

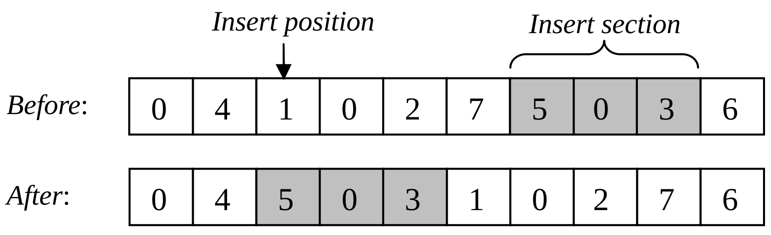


Figure 5. Random insertions of subsequences

5. Reversing a subsequence

   A subsequence of consecutive customers and depot of random length is selected and then the order of the corresponding customers and depot is reversed, as shown in Figure 6.

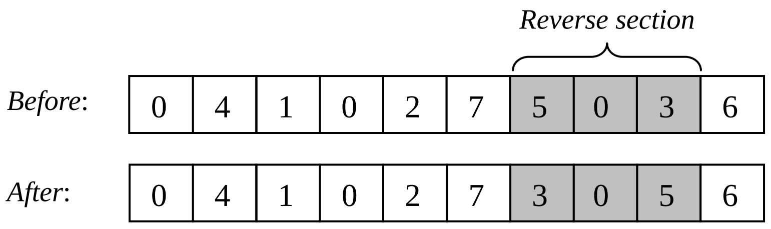


Figure 6. Reversing a subsequence

6. Random swaps of reversed subsequences

   This operator is a combination of two previously mentioned operators. Two subsequences of customers and depot of random lengths are chosen and swapped. Then each of the swapped subsequences may be reversed with a probability of 50%. An example of this operator is displayed in Figure 7.

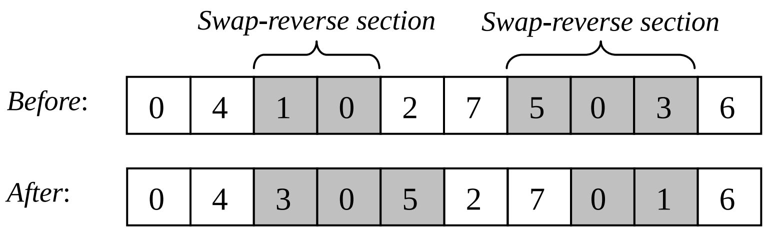


Figure 7. Random swaps of reversed subsequences

7. Random insertions of reversed subsequences

   A subsequence of customers and depot of random length starting from position i is relocated to position j. Then the relocated subsequence has a 50% chance of being reversed. Figure 8 shows an example of this operator where $i = 7$ and $j = 3$, and the subsequence is reversed.

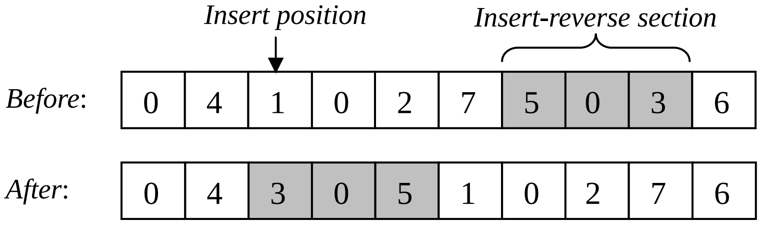


Figure 8. Random insertions of reversed subsequences

3.5 Selection of food sources

At each iteration of the algorithm, each onlooker selects a food source randomly. In order to drive the selection process towards better food sources, we have implemented a roulette-wheel selection method for randomly selecting a food source. The probability of choosing the food source x_i is then defined as $p(x_i) = \frac{f(x_i)}{\sum_{i=1}^{\tau} f(x_i)}$, $i = 1, \ldots, \tau$ where $f(x_i) = \frac{1}{z(x_i)}$.

3.6 An enhanced artificial bee colony algorithm

It is shown in the literature that the basic ABC algorithm is capable of solving certain problems with great success. However, according to our computational experiments (see Section 4) this does not apply to the CVRP. Therefore, in this section, we will propose an enhanced version of the ABC algorithm to improve the performance of the basic ABC algorithm for solving the CVRP.

Step 4d of the basic ABC algorithm states that a food source x_i will only be replaced by a neighbor solution $\hat{x}$ if $\hat{x}$ is of a better quality than x_i, where $\hat{x}$ is the best neighbor solution found by the onlookers in Step 4b. In the refined version, this condition is altered so that if the best neighbor solution $\hat{x}$ found by all the onlookers associated with food source i is better than the food source x_i, then $\hat{x}$ will replace the food source x_j that possesses the following two properties: 1) x_j has not been improved for the largest number of iterations among all the existing food sources and 2) x_j is worse than $\hat{x}$. In this way, potential food sources (i.e., the food sources which can produce better neighbor solutions) will be given opportunities to be further explored whereas non-potential food sources (i.e., the food sources which have not been improved for a relatively large number of iterations and are worse than new neighbor solutions) are excluded.

Step 4e of the basic ABC algorithm states that if no improving neighbor solutions of the food source x_i have been identified during the last *limit* successive iterations, then x_i is replaced by a randomly generated solution (using the procedure described in Section 3.3). In the refined version, this is modified so that x_i is replaced by a new neighbor solution $\hat{x}$ of x_i. The quality of $\hat{x}$ can be worse or better than x_i. This modification may be beneficial as it prevents the search from searching in bad regions of the solution space with no control of the quality of the food sources.

The steps of the refined ABC algorithm are summarized below:

```
1. Randomly generate a set of solutions as initial food sources $x_i$, $i = 1, \dots, \tau$.
   Set $F = \cup_{i=1}^{\tau} \{x_i\}$, where $F$ is the set of food sources. Assign each employed bee to a food
   source.
2. Evaluate the fitness $f(x_i)$ of each of the food sources $x_i$, $i = 1, \dots, \tau$.
3. Set $v = 0$ and $l_1 = l_2 = \dots = l_\tau = 0$.
4. Repeat
      a. For each food source $x_i$
            i. Apply a neighborhood operator on $x_i \rightarrow \tilde{x}$.
            ii. If $f(\tilde{x}) > f(x_i)$, then replace $x_i$ with $\tilde{x}$ and $l_i = 0$, else $l_i = l_i + 1$.
      b. Set $G_i = \varnothing,\ i = 1, \dots, \tau$.
      c. For each onlooker
            i. Select a food source $x_i$ using the fitness-based roulette wheel selection
               method.
            ii. Apply a neighborhood operator on $x_i \rightarrow \tilde{x}$.
      d. $G_i = G_i \cup \{\tilde{x}\}$
      e. For each food source $x_i$ and $G_i \neq \varnothing$
            i. Set $\hat{x} \in \arg\max_{\sigma \in G_i} f(\sigma)$.
            ii. If $f(x_i) < f(\hat{x})$, then select $\tilde{x}_j \in F$ with
                $j \in \arg\max_{i=1,\dots,|F|} \{l_i \mid f(\hat{x}) > f(x_i)$ and $x_i \in F\}$, replace $\tilde{x}_j$ with $\hat{x}$, and
                $l_i = 0$, else $l_i = l_i + 1$.
      f. For each food source $x_i$
            i. If $l_i = limit$, then apply a neighborhood operator on $x_i \rightarrow \tilde{x}$ and replace $x_i$
               with $\tilde{x}$.
      g. $v = v + 1$
5. Until $(v = MaxIterations)$
```

4. Computational experiments

We have tested both the original and the refined ABC heuristics on two standard benchmark instance sets. These include the fourteen classical Euclidean VRP and distance-constrained VRP instances described in Christofides and Eilon (1969) and Christofides et al. (1979) and the twenty large scale instances described in Golden et al. (1998). All experiments were performed on a 1.73 GHz computer, and the heuristics were coded in Visual C++ 2003.

The number of employed bees and the number of onlookers are set to be equal to the number of food sources (τ), which is 25. This is based on Karaboga and Basturk (2008), which set the number of employed bees to be equal to the number of onlookers to reduce the number of parameters, and find that the colony size (i.e., the total number of employed bees and onlookers) of 50 can provide an acceptable convergence speed for search.

To reduce the number of parameters tuned, we let $\alpha = \beta$. We then evaluated the performance of the heuristics using different values of α in the interval [0.0001, 1] and δ = 0.001. It is found that the performance of the heuristics is insensitive over this range for many test instances, and for a majority of the test instances setting $\alpha = \beta$ to 0.1 seems to be the best choice. Based on these values of α and β, we tested the best value of δ over the range [0.0001, 1]. The results show that the performance of the heuristics varies significantly in this interval for many test instances, and good solutions are obtained with δ = 0.001. Therefore, we set $\alpha = \beta = 0.1$ and δ = 0.001 for all further experiments.

4.1 Comparison of neighborhood operators

In order to evaluate the effectiveness of each neighborhood operator, we have first tested the original ABC heuristic on instance vrpnc1 by considering each neighborhood operator at a time, including: 1. random swaps (*swap*); 2. random swaps of subsequences (*section_swap*); 3. random insertions (*insert*); 4. random insertions of subsequences (*section_insert*); 5. reversing a subsequence (*reverse*); 6. random swaps of reversed subsequences (*swap_reverse*); and 7. random insertions of reversed subsequences (*insert_reverse*). The results are reported in Table 1. The best known solution reported in the literature for this instance is 524.61. Each of the seven variants was run 20 times, and each time the algorithm was run for 100,000 iterations. From the table, one can observe that the different neighborhood operators contribute differently. It can be seen that the ABC heuristic with the operator of randomly swapping reversed subsequences achieves the best average objective value, while random insertions and random insertions of subsequences did not yield promising results. Even though the variant of incorporating the operator of randomly swapping reversed subsequences did not lead to the best known solution, the average deviation from the best known solution is less than 1.5%. Reversing a subsequence is another operator which also yields good performance, and uses less computation time than the operator of randomly swapping reversed subsequences.

Table 1. Experimental results by different operators for instance vrpnc1

Operator	*swap*	*section_ swap*	*insert*	*section_ insert*	*reverse*	*swap_ reverse*	*insert_ reverse*	*combined*
min.[a]	576.47	550.94	758.12	623.79	528.56	527.82	534.41	526.97
max [b]	627.52	597.14	810.71	655.81	543.83	536.64	556.91	535.43
avg [c]	610.81	567.34	784.09	640.69	535.57	532.44	542.58	531.74
std.dev.[d]	15.98	10.89	14.01	8.60	4.62	4.43	6.01	3.58
gap [e]	16.43%	8.15%	49.46%	22.13%	2.09%	1.49%	3.43%	1.36%
run time [f]	0.24	0.37	0.30	0.40	0.32	0.62	0.48	0.42

[a] Minimum objective value obtained in 20 runs

[b] Maximum objective value obtained in 20 runs

[c] Average objective value of 20 runs

[d] Standard deviation of objective values of 20 runs

[e] Average percentage deviation from the best known solution

[f] Average CPU time in minutes for each run

The search using only the operator of randomly swapping reversed subsequences may be too diversified, and may not lead to promising regions of the solution space. Thus, instead of using only one operator we will use a combination of several operators. A number of different combinations of operators have been tried and experimented with. The combination of the following operators seems to yield the most promising results: random swaps (*swap*), reversing a subsequence (*reverse*) and random swaps of reversed subsequences (*swap_reverse*). This seems reasonable as two of the operators were identified to yield the best results from Table 1, while the search using the third operator (i.e., random swapping) is not as diversified as the searches using the two other operators. Equal probabilities are associated with the operators being selected. Using this combination of operators, the heuristic was run and obtained an average deviation of 1.36% from the best known solution, as shown in the last column (*combined*) of Table 1. In addition, it is also faster than the variant using randomly swapping of reversed subsequences (*swap_reverse*) as the neighborhood operator.

Figure 9 shows the plot of the best objective solution values found by the heuristics using the operator of reversing a subsequence (*reverse*), the operator of randomly swapping of reversed subsequences (*swap_reverse*) and the combination of the operators for randomly swapping two positions, reversing a subsequence, and randomly swapping reversed subsequences (*combined*) during one run on instance vrpnc1. It can be observed from the figure that the variant using the operator of reversing a subsequence converges faster than the other two. However, the solution quality is not good as reflected by the objective value, which is the highest among the three operators. The operator of randomly swapping reversed subsequences converges at a later stage but achieves a better solution. The combined approach leads to the best solution quality than the other two, and the convergence rate is somewhere between the other two. This phenomenon was also observed when the different variants of the heuristic were experimented on the rest of the test instances. For this reason, the combination of the three different operators will be adopted in the heuristic to generate neighbor solutions.

4.2 Calibrating *limit*

As mentioned earlier, a food source x_i will be abandoned if no improving neighbor solutions $\hat{x}$ can be found in the neighborhood of x_i for consecutive $limit$ iterations. Karaboga (2009) has shown that this parameter is important regarding the performance of the ABC algorithm on solving function optimization problems. Hence, we will also study the effect of this parameter on the performance of the ABC algorithm for solving the CVRP.

The value of the parameter $limit$ was determined by running every benchmark instance 20 times for each of the predetermined values of $limit$. This calibrating process is important because if too few iterations are spent on exploring the neighborhood of a food source, the neighborhood may not be fully explored and the search will be too diversified. On the contrary, if too many iterations are spent on exploring the neighborhood, then the search will tend to focus on a few portions of the search space and will miss out other potential regions of the search space. Experiments on all the test instances show that the most appropriate value for the parameter $limit$ is proportional to the number of customers (n), and this value is approximately equal to $50n$. Due to space limitation, we only show the average objective function values of instance vrpnc1 with 50 customers for the different values of $limit$ in Figure 10. From this figure, we can easily see that $limit = 2,500$ yields the best results. Thus $limit = 50n$ was used in all the experiments reported in the remainder of this section.

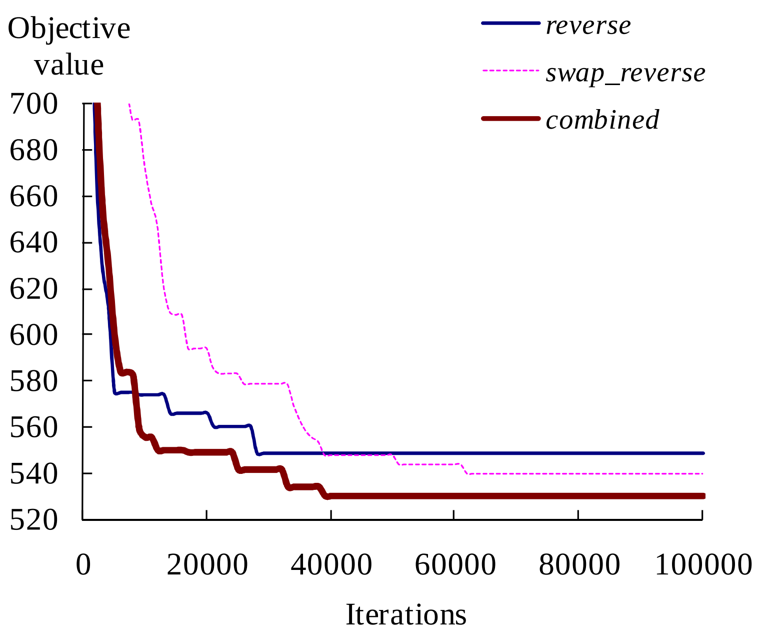


Figure 9. Converging processes for *reverse*, *swap_reverse*, and *combined*

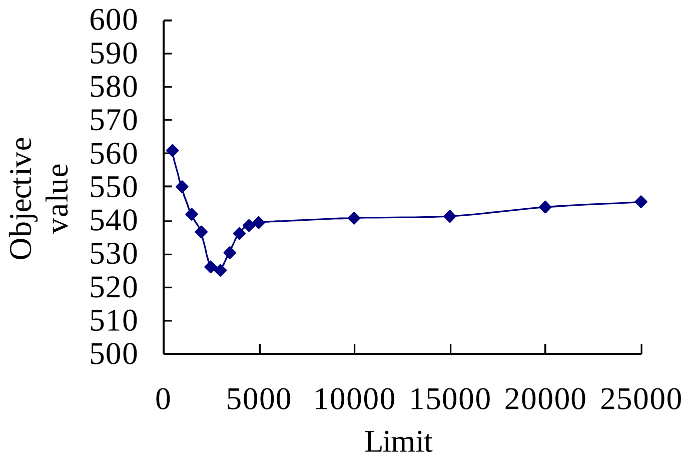


Figure 10. Effect of the parameter *limit*

4.3 Original ABC vs. enhanced ABC

According to some preliminary tests (not shown here) and the results shown in Table 1, the results of the CVRP obtained from the basic ABC algorithm are not very close to the best known result on average, even if we used many different combinations of operators. Therefore, we aim to improve the performance of the algorithm and propose an enhanced version. This version has been depicted in Section 3.6. The performance of this enhanced version was compared with that of the basic version. Both versions were implemented by using the combination of the three previously described operators as well as setting *limit* equal to $50n$, and assessed by the set of 14 classical instances. Each test instance was run 20 times using each of the ABC heuristics. It was found that the speed of convergence of the two heuristics depends on n. Moreover, it was found that setting the termination condition ($MaxIterations$) to $2000n$ iterations is sufficient for the two heuristics to converge. The best results, the average results, the average CPU times obtained by the two versions for each test instance are also reported in Table 2, where the solution values recorded for both the best and average results are rounded to the nearest integer. According to this table, the enhanced version obtained better solutions than the original version in all test instances in terms of both the average and the best results in 20 runs. The mean percentage improvement of the average (best) results of all test instances is 4.16% (3.53%). The largest percentage improvement of the average (best) result is 12.29% (11.32%), indicating that the enhanced version can produce much better solutions than the basic version. From the table, we can observe that the enhanced ABC heuristic requires more computational effort than the original version. This is due to the modification of step 4d of the enhanced heuristic.

Table 2. Comparison of experimental results between the original and enhanced ABC heuristics

Instance	Original ABC			Enhanced ABC			Improvement[d]
	Best[a]	*Avg.*[b]	*CPU*[c]	*Best*[a]	*Avg.*[b]	*CPU*[c]	
vrp_nc1	526.97	531.74	0.42	524.61	526.23	0.65	1.04%
vrp_nc2	865.23	885.78	0.85	836.74	842.97	1.33	4.83%
vrp_nc3	842.47	850.20	0.94	831.16	834.61	1.68	1.83%
vrp_nc4	1065.71	1087.29	5.17	1031.69	1061.80	8.17	2.34%
vrp_nc5	1382.85	1398.04	5.84	1320.24	1331.24	9.72	4.78%
vrp_nc6	560.33	570.48	0.44	555.43	558.09	0.70	2.17%
vrp_nc7	945.56	958.87	1.86	909.68	916.58	2.35	4.41%
vrp_nc8	887.71	897.44	1.94	865.94	876.12	2.54	2.38%
vrp_nc9	1235.37	1265.80	4.88	1170.25	1192.64	6.93	5.78%
vrp_nc10	1497.92	1517.24	8.59	1415.23	1434.05	12.83	5.48%
vrp_nc11	1183.92	1203.29	2.20	1049.91	1055.41	3.47	12.29%
vrp_nc12	843.17	854.32	2.07	819.56	821.37	2.90	3.86%
vrp_nc13	1592.88	1623.29	3.53	1545.98	1558.28	4.97	4.00%
vrp_nc14	881.41	895.27	2.62	866.37	868.60	3.35	2.98%

[a] Best result obtained in 20 runs

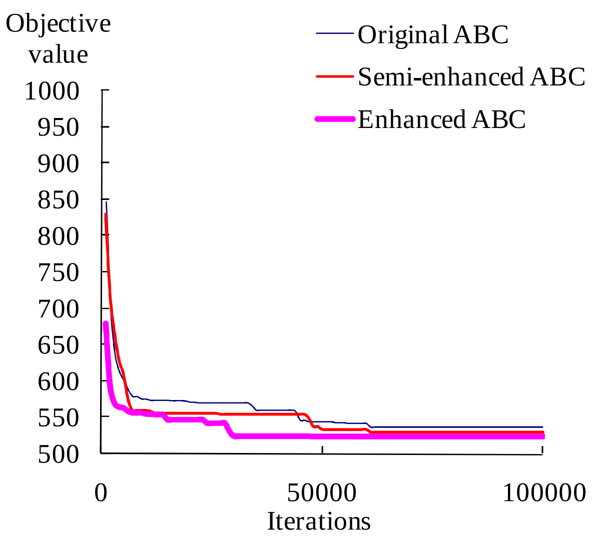
[b] Average result obtained in 20 runs

[c] Average CPU run time in minutes in 20 runs

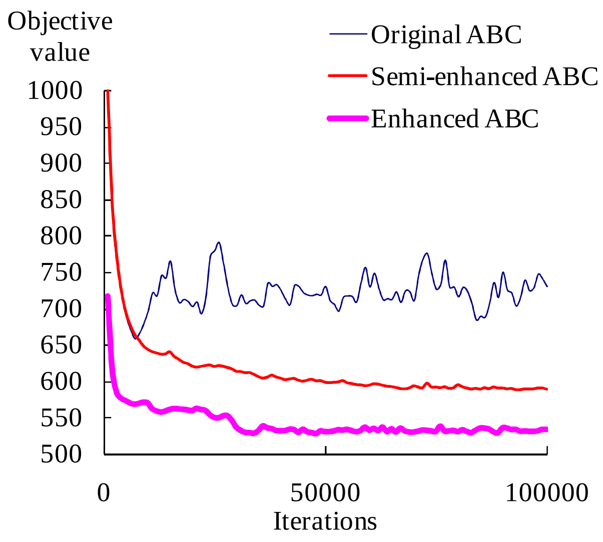
[d] Percentage improvement of the average result obtained by the enhanced ABC over the original ABC

To clearly illustrate the effect of the modifications in steps 4d and 4e on the improvement of solution quality, we plot figures 11a and 11b. Figure 11a shows the plot of the best objective values found by the original ABC heuristic, the semi-enhanced ABC heuristic (where only step 4e is enhanced) and the enhanced ABC heuristic (both the steps 4d and 4e are enhanced) during one run on instance vrpnc1. It can be observed from the figure that the original and semi-enhanced ABC heuristics converge at the same rate, while the semi-enhanced version obtains a better solution. The enhanced variant converges at an earlier stage with a better solution than the other two variants.

Figure 11b shows the plot of the average objective values of the τ food sources obtained by the original, semi-enhanced and enhanced ABC heuristics at the different iterations during one run. The behaviors of the semi-enhanced and enhanced ABC heuristics are similar where the values are gradually lowered. On the opposite, the average objective values of the original ABC heuristic do not show this pattern and some fluctuations can be observed. Around 10,000 iterations, the original ABC heuristic has identified a minimum average objective value. However, the algorithm has not been able to improve this further but has given some fluctuation results. This is due to step 4e where a food source x_i will be replaced by a randomly generated solution if no improving neighbor solutions $\hat{x}$ have been encountered for consecutive *limit* iterations. The effect is that after most food sources have not been improved for *limit* iterations, the original ABC heuristic randomly generates new solutions in which most of the new solutions have large objective values compared with the minimum average objective value. The outcome is that the average objective value increases after reaching the minimum average. The average objective value decreases slightly afterwards because the heuristics finds most of the food sources near local minima. However, the average value increases again later due to creating many new food sources to avoid trapping in local minima. This process is repeated and this is why the fluctuation can be observed. On the contrary, the average objective values of the enhanced ABC heuristic are gradually lowered due to the fact that a food source is replaced by a neighbor solution and not by a randomly generated solution, where the neighbor solutions have higher chances to have closer objective values to the minimum average objective value especially if the local objective surface is not too mountainous.



(a) Best solution



(b) Average solution

Figure 11. Converging processes of the basic ABC heuristic and its variants

In order to evaluate the effect of the modification made in step 4d on the average performance, we compared the average objective values of the semi-enhanced ABC heuristic with those of the enhanced one in figure 11b. It can be observed that the average objective values of the semi-enhanced variant are much worse than those of the enhanced variant, which indicates that the modification made in step 4d significantly improves the quality of the search.

4.4 Computational results

Table 3 lists the characteristics of the 14 classical instances. For each test instance, the table indicates the number of customers (n), the vehicle capacity (Q), the service time (s) for each customer, the route maximum length (L) and the number of vehicle routes (m). The table also gives for each instance the best known solution so far. In addition, the table provides information about the average and best solutions obtained with the enhanced ABC heuristic. It can be seen from the table that the enhanced ABC heuristic has matched six of the 14 best known solutions, and the average deviation from the best solutions is 0.46%.

Table 3. Enhanced ABC results for the classical instances

Instance	*n*	*Q*	*s*	*L*	*m*	Best known solution	ABC average [d]	ABC best [e]	DFB [f] (%)
vrpnc1	50	160	0	∞	5	524.61[a]	526.23	**524.61**	0.00
vrpnc2	75	140	0	∞	10	835.26[a]	842.97	836.74	0.18
vrpnc3	100	200	0	∞	8	826.14[a]	834.61	831.16	0.61
vrpnc4	150	200	0	∞	12	1028.42[a]	1061.80	1031.69	0.32
vrpnc5	199	200	0	∞	17	1291.29[b]	1331.24	1320.24	2.24
vrpnc6	50	160	10	200	6	555.43[a]	558.09	**555.43**	0.00
vrpnc7	75	140	10	160	11	909.68[a]	916.58	**909.68**	0.00
vrpnc8	100	200	10	230	9	865.94[a]	876.12	**865.94**	0.00
vrpnc9	150	200	10	200	14	1162.55[a]	1192.64	1170.25	0.66
vrpnc10	199	200	10	200	18	1395.85[c]	1434.05	1415.23	1.39
vrpnc11	120	200	0	∞	7	1042.11[a]	1055.41	1049.91	0.75
vrpnc12	100	200	0	∞	10	819.56[a]	821.37	**819.56**	0.00
vrpnc13	120	200	50	720	11	1541.14[a]	1558.28	1545.98	0.31
vrpnc14	100	200	90	1040	11	866.37[a]	868.60	**866.37**	0.00
						Average			0.46

[a] Obtained from Taillard (1993)

[b] Obtained from Mester and Bräysy (2007)

[c] Obtained from Rochat and Taillard (1995)

[d] Average solution obtained by enhanced ABC in 20 runs

[e] Best solution obtained by enhanced ABC in 20 runs

[f] Deviation of ABC best from the best known solution

Table 4 provides a comparison on the results obtained on the 14 classical instances by some of the metaheuristics for the CVRP. It is clear that the enhanced ABC heuristic performs better than some of the well-known tabu search heuristics such as Taburoute (Gendreau et al., 1994), the ejection chain method (Rego and Roucairol, 1996), the subpath ejection chain method (Rego, 1998), the granular tabu search (Toth and Vigo, 2003) and the Unified Tabu Search (Cordeau et al., 2001). In addition, the enhanced ABC heuristic outperforms the simulated annealing heuristic by Osman (1993), the saving-based ant system of Bullnheimer et al. (1999) and Reimann et al. (2004), and the hybrid genetic algorithm of Berger and Barkaoui (2003). Finally, the enhanced ABC heuristic also surpasses the path relinking approach of Ho and Gendreau (2006), the particle swarm optimization method by Ai and Kachitvichyanukul (2009) and the hybrid eletromagnetism-like algorithm of Yurtkuran and Emel (2010). Outstanding results were obtained by the adaptive memory procedure of Rochat and Taillard (1995) and by the memetic heuristic by Nagata and Bräysy (2009). The performances of the honey bees mating optimization algorithm (Marinakis et al., 2010) and the

AGES algorithm (Mester and Bräysy, 2007) are also excellent.

Table 4. Comparison of computational results of different methods for the classical instances

Reference	ADFB [a]	CPU time [b]	Computer
Rochat and Taillard (1995)	0.00	N/A	Silicon Graphics 100 MHz
Nagata and Bräysy (2009)	0.00	13.80	Opteron 2.4 GHz
Mester and Bräysy (2007)	0.03	2.80	Pentium IV 2.8 GHz
Marinakis et al. (2010)	0.03	0.80	Pentium M750 1.86 GHz
Taillard (1993)	0.05	N/A	Silicon Graphics 100 MHz
Prins (2009)	0.07	0.27	Pentium IV 2.8 GHz
Pisenger and Ropke (2007)	0.11	17.50	Pentium IV 3 GHz
Chen et al. (2010)	0.13	10.90	Pentium IV 2.93 GHz
Yu et al. (2009)	0.14	2.91	Pentium 1000 MHz
Tarantilis (2005)	0.20	5.63	Pentium 400 MHz
Derigs and Kaiser (2007)	0.21	5.84	Celeron 2.4 GHz
Tarantilis and Kiranoudis (2002)	0.23	5.22	Pentium 2 400 MHz
Ergun et al. (2006)	0.23	28.91	Pentium 733 MHz
Prins (2004)	0.24	5.19	Pentium 1 GHz
Lin et al. (2009)	0.35	8.21	Pentium IV 2.8 GHz
Szeto et al.	**0.46**	4.40	Pentium 1.73 GHz
Reimann et al. (2004)	0.48	3.63	Pentium 900 MHz
Berger and Barkaoui (2003)	0.49	21.25	Pentium 400 MHz
Ho and Gendreau (2006)	0.54	4.13	Pentium 2.53 GHz
Rego and Roucairol (1996)	0.55	24.65	4 Sun Sparc IPC
Cordeau et al. (2001)	0.56 [c]	24.62	Pentium IV 2 GHz
Baker and Ayechew (2003)	0.56	29.11	Pentium 266 MHz
Toth and Vigo (2003)	0.64	3.84	Pentium 200 MHz
Gendreau et al. (1994)	0.86	46.80	Silicon Graphics 36 MHz
Ai and Kachitvichyanukul (2009)	0.88	2.72	Intel P4 3.4 GHz
Yurtkuran and Emel (2010)	1.04	2.20	Intel Core2 Duo, 2 GHz
Bullnheimer et al. (1999)	1.51	18.44	Pentium 100 MHz
Rego (1998)	1.54	2.32	HP 9000/712
Osman (1993)	2.11	151.35	VAX 8600

[a] Average deviation from best known results

[b] Average computing time in minutes

[c] Computational results obtained from Cordeau et al. (2005)

Many of the heuristics within the 0.1% average deviation from best known solutions are population-based methods combined with local search, and many of the methods that yielded an average deviation of less than 0.4% are hybrids of various techniques. The implemented version of the heuristic is a pure artificial bee colony algorithm, and is also one of the effective algorithms for the CVRP. It is not easy to compare running times for the various metaheuristics due to different computing environments. We believe that the reported CPU time for the enhanced ABC heuristic is reasonable and acceptable.

Table 5. Enhanced ABC results for the large-scale instances

Instance	*n*	*Q*	*s*	*L*	*m*	Best known solution	ABC solution	DFB [d] (%)
kelly01	240	550	0	650	10	5626.81 [a]	5861.14	4.16
kelly02	320	700	0	900	11	8431.66 [a]	8720.37	3.42
kelly03	400	900	0	1200	10	11036.22 [b]	11572.69	4.86
kelly04	480	1000	0	1600	10	13592.88 [a]	13829.02	1.74
kelly05	200	900	0	1800	5	6460.98 [b]	6593.17	2.05
kelly06	280	900	0	1500	7	8404.26 [a]	8729.70	3.87
kelly07	360	900	0	1300	9	10156.58 [a]	10533.43	3.71
kelly08	440	900	0	1200	11	11643.90 [c]	12054.77	3.53
kelly09	255	1000	0	∞	14	580.42 [a]	593.35	2.23
kelly10	323	1000	0	∞	16	738.49 [a]	750.77	1.66
kelly11	399	1000	0	∞	18	914.72 [a]	932.81	1.98
kelly12	483	1000	0	∞	19	1106.76 [a]	1138.32	2.85
kelly13	252	1000	0	∞	26	857.19 [a]	872.45	1.78
kelly14	320	1000	0	∞	30	1080.55 [a]	1109.48	2.68
kelly15	396	1000	0	∞	33	1342.53 [a]	1367.37	1.85
kelly16	480	1000	0	∞	37	1620.85 [a]	1651.31	1.88
kelly17	240	200	0	∞	22	707.76 [a]	710.42	0.36
kelly18	300	200	0	∞	27	995.13 [a]	1019.64	2.46
kelly19	360	200	0	∞	33	1365.97 [a]	1377.54	0.85
kelly20	420	200	0	∞	38	1820.02 [a]	1850.59	1.68
						Average		2.48

[a] Obtained from Nagata and Bräysy (2009)

[b] Obtained from Prins (2004)

[c] Obtained from Prins (2009)

[d] Deviation from the best known solution

The enhanced ABC heuristic was also tested on the 20 large-scale instances by Golden et al. (1998). Table 5 lists the characteristics of these instances together with the average solutions and best solutions obtained by the enhanced ABC heuristic. The average deviation from best known solutions is 2.31%. Table 6 provides a comparison on the results with some of the metaheuristics published in the literature. The enhanced ABC heuristic outperforms the deterministic annealing algorithm described in Golden et al. (1998), the granular tabu search by Toth and Vigo (2003), the very large neighborhood search by Ergun et al. (2006), the path relinking approach by Ho and Gendreau (2006) and the variable neighborhood search procedure by Kytöjoki et al. (2007). As with the classical instances, population-based heuristics combined with local search also generated the best results for the large-scale instances. From the table we notice that the memetic algorithm by Nagata and Bräysy (2009) and the AGES algorithm by Mester and Bräysy (2007) obtained much better solutions than the rest of the methods.

Table 6. Comparison of computational results of different methods for the large-scale instances

Reference	ADFB[a]	CPU time[b]	Computer
Nagata and Bräysy (2009)	0.02	355.90	Opteron 2.4 GHz
Mester and Bräysy (2007)	0.18	24.40	Pentium IV 2.8 GHz
Prins (2009)	0.48	7.27	Pentium IV 2.8 GHz
Marinakis et al. (2010)	0.58	4.06	Pentium M750 1.86 GHz
Pisinger and Ropke (2007)	0.67	107.67	Pentium IV 3 GHz
Chen et al. (2010)	0.74	284.40	Pentium IV 2.93 GHz
Reimann et al. (2004)	0.78	49.33	Pentium 900 MHz
Tarantilis (2005)	0.78	45.58	Pentium 400 MHz
Derigs and Kaiser (2007)	0.86	113.34	Celeron 2.4 GHz
Prins (2004)	1.09	66.90	Pentium 1 GHz
Li et al. (2005)	1.24	N/A	N/A
Cordeau et al. (2001)	1.64[c]	56.11	Pentium 2 GHz
Lin et al. (2009)	1.79	118.98	Pentium IV 2.8 GHz
Szeto et al.	**2.48**	32.38	Pentium 1.73 GHz
Ho and Gendreau (2006)	2.74	39.96	Pentium 2.53 GHz
Toth and Vigo (2003)	3.06	17.55	Pentium 200 MHz
Ergun et al. (2006)	3.95	137.95	Pentium 733 MHz
Golden et al. (1998)	4.10	37.20	Pentium 100 MHz
Kytöjoki et al. (2007)	4.67	0.02	AMD Athlon64 3000+

[a] Average deviation from best known results

[b] Average computing time in minutes

[c] Computational results obtained from Cordeau et al. (2005)

5. Conclusions

In this paper, we presented an artificial bee colony (ABC) heuristic for the capacitated vehicle routing problem (CVRP). The ABC heuristic is a fairly new approach, and it is based on the foraging behavior of honey bees. An enhanced version of the ABC heuristic has been developed to improve the original heuristic for solving the CVRP. Computational results show that the enhanced ABC heuristic is able to produce much better solutions than the original one and uses slightly little more computation time. The results also show that the enhanced ABC can produce good solutions when compared with existing heuristics for solving the CVRP. These results seem to indicate that the enhanced heuristic is an alternative to solve the capacitated vehicle routing problem.

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