| Fourier 变换 | |
|---|---|
| 定义 | |
| | $\mathcal{F}[f(t)] = F(w) = \int_{-\infty}^{+\infty} f(t)\, e^{-jwt} dt$ <br> $\mathcal{F}^{-1}[F(w)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(w)\, e^{jwt} dw$ |
| 主要性质 | |
| | $\mathcal{F}[f(t \pm t_0)] = e^{\pm jwt_0} F(w)$ |
| | $\mathcal{F}[e^{\pm jw_0 t} f(t)] = F(w \mp w_0)$ <br> 等价于 <br> $\mathcal{F}^{-1}[F(w \mp w_0)] = e^{\pm jw_0 t} f(t)$ |
| | $\mathcal{F}[f(at)] = \frac{1}{\lvert a \rvert} F(\frac{w}{a})$ |
| 微分性质 | $\mathcal{F}[f^{(n)}(t)] = (jw)^n F(w)$ <br> $F^{(n)}(w) = (-j)^n \mathcal{F}[t^n f(t)]$ |
| 积分性质(了解) | $\mathcal{F}[\int_{-\infty}^{t} f(t)\, dt] = \frac{F(w)}{jw}$ <br> 或 <br> $\mathcal{F}[\int_{-\infty}^{t} f(t)\, dt] = \frac{F(w)}{jw} + \pi F(0)\delta(w)$ |

| 常用 Fourier 变换对 | |
|---|---|
| $f(t)$ | $F(w)$ |
| $\delta(t)$ | 1 |
| 1 | $2\pi\delta(w)$ <br> $\int_{-\infty}^{+\infty} e^{-jwt}\, dt = 2\pi\delta(w)$ |
| $e^{jw_0 t}$ | $2\pi\delta(w - w_0)$ |
| $\delta(t - t_0)$ | $e^{-jwt_0}$ |
| $u(t)$[Heaviside 函数] | $\pi\delta(w) + \frac{1}{jw}$ |
| $sgn(t)$ | $\frac{2}{jw}$ |
| $sin(w_0 t)$ | $\frac{\pi}{j}[\delta(w - w_0) - \delta(w + w_0)]$ |
| $cos(w_0 t)$ | $\pi[\delta(w - w_0) + \delta(w + w_0)]$ |
| $Dirichlet$积分：$\int_0^{+\infty} \frac{sint}{t} dt = \frac{\pi}{2}$ | |

| Laplace 变换 | |
|---|---|
| 定义 | |
| | $$F(s)=\mathcal{L}[f(t)]=\int_{0}^{+\infty} f(t)\,e^{-st}dt$$ |
| 性质 | |
| | $$\mathcal{L}[e^{at}f(t)]=F(s-a)$$ |
| | $$\mathcal{L}[f(t-\tau)u(t-\tau)]=e^{-s\tau}F(s)$$<br>等价于<br>$$\mathcal{L}^{-1}[e^{-s\tau}F(s)]=f(t-\tau)u(t-\tau)$$ |
| | $$\mathcal{L}[f(at)]=\frac{1}{a}F\left(\frac{w}{a}\right),a>0$$ |
| 微分性质 | $$\mathcal{L}[f'(t)]=sF(s)-f(0)$$<br>一般地 $$\mathcal{L}\left[f^{(n)}(t)\right]=s^{n}F(s)-\sum_{i=0}^{n-1}s^{n-1-i}f^{(i)}(0)$$<br>$$F^{(n)}(s)=(-1)^{n}\mathcal{L}[t^{n}f(t)]$$ |
| 积分性质(了解) | $$\mathcal{L}[\int_{0}^{t} f(t)\,dt]=\frac{F(s)}{s}$$<br>一般地<br>$$\mathcal{L}[\int_{0}^{t} dt\int_{0}^{t} dt\cdots\int_{0}^{t} f(t)\,dt]=\frac{F(s)}{s^{n}}$$<br>另外<br>$$\mathcal{L}\left[\frac{f(t)}{t^{n}}\right]=\int_{s}^{+\infty} ds\int_{s}^{+\infty} ds\cdots\int_{s}^{+\infty} F(s)ds$$<br>注：由$\mathcal{L}\left[\frac{f(t)}{t}\right]=\int_{s}^{+\infty} F(s)ds$，取$s=0$，则有$\int_{0}^{+\infty}\frac{f(t)}{t}dt=\int_{0}^{+\infty} F(s)ds$ |

| 常用 Laplace 变换对 | |
|---|---|
| $f(t)$ | $F(s)$ |
| $u(t)$ | $\frac{1}{s}$ |
| $\delta(t)$ | $1$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $sin(kt)$ | $\frac{k}{s^{2}+k^{2}}$ |
| $cos(kt)$ | $\frac{s}{s^{2}+k^{2}}$ |
| $t^{m}$ | $\frac{\Gamma(m+1)}{s^{m+1}},\quad m>-1$<br>特别地，$m$ 为整数，则为$\frac{m\,!}{s^{m+1}}$. |