# Throughput Maximization of UAV Networks

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Abstract—In this paper we study the deployment of multiple unmanned aerial vehicles (UAVs) to form a temporal UAV network for the provisioning of emergent communications to affected people in a disaster zone, where each UAV is equipped with a lightweight base station device and thus can act as an aerial base station for users. Unlike most existing studies that assumed that a UAV can serve all users in its communication range, we observe that both computation and communication capabilities of a single lightweight UAV are very limited, due to various constraints on its size, weight, and power supply. Thus, a single UAV can only provide communication services to a limited number of users. We study a novel problem of deploying K UAVs in the top of a disaster area such that the sum of the data rates of users served by the UAVs is maximized, subject to that (i) the number of users served by each UAV is no greater than its service capacity; and (ii) the communication network induced by the K UAVs is connected. We then propose a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximation algorithm for the problem, improving the current best result of the problem by five times (the best approximation ratio so far is  $\frac{1-1/e}{5(\sqrt{K}+1)}$ ), where e is the base of the natural logarithm. We finally evaluate the algorithm performance via simulation experiments. Experimental results show that the proposed algorithm is very promising. Especially, the solution delivered by the proposed algorithm is up to 12% better than those by existing algorithms.

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#### I. Introduction

It is estimated that the annual loss incurred by natural disasters (e.g., earthquakes, tsunamis, flooding, etc.) is about US\$115 billion in the past 30 years. And even worse, on average 48 thousand people died in disasters per year [27]. When a disaster event occurs, existing communication and transportation infrastructures may have been totally destroyed already. It is well recognized that the first 72 hours after the disaster are the golden time window for people life rescues, and search and rescue operations must be conducted quickly and efficiently [10], [19]. To rescue the people trapped in the disaster zone, it is urgent to have temporarily emergent communications to help them get out from there as soon as possible.

On the other hand, wireless communications by utilizing UAVs recently have attracted a lot of attentions [8], [10], [12], [20], [24], [25], [34], [37]–[39], [42], [44], [46]. Unlike terrestrial communication infrastructures, low-altitude UAVs are more cost-effective, swift and flexible for on-demand deployments [43], where UAVs can work as aerial base stations by attaching lightweight base station devices [7], [26]. Several mobile operators, including AT&T and Verizon in the United States, have conducted trials with LTE base stations mounted on UAVs [26]. It is recognized that a UAV network composed of multiple UAVs is perfectly applicable for temporary and unexpected burst communication scenarios, such as natural disaster reliefs, concerts, and traffic congestion [3], where multiple UAVs can be temporarily deployed in the top of a disaster area to provide efficient communication services to ground users. Another advantage of UAV communications is that, they usually enjoy higher Line-of-Sight (LoS) link opportunity with ground users, due to high heights of UAVs, thereby having higher data rates and larger communication ranges [1]. Fig. 1 illustrates a UAV network with four UAVs, and the UAVs provide communication services (e.g., LTE or WiFi) to the ground people in a disaster area. With the aid of the UAV network, the people can send and receive emergent data, such as voice and video, to/from a rescue team nearby, thereby reducing their injuries and saving lives.

The deployment of UAV networks recently has gained lots of attentions [21], [30], [40], [45]. For example, Zhao *et al.* [45] presented a motion control algorithm for

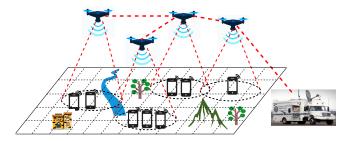


Fig. 1. A UAV network that provides services for ground users in a disaster area, where the network is connected to the Internet via an emergency communication vehicle.

deploying K UAVs to cover as many users as possible while maintaining the connectivity of the K UAVs. Liu  $et\ al.$  [21] considered a similar problem and proposed a deep reinforcement learning (DRL) based algorithm. Yang  $et\ al.$  [40] investigated the problem of scheduling multiple UAVs to fairly provide communication services to mobile ground users for a given period, by using the DRL method, too.

In spite of the aforementioned studies on the deployment of UAVs, most of them did not consider the service capacities of UAVs and assumed that each UAV is capable of serving all users within its communication range. On the other hand, due to the constraint on the payload of a UAV, e.g., the maximum payload a DJI Matrice M300 RTK UAV is only 2.7 kg [23], the computation capacity of the base station device mounted on a UAV is limited [6], [7], [26]. Then, the service capacity of each UAV, i.e., the maximum number of users that the UAV can serve, is limited too. Furthermore, it is shown that users may not be uniformly distributed in a monitoring area. It is very likely that there are many people at a small portion of places while only a few people at the other places in the monitoring area. That is, the human density follows the power law [31]. For example, after an earthquake, many people may stay in a public plaza without surrounding buildings for their safeties. It can be seen that only one UAV may be deployed in the top of a dense location with many people by existing studies [21], [40], [45], as the people are within the transmission range of the UAV. However, only a portion of the people can be served by the UAV due to its limited service capacity. A simple solution to address this issue is to deploy multiple UAVs to cover such a dense location with many people. However, a challenge lies in that there may be multiple such dense locations in a disaster zone which are far away from each other, while the communication network formed by the deployed UAVs at different locations may not be connected.

In this paper, we consider that each UAV can provide communication service to limited numbers of users simultaneously. We study a novel connected maximum throughput problem, which is to deploy K UAVs for serving people in a disaster zone, such that the sum of the data rates of users served is maximized, subject to that (i) the number of users served by each UAV is no greater than its service capacity; and (ii) the communication subnetwork induced by the K UAVs is connected.

Tackling this defined problem poses the following challenges: (1) Among potential hovering locations for the UAVs, which K locations should be chosen such that the sum of the data rates of users is maximized, subject to that each UAV has a limited service capacity. (2) How to ensure that the communication network induced by the K UAVs at the K identified locations is connected. (3) Considering that the problem is NP-hard, it is extremely difficult to develop an approximation algorithm with a provable approximation ratio for the problem, as not only do we need finding the K locations for the K UAVs to maximize the sum of user data rates, but also we must ensure that the communication network induced by the K UAVs is connected.

The novelty of this paper lies in not only incorporating the service capacities of UAVs into consideration but also proposing a performance-guaranteed approximation algorithm to the connected maximum throughput problem in a UAV network. Specifically, the proposed algorithm delivers a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximate solution to the problem, which significantly improves the best  $\frac{1-1/e}{5(\sqrt{K}+1)}$ -approximate solution so far by five times [18], where K is the number of UAVs, e is the base of the natural logarithm.

The main contributions of this paper can be summarized as follows.

- We first formulate a novel connected maximum throughput problem for a UAV network that consists of K UAVs.
- We then devise a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximation algorithm for the problem. Particularly, the approximation ratio of the algorithm is only  $\frac{1-1/e}{7}$  when K (=50) UAVs are deployed.
- We finally evaluate the proposed algorithm performance via simulation experiments. Experimental results show that the algorithm is very promising. Especially, the sum of the data rates of users served by the proposed algorithm is up to 12% larger than those by existing algorithms.

The rest of this paper is organized as follows. Section II reviews related work. Section III introduces the system model and defines the problem precisely. Section IV proposes an efficient algorithm for a subproblem of the connected maximum throughput problem that will serve as a subroutine of the proposed approximation algorithm. Section V devises an approximation algorithm for the problem, and Section VI analyzes the approximation ratio of the approximation algorithm. Section VII evaluates the performance of the proposed algorithm via simulation experiments, and Section VIII concludes this paper.

## II. RELATED WORK

The utilization of UAVs as aerial base stations has attracted a lot of attentions recently. For example, Zhao  $et\ al.$  [45] proposed a distributed motion control algorithm to deploy a fixed number K of UAVs to serve as many as users, while guaranteeing the connectivity of the UAV network. Liu  $et\ al.$  [21] designed an improved algorithm for a similar problem, by adopting a deep reinforcement learning. Yang  $et\ al.$  [40] studied a problem of scheduling multiple UAVs to provide communications to users within a period

in a fair way, by using the deep reinforcement method, too. Shi et al. [30] investigated a problem of finding the flying tours of UAVs for a period so that the average UAVto-user pathloss is minimized. They decoupled the problem into several local optimization subproblems, and solved the subproblems independently. Selim and Kamal [29] proposed a UAV network infrastructure, which consists of three types of UAVs: tethered backhaul UAVs that provide high capacity backhauling, untethered communication UAVs that provide communication service to ground users, and untethered powering UAVs for charging communication UAVs. They studied a problem of finding the placement locations of UAVs such that the energy consumption of the UAVs is minimized, where each user has a minimum data rate. Also, Alzidaneen et al. [2] considered a network that consists of UAVs and tethered balloons, where UAVs can communicate with ground users while UAVs access the Internet by directly communicating with the balloons. It can be seen that the coverage area of the network is limited, since UAVs must be within the communication range of the tethered balloons. They studied a problem of finding the hovering locations of UAVs, the association between UAVs and users, and the association between UAVs and balloons, such that the sum of the data rates of all users is maximized. However, most of these mentioned studies did not take the service capacity of each UAV into consideration, and assumed that a UAV can serve all users in its communication range. On the other hand, it is not uncommon that both the computation and communication capabilities of a UAV are limited, due to various constraints on its size, weight, and power supply [6], [7]. In this paper, we assumed that each UAV can provide communication services to limited numbers of users simultaneously.

On the other hand, there are several studies on maximizing submodular functions subject to the connectivity constraint. For example, Kuo et al. [18] considered the problem of deploying K wireless routers in a network such that a submodular function of the deployed routers is maximized, subject to the connectivity constraint that the subnetwork induced by the Krouters is connected. They proposed a  $\frac{1-1/e}{5(\sqrt{K}+1)}$ -approximation algorithm, which is inferior to the approximation ratio  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ of the algorithm obtained in this paper, where e is the base of the natural logarithm. Khuller et al. [16], [17] studied a problem of choosing K nodes in a network so that a special submodular function of the chosen nodes is maximized, subject to the connectivity constraint, where f is a special submodular if (i) f is submodular; and (ii)  $f(A \cup B) =$ f(A) + f(B) if  $N(A) \cap N(B) = \emptyset$  for any  $A, B \subset V$ , and N(X) denotes the neighborhood of a set X, including X itself. They designed a  $\frac{1-1/e}{12}$ -approximation algorithm. However, the objective function of the problem considered in this paper may not be a special submodular function. In addition, Huang et al. [13] investigated the problem of placing K sensors to monitor targets so that the number of targets covered by the K sensors is maximized and the network formed by the K placed sensors is connected, where a target is covered by a sensor if their Euclidean distance is no more than a given sensing range  $R_s$ , and two sensors can communicate

with each other if their Euclidean distance is no greater than a given communication range  $R_c$ , and  $R_s \leq R_c$ . They proposed a  $\frac{1-1/e}{8(\lceil 2\sqrt{2}\alpha\rceil+1)^2}$ -approximation algorithm, where  $\alpha=\frac{R_s}{R_c}$ , and the ratio thus is between  $\frac{1-1/e}{128}$  and  $\frac{1-1/e}{32}$ , as  $0<\alpha\leq 1$ . Yu et al. [41] recently improved the ratio to  $\frac{1-1/e}{8(\lceil \frac{1}{\sqrt{3}}\alpha\rceil+1)^2}$ . It can be seen that both the approximation ratios in [13] and [41] are  $\frac{1-1/e}{8(3+1)^2}=\frac{1-1/e}{128}$ , when  $\alpha=\frac{R_s}{R_c}=1$ , which indicates that the performance of the solutions delivered by the both algorithms may be far from the optimal solution. Therefore, the both algorithms in [13] and [41] are applicable to the case with many to-be-placed sensors, i.e., the value of K is very large, e.g., K=10,000. Notice that there are usually tens or hundreds of UAVs to-be-deployed in a real UAV network, and the approximation ratio  $\frac{1-1/e}{|\sqrt{K}|}$  in this case is much larger than  $\frac{1-1/e}{128}$ . For example,  $\frac{1-1/e}{|\sqrt{K}|}=\frac{1-1/e}{10}>\frac{1-1/e}{32}$ , when K(=100) UAVs are to be deployed.

Although the proposed  $\frac{1-1/e}{|\sqrt{K}|}$ -approximation algorithm in this paper are motivated by the  $\frac{1-1/e}{5(\sqrt{K}+1)}$ -approximation algorithm algorithm in this paper are motivated by the  $\frac{1-1/e}{5(\sqrt{K}+1)}$ -approximation algorithm.

rithm in [18], the main technical differences between them are twofold. (i) One difference is that the algorithm in [18] shows that a tree  $T^*$  that covers the optimal K location nodes can be decomposed into  $5(\sqrt{K}+1)$  subtrees  $T_1, T_2, \dots, T_{5(\sqrt{K}+1)}$ so that the number of nodes in each subtree is no more than  $\lfloor \sqrt{K} \rfloor$ . Then, there must be one subtree, say  $T_l$ , among the  $5(\sqrt{K}+1)$  subtrees such that the sum of the data rates of users served by UAVs deployed at the location nodes in the subtree is no less than  $\frac{1}{5(\sqrt{K}+1)}$  of the sum of the data rates for the optimal tree  $T^*$ . In contrast, in this paper we show that the tree  $T^*$  can be decomposed into  $\lfloor \sqrt{K} \rfloor$  subtrees  $T_1^\#, T_2^\#, \dots, T_{\lfloor \sqrt{K} \rfloor}^\#$ , such that the number of nodes in each subtree is no greater than  $2|\sqrt{K}|$ , see Lemma 3 in Section VI-A. It thus can be seen that there must be one subtree, say  $T_{l}^{\#}$ , among the  $|\sqrt{K}|$  subtrees, such that the sum of the data rates of users served by UAVs at the location nodes in  $T_l^\#$  is no less than  $\frac{1}{\lfloor \sqrt{K} \rfloor}$  of the sum of the data rates for the optimal tree  $T^*$ . Notice that the number  $2|\sqrt{K}|$  of nodes in  $T_l^\#$  is much larger than the number  $|\sqrt{K}|$  of nodes in  $T_l$ .

(ii) The other difference is that the algorithm in [18] finds a (1-1/e)-approximate tree  $T'_l$  of  $T_l$  with  $\lfloor \sqrt{K} \rfloor$  nodes, by observing that, for any node v in  $T_l$ , the minimum number of hops between any node u in  $T_l$  and v is no greater than  $\lfloor \sqrt{K} \rfloor -1$ , where e is the base of the natural logarithm. Then,  $T'_l$  is a  $\frac{1-1/e}{5(\sqrt{K}+1)}$ -approximate solution. Contrarily, we find a (1-1/e)-approximate tree  $T''_l$  of  $T^\#_l$  with  $2\lfloor \sqrt{K} \rfloor$  nodes, by showing that there is a special node v' in  $T^\#_l$  such that the sum of the minimum numbers of hops between nodes in  $T^\#_l \setminus \{v'\}$  and v' is no greater than K-1, see Lemma 4 in Section VI-B. Then,  $T''_l$  is a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximate solution.

We also note that Cadena *et al.* [5] proposed a  $\frac{1-1/e}{5K^{(d-1)/(2d-1)}}$ -approximation algorithm for the problem of finding K connected nodes in a metric graph, such that a submodular function is maximized, where d is the doubling dimension of the graph. The proposed algorithm in this paper

exhibits some advantages over the one in [5] as follows. First, the approximation ratio  $\frac{1-1/e}{\sqrt{K}}$  of the proposed algorithm is better than the approximation ratio  $\frac{1-1/e}{5K(d-1)/(2d-1)}$  of the algorithm in practical applications. In a two dimensional Euclidean space, the doubling dimension d is  $\log_2 7=2.8$  [35]. Then,  $\frac{d-1}{2d-1}=0.39$  and the approximation ratio of the algorithm by Cadena et~al. [5] is  $\frac{1-1/e}{5K^{(d-1)/(2d-1)}}=\frac{1-1/e}{5K^{0.39}}.$  It can be seen that the approximation ratio  $\frac{1-1/e}{\sqrt{K}}$  in this paper is larger than  $\frac{1-1/e}{5K^{0.39}}$  when  $K\leq 5\frac{1}{0.11}=2,257,549.$  In a UAV network, there are only tens of, or hundreds of to-be-deployed UAVs. On the other hand, the ratio  $\frac{1-1/e}{\sqrt{K}}$  is smaller than  $\frac{1-1/e}{5K^{0.39}}$  when K>2,257,549. However, it is unlikely to deploy more than two million UAVs.

Second, the algorithm of Cadena et~al.~[5] is only applicable to metric graphs, in which the value of the doubling dimension d is small. For a non-metric graph, the value of d may be very large. In this case, the approximation ratio  $\frac{1-1/e}{5K^{(d-1)/(2d-1)}}$  approaches to  $\frac{1-1/e}{5K^{1/2}}=\frac{1-1/e}{5\sqrt{K}}$ , when d is very large. On the other hand, the proposed algorithm in this paper is applicable to non-metric graphs and its approximation ratio  $\frac{1-1/e}{\sqrt{K}}$  still holds.

#### III. PRELIMINARIES

In this section, we first introduce the system and channel models, then define the problem.

#### A. System Model

When a disaster (e.g., an earthquake or a flooding) occurs, the communication and transportation infrastructures may have been destroyed. To rescue the people trapped in the disaster area, it is urgent to have temporarily emergent communications to help them get out from there. A promising solution is to deploy multiple UAVs to form a network.

Fig. 1 shows a UAV network in which four UAVs work as base stations to provide communication services (e.g., LTE or WiFi) to affected people in a disaster zone. Assume that at least one of the UAVs serves as a *gateway UAV*, which is connected to the Internet, with the help of an emergency communication vehicle or satellites, see Fig. 1. It can be seen that once a trapped people can communicate with a nearby UAV using his smartphone, the people can send and receive critical voice, video, and data to/from the rescue team, with the help of the UAV network.

The disaster area can be treated as a 3D space with length L, width W, and height H, e.g., L=W=3 km and H=500 m. Assume that there is a set U of n users  $u_1,u_2,\ldots,u_n$  on the ground of the disaster area, i.e.,  $U=\{u_1,u_2,\ldots,u_n\}$ . We also assume that each user  $u_i$  has a minimum data rate requirement  $b_{min}^i$ , e.g.,  $b_{min}^i=2$  kbps.

Denote by  $(x_i, y_i, 0)$  the coordinate of a user  $u_i$  with  $1 \le i \le n$ . We assume that the locations of users are known, which can be obtained by one of the following methods. On one hand, user smartphones usually are equipped with GPS modules, and each user can send his location information to a UAV within the communication range of his smart phone,

when the UAV flies over the disaster area. On the other hand, if users do not know their locations, since most UAVs are equipped with GPS modules, a few UAVs can fly over the disaster area and estimate user locations, by first taking photos for users with their on-board cameras (each photo is tagged with the location information where it was taken), then inferring user locations by applying an existing target detection method [14], [15]. In addition, if some users are not in Line-of-Sight (LoS) of UAVs and thus cannot be seen by the UAVs, the users can broadcast a probe request with their wireless communication devices. The UAVs then can estimate the locations of the users by the received radio signal strength index (RSSI) measurements [9], [32].

We consider the deployment of no more than K UAVs to provide communication services (e.g., LTE or WiFi) to affected users in a monitoring area. Each UAV k is equipped with a lightweight base station device and can act as an aerial base station with  $1 \le k \le K$  [7]. Due to the constraint on the payload of a UAV, e.g., the maximum payload a DJI Matrice M300 RTK UAV is only 2.7 kg [23], the computation capacity of the base station device mounted on the UAV is very limited [6], [7], [26]. Denote by C the service capacity of each UAV, which means that a UAV can provide communication services to C users simultaneously, e.g., C = 100 users.

We assume that all UAVs hover at the same altitude h, which is the optimal altitude for the maximum coverage from the sky [1], [45], e.g., h = 300 m. It can be seen that there are infinite numbers of potential hovering locations for the UAVs, which however makes their placements intractable. For the sake of convenience, we here only consider a finite number of potential hovering locations for the UAVs, by dividing their hovering plane at altitude h into equal size squares with side length  $\delta$ , e.g.,  $\delta = 50$  meters. For the sake of convenience, we assume that both length L and width W are divisible by  $\delta$ . Thus, the hovering plane of the UAVs can be partitioned into  $m=\frac{L}{\delta}\times\frac{W}{\delta}$  grids. We further assume that each UAV hovers only at the center of a grid but do not allow two or more UAVs to hover at the same grid to avoid collisions [45]. Denote by  $v_1, v_2, \dots, v_m$  the center locations of the m grids. Let V be the set of the m potential hovering locations, i.e.,  $V = \{v_1, v_2, \dots, v_m\}$ . Table I lists the notations used in this paper.

#### B. Channel Models

We consider both UAV-to-UAV and UAV-to-user channel models as follows. UAV-to-UAV channels are mainly dominated by Line-of-Sight (LoS) links, and can be modelled as the free space path loss [1]. Denote by  $R_{uav}$  the communication range of a UAV, i.e., two UAVs can communicate with each other if their Euclidean distance is no greater than  $R_{uav}$ .

On the other hand, the UAV-to-user channel model is more complicated, which must consider both LoS and NLoS (Non-Line-of-Sight) links [1], [45]. Denote by  $PL_{i,j}^{LoS}$  and  $PL_{i,j}^{NLoS}$  the average pathlosses of LoS and NLoS for a user  $u_i$  from UAV j, respectively. Following the work in [1], we have

$$PL_{i,j}^{LoS} = 20 \log_{10} \frac{4\pi f_c d_{i,j}}{c} + \eta_{LoS},$$
 (1)

TABLE I NOTATION TABLE

$U = \{u_1, u_2, \dots, u_n\}$	set of $n$ ground users $u_1, u_2, \ldots, u_n$		
$b_{min}^i$	minimum data rate of a user $u_i$		
$(x_i, y_i, 0)$	coordinate of user $u_i$ with $1 \le i \le n$		
K	number of UAVs		
C	service capacity of each UAV		
$B_w$	bandwidth of each user		
L, W, H	length, width, and height of the disaster		
	area		
h	hovering altitude of UAVs		
δ	side length of a square		
$m = \frac{L}{\delta} \times \frac{W}{\delta}$	number of squares in the plane at alti-		
	tude h		
$v_1, v_2, \ldots, v_m$	center locations of the $m$ squares		
$V = \{v_1, v_2, \dots, v_m\}$	set of m potential UAV locations		
$R_{uav}$	transmission range between two UAVs		
$R_{user}$	transmission range between a user and		
	a UAV		
$d_{i,j}$	Euclidean distance between user $u_i$ and		
	the UAV at location $v_j$		
$y_j \in \{0, 1\}$	whether a UAV is deployed at location		
	$\mid v_j \mid$		
$x_{ij} \in \{0, 1\}$	whether user $u_i$ is served by a UAV		
	deployed at location $v_j$		
$r_{i,j}$	data rate of user $u_i$ from the UAV at $v_j$		
$G = (U \cup V, E)$	UAV network		
$S(\subseteq V)$ $f(S)$	a set of UAV hovering locations		
f(S)	maximum sum of the data rates of users		
	served by the UAVs deployed at loca-		
	tions in S		
M	a maximum weighted matching in		
	$\operatorname{graph} G$		
T	a minimum spanning tree in graph G		
w(T)	weighted sum of edges in tree $T$ , i.e.,		
	$w(T) = \sum_{e \in T} w(e)$		

$$PL_{i,j}^{NLoS} = 20 \log_{10} \frac{4\pi f_c d_{i,j}}{c} + \eta_{NLoS},$$
 (2)

where  $f_c$  is the radio frequency,  $d_{i,j}$  is the Euclidean distance between user  $u_i$  and UAV j, c is the speed of light,  $\eta_{LoS}$  and  $\eta_{NLoS}$  are the average shadow fadings for LoS and NLoS links, respectively, and the value of the pair  $(\eta_{LoS}, \eta_{NLoS})$  is (0.1 dB, 21 dB), (1 dB, 20 dB), (1.6 dB, 23 dB), (2.3 dB, 34 dB) for suburban, urban, dense urban, and highrise urban environments, respectively.

Let  $P_t$  (in dB) and  $g_t$  (in dB) be the signal transmission power and antenna gain of each UAV, respectively. Then, the LoS signal-to-noise ratio (SNR) and the NLoS SNR for user  $u_i$  from UAV j are

$$SNR_{i,j}^{LoS} = 10^{\frac{P_t + g_t - PL_{i,j}^{LoS} - P_N}{10}},$$

$$SNR_{i,j}^{NLoS} = 10^{\frac{P_t + g_t - PL_{i,j}^{NLoS} - P_N}{10}},$$
(3)

$$SNR_{i,j}^{NLoS} = 10^{\frac{P_t + g_t - PL_{i,j}^{NLoS} - P_N}{10}},$$
 (4)

respectively, where  $P_N$  (in dB) is the noise power [40].

Denote by  $p_{i,j}^{LoS}$  and  $p_{i,j}^{NLoS}$  the probabilities of an LoS link and an NLoS link between user  $u_i$  and UAV j, respectively, where  $p_{i,j}^{NLoS} + p_{i,j}^{LoS} = 1$ , and the value of probability  $p_{i,j}^{LoS}$ depends on the UAV altitude h and the horizontal distance between user  $u_i$  and UAV j [1]. Then, the expected data rate  $r_{i,j}$  of user  $u_i$  served by UAV j is

$$r_{i,j} = p_{i,j}^{LoS} \cdot B_w \cdot \log_2(1 + SNR_{i,j}^{LoS}) + p_{i,j}^{NLoS} \cdot B_w \cdot \log_2(1 + SNR_{i,j}^{NLoS})$$
 (5

where  $B_w$  is the wireless channel bandwidth [40].

Denote by  $R_{user}$  the communication range between a UAV and a user [1]. Note that  $R_{user}$  usually is smaller than  $R_{uav}$  [21].

#### C. Spectrum Allocations

To provide communication service to multiple users at the same time, we assume that the OFDMA technique is adopted by each UAV. Denote by  $B_{UAV}$  the spectrum segment available for each UAV. Following the LTE standard [28], the spectrum segment used by a base station is one of the values among 1.4, 3, 5, 10, 15, and 20 MHz. In this paper, we adopt that  $B_{UAV} = 20$  MHz [28]. To ensure fair sharing of the communication bandwidth among users, assume that each user is allocated with the same amount of bandwidth  $B_w$ , e.g.,  $B_w = 180$  kHz [28], and at most C = 100 users can access the UAV at the same time, since a resource block is the minimum unit of transmission and is 180 kHz wide [28] (notice that  $20~MHz - 180~kHz \times 100 = 2~MHz$  in the total 20 MHz bandwidth cannot be used).

Since some users may be within the transmission ranges of multiple UAVs, to reduce the interference of such users, two adjacent UAVs can be allocated with different spectrum segments [28]. On the other hand, to reuse the frequency as efficiently as possible, the same spectrum segment can be reused by two UAVs if they are not within the communication range of each other. Note that the problem of allocating the minimum number of spectrum segments is equivalent to the vertex coloring problem in graphs [4]. Specifically, a graph  $G_{spectrum} = (S, E_s)$  is first constructed, where S is the set of hovering locations of the K UAVs, and there is an edge  $(v_i, v_j)$ in  $E_s$  between two locations  $v_i$  and  $v_j$  if their Euclidean distance  $d_{i,j}$  is no more than twice the communication range  $R_{user}$  of a ground user, i.e.,  $d_{i,j} \leq 2R_{user}$ . The vertex coloring problem in  $G_{spectrum}$  is to use the minimum number of colors to color vertices in the graph, such that no two adjacent vertices are colored with the same color. The vertex coloring problem however is NP-hard, and the algorithm in [4] can be applied to find an approximate solution to the problem in polynomial time.

## D. Problem Definition

We use an undirected graph  $G = (U \cup V, E)$  to represent the UAV network, where U is the set of users on the ground of the monitoring area, V is the set of potential hovering locations of UAVs at altitude h. There is an edge  $(u_i, v_i)$  in E between a user  $u_i$  and a hovering location  $v_i$  if the Euclidean distance between them is no greater than  $R_{user}$ , and there is an edge  $(v_j, v_k)$  in E between two hovering locations  $v_j$  and  $v_k$  if their distance is no more than  $R_{uav}$ . Notice that the number of available UAVs just after a disaster may be very limited and thus they may not be able to serve all users. In addition, it usually takes time, e.g., one or two days, to purchase new UAVs and install base station devices for rescuing. However, it is very urgent to provide communication services to users. Then, an important problem is to serve as many users as possible by the available UAVs.

In this paper we consider the connected maximum throughput problem in G, which is to choose no more than K hovering locations among all potential hovering locations in V for placing the UAVs, such that the network throughput, i.e., the sum of the data rates of users served by the deployed UAVs, is maximized, subject to that (i) each user  $u_i \in U$  can be served by at most one UAV within its communication range  $R_{user}$ ; (ii) the data rate of user  $u_i$  is no less than its minimum data rate  $b_{min}^i$  if user  $u_i$  is served by a UAV; (iii) the number of users served by each UAV is no more than its service capacity C; and (iv) the communication network induced by the deployed UAVs is connected. Notice that we also provide an Integer Programming for the problem in the supplementary file.

To deal with the defined connected maximum throughput problem, we here define another problem: the maximum assignment problem, which will serve as an important subroutine for the original problem. Given a subset  $S \subseteq V$  of hovering locations with  $|S| \leq K$  such that a UAV has already been deployed at each location in S, the problem is how to assign users in U to the UAVs at locations in S so that the sum of the data rates of users is maximized, subject to the service capacity C on each UAV and the minimum data rate  $b_{min}^i$  of each user  $u_i$ . Notice that the subnetwork induced by the |S| UAVs of the maximum assignment problem may not necessarily be connected. For any subset  $S \subseteq V$ , denote by f(S) the maximum sum of the data rates of users served by UAVs deployed at locations in S.

#### E. Submodular Functions

Let V be a set of finite elements and f a function with  $f: 2^V \mapsto \mathbb{R}^{\geq 0}$ . For any two subsets A and B of V with  $A \subseteq B$  and any element  $v \in V \setminus B$ , f is submodular if  $f(A \cup \{v\}) - f(A) \geq f(B \cup \{v\}) - f(B)$  [33], and f is monotone submodular if  $f(A) \leq f(B)$ .

# IV. OPTIMAL ALGORITHM FOR THE MAXIMUM ASSIGNMENT PROBLEM

In this section, we propose an exact algorithm for the maximum assignment problem, which calculates the maximum sum f(S) of data rates of users served by UAVs at locations in a given subset  $S \subseteq V$ , assuming that a UAV has already been deployed at each location in S. This algorithm will serve as a subroutine for the connected maximum throughput problem later. We also show an important property of function f(S), that is, f(S) is nondecreasing and submodular. This property is a cornerstone of the proposed algorithm for the connected maximum throughput problem later.

## A. An Optimal Algorithm for Calculating f(S)

The basic idea behind the algorithm is to reduce the problem to the maximum weighted matching problem in an auxiliary bipartite graph, and an optimal matching to the problem in the auxiliary graph in turn returns an optimal solution to the maximum assignment problem.

An auxiliary bipartite graph  $G_S = (U \cup S', E_S; \rho : E_S \mapsto \mathbb{R}^{\geq 0})$  is constructed, where U is the set of users, there are

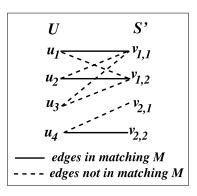


Fig. 2. An illustration of the construction of graph  $G_S$ , where the service capacity of each UAV is C=2, and users  $u_1,u_2$ , and  $u_4$  are matched in the maximum weighted matching M.

C 'virtual' hovering locations  $v_{j,1}, v_{j,2}, \ldots, v_{j,C}$  in S' for each real location  $v_j \in S$ , and C is the service capacity of each UAV. There is an edge  $(u_i, v_{j,l})$  in  $E_S$  between a user  $u_i$  and each virtual location  $v_{j,l}$  with  $1 \leq l \leq C$  if the Euclidean distance between user  $u_i$  and  $v_j$  is no more than the communication range  $R_{user}$  of user  $u_i$ , i.e.,  $d_{i,j} \leq R_{user}$ , and the data rate  $r_{i,j}$  is no less than the minimum data rate  $b_{min}^i$  of user  $u_i$ , i.e.,  $r_{i,j} \geq b_{min}^i$ . Finally, the weight  $\rho(u_i, v_{j,l})$  of edge  $(u_i, v_{j,l})$  is the data rate  $r_{i,j}$  of user  $u_i$  if  $u_i$  is served by a UAV at  $v_j$ . Fig. 2 illustrates such a graph  $G_S$  with C=2.

Having constructed graph  $G_S$ , a maximum weighted matching M in  $G_S$  then is found, by applying an algorithm in [11], where M is a matching in  $G_S$  such that the weighted sum of edges in M, i.e.,  $\sum_{e\in M} \rho(e)$ , is maximized. Fig. 2 shows such a maximum weighted matching M. A solution to the maximum assignment problem is obtained from the matching, where a user  $u_i$  is assigned to the UAV at  $v_j$  if  $u_i$  is matched to a virtual node  $v_{j,l}$  in M. For example, it can be seen from Fig. 2 that both users  $u_1$  and  $u_2$  are assigned to  $v_1$ , and  $v_2$  is assigned to  $v_2$ . However,  $v_3$  is not assigned to anyone. Then, the sum of the data rates of users served by UAVs is equal to the weighted sum  $\rho(M)$  of edges in matching  $v_1$ , i.e.,  $v_2$ ,  $v_3$ ,  $v_4$ ,  $v_5$ ,  $v_6$ ,  $v_7$ ,  $v_8$ ,  $v_8$ ,  $v_8$ ,  $v_9$ ,

The algorithm for the maximum assignment problem is presented in Algorithm 1.

## B. Algorithm Analysis

Lemma 1: Given a UAV network  $G = (U \cup V, E)$ , a subset  $S \subset V$  of hovering locations with a UAV deployed at each location in S, and the service capacity C of each UAV, there is an algorithm, Algorithm 1, for the maximum assignment problem in G, which delivers an optimal solution in time  $O((KC + n)^2 \log(KC + n))$ , where K = |S| and N = |U|.

*Proof:* It can be seen that the value of the optimal solution to the maximum assignment problem in G is equal to the weighted sum of the edges of the maximum weighted matching M in  $G_S$  [11]. Since the algorithm in [11] delivers a maximum weighted matching in  $G_S$ , Algorithm 1 delivers an optimal solution.

We analyze the time complexity of Algorithm 1. Denote by  $n_S$  and  $m_S$  the number of nodes and edges in  $G_S$ ,

Algorithm 1 Algorithm for the Maximum Assignment Problem

**Require:** a set U of users, a set S of hovering locations with a UAV at each location, and the service capacity C of each UAV

**Ensure:** An optimal solution to the maximum assignment problem

- 1: Construct an auxiliary bipartite graph  $G_S = (U \cup S', E_S; \rho : E_S \mapsto \mathbb{R}^{\geq 0})$ , where there is an edge  $(u_i, v_{j,l})$  in  $E_S$  between a user  $u_i$  and each virtual location  $v_{j,l}$  with  $1 \leq l \leq C$ , if the Euclidean distance between user  $u_i$  and  $v_j$  is no more than the communication range  $R_{user}$  of user  $u_i$  and the data rate  $r_{i,j}$  is no less than the minimum data rate  $b^i_{min}$  of user  $u_i$ , and  $\rho(u_i, v_{j,l})$  is the data rate  $r_{i,j}$  of user  $u_i$  by the UAV at  $v_j$ ;
- 2: Find an optimal maximum weighted matching M in  $G_S$ , by invoking the algorithm in [11];
- 3: For each user  $u_i \in U$ , assign it to the UAV at  $v_j$  if  $u_i$  is matched to a virtual node  $v_{j,l}$  in M.

respectively. Following the construction of  $G_S$ , it can be seen that  $n_S = KC + n$  and  $m_S = O(Cn)$ , since the number of UAVs within the communication range of each user is limited. Since the time complexity of the algorithm in [11] is  $O(n_S^2 \log n_S + n_S m_S)$ , the time complexity of Algorithm 1 is  $O((KC+n)^2 \log(KC+n) + (KC+n)O(Cn)) = O((KC+n)^2 \log(KC+n))$ , since  $C = O(\log(KC+n))$ . The lemma then follows.

## C. Submodularity of Function f(S)

Lemma 2: Given any subset S of V, let f(S) be the maximum sum of the data rates of users served by the UAVs at the hovering locations in S, which can be calculated by Algorithm 1. Then, function f(S) is nondecreasing and submodular.

*Proof:* The proof is contained in the supplementary file.

## V. APPROXIMATION ALGORITHM FOR THE CONNECTED MAXIMUM THROUGHPUT PROBLEM

In this section, we study the connected maximum throughput problem. We first provide the basic idea of the proposed approximation algorithm. We then devise a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximation algorithm for the problem, where e is the base of the natural logarithm and K is the number of UAVs.

## A. Basic Idea

The basic idea behind the proposed algorithm is as follows. Given any location  $v_j \in V$ , the algorithm first identifies a subset  $V_j$  of hovering locations around  $v_j$ , such that the sum of the data rates of users served by the UAVs at the identified locations is maximized, while ensuring that the sum of shortest distances between  $v_j$  and nodes in  $V_j$  is no more than K-1, where the shortest distance between  $v_l$  and a node  $v_k \in V_j$  is the minimum number of hops between them in G.

Since the subnetwork induced by nodes in  $V_j \cup \{v_j\}$  may not be connected, ensuring these nodes to be connected is done through adding relaying nodes among them while keeping the number of nodes in the resulting connected component is no greater than K.

#### B. Approximation Algorithm

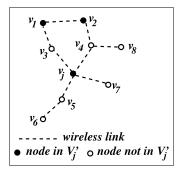
For each hovering location  $v_j \in V$ , the algorithm finds a set  $S_j'$  of K location nodes such that the induced graph  $G[S_j']$  of G by the nodes in  $S_j'$  is connected, where  $v_j$  is contained in  $S_j'$ . The solution to the connected maximum throughput problem then is such a set  $S_{j^*}'$  that the sum of the data rates of users served by the UAVs deployed at locations in  $S_{j^*}'$  is maximized, i.e.,  $j^* = \arg\max_{1 \le j \le m} \{f(S_j')\}$ , where m is the number of hovering locations in V. In the following, we show how to find set  $S_j'$  for each  $v_j \in V$ .

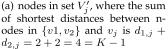
For each hovering location  $v_j \in V$ , we first find the shortest distance  $d_{k,j}$  (in terms of numbers of hops) in G between  $v_j$  and each hovering location  $v_k$  in  $V \setminus \{v_j\}$ , by applying a Breadth-First-Search starting from  $v_j$ .

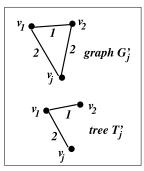
We then consider a constrained maximum throughput problem, which is to find a subset  $V_j$  of  $V\setminus\{v_j\}$  such that the sum of the data rates of users served by the UAVs deployed at the locations in  $V_j\cup\{v_j\}$  is maximized, subject to that the sum of the shortest distances between the nodes in  $V_j$  and  $v_j$  is no greater than K-1, i.e.,  $\sum_{v_k\in V_j}d_{k,j}\leq K-1$ , where  $d_{k,j}$  is the minimum number of hops between a node  $v_k\in V_j$  and  $v_j$  in G. We later show that this problem can be cast as a submodular function maximization problem subject to a knapsack constraint. Then, we can find a (1-1/e)-approximate solution  $V_j$  to the constrained maximum throughput problem, by applying the algorithm in [33], where e is the base of the natural logarithm. Let  $V_j'=V_j\cup\{v_j\}$ , see Fig. 3(a).

Notice that the induced graph  $G[V'_j]$  of G by the nodes in  $V'_j$  may not be connected. The rest is to find a connected subgraph  $G_j$  of G, such that the nodes in  $V'_j$  are contained in  $G_j$  and  $G_j$  contains no more than K nodes.

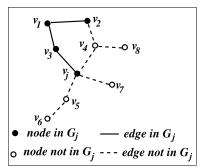
A graph  $G'_{i} = (V'_{i}, E'_{i})$  is first constructed from set  $V'_{i}$ , where there is an edge  $(v_k, v_l)$  in  $E'_i$  between any two nodes  $v_k$  and  $v_l$  in  $V_i'$ , and its edge weight  $w(v_k, v_l)$  is the minimum number of hops between them in G. A minimum spanning tree (MST)  $T'_i$  in  $G'_i$  is then found, see Fig. 3(b). There is an important property, that is, the weighted sum of the edges in  $T'_i$ is no greater than K-1, i.e.,  $w(T_j')=\sum_{(v_k,v_l)\in T_j'}w(v_k,v_l)\leq K-1$ , which will be shown later. Denote by  $n_j$  the number of nodes in tree  $T'_i$ . Then, there are  $(n_j - 1)$  edges in  $T'_i$ . For each edge  $(v_k, v_l)$  in tree  $T'_i$ , there is a corresponding shortest path  $P_{k,l}$  in graph G between nodes  $v_k$  and  $v_l$ . A connected subgraph  $G_i$  of G then can be obtained from  $T'_i$ , which is the union of the  $(n_j - 1)$  shortest paths in G, i.e.,  $G_j =$  $\{P_{k,l} \mid (v_k, v_l) \in T_i'\}$ . For example, Fig. 3(c) shows a graph  $G_j$  constructed from the tree  $T'_j$  in Fig. 3(b), where  $P_{1,j}$  $v_1 - v_3 - v_j$  and  $P_{1,2} = v_1 - v_2$ . Following the construction of  $G_j$ , it can be seen that the number of edges in  $G_j$  is no more than the weighted sum  $w(T'_i)$  of the edges in  $T'_i$ , as the weight  $w(v_k, v_l)$  of each edge  $(v_k, v_l)$  in  $T'_i$  is the shortest distance between nodes  $v_k$  and  $v_l$  in G. The number of nodes



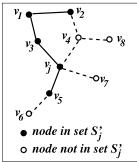




(b) graph  $G_j' = (V_j', E_j')$  and the minimum spanning tree  $T_j'$  in  $G_j'$ 



(c) graph  $G_j$ , where  $S_j$  is the set of nodes in  $G_j$ 



(d) nodes in set  $S'_j$ , which is obtained by adding node  $v_5$  to  $S_j$ 

Fig. 3. An illustration of the execution of the approximation algorithm, where K=5.

in  $G_j$  thus is no more than  $w(T_j') + 1 \le K - 1 + 1 = K$ , since  $G_j$  is connected.

Denote by  $S_j$  the node set of graph  $G_j$ . Then,  $|S_j| \leq K$ . We construct a set  $S_j'$  that contains no more than K nodes and the nodes in  $S_j$  are contained in  $S_j'$  as well. If  $|S_j| = K$ , then  $S_j' = S_j$ . Otherwise  $(|S_j| < K)$ , let  $S_j' = S_j$  initially. We add nodes to  $S_j'$  one by one until  $S_j'$  contains exactly K nodes or the marginal gain of adding any node is zero, such that each added node has the maximum marginal gain, subject to that the node is connected with a node already in  $S_j'$ . For example, Fig. 3(d) shows that node  $v_5$  is added to  $S_j'$  and  $|S_j'| = K = 5$ .

It can be seen that the induced subgraph  $G[S'_j]$  by the nodes in set  $S'_j$  is connected.

The algorithm for the problem is presented in Algorithm 2.

## C. Redeployment of UAVs With User Mobility

Users may move around in a disaster area. It can be seen that an optimal deployment of UAVs may become sub-optimal after a period of time, due to user mobility. In this user mobility scenario, we invoke the proposed algorithm to calculate the updated optimal deployment locations of the K UAVs at each time slot, e.g., 2 minutes. In the beginning of each time slot, we first calculate the new deployment locations of the UAVs with the most recent location information of users, by invoking the proposed algorithm, taken by the on-board cameras of the UAVs [14], [15]. If the network throughput under the previous UAV deployment locations is only slightly worse than the one under this new UAV deployment locations, e.g., no more than 5% smaller, the K UAVs do not fly to their new deployment locations, since frequent redeployments of UAVs will consume large amounts of energy of UAVs. Otherwise (the previous network throughput is at least 5% smaller than the new network throughput), the UAVs will fly to their new locations.

#### D. The Energy Issue of UAVs

To provide uninterrupted communication services to users in a disaster area for a critical period, e.g., within 72 hours, we assume that the UAV communication network consists of (1) K, e.g., 30, communication UAVs; (2)  $K_{sUAV}$  standby communication UAVs; and (3)  $K_{battery}$  standby UAV batteries which can be simultaneously charged at a nearby service center, where the cost of a UAV battery usually is much cheaper than the cost of a UAV. When some communication UAVs run out of energy, the standby UAVs first replace the communication UAVs by flying to the service hovering locations of the communication UAVs, and the communication UAVs then return to the service center to replace their batteries. The communication UAVs act as new standby UAVs for later UAV replacements. The detached UAV batteries can be recharged at the service center. By doing so, there are always K communication UAVs deployed to provide communication services to ground users.

Given the number of K communication UAVs, the number  $K_{sUAV}$  of standby communication UAVs and the number  $K_{battery}$  of standby UAV batteries are calculated as follows. We first calculate the number  $K_{sUAV}$  of standby communication UAVs. Denote by  $T_{max}$  the maximum operation time of a fully charged UAV, e.g.,  $T_{max} = 55$  minutes for a DJI Matrice M300 RTK UAV [23]. Also, denote by  $\eta$  the flying speed of a UAV, e.g.,  $\eta = 23 \ m/s$  [23]. Assume that the service center is located at the center of the disaster area. Then, the maximum distance  $d_{max}$  between the service center and a UAV service location in the sky is  $d_{max} = \sqrt{(L/2)^2 + (W/2)^2 + h^2}$ , where L and W are the length and width of the disaster area, and h is the UAV hovering altitude. It can be seen that the flying time  $t_{fly}$  of a UAV between the service center and a service location in the sky for a round trip is no greater than  $t_{fly} = 2 \frac{d_{max}}{\eta}$ , and  $t_{fly}$  is the maximum time for replacing one communication UAV. Then, one standby UAV is able to replace  $\lfloor \frac{T_{max}}{t_{fly}} \rfloor$  communication UAVs within the maximum UAV operation time  $T_{max}$ .

To provide uninterrupted communication services for K communication UAVs, the number of  $K_{sUAV}$  of standby communication UAVs needed is  $K_{sUAV} = \lceil \frac{K}{\lfloor \frac{T_{max}}{t_{fly}} \rfloor} \rceil$ . For example, assume that both the length L and width W of the disaster area are 3 km [45], i.e., L = W = 3 km, and the UAV hovering altitude h is 300 m [1]. Then,  $d_{max} = \sqrt{(L/2)^2 + (W/2)^2 + h^2} = 2{,}142$  meters,

**Algorithm 2** Approximation Algorithm for the Connected Maximum Throughput Problem (ApproAlg)

**Require:** a set U of users, a set V of potential hovering locations, and K UAVs with the service capacity C of each UAV

**Ensure:** A solution to the connected maximum throughput problem

- 1: Let D be the maximum number between  $2\lfloor \sqrt{K-1} \rfloor$  and the largest odd number no more than  $\lfloor \sqrt{4K-3} \rfloor$ ;
- 2: Let  $V' \leftarrow \emptyset$ ; /\* the set of hovering locations \*/
- 3: **for** each location  $v_i \in V$  **do**
- 4: Calculate the shortest distance between each location in  $V \setminus \{v_j\}$  and  $v_j$  in G, by using a Breadth-First-Search starting from  $v_j$ , where the cost of each edge is one;
- 5: Find a set  $V_j$  of locations for the constrained maximum throughput problem, by invoking the algorithm in [33];
- 6: Let  $V'_{i} = V_{j} \cup \{v_{j}\};$
- 7: Construct a graph  $G'_j = (V'_j, E'_j)$ , where there is an edge  $(v_k, v_l) \in E'_j$  between any two nodes  $v_k$  and  $v_l$  in  $V'_j$ , and its edge weight  $w(v_k, v_l)$  is the minimum number of hops between  $v_k$  and  $v_l$  in G;
- 8: Find a Minimum Spanning Tree (MST)  $T'_i$  in  $G'_i$ ;
- 9: Construct a subgraph  $G_j$  of G, where  $G_j = \{P_{k,l} \mid (v_k, v_l) \in T_j'\}$  and  $P_{k,l}$  is the shortest path in G between nodes  $v_k$  and  $v_l$ . Let  $S_j$  be the set of nodes in  $G_j$ ;
- 10: Let  $S'_j = S_j$  initially, and continue to add nodes in  $V \setminus S'_j$  to  $S'_j$  until  $S'_j$  has K nodes, such that each added node has the maximum positive marginal gain, subject to that the node is connected to a node already in  $S'_i$ ;
- 11: **if**  $f(S'_i) > f(V')$  **then**
- 12: /\* Find a better set of hovering locations \*/
- 13: Let  $V' \leftarrow S'_i$ ;
- 14: **end if**
- 15: end for
- 16: Assign users in U to the UAVs at the hovering locations in V' by invoking Algorithm 1;
- 17: **return** the hovering locations in V' and the assignment of users in U.

$$t_{fly} = 2\frac{d_{max}}{m} = 186.3$$
 seconds, and  $K_{sUAV} = \lceil \frac{K}{\lfloor \frac{T_{max}}{t_{fly}} \rfloor} \rceil = \lceil \frac{K}{\lfloor \frac{50000}{150000} \rfloor} \rceil = \lceil \frac{K}{17} \rceil = 2$  when  $K = 30$ .

We then calculate the number  $K_{battery}$  of standby UAV batteries. Denote by  $T_{charge}$  the time for fully charging a UAV battery, i.e.,  $T_{charge} = 60$  minutes [23]. Then, the number of standby UAV batteries is  $K_{battery} = \lceil \frac{T_{charge}}{T_{max}} \cdot K \rceil$ , where  $T_{max}$  is the maximum operation time of a fully charged UAV. For example,  $K_{battery} = \lceil \frac{T_{charge}}{T_{max}} \cdot K \rceil = \lceil \frac{60}{55} \cdot K \rceil \le \lceil 1.1K \rceil = 33$  when K = 30. Notice that the cost of a UAV battery usually is much cheaper than the cost of a UAV.

## VI. ANALYSIS OF THE APPROXIMATION ALGORITHM

In this section, we analyze the approximation ratio of the proposed algorithm, Algorithm 2, for the connected maximum throughput problem. Denote by  $V^{\ast}$  and OPT the

optimal solution and its value, respectively, i.e.,  $OPT = f(V^*)$ . Following the definition of the connected maximum throughput problem, the induced graph  $G[V^*]$  by the nodes in  $V^*$  is connected. It can be seen that there is a spanning tree T in  $G[V^*]$ , assuming that the cost of each edge is one. The roadmap of the approximation ratio analysis is as follows.

We first show that tree T can be decomposed into  $\Delta$  subtrees  $T_1, T_2, \ldots, T_\Delta$  such that the number of nodes in each subtree is no more than D and  $\Delta \leq \lfloor \sqrt{K} \rfloor$  in Lemma 3 of Section VI-A, where D be the maximum integer between  $2\lfloor \sqrt{K-1} \rfloor$  and the largest odd number no more than  $\lfloor \sqrt{4K-3} \rfloor$ . Due to the submodularity of the objective function by Lemma 2 in Section IV-C, it can be seen that there is a subtree, say  $T_l$ , among the  $\Delta$  subtrees such that the sum of the data rates of users served by the UAVs in  $T_l$  is no less than  $\frac{1}{\Delta}$  of that in the original tree T, i.e.,  $f(T_l) \geq \frac{f(T)}{\Delta} = \frac{OPT}{\Delta} \geq \frac{OPT}{\lfloor \sqrt{K} \rfloor}$ , where f(T) = OPT.

We then prove that there is a node  $v_l$  in  $T_l$  such that the sum of the shortest distances between  $v_l$  and the nodes in  $T_l \setminus \{v_l\}$  is no more than K-1, i.e.,  $\sum_{v_k \in T_l \setminus \{v_l\}} d_{k,l} \leq K-1$ , see Lemma 4 in Section VI-B. Denote by  $V_l^*$  the optimal solution to the constrained maximum throughput problem with respect to node  $v_l$ . It can be seen that the nodes in  $T_l \setminus \{v_l\}$  form a feasible solution to the constrained maximum throughput problem with respect to node  $v_l$ , and thus  $f(T_l) \leq f(V_l^* \cup \{v_l\})$ . Therefore, we obtain a non-trivial upper bound on the optimal solution OPT, i.e.,  $OPT \leq \lfloor \sqrt{K} \rfloor \cdot f(T_l) \leq \lfloor \sqrt{K} \rfloor \cdot f(V_l^* \cup \{v_l\})$ , see Lemma 5 in Section VI-B.

On the other hand, we are able to find a (1-1/e)-approximate solution  $V_l$  to the constrained maximum throughput problem with respect to node  $v_l$ , which implies that  $f(V_l \cup \{v_l\}) \geq (1-1/e) \cdot f(V_l^* \cup \{v_l\}) \geq (1-1/e) \cdot \frac{OPT}{\lfloor \sqrt{K} \rfloor}$ . In addition, we can obtain a set  $S_l$  of K nodes including nodes in  $V_l \cup \{v_l\}$  such that the induced subgraph by  $S_l$  in G is connected, since  $\sum_{\substack{v_k \in V_l \setminus \{v_l\} \\ \lfloor \sqrt{K} \rfloor}} d_{k,l} \leq K-1$ . Then,  $f(S_l) \geq f(V_l \cup \{v_l\}) \geq \frac{1-1/e}{\lfloor \sqrt{K} \rfloor} \cdot OPT$ , as f(.) is a nondecreasing function.  $S_l$  thus is a  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximate solution to the connected maximum throughput problem, see Theorem 1 in Section VI-C.

#### A. Bound the Number of Decomposed Subtrees

It can be seen that there are K-1 edges in tree T since  $|V^*|=K$ . Let B=D-1, where D is the maximum number between  $2\lfloor \sqrt{K-1}\rfloor$  and the largest odd number no more than  $\lfloor \sqrt{4K-3}\rfloor$ . Following the work due to Xu *et al.* [36], tree T can be decomposed into  $\Delta$  edge-disjoint subtrees  $T_1, T_2, \ldots, T_{\Delta}$ , such that the number of edges in each subtree is no more than B, where  $\Delta \leq \lfloor \frac{K-1}{(B+1)/2} \rfloor = \lfloor \frac{2(K-1)}{D} \rfloor$ . Then, the number of nodes in each of the  $\Delta$  subtrees is no more than B+1=D.

We show that  $\Delta \leq |\sqrt{K}|$  by the following lemma.

Lemma 3: Given a tree T with K nodes and  $K \geq 2$ , let D be the maximum integer between  $2\lfloor \sqrt{K-1} \rfloor$  and the largest odd number no more than  $\lfloor \sqrt{4K-3} \rfloor$ . Then, tree T can be decomposed into  $\Delta$  subtrees  $T_1, T_2, \ldots, T_{\Delta}$  such that the number of nodes in each subtree is no more than D and  $\Delta \leq \lfloor \sqrt{K} \rfloor$ .

TABLE II					
The Change of the Value of $D$ With the Change of $b$					

value of b	0	[1, a]	[a+1, 2a]
value of $2\lfloor \sqrt{K-1} \rfloor$	2(a-1)	2a	2a
value of $\lfloor \sqrt{4K-3} \rfloor$	2a - 1	2a	2a + 1
maximum odd number no more than $\lfloor \sqrt{4K-3} \rfloor$	2a-1	2a - 1	2a + 1
value of D	2a-1	2a	2a + 1

Proof: Following the work due to Xu et al. [36] ( see Lemma 1 in [36]), the number of nodes in each subtree is no more than D. The rest is to show that  $\Delta \leq |\sqrt{K}|$ . We assume that  $K = a^2 + b$ , where a is the largest integer so that  $a^2 \le K$ and  $b = K - a^2$ . It can be seen that  $a \ge 1$  as  $K \ge 1$ , and  $b \le 2a$ . Otherwise  $(b \ge 2a + 1)$ , we have  $K = \vec{a^2} + \vec{b} \ge 2a$  $a^2 + 2a + 1 = (a+1)^2$ . This contradicts the definition of a, as (a+1) is larger than a and  $(a+1)^2 < K$ .

Table II shows the change of the value of D with the change of b. For example, when b = 0, we have  $2\lfloor \sqrt{K-1} \rfloor =$  $2|\sqrt{a^2-1}| = 2(a-1)$ , as  $K = a^2 + b$  and a is a positive integer. Also, when  $1 \le b \le a$ , we conclude that  $2\lfloor \sqrt{K-1} \rfloor = 2\lfloor \sqrt{a^2+b-1} \rfloor \ge 2\lfloor \sqrt{a^2} \rfloor = 2a \text{ as } b \ge 1,$  while  $2\lfloor \sqrt{K-1} \rfloor = 2\lfloor \sqrt{a^2+b-1} \rfloor \le 2\lfloor \sqrt{a^2+a-1} \rfloor = 2 \lfloor \sqrt{a^$ 2a, since  $b \leq a$ .

We distinguish into three cases: (1) b = 0; (2)  $1 \le b \le a$ ; and (3)  $a + 1 \le b \le 2a$ .

Case (1) where b=0, we know that  $K=a^2$  and D=2a-1. Then,

$$\Delta \le \lfloor \frac{2(K-1)}{D} \rfloor,$$

$$= \lfloor \frac{2(a^2-1)}{2a-1} \rfloor, \text{ as } D = 2a-1 \text{ and } K = a^2$$

$$= \lfloor a+1 - \frac{a+1}{2a-1} \rfloor$$

$$\le a, \text{ as } 1 - \frac{a+1}{2a-1} < 1$$

$$= \lfloor \sqrt{K} \rfloor, \text{ as } \lfloor \sqrt{K} \rfloor = a. \tag{6}$$

Case (2) where  $1 \le b \le a$ , we know that D = 2a and  $K = a^2 + b < a^2 + a$ . Then,

$$\Delta \le \lfloor \frac{2(K-1)}{D} \rfloor$$

$$\le \lfloor \frac{a^2 + a - 1}{a} \rfloor, \text{ as } D = 2a \text{ and } K \le a^2 + a$$

$$= \lfloor a + 1 - \frac{1}{a} \rfloor$$

$$= a, \text{ as } 1 - \frac{1}{a} < 1$$

$$= \lfloor \sqrt{K} \rfloor, \text{ as } |\sqrt{K}| = a.$$
(7)

Case (3) where  $a + 1 \le b < 2a$ , we know that D =2a + 1. Also, following the work due to Xu et al. [36] (see Lemma 1 in [36]), the number of edges in each of the first  $\Delta-2$  obtained subtrees  $T_1,T_2,\ldots,T_{\Delta-2}$  is no less than  $\lceil \frac{B+1}{2} \rceil = \lceil \frac{D}{2} \rceil = a+1$ , while the sum of the numbers of edges in the last two subtrees  $T_{\Delta-2}$  and  $T_{\Delta}$  is no less than B+1=D=2a+1, where B=D-1. We thus have

$$(\Delta - 2)(a+1) \le K - 1 - (2a+1)$$
  
  $\le a^2 - 2$ , as  $K \le a^2 + 2a$ . (8)

Therefore,  $\Delta \leq a+1-\frac{1}{a+1}$ . Notice that  $\Delta$  is an integer and  $1 - \frac{1}{a+1} < 1$ , then

$$\Delta \le a = |\sqrt{K}|. \tag{9}$$

Combining Ineq. (6), (7), and (9), we have  $\Delta \leq \lfloor \sqrt{K} \rfloor$  for each K with  $K \geq 2$ . The lemma then follows.

## B. An Upper Bound on the Optimal Solution

In the following we provide a non-trivial upper bound on the optimal solution to the connected maximum throughput problem, which will be used in the approximation ratio analysis.

For each subtree  $T_l$  with  $1 \le l \le \Delta$ , let  $P_l$  be the longest path in  $T_l$ , where the length of a path in  $T_l$  is the number of edges in the path. Also, let  $v_l$  be a middle node of path  $P_l$ . For node  $v_l \in V$ , denote by  $V_l^*$  the optimal solution to the constrained maximum throughput problem. In the following, we first show that the sum of the shortest distances between nodes in  $V(T_l) \setminus \{v_l\}$  and  $v_l$  is no more than K-1. Then,  $V(T_l)\setminus\{v_l\}$  is a feasible solution to the constrained maximum throughput problem. Therefore,  $f(T_l) = f((T_l \setminus \{v_l\}) \cup$  $\{v_l\}$ )  $\leq f(V_l^* \cup \{v_l\}).$ 

Lemma 4: Given any tree  $T_l$  in graph G with no more than D nodes, let  $P_l$  be the longest path in  $T_l$  and  $v_l$  be a middle node of path  $P_l$ , where D is the maximum integer between  $2|\sqrt{K-1}|$  and the largest odd number no more than  $|\sqrt{4K-3}|$ . Then, the sum of the shortest distances between nodes in  $V(T_l) \setminus \{v_l\}$  and  $v_l$  in G is no more than K-1, i.e., 

file.

Lemma 5: Denote by  $V^*$  and OPT the optimal solution and its value of the connected maximum throughput problem, i.e.,  $OPT = f(V^*)$ . For each location  $v_i \in V$ , denote by  $V_i^*$ the optimal solution to the constrained maximum throughput problem. Then,  $OPT \leq \lfloor \sqrt{K} \rfloor \cdot \max_{v_i \in V} \{f(V_i^* \cup \{v_j\})\}.$ 

*Proof:* It can be verified that this claim holds when K = 1. In the following, we assume that  $K \geq 2$ .

Recall that, by Lemma 3, tree T can be decomposed into  $\Delta$  edge-disjoint subtrees  $T_1, T_2, \dots, T_{\Delta}$ , such that the number of nodes in each subtree is no more than D and  $\Delta \leq |\sqrt{K}|$ . In addition, for each subtree  $T_l$  with  $1 \leq l \leq \Delta$ , let  $P_l$  be the longest path in  $T_l$  and  $v_l$  be a middle node of path  $P_l$ . For node  $v_l \in V$ , denote by  $V_l^*$  the optimal solution to the constrained maximum throughput problem. Recall that the sum of the shortest distances between nodes in  $V(T_l) \setminus \{v_l\}$  and  $v_l$  is no more than K-1, by Lemma 4. Then,  $V(T_l)\setminus\{v_l\}$ is a feasible solution to the constrained maximum throughput problem. Therefore,

$$f(T_l) = f((V(T_l) \setminus \{v_l\}) \cup \{v_l\})$$
  
 
$$\leq f(V_l^* \cup \{v_l\}) \leq \max_{v_i \in V} \{f(V_j^* \cup \{v_j\})\}.$$
 (10)

On the other hand, we have

$$OPT = f(V^*) \leq \sum_{l=1}^{\Delta} f(T_l), \text{ as } f(.) \text{ is a submodular}$$

$$= \Delta \cdot \max_{v_j \in V} \{ f(V_j^* \cup \{v_j\}) \}, \text{ by Ineq. (10)}$$

$$\leq \lfloor \sqrt{K} \rfloor \cdot \max_{v_j \in V} \{ f(V_j^* \cup \{v_j\}) \}, \text{ by Lemma 3.} \quad (11)$$

The lemma then follows.

## C. The Approximation Ratio Analysis

We finally analyze the approximation ratio of the proposed algorithm by the following theorem.

Theorem 1: Given a UAV network  $G=(U\cup V,E)$  and K UAVs with the service capacity constraint C on each UAV, there is a  $\frac{1-1/e}{\lfloor \sqrt{K}\rfloor}$ -approximation algorithm, Algorithm 2, for the connected maximum throughput problem with a time complexity  $O(Km^3n^2\log n)$ , where e is the base of the natural logarithm, n=|U|, and m=|V|.

*Proof:* The feasibility of set V' is proved in in the supplementary file. The rest is to analyze its approximation ratio.

Following Lemma 2, the constrained maximum coverage problem can be cast as a submodular function maximization problem, subject to a knapsack constraint. Then, the algorithm in [33] delivers a (1-1/e)-approximate solution  $V_j$  to the constrained maximum coverage problem with respect to node  $v_j$ , i.e.,

$$f(V_j \cup \{v_j\}) \ge (1 - 1/e) \cdot f(V_j^* \cup \{v_j\}), \quad \forall v_j \in V,$$
(12)

assuming that  $V_j^*$  is the optimal solution.

Assume that  $v_l = \arg\max_{v_j \in V} \{f(V_j^* \cup \{v_j\})\}$ , i.e.,  $f(V_l^* \cup \{v_l\}) = \max_{v_j \in V} \{f(V_j^* \cup \{v_j\})\}$ , where  $1 \leq l \leq m$  and m = |V|.

Following Algorithm 2, a set  $S'_l$  of K nodes is found such that the induced graph  $G[S'_l]$  is connected and the nodes in  $V'_l (= V_l \cup \{v_l\})$  are contained in  $G[S'_l]$ . We then have

$$f(V') = \max_{v_j \in V} \{ f(S'_j) \} \ge f(S'_l) \ge f(V_l \cup \{v_l\})$$

$$\times \text{ as } V_l \cup \{v_l\} \subseteq S'_l \text{ and } f(.) \text{ is nondecreasing}$$

$$\ge (1 - 1/e) \cdot f(V_l^* \cup \{v_l\}), \text{ by Ineq. (12)}$$

$$= (1 - 1/e) \cdot \max_{v_j \in V} \{ f(V_j \cup \{v_j\}) \}$$

$$\ge \frac{1 - 1/e}{|\sqrt{K}|} \cdot OPT, \text{ by Lemma 5.}$$
(13)

The time complexity of Algorithm 2 is analyzed as follows. The running time of Algorithm 2 is dominated by Step 5, which finds a set  $V_j$  of locations for the constrained maximum throughput problem with respect to each node  $v_j$ , by invoking the algorithm in [33]. Notice that the algorithm in [33] invokes  $O(Km^2)$  times of Algorithm 1. Since the time complexity of is Algorithm 1 is  $O((KC+n)^2\log(KC+n))$  by Lemma 1, the time complexity of Algorithm 2 is  $m\cdot O(Km^2)\cdot O((KC+n)^2\log(KC+n)) = O(Km^3n^2\log n)$ , since the maximum number KC of users

that can be served by K UAVs usually is in the order of the number n of to-be-served users, i.e., KC = O(n).

#### VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed algorithm. We also study the impact of important parameters on the algorithm performance.

## A. Experimental Environment Settings

We consider a disaster area in a  $3 \times 3 \ km^2$  Euclidean space [45]. Assume that there are from 500 to 3,000 users located in the disaster area, and the human density follows the fat-tailed distribution, that is, most of people are crowded at a few places, while a small portion of people are sparsely distributed at rest places [31]. Also, assume that the number K of to-be-deployed UAVs is from 10 to 50. Then, the approximation ratio of the proposed algorithm is at least  $\frac{1-1/e}{\lfloor \sqrt{50} \rfloor} = \frac{1-1/e}{7}$  following Theorem 1, where e is the base of the natural logarithm. Furthermore, the service capacity C of each UAV varies from 50 users to 300 users. In addition, assume that the K UAVs hover at an altitude h=300 m [1]. The communication range between two UAVs is  $R_{uav}=600$  m, whereas the communication range between a UAV and a ground user is  $R_{user}=500$  m [45].

The transmission power  $P_t$  of each UAV is -6 dB, the antenna gain  $g_t$  is 5 dB, and the noise power  $P_N$  is -105 dB [40]. Also, the transmission bandwidth  $B_w$  is 180 kHz, radio frequency  $f_c$  is 2.5 GHz, and speed c of light is  $3 \times 10^8$  m/s. We consider an urban environment. Then, the average shadow fadings for LoS and NLoS links are  $\eta_{LoS}=1$  dB and  $\eta_{NLoS}=20$  dB, respectively [1]. Finally, the LoS probability  $p_{i,j}^{LoS}$  of a user  $u_i$  served by a UAV at location  $v_j$  is  $p_{i,j}^{LoS}=\frac{1}{1+a\cdot\exp(-b(\theta-a))}$ , where a=9.611725, b=0.158062, and  $\theta$  is the elevation angle of user  $u_i$  for the UAV deployed at  $v_j$  at altitude h [1].

To evaluate the algorithm performance, we compare with three state-of-the-art algorithms as follows. (i) Algorithm MotionCtrl [45] delivers a motion control solution to deploy K UAVs to serve as many user as possible, while guaranteeing the UAV network connectivity. (ii) Algorithm MCS [18] finds a  $\frac{1-1/e}{5(\sqrt{K+1})}$ -approximate solution to a problem of placing K wireless routers so that the value of a generalized submodular function over the placed routers is maximized, subject to the connectivity constraint. (iii) Algorithm GreedyLabel [16] first assigns a profit for each node in a greedy strategy, then finds a connected subgraph with K nodes such that the profit sum of the nodes is maximized. All algorithms are implemented by the programming language C. All experiments are performed on a powerful server, which contains an Intel(R) Core(TM) i9-9900K CPU with 8 cores and each core having a maximum turbo frequency of 5 GHz, and 32 GB RAM. Notice that we implement the proposed algorithms in parallel by adopting multiple threads. The value in each figure is the average of the results out of 200 problem instances with the same network size.

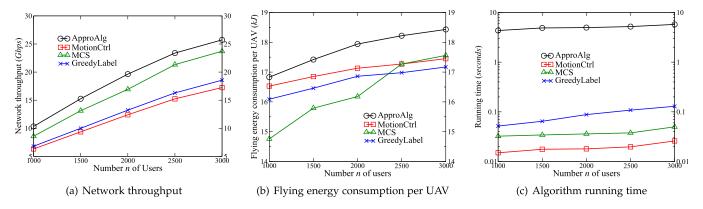


Fig. 4. The performance of different algorithms by increasing the number of to-be-served users n from 500 to 3,000, when the number of UAVs K = 30.

## B. Algorithm Performance

We first evaluated the algorithm performance by increasing the number n of users from 500 to 3,000, when there are K = 30 UAVs in the network. Fig. 4(a) shows that the network throughput by algorithm ApproAlg is about from 8.5% to 12% larger than those by algorithms MotionCtrl, MCS, and GreedyLabel. For example, the network throughput by the four algorithms ApproAlg, MotionCtrl, MCS, and GreedyLabel are 25.8, 17.2, 23.7, and 18.5 Gbps, respectively when there are 3,000 users in the disaster area. Fig. 4(a) also demonstrates that the network throughput by each of the four algorithms becomes larger where there are more users in the network. Fig. 4(b) plots the average amount of energy consumed by the K UAVs for flying from the service center in the disaster area to their service hovering locations, from which it can be seen that the average UAV flying energy consumption by each algorithm becomes larger, as the KUAVs need to be more sparsely deployed in a larger network, which incurs a longer flying distance. Fig. 4(c) illustrates the running times of the four comparison algorithms. It can be seen that the running time of algorithm ApproAlg is around five seconds, which is longer than those of the other three mentioned algorithms. Notice that such a short delay of a few seconds by algorithm ApproAlg is acceptable in real UAV networks, as the network throughput by the algorithm is up to 12% larger than those by the other three algorithms.

We then studied the algorithm performance by varying the number K of UAVs from 10 to 50 when there are n=3,000 users. Fig. 5 plots that the network throughput by each algorithm increases with more UAV deployments, as more users can be served. In addition, the network throughput by algorithm ApproAlg is from 5% to 9% higher than those by the other three algorithms. For example, the network throughput by the four algorithms ApproAlg, MotionCtrl, MCS, and GreedyLabel are 30.1, 20.1, 27.4, and 21.5 Gbps, respectively, when there are K=40 UAVs.

We thirdly investigated the performance of various algorithms by varying the service capacity C of each UAV from 50 users to 300 users when there are n=3,000 users and K=30 UAVs in the monitoring area. Fig. 6 demonstrates that the network throughput by each of the four algorithms increases with the growth of the service capacity C, as less

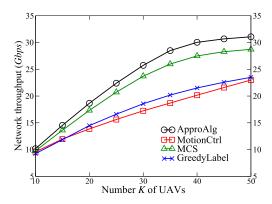


Fig. 5. The performance of different algorithms by varying the number of UAVs K from 10 to 50, when n=3,000 users.

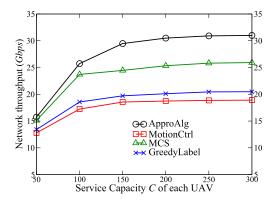


Fig. 6. The performance of different algorithms by increasing the service capacity C of each UAV from 50 users to 300 users, when there are n=3,000 users and K=30 UAVs.

numbers of UAVs are needed to serve the users in places with high human densities and more UAVs thus can be used to serve the users in other places. Fig. 6 shows that the network throughput by each algorithm only slightly increases when the service capacity C of each UAV is larger than 150 users, as there are a small portion of users located at other locations in the monitoring area, which needs several relaying UAVs to serve them [16], [22].

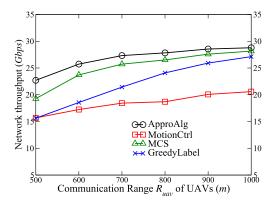


Fig. 7. The performance of different algorithms by varying the UAV communication range  $R_{uav}$  from 500 m to 1,000 m while fixing  $R_{user}=500$  m, when  $n=3,000,\,K=30$  and C=100.

We finally evaluated the algorithm performance by increasing the UAV communication range  $R_{uav}$  from 500 m to 1,000 m while fixing the user communication range  $R_{user}$ at 500 m, when n = 3,000, K = 30 and C = 100. Fig. 7 demonstrates that the network throughput by each of the four algorithms ApproAlg, MotionCtrl, MCS, and GreedyLabel increases with the growth of the UAV communication range  $R_{uav}$ . The rationale behind the phenomenon is that less numbers of relaying UAVs are needed when the UAV communication range  $R_{uav}$  is larger, and more UAVs thus can be used to serve users, thereby bringing about higher throughput. Fig. 7 also shows the difference between the network throughput by algorithms ApproAlg, MotionCtrl, MCS, and GreedyLabel. For example, the network throughput by algorithm ApproAlg is about 18% larger than that by algorithm MCS when the UAV communication range  $R_{uav}$  is 500 m, while the network throughput by algorithm ApproAlg is only about 2.5% higher than that by algorithm MCS when  $R_{uav} = 1,000 \text{ m}.$ 

#### VIII. CONCLUSION

In this paper, we studied the problem of deploying a communication network that consists of K UAVs to provide temporarily emergent services for people trapped in a disaster area. Under the assumption that the service capacity of each UAV is limited and each of them can only serve limited numbers of users, we investigated the problem of deploying K UAVs as aerial base stations in the top of the disaster area, such that the sum of the data rates of users served by the UAVs is maximized, subject to that (i) the number of users served by each UAV is no greater than its service capacity; and (ii) the communication network induced by the K UAVs is connected. We devised a novel  $\frac{1-1/e}{\lfloor \sqrt{K} \rfloor}$ -approximation algorithm for the problem, where e is the base of the natural logarithm. We also evaluated the performance of the proposed algorithm via simulation experiments. Experimental results showed that the proposed algorithm is very promising. Especially, the network throughput delivered by the proposed algorithm is up to 12% higher than those by existing algorithms.

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