EE559- Mathematical Pattern Recognition

Homework #2

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Problem 2:

Part B

Using Wine Data Set, I have implemented One vs Rest mechanism.
Considering one class at a time and combining other two into 1 group yield me the result i.e. the decision boundary line on the graph.

Below are the 6 graphs of which 3 are training data set and rest of the testing data set.

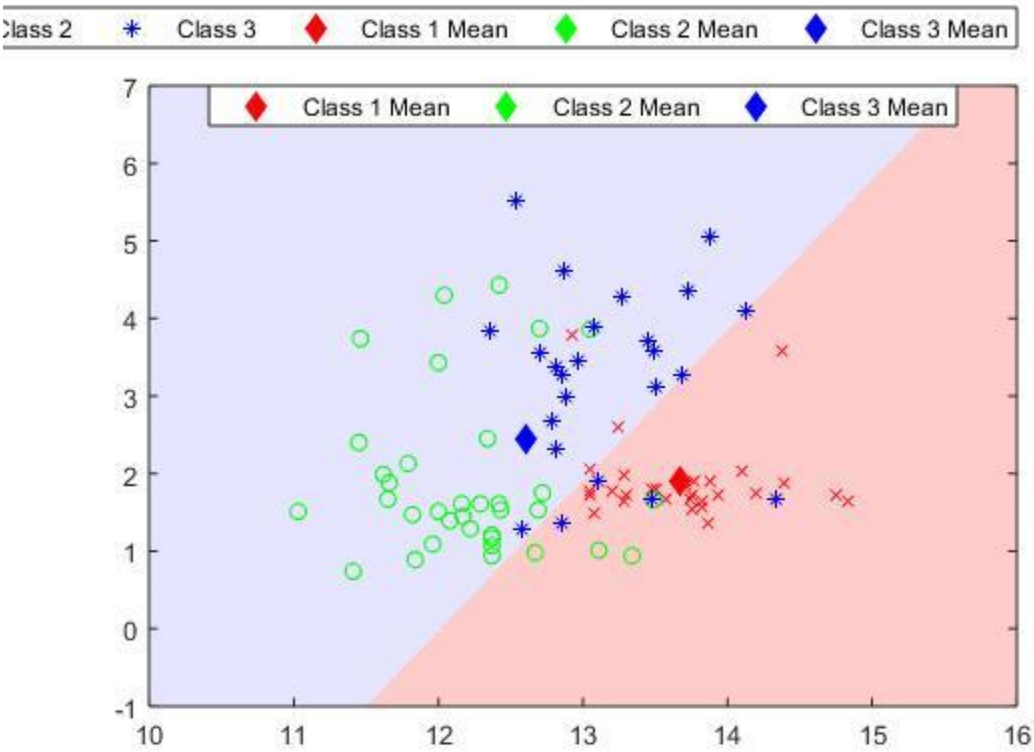

Fig 1. Training Plot- Class1 vs (2 and 3)

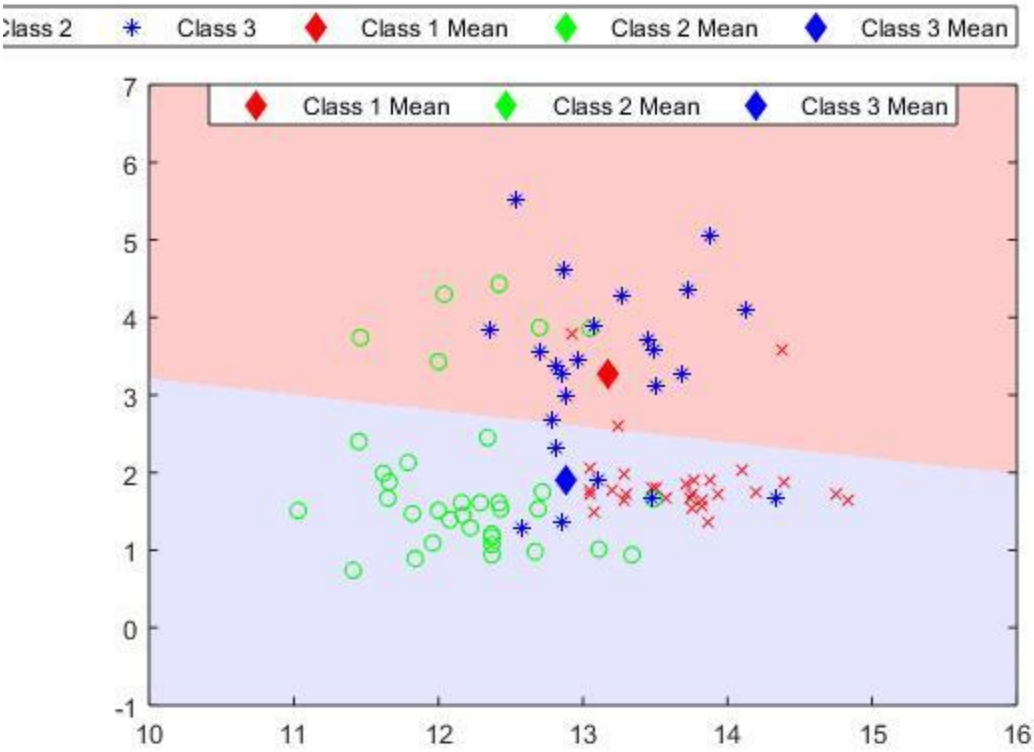

Fig 2. Training Plot- Class3 vs (1 and 2)

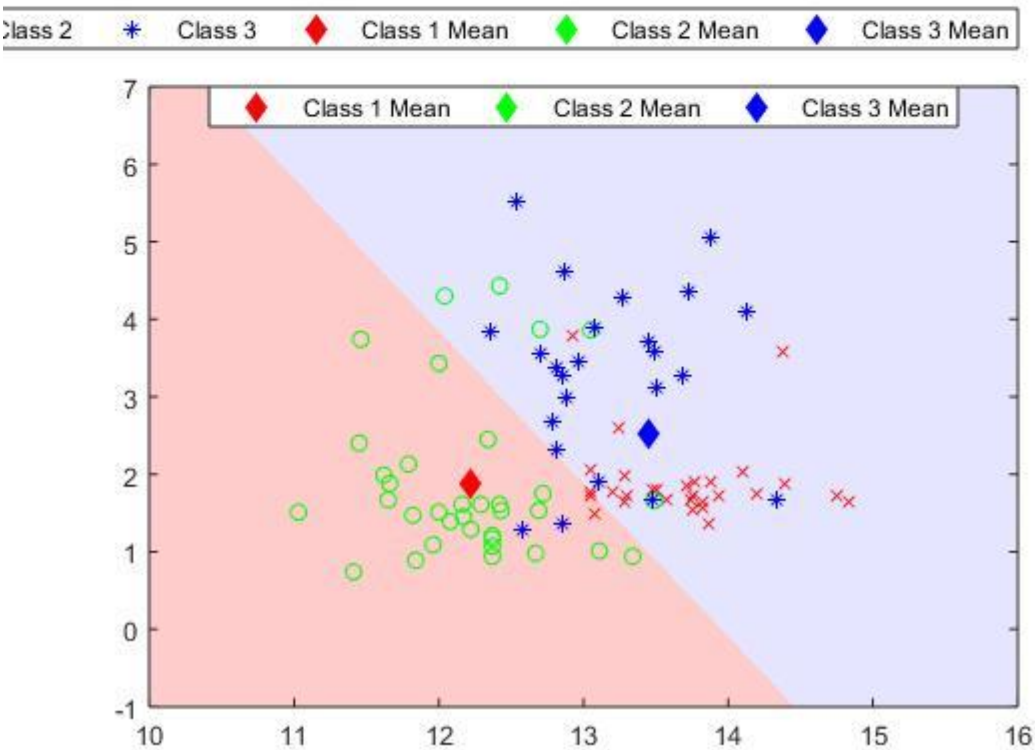

Fig 3. Training Plot- Class2 vs (1 and 3)

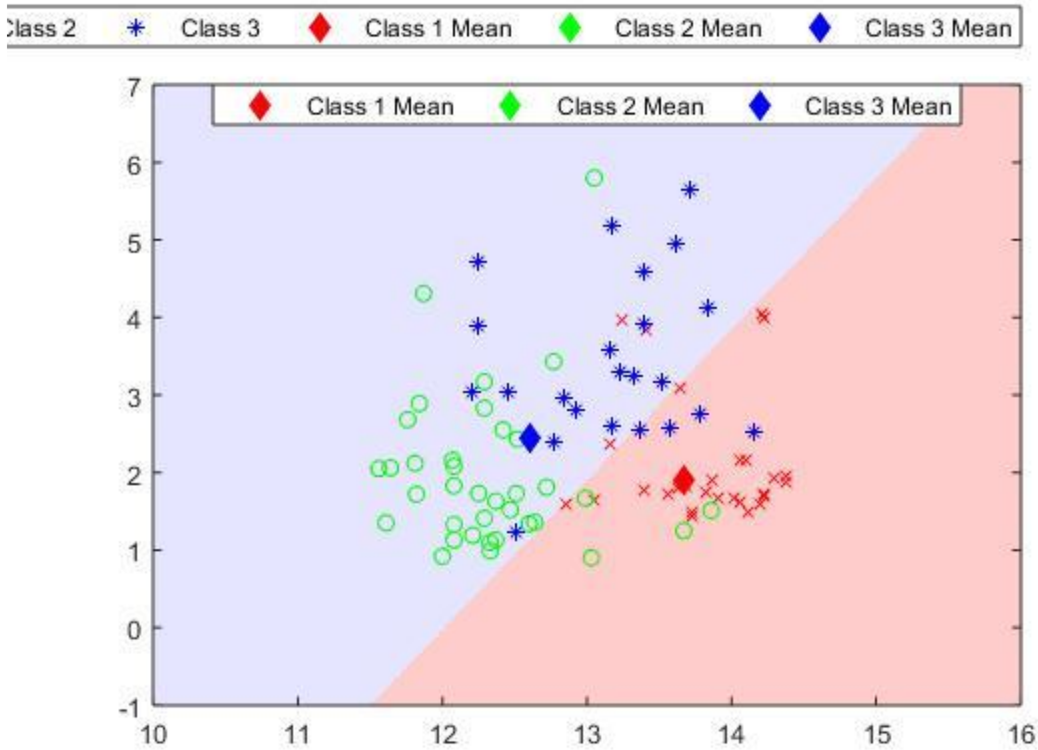

Fig 4. Testing Plot- Class1 vs (2 and 3)

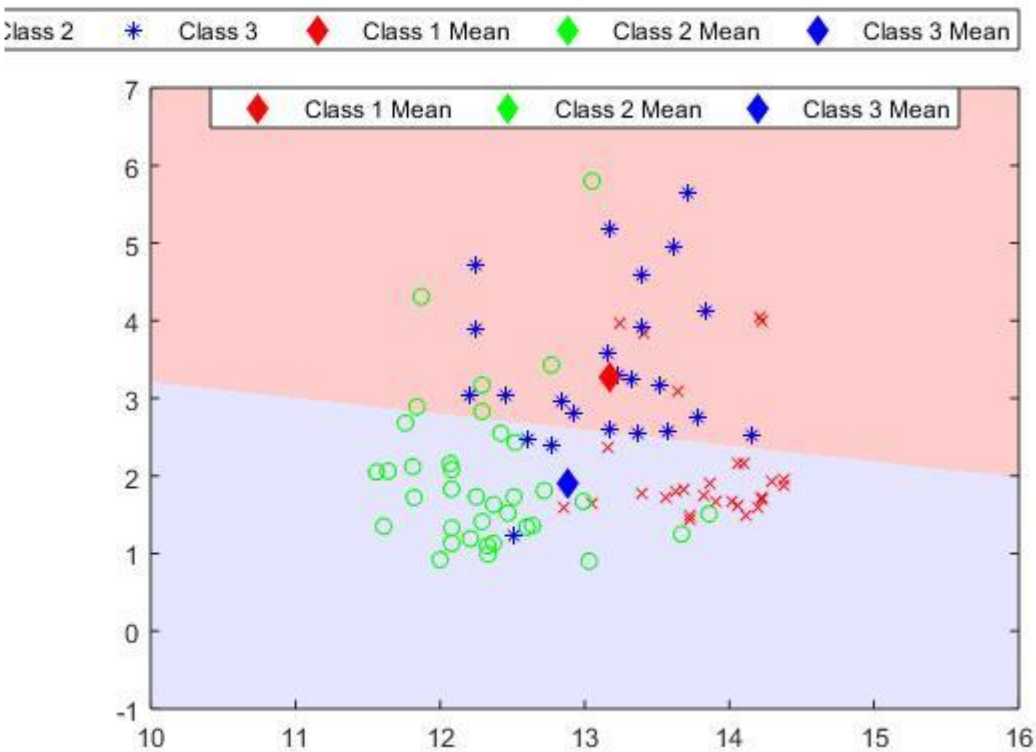

Fig 5. Testing Plot- Class 3 vs (1 and 2)

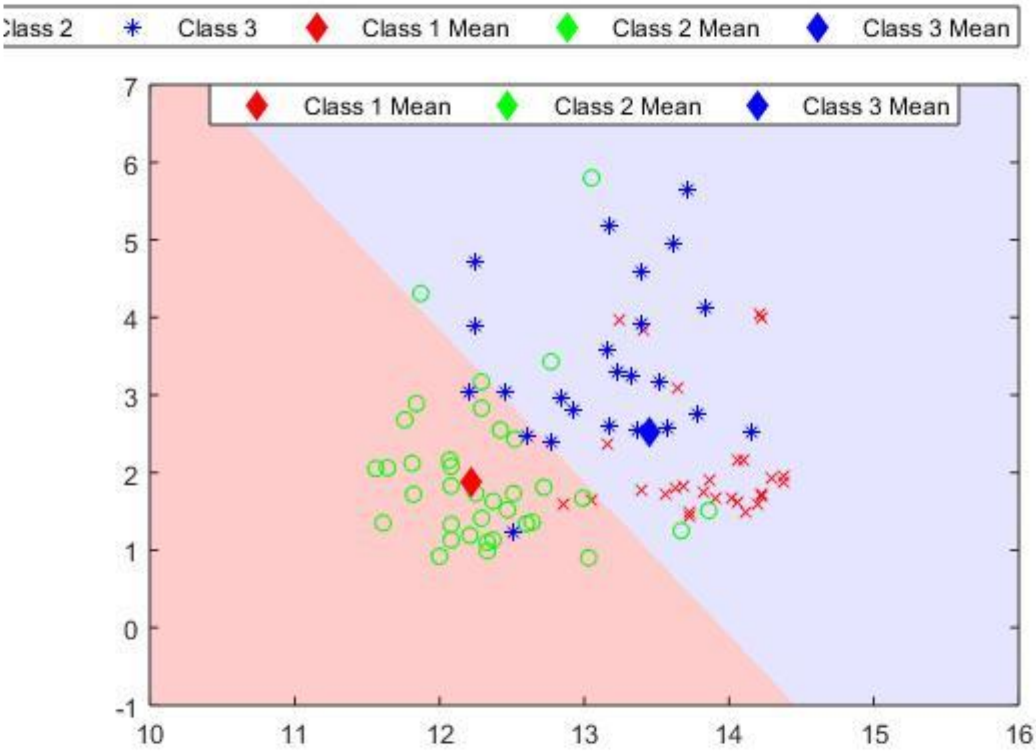

Fig 6. Testing Plot- Class 2 vs (1 and 3)

Fig 1,2 and 3 shows training data set graphs. We can see that decision boundary plotted say in fig 1, clearly shows that decision line separates class 1 from class 2 and 3. Similarly goes for Fig 2 and 3.

For testing data, I have used the same Sample Mean which is a 2*2 matrix in each of the 3 cases to implement. Passing testing feature data, label of training data and Sample Mean gives me the same output which gives decision boundary lines in One vs Rest Approach.

Code Approach:

I have extracted feature 1and 2 who belongs to class 1, stored it in a variable and calculated mean for the same. For class 2 and 3, I have extracted feature data set combined, then going forward to calculate mean for combined data set and hence making 2*2 sample mean matrix.

Part C

Combing all the 3 plots of testing into 1 and then applying One vs Rest decision rule, we get 3 regions say Γ1, Γ2 and Γ3.

The graphs of the training and testing are as follows:

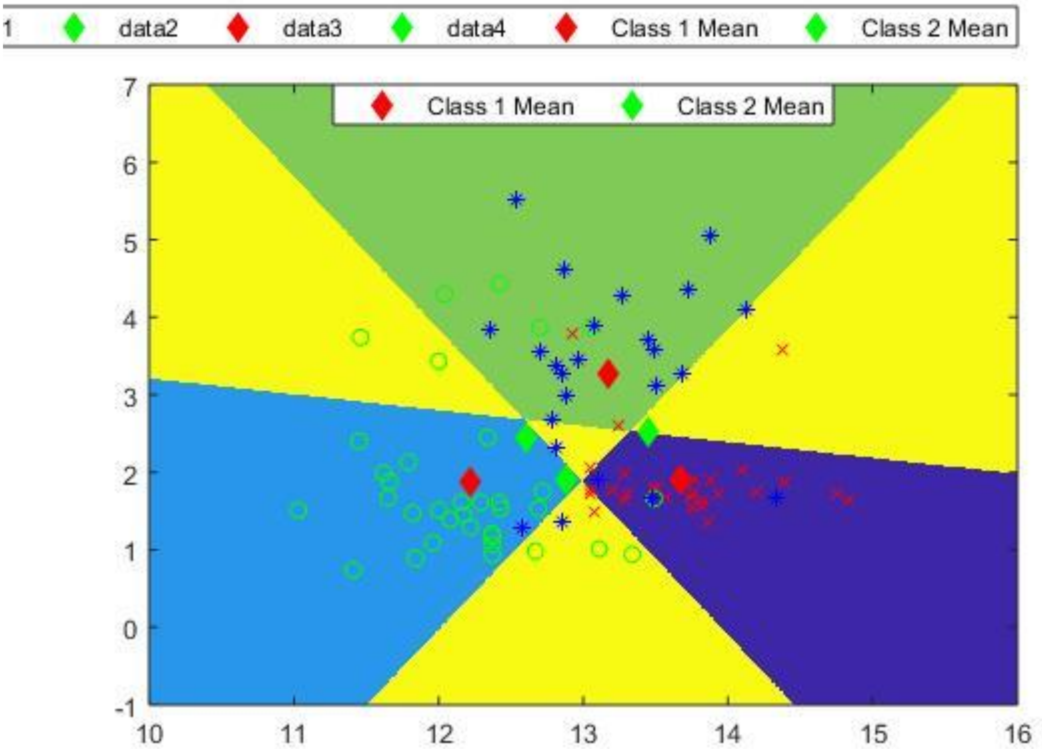

Fig 7. Training Plot- One vs Rest
(Final Regions)

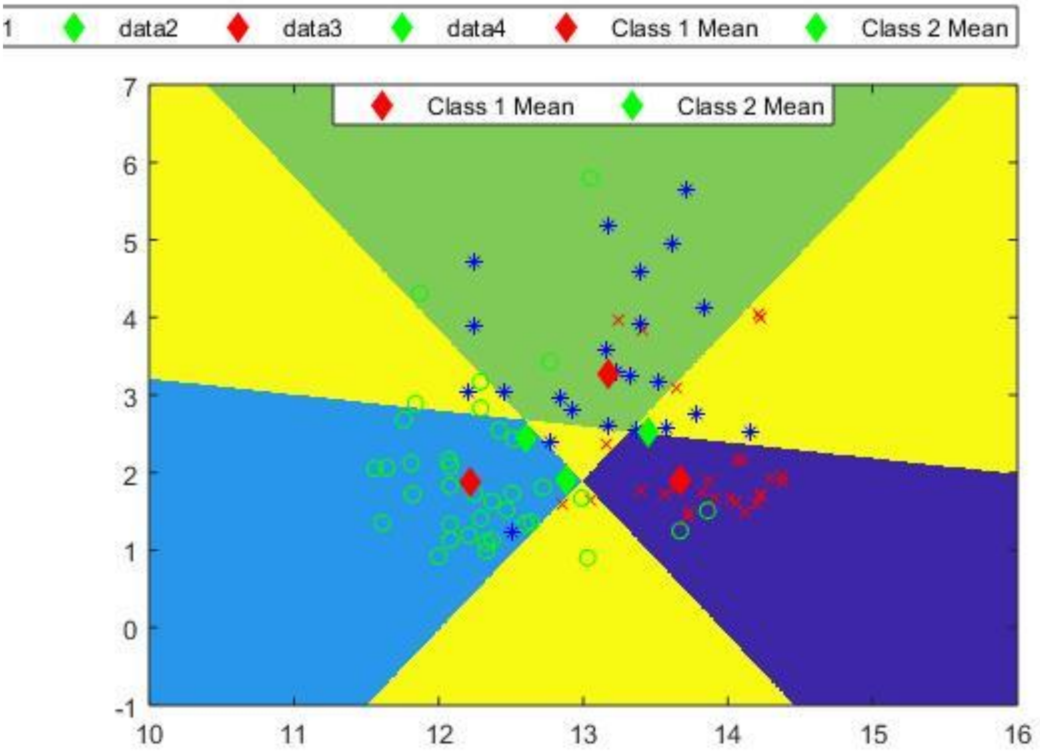

Fig 8. Testing Plot- One vs Rest
(Final Regions)

Regions:

Yellow region- Indeterminate Region

Dark Blue Region- Class 1

Green Region- Class 3

Light Blue Region- Class 2

Part A

Classification Accuracy- It is defined as the number of correctly classified points divided by total number of datapoints.

For training data, we get Classification Accuracy= 74.17%

For testing data, we get Classification Accuracy= 71.53%

Classification Accuracy is calculated as follows:

I have taken a count of all the points that lie in $\Gamma 1$, $\Gamma 2$ and $\Gamma 3$. These points are the correctly classified points.

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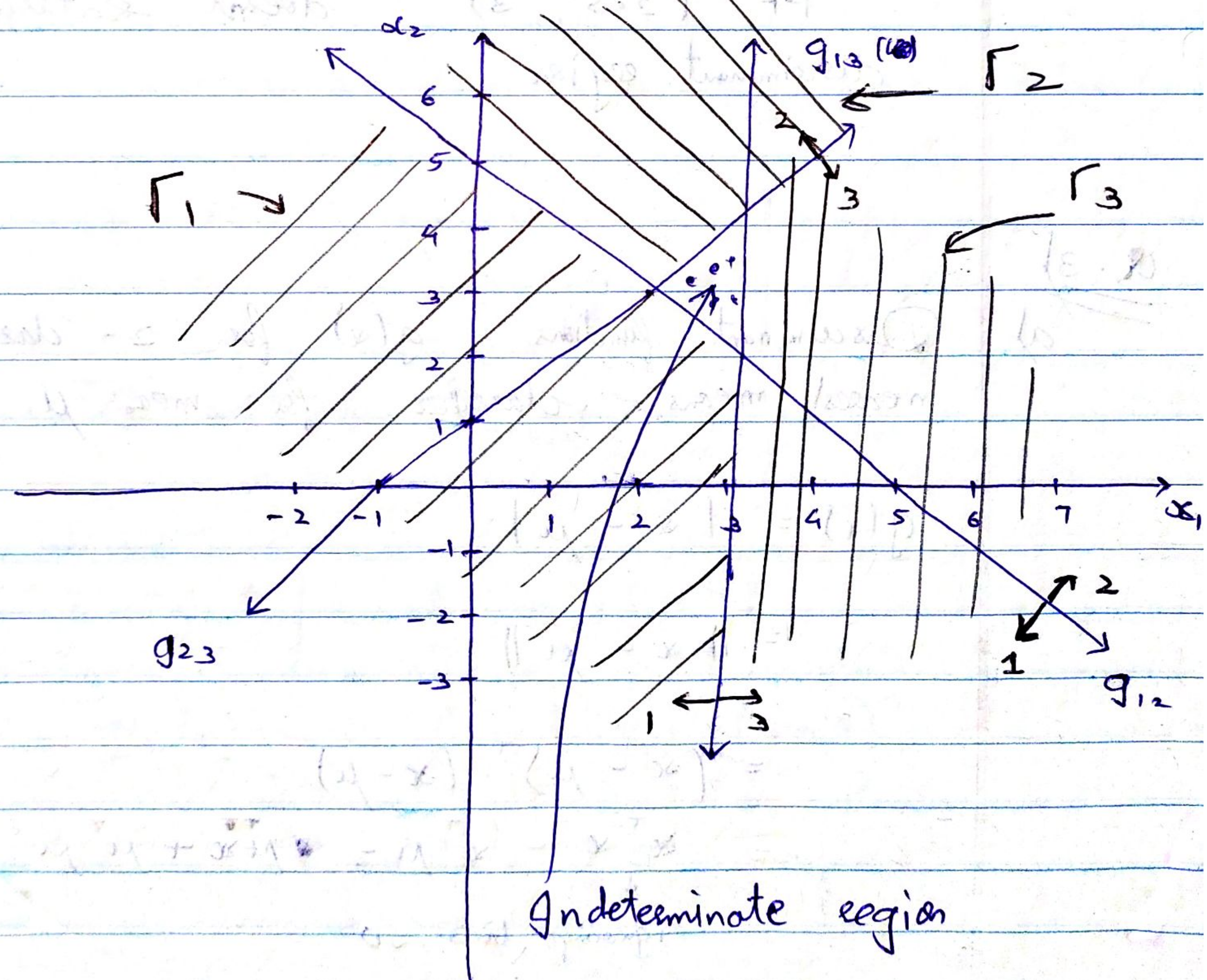
EE 559 HW # 2

Q.1)

$g_{12}(\underline{x}) = -x_1 - x_2 + 5$
$g_{13}(\underline{x}) = -x_1 + 3$
$g_{23}(\underline{x}) = -x_1 + x_2 - 1$

Decision Rule.

$x \in S_k$ iff $g_{kj}(x) > 0 \quad \forall j \neq k$.



Indeterminate region $\rightarrow$ $\underline{\underline{(2.5, 3)}}$

$(0, 0) \rightarrow \Gamma_1$ (Class 1)

$(4, 1) \rightarrow$ On the decision boundary of g_{12}
Γ_3 (Class 3)

$(1, 5) \rightarrow \Gamma_2$ (Class 2)

Indeterminate region.
pt $(2.5, 3)$ doesn't satisfy the discriminant region

Q. 3)

a) Discriminant functions $g(x)$ for 2 - class nearest means classifier, for mean μ_1 & μ_2

$$g(x) = |x - \mu|$$

$$= \| x - \mu \|$$

$$= (x - \mu)^T (x - \mu)$$

$$= x^T x - x^T \mu - \mu^T x + \mu^T \mu$$

ignoring bias ≈ 0

$$= - x^T \mu - x^T \mu + \mu^T \mu$$

$$= - 2 x^T \mu + \mu^T \mu$$

$$= x^T \mu - \frac{\mu^T \mu}{2}$$

$$g_1(x) = x^T \mu_1 - \frac{\mu_1^T \mu_1}{2}$$

$$g_2(x) = x^T \mu_2 - \frac{\mu_2^T \mu_2}{2}$$

Now $g_{12}(x) = g_1(x) - g_2(x)$

$$= x^T \mu_1 - \frac{\mu_1^T \mu_1}{2} - x^T \mu_2 + \frac{\mu_2^T \mu_2}{2}$$

$$= x^T \mu_1 - x^T \mu_2 - \left(\frac{\mu_1^T \mu_1}{2} - \frac{\mu_2^T \mu_2}{2} \right)$$

$$\underline{\underline{g_{12}(x) = x^T (\mu_1 - \mu_2) - \frac{1}{2} (\mu_1^T \mu_1 - \mu_2^T \mu_2)}}$$

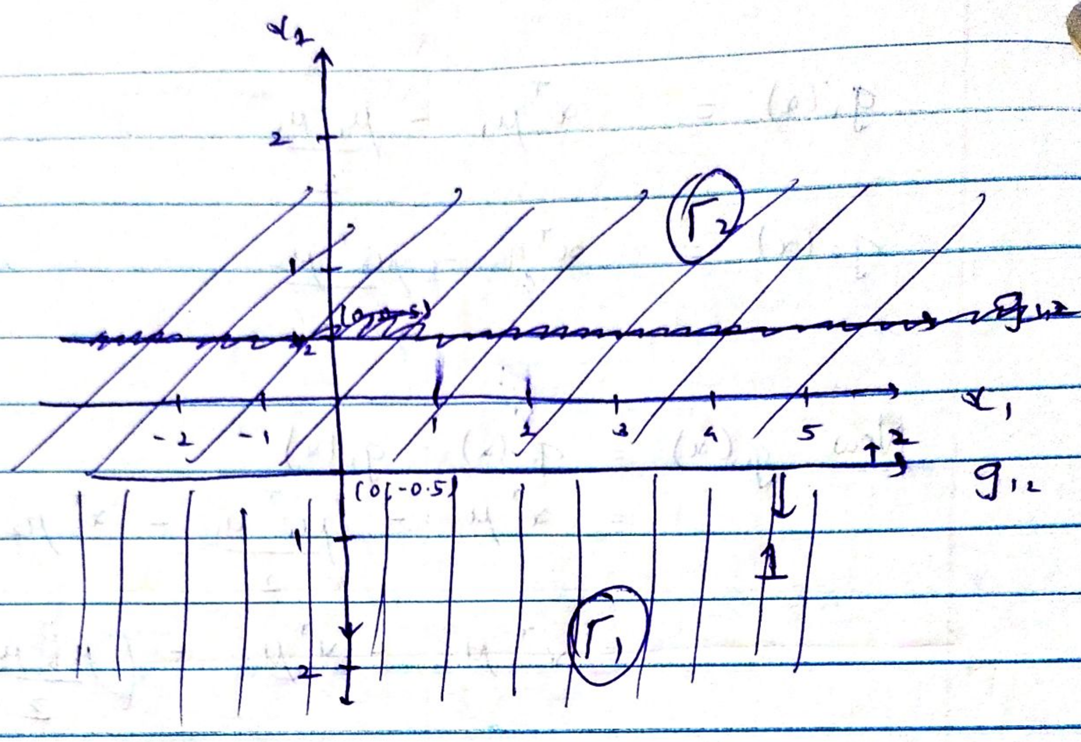
The classification <u>is linear.</u>

b) $\mu_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ & $\mu_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\rightarrow$ $g_{12}(x) = x^T \begin{bmatrix} 0 \\ -3 \end{bmatrix} - \frac{1}{2}(4 - 1)$

$$= (x_1 \; x_2) \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \frac{1}{2}(3)$$

$$g_{12}(x) = -3x_2 - \frac{3}{2}$$



c) Continuing from part a, we get.

$$g_1(x) = x^T \mu_1 - \frac{\mu_1^T \mu_1}{2}$$

$$g_2(x) = x^T \mu_2 - \frac{\mu_2^T \mu_2}{2}$$

$$g_3(x) = x^T \mu_3 - \frac{\mu_3^T \mu_3}{2}$$

The classifier is linear.

d)

$$\mu_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mu_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$g_{12} = g_1(x) - g_2(x)$$

$$= x^T(\mu_1 - \mu_2) - \frac{1}{2}(\mu_1^T \mu_1 - \mu_2^T \mu_2)$$

Sub μ_1 & μ_2

$$g_{12} = x^T \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \frac{1}{2}(3)$$

$$= (x_1 \quad x_2) \begin{pmatrix} 0 \\ -3 \end{pmatrix} - \frac{3}{2}$$

$$= \underline{-3x_2 - \frac{3}{2}}$$

$$g_{13} = x^T(\mu_1 - \mu_3) - \frac{1}{2}(\mu_1^T \mu_1 - \mu_3^T \mu_3)$$

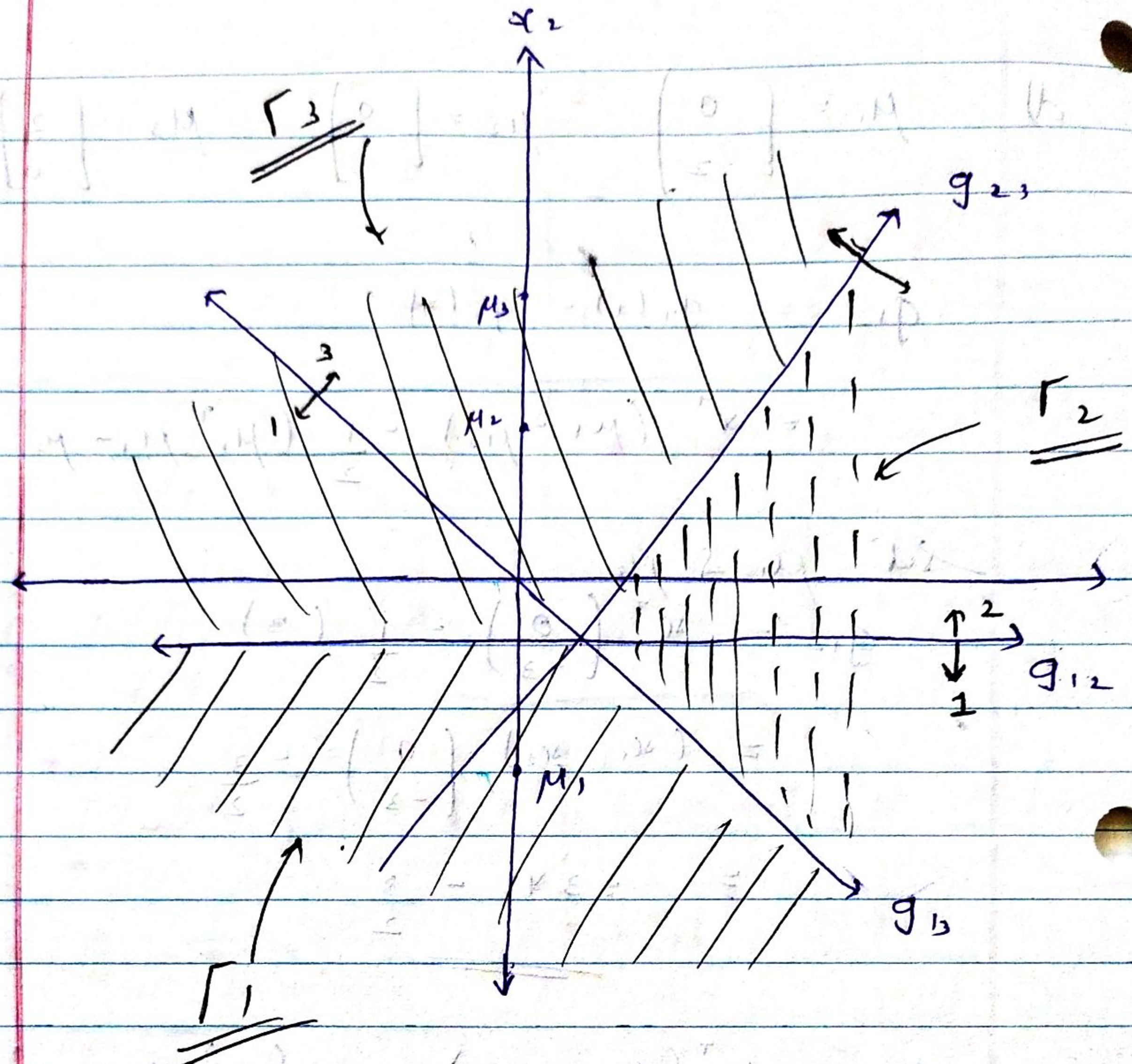
$$= (x_1, x_2) \begin{pmatrix} -2 \\ -2 \end{pmatrix} - \frac{1}{2}(4 - 4)$$

$$g_{13} = \underline{-2x_1 - 2x_2} + 0$$

$$g_{23} = x^T(\mu_2 - \mu_3) - \frac{1}{2}(\mu_2^T \mu_2 - \mu_3^T \mu_3)$$

$$= (x_1, x_2) \begin{pmatrix} -2 \\ 1 \end{pmatrix} - \frac{1}{2}(1 - 4)$$

$$= \underline{-2x_1 + x_2} + \frac{3}{2}$$



x_2
Γ_3
g_{23}
μ_3
3
1
μ_2
Γ_2
x_1
2
g_{12}
1
μ_1
g_{13}
Γ_1