Classification in Vehicle Routing and Scheduling

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I. INTRODUCTION

The routing and scheduling of vehicles and their crews is an area of importance to both operations researchers and transportation planners. Recent research in this field includes significant breakthroughs in problem formulations and in the construction, analysis, and implementation of solution procedures. These advances have important implications for future research in routing and scheduling, as well as for immediate use in the design of mass transit systems and other applications. From a practical standpoint, the effective routing and scheduling of vehicles and crews can save government and industry many millions of dollars a year by increasing productivity, aiding long-range planning, assisting in contract negotiation, and in controlling the financial impact of adverse weather conditions on vehicle utilization.

In keeping with the importance of this area, an International Workshop on the Routing and Scheduling of Vehicles and Crews, sponsored in part by the National Science Foundation, was held at the University of Maryland at College Park on June 5-7, 1979. The papers in this Special Issue of *Networks* are polished versions of the presentations made at the Workshop.

First, we need to define what we mean by routing and scheduling of vehicles. A vehicle *route* is a sequence of pickup and/or delivery points which the vehicle must traverse in order, starting and ending at a depot or domicile. A vehicle *schedule* is a sequence of pickup and/or delivery points together with an associated set of arrival and departure times. The vehicle must traverse the points in the designated order at the specified times. When arrival times at nodes and/or arcs are fixed in advance, we refer to the problem as a scheduling problem. When arrival times are unspecified, the problem is a straightforward routing problem. When time windows and/or precedence relationships exist so that both routing and scheduling functions need to be performed, we view the problem as a combined vehicle routing and scheduling problem. These combined routing and scheduling problems often arise in practice and are representative of many real-world applications.

In Sec. II of this paper, we discuss numerous variations of the vehicle routing and scheduling problem and provide a taxonomy for these problems. In Sec. III, we classify current solution strategies for vehicle routing. In Sec. IV, we demonstrate a hierarchy

NETWORKS, Vol. 11 (1981) 97–108

CCC 0028-3045/81/020097-12$01.20

of scheduling problems, moving from the very simple to the extremely complex. In Sec. V, we describe three of the many possible combined routing and scheduling problems. Throughout our discussion, we relate the themes we touch upon briefly to other papers in this Special Issue of *Networks*, which focus in more depth on that particular concept. It is not our intention to survey the field of vehicle routing and scheduling in this paper. Such an undertaking would require considerably more space than is available to us. Rather our goal is to better organize the many and diverse routing and scheduling problems that have been studied.

II. A TAXONOMY FOR VEHICLE ROUTING AND SCHEDULING PROBLEMS

We first outline a number of characteristics that describe any vehicle routing and scheduling problem. A specific vehicle routing or scheduling problem can then be classified on the basis of these characteristics in rather obvious ways. The utility of this taxonomy is that it can help the analyst to identify the type of problem that he is confronting. If the characteristics define a well-known problem, then existing algorithms can be applied to solve the problem. On the other hand, if the problem has not received much research attention, then a new routing problem has been described. The outline below is an offshoot of earlier efforts by Bodin [6], Golden [24], and Golden et al. [27]. As can be seen, category A in our taxonomy serves a principal role.

A. time to service a particular node or arc
   1. time specified and fixed in advance (pure vehicle scheduling problem)
   2. time windows (combined vehicle routing and scheduling problem)
   3. time unspecified (in this case, we have a vehicle routing problem unless there are precedence relationships as well, in which case we have a combined vehicle routing and scheduling problem)

B. number of domiciles
   1. one domicile
   2. more than one domicile

C. size of vehicle fleet available
   1. one vehicle
   2. more than one vehicle

D. type of fleet available
   1. homogeneous case (all vehicles the same)
   2. heterogeneous case (not all vehicles the same)

E. nature of demands
   1. deterministic
   2. stochastic

F. location of demands
   1. at nodes (not necessarily all)
   2. on arcs (not necessarily all)
   3. mixed

G. underlying network
   1. undirected
   2. directed
   3. mixed

H. vehicle capacity constraints
  1. imposed–all the same
  2. imposed–not all the same
  3. not imposed
I. maximum vehicle route-times
  1. imposed–all the same
  2. imposed–not all the same
  3. not imposed
J. costs
  1. variable or routing costs
  2. fixed operating or vehicle acquisition costs (capital costs)
K. operations
  1. pickups only
  2. drop-offs only
  3. mixed
L. objective
  1. minimize routing costs incurred
  2. minimize sum of fixed and variable costs
  3. minimize number of vehicles required
M. other (problem-dependent) constraints

Obviously, this framework includes a vast number of possible variations on the standard Dantzig-Ramser [16] vehicle routing problem. For example, the problem that arises when there is a single domicile, a single vehicle of unlimited capacity, demands of unity which need to be collected at each node of an undirected network, routing costs only, and an objective function which minimizes total distance traveled is a traveling salesman problem. Also, there exist situations in which two routing or scheduling problems may seem *almost* identical in terms of the characteristics listed above and yet their computational complexity may differ significantly. For example, as Lenstra and Rinnooy Kan illustrate in their article in this issue, although the Chinese postman problem (CPP) on undirected and directed networks is solvable by a polynomial-bounded algorithm, the mixed-CPP is NP-hard. An interesting and recent survey of results on the mixed postman problem is given by Minieka [38]. In addition, a number of the well-known single vehicle routing problems are mentioned by name in the Lenstra-Rinnooy Kan article in this issue.

III. CLASSIFICATION OF SOLUTION STRATEGIES

Most solution strategies for vehicle routing problems can be classified as one of the following approaches:

1. cluster first–route second,
2. route first–cluster second,
3. savings/insertion,
4. improvement/exchange,
5. mathematical-programming-based,
6. interactive optimization, or
7. exact procedures.

The first four and last approaches have been used extensively in the past. The other two approaches represent relatively recently developed ideas. A more general framework for heuristic algorithms is given in the article by Ball and Magazine in this issue.

Cluster first–route second procedures group or cluster demand nodes and/or arcs first and then design economical routes over each cluster as a second step. Examples of this idea are given by Gillett and Miller [23], Gillett and Johnson [22], and Karp [32] for the standard single depot vehicle routing problem.

Route first–cluster second procedures work in the reverse sequence. First, a large (usually infeasible) route or cycle is constructed which includes all of the demand entities (that is, nodes and/or arcs). Next, the large route is partitioned into a number of smaller, but feasible, routes. Golden et al. [25] provide an algorithm that typifies this approach for a heterogeneous fleet size vehicle routing problem. Newton and Thomas [39] and Bodin and Berman [7] use this approach for routing school buses to and from a single school, and Bodin and Kursh [8], [9] utilize this approach for routing street sweepers. See also the work of Stern and Dror [48].

Savings or *insertion* procedures build a solution in such a way that at each step of the procedure (up to and including the penultimate step) a current configuration that is possibly infeasible is compared with an alternative configuration that may also be infeasible. The alternative configuration is one that yields the largest savings in terms of some criterion function, such as, total cost, or that inserts least expensively a demand entity not in the current configuration into the existing route or routes. The procedure eventually concludes with a feasible configuration. Examples of savings/ insertion procedures for single depot node and arc routing problems are described by Clarke and Wright [14], Golden et al. [26], Norback and Love [40], and Golden and Wong [28]. Hinson and Mulherkar [31] use a variant of this procedure for routing airplanes.

Improvement or *exchange* procedures such as the well-known branch exchange heuristic developed by Lin [35] and Lin and Kernighan [36] and extended by Christofides and Eilon [12] and Russell [44] always maintain feasibility and strive towards optimality. At each step, one feasible solution is altered to yield another feasible solution with a reduced overall cost. This procedure continues until no additional cost reductions are possible. Bodin and Sexton [10] modify this approach in order to schedule minibuses for the subscriber dial-a-ride problem.

Mathematical programming approaches include algorithms that are directly based on a mathematical programming formulation of the underlying routing problem. An excellent example of mathematical-programming-based procedures is given in the Fisher and Jaikumar article "A Generalized Assignment Heuristic For Vehicle Routing" contained in this issue. They formulate the Dantzig-Ramser vehicle routing problem as a mathematical program in which two interrelated components are identified. One component is a traveling salesman (routing) problem and the other is a generalized assignment (packing) problem. Their heuristic attempts to take advantage of the fact that these two problems have been studied extensively and powerful mathematical programming approaches for their solution have already been devised. Christofides et al. [13] and Stewart and Golden [49] discuss Lagrangean relaxation procedures for the routing of vehicles. In addition, the article by Christofides, Mingozzi, and Toth in this issue represents a mathematical-programming-based (in particular, dynamic programming) approach for obtaining lower bounds in a variety of combinatorial optimization problems related to vehicle routing. Further discussion on this topic can be found in the article by Magnanti in this issue.

Interactive optimization is a general-purpose approach in which a high degree of human interaction is incorporated into the problem-solving process. The idea is that the experienced decision-maker should have the capability of setting and revising parameters and injecting subjective assessments based on knowledge and intuition into the optimization model. This almost always increases the likelihood that the model will eventually be implemented and used. Some early adaptations of this approach to vehicle routing problems are presented by Krolak, Felts, and Marble [33] and Krolak, Felts, and Nelson [34]. The paper by Jarvis and Ratliff in this issue introduces several rather novel interactive optimization heuristics.

Exact procedures for solving vehicle routing problems include specialized branch and bound and cutting plane algorithms. Some of the more effective exact approaches are described by Held and Karp [29], [30], Crowder and Padberg [15], and Christofides et al. [13].

IV. HIERARCHY OF VEHICLE SCHEDULING PROBLEMS

In this section, we demonstrate a hierarchy of scheduling problems, progressing from the very simple to the more complex. In each situation, we both describe the constraints which are added to the previous problem in order to make it more realistic and complex and present a real-world application. Due to the fact that vehicle scheduling problems have not received nearly as much attention in the literature as have vehicle routing problems, we discuss these scheduling problems in some detail.

A. Some Simple Vehicle Scheduling Problems

Suppose we are given a fleet of vehicles that are housed at a single domicile and a set of n tasks denoted $t_1, t_2, \ldots, t_n$. Task t_i has a fixed time of completion T_i, a duration D_i, a start location S_i, and an end location E_i. For example, if $T_i = 12{:}30$ and $D_i =$ one hour, then task t_i must begin at 11:30. The time to deadhead between any pair of points U and V is given by $\mathrm{DH}(U, V)$. It is assumed that task t_i can be followed by task t_j in a vehicle schedule if $T_j - D_j > T_i + \mathrm{DH}(E_i, S_j)$. The problem is to schedule the vehicles in order to minimize the number of vehicles and service each of the required tasks. A directed acyclic network can be constructed in which each task t_i is represented by a node and an arc (i, j) exists if $T_j - D_j > T_i + \mathrm{DH}(E_i, S_j)$. The problem then reduces to one of finding the minimum number of paths in this network from a start point S to an end point E which cover all the nodes. Each of these paths is interpreted as a vehicle schedule. This problem, which is sometimes referred to as a Dilworth decomposition problem [20], can be solved using either a maximum flow or a minimum cost flow algorithm.

A variant of the Dilworth problem is utilized in a portion of the RUCUS program [5] which was developed for scheduling vehicles and crews for mass transit systems. In this program, the network is the same as before except that there is the cost $c_{ij} = \mathrm{DH}(E_i, S_j)$ assigned to arc (i, j). The problem is to cover all the nodes in the network with a given number of paths (specified *a priori*) from S to E and the objective is to minimize total deadhead time (or some other function of the arc costs) over all the vehicle schedules. This problem can also be solved using a minimum cost flow algorithm.

The two scheduling problems discussed so far suggest a two-step procedure for determining effective vehicle schedules. First, the Dilworth problem is solved. The solution minimizes the number of vehicles (or, equivalently, the capital outlay) needed to cover all the tasks. Next, fixing the number of vehicles, the minimum cost flow problem associated with the RUCUS program can be solved. This solution then minimizes the operating cost while ensuring a minimal fleet size (by setting the costs judiciously, this can be accomplished in a single step). Natural variations on this theme in which tradeoffs between operating and capital costs are investigated can be handled expeditiously.

B. The Multiple Depot Vehicle Scheduling Problem

A straightforward extension of the problem just discussed is to allow vehicles to reside at more than one depot and to seek the minimum number of vehicles needed to cover all the tasks. Lenstra and Rinnooy Kan, at the Workshop, proved that the problem is NP-hard.

There are two obvious approaches to this problem. In the first approach (a cluster first–schedule second approach), we arbitrarily divide the tasks into d subsets where each subset corresponds to a depot. We then solve a Dilworth problem for each depot. If the minimum or maximum capacity of any depot (in terms of number of vehicles) is violated, the initial partitioning of the tasks is revised and new Dilworth problems must be solved. This process continues in this manner until a satisfactory solution is found.

In the second approach (a schedule first–cluster second approach), we solve a Dilworth problem over the entire network disregarding the depot that would house each vehicle. We then have the minimum number of vehicles needed to service all the required tasks. Next, we assign each vehicle schedule to a depot. The objective in this assignment is to minimize total deadhead time and the constraints are on the minimum and maximum number of vehicles which can be housed at each depot. This latter problem can be viewed as a simple transportation problem. This approach is very reasonable when the travel times to and from the depot are significantly less than the time the vehicles spend on their schedules. This approach has been utilized by Bodin et al. [11] to estimate the cost of mass transit systems.

We note the similarity in algorithmic philosophy between the multidepot approaches in vehicle routing, as described by Golden et al. [27], and the multi-depot approaches described above. We believe that both the multi-depot routing and the multi-depot scheduling problems are promising areas for future research.

C. Length of Path Constraints

In the previous two subsections, it was assumed that there was no restriction on the length of a path (or vehicle schedule). However, in many real-world situations, since vehicles must refuel and for other reasons, an upper bound on this length does, in fact, exist. For example, a vehicle can travel only so many miles or for so many hours before needing to refuel. This path-length-constrained problem is NP-hard (see Ball [2]). Matching-based heuristics and heuristics based on the minimum cost flow problem have been applied by Ball et al. [3]. In the next subsection, we will see that an even more complicating set of path length constraints arises in the problem of developing crew schedules for mass transit systems.

D. Crew Scheduling For Mass Transit Systems

The scheduling of crews for mass transit systems has a structure similar to that of the vehicle scheduling problem described in the previous subsection, and so we discuss it here briefly. This problem is more complex, however, than the former problem because of the workrules and costs which govern the formation of crew schedules.

There are three basic types of acceptable crew schedules: straight shifts, split shifts, and tripper shifts. A *straight shift* is a crew schedule with a planned break sufficiently long so as to allow the crew a local lunch stop (usually 30 to 60 min in duration). A *split shift* is a crew schedule with a considerably longer break (usually 3 to 5 h). A *tripper shift* is a crew schedule for a part-time worker. Generally, tripper shifts incur high penalties and are to be avoided if possible. A *piece of work* is a segment of a crew schedule and vehicle schedule in which the crew remains with the vehicle. A tripper shift generally consists of one piece; a straight shift and split shift are generally comprised of from two to four pieces of work. A mathematical representation of the crew scheduling problem is presented by Ball et al. [3].

Typically, mass transit crews are scheduled manually. Computer aided procedures have received limited use due to the size of the problem that must be solved, the mediocre results obtained to date, the tremendous computational effort required to find an adequate solution, and the inflexibility of available computer packages. Moreover, seasonal variations and changes in demand patterns require that new schedules be generated several times a year and it takes planners a significant amount of time to produce a new set of crew and vehicle schedules. With this in mind, it is nearly impossible to alter the schedules on short notice and to address important sensitivity analysis issues. While it is generally believed that proficient schedulers can produce cost-effective manual schedules, it is well-recognized that several years of experience are usually required before a scheduler becomes truly expert.

In recent work, Ball et al. [3] describe a computerized system that generates a low-cost set of crew and vehicle schedules quickly and efficiently. Their approach makes repetitive use of the weighted matching problem and a recently implemented algorithm developed by Derigs [17] and Derigs and Kazakides [18]. This approach to crew scheduling appears to hold great promise for solving a wide variety of large and difficult combinatorial optimization problems.

E. The Airplane and Air Crew Scheduling Problem

The scheduling problems which have received perhaps the most attention in recent years are the air crew and airplane scheduling problems. As with the transit scheduling problem, a fixed timetable is assumed. Since the operating costs for the planes are several times greater than the crew costs, the scheduling of the planes is carried out first. Ball et al. [3] propose to schedule transit crews before generating vehicle schedules since transit crew operating costs are greater than vehicle operating costs and as a general rule the planner should always first schedule that component which accounts for the larger cost contribution.

Recent work on the airplane scheduling problem has been done by Hinson and Mulherkar [31]. Their procedure is an adaptation of the Clarke and Wright savings approach. They utilize a modified savings function since trips have definite starting and ending times in their problem (this savings function is similar to the one suggested by Stewart and Golden [49] for the subscriber bus routing problem) and they randomly sample from the savings list. In this way, they can obtain numerous solutions and merge these solutions together in order to derive good final results. They report 2 to 3% improvements over the standard Clarke and Wright procedure on typical problems. Obviously, their procedure is more burdensome computationally than the Clarke and Wright algorithm.

The crew scheduling problem is generally broken down into two parts—generating pairings and constructing bid lines. A *pairing* is a collection of trips that a crew must complete that begin and end at the same domicile. The *bid lines* are sets of pairings that represent the monthly work schedules for the crews. Since both problems have complicated workrules for constraints, the construction of an effective network flow model seems unlikely. With multiple domiciles, the problems become even more intractable. In the past, for airlines to come up with reasonable solutions, vast expenditures in computer time and programming effort have been required.

The appropriate trip pairing algorithm to employ depends upon the size of problem to be solved and the number of domiciles involved. Baker [1] utilizes a set of heuristics similar to some of the aforementioned vehicle routing and scheduling approaches to solve the pairing problem. He has been able to solve a single domicile/900-trip crew scheduling problem for Federal Express Corporation to within 1% of the best known result. Baker [1] has also solved a single domicile/3200-trip problem and a multiple domicile problem of several hundred trips.

Although less work has transpired on bid line construction, Finnegan [19] makes repeated use of a matching algorithm to develop bid lines, given a set of pairings.

In this issue of *Networks*, the article by Marsten and Shepardson describes solution procedures to both the pairing and bid lines problems based on set partitioning ideas. Their research extends earlier work by Marsten, Muller, and Killion [37].

F. Time Windows

To this point, the scheduling problem has always assumed that the starting and ending times for each task were specified. Complications arise, however, if there exists a time window in which a task must be carried out; that is to say, task t_i must be completed between 11:30 and 12:00. With no time windows, the set of tasks that can follow any particular task can be specified a priori and an acyclic network, as described earlier for the Dilworth problem, can be constructed. With time windows, the set of feasible tasks that can follow a given task cannot be specified beforehand, and hence this acyclic network cannot be formed. The only literature in this area that we are aware of are the works of Orloff [41] and Wren and Smith [50].

When time windows or precedence relationships are present, the problem involves a combination of routing and scheduling components. Routes must be designed in order to minimize total transportation costs but, at the same time, scheduling must be performed in order to ensure feasibility. We address this class of problems next.

V. COMBINED ROUTING AND SCHEDULING PROBLEMS

Combined routing and scheduling problems are extremely complex. There are, in fact, so many potential, complicating factors that we do not even dare, in this section, to propose a detailed classification scheme or a hierarchy of problems. Instead we focus on specific examples in order to illustrate the myriad of difficulties that can arise when routing and scheduling operations must be performed simultaneously. The three examples that we describe are

1. school bus routing and scheduling,
2. subscriber dial-a-ride routing and scheduling,
3. routing and scheduling with full loads and time windows.

Another interesting example is discussed at length by Russell and Igo [45]. See also the article by Schrage in this issue.

A. School Bus Routing and Scheduling

In the school bus routing and scheduling problem, there are a number of schools and each one has a set of bus stops associated with it. In addition, there are a given number of students associated with each bus stop. Each school has a fixed start and finish time with a time window about each of these for the delivery of students to the school in the morning and the pickup of students from the school in the afternoon. The problem is to minimize the number of buses used or total transportation costs while servicing all the students and satisfying all of the time windows.

Much thought has gone into the analysis of this problem (e.g., see Newton and Thomas [39], Bodin and Berman [7], Rousseau et al. [43], and Orloff [41]) although most of the effort has been directed at the routing aspect. A general solution procedure can be sketched as follows:

(i) Determine a reasonable set of routes for each of the schools.
(ii) Schedule the buses, starting from the beginning of the day. That is, start with all schools opening at 7:30 AM. Next, consider all schools that begin their days at 8:00 AM and so on.
(iii) Improve routes and/or schedules.

B. Subscriber Dial-a-Ride Routing and Scheduling

In the subscriber dial-a-ride environment, customers call in advance requesting service. Each customer specifies a pickup point, a delivery point, and a desired time of pickup and/or delivery. The problem is to develop a set of routes and schedules for the vehicles in a fleet of fixed size in order to minimize total customer dissatisfaction or inconvenience. This problem is characterized by a set of precedence relationships (each customer's origin must precede his destination) and time windows at each point (in particular, to pickup a customer too early or deliver him too late is unacceptable).

The single vehicle problem has been studied by Psaraftis [42] and by Sexton [46]. The multiple vehicle case has been analyzed by Bodin and Sexton [10]. For additional results in this area, see Stein [47] and Gavish and Srikanth [21].

C. Routing and Scheduling with Full Loads and Time Windows

In this problem, a set of demands is specified for a number of origin-destination pairs. Each demand is a full trailer which must be loaded onto a tractor at an origin and unloaded at a destination. These stops must satisfy prespecified time window constraints and the goal is to design routes and schedules for the fleet of tractors. In most cases, the objective is to minimize total transportation costs or the number of tractors used. In the real-world instance of this problem described by Ball et al. [4], a "bang-for-buck" objective was specified by management. Three algorithmic approaches were experimented with. In these experiments, a greedy insertion procedure outperformed the other heuristics with respect to the bang-for-buck objective function (see [4]). However, all three approaches gave reasonable results with respect to the objective of minimizing total transportation costs.

VI. CONCLUDING REMARKS

In this paper, we have attempted to classify the many vehicle routing and scheduling problems in a reasonable and useful way. A few of these problems have been looked at in some detail while others have been mentioned just briefly. In the remaining portion of this Special Issue, other papers highlight some of the more significant recent developments in this field. It is our hope that the classification scheme and discussion presented in this paper along with the other papers in this issue will lead to new insights and suggest important new research topics in the area of vehicle routing and scheduling.

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