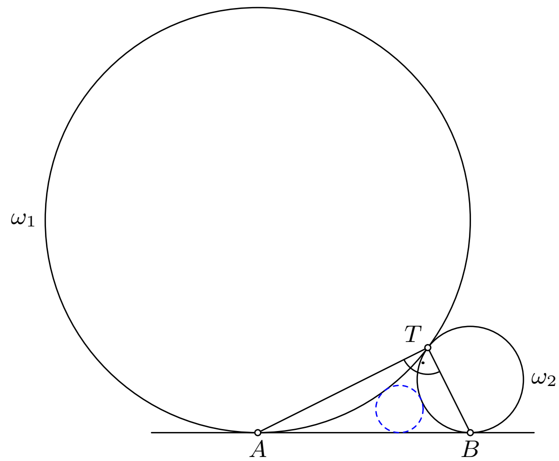
1. (free points!) Circles ω_1, ω_2 are externally tangent at T. Points A and B are the touchpoints of a common tangent line k, as in the picture below. Denote by r_1, r_2 the radii of ω_1, ω_2 respectively.
   (a) Calculate $|AB|$.
   (b) Prove that $\sphericalangle ATB = 90°$.
   (c) Let r denote the radius of a circle ω, which is externally tangent to both ω_1, ω_2 and to line k. Find r.



Rys. 1

2. Circles ω_1, ω_2 are internally tangent at T. A line through T meets ω_1, ω_2 at A, B respectively. Prove that the tangents to ω_1, ω_2 at A and B are parallel.

3. Points D, E are chosen on sides of AC and BC of a triangle ABC so that the line DE is parallel to the tangent line to the circumcircle of ABC at C. Prove that $ABDE$ is cyclic.

   Remark: We then say that DE is **antiparallel** to AB with respect to AC and BC.

4. Circles ω_1, ω_2 intersect at A and B. Choose a point C on ω_1 different from A and B. Lines CA i CB intersect ω_2 at P, Q respectively. Prove that PQ is parallel to the tangent line to ω_1 at C.

5. Let AD and BE be the altitudes of a triangle ABC. Point M is the midpoint of AB. Prove that the lines MD and ME are tangent to the circumcircle of CDE.

6. Let $ABCDE$ be a convex pentagon in which

$$\sphericalangle BAC + \sphericalangle ADB = \sphericalangle AEC\,.$$

   Prove that the circumcircles of ABD and ACE are tangent.

7. Two circles ω_1, ω_2 intersect at K, M. Let A and B denote the touchpoints of a common tangent line to ω_1, ω_2. Show that

$$\sphericalangle AMB + \sphericalangle AKB = 180°\,.$$

8. Let ABC be a triangle. The bisector of $\sphericalangle BAC$ meets the side BC at point D and meets the circumcircle of triangle ABC at point E. The line through B that is parallel to line CE meets side AC at point F.

(a) Prove that $ABDF$ is cyclic.

(b) Prove that line BE is tangent to the circumcircle of triangle ADF at point B.

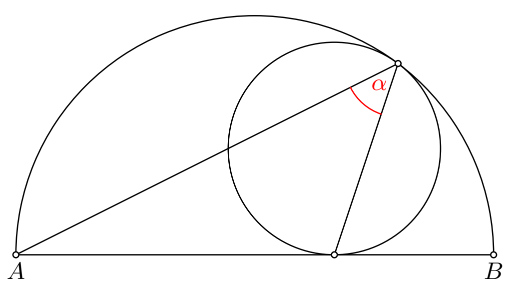
9. ★ Two circles o_1, o_2 are externally tangent at D. A tangent line to o_1 at A meets the circle o_2 at two different points B and C. Prove that A is equidistant to BD i CD.

10. ★ Two circles Ω, ω are internally tangent at T. A segment AB of Ω is tangent to ω at C. Let line CT meet Ω at two different points T, M. Prove that M is the midpoints of arc AB of the circle Ω.

11. ★ Circles ω_1, ω_2 are internally tangent at T. A line meets ω_1 at points A and B and meets ω_2 at C and D. Prove that
$$\sphericalangle ATC = \sphericalangle BTD\,.$$

12. Let $ABCD$ be square. Let P lie inside $ABCD$. Prove that if
$$\sphericalangle CAP = \sphericalangle DCP = 15^\circ$$
then triangle ABP is equilateral.

13. ★ A circle ω is tangent to a semicircle with diameter AB (see the picture below). Find α.



Rys. 2