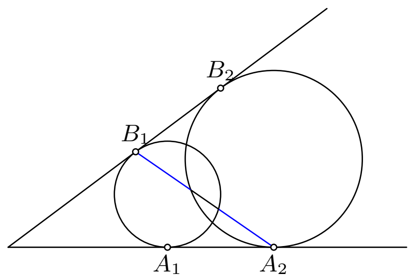
1. Circles o_1, o_2 intersect at A and B. Line k is a common tangent line to circles o_1 and o_2. Let C and D denote the points of tangency. Prove that the midpoint of CD lies on AB.

2. Let ω_1, ω be two different circles. Lines k and l are externally tangent to ω_1, ω_2 (see the picture below). Prove that line B_1A_2 cuts on ω_1 and ω_2 equal chords.



Rys. 1

3. Let ABC be an equilateral triangle. A circle ω meets lines AB, BC, CA in points K, L; M, N; P, Q respectively. Points K, L, M, N, P, Q lie on ω in this order. Show that

$$AK + BM + CP = AQ + BL + CN\,.$$

4. Let ABC ba a triangle with $\sphericalangle BAC = 2 \cdot \sphericalangle ABC$. Prove that

$$BC^2 = AC^2 + AC \cdot AB\,.$$

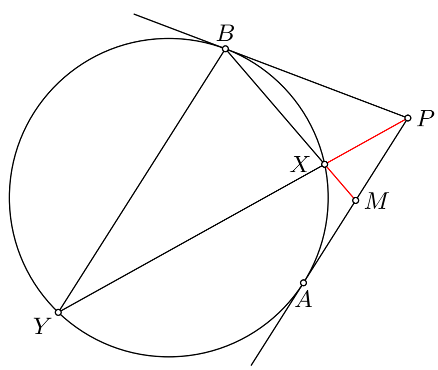
5. ★ Points A and B lie on circle ω. Tangents to circle ω at A and B meet at P. Let M be the midpoint of AP. Line MB meets ω at X (see the picture below).

   (a) Prove that

   $$PX = 2 \cdot XM\,.$$

   (b) Let PX meet ω at Y. Show that

   $$BY \parallel PA\,.$$



Rys. 2

6. ★ Point P lies inside circle ω. A line through P meets ω at X, Y. Denote by x, y the distances from P to the tangent lines to ω at X i Y. Prove that the expression

$$\frac{1}{x} + \frac{1}{y}$$

is independent of the choice of line through P.

7. ★ Let ω be a circle let k be a line disjoint from ω. From varying point X on k construct tangent lines touching ω at Y and Z. Prove that for all points X constructed lines YZ are concurrent.

   Remark: The point of concurrrency is called the **pole** of k with respect to ω.

8. ★ Let ABC be a triangle. Point F lies on AB in such a way that CF is an altitude of ABC. Points D and E lie on BC and AC respectively so that $DF \perp BC$ and $EF \perp AC$. Line DE meets the circumcircle of ABC at X and Y. Line CF meets the circumcircle of ABC at C and Z. Prove that F is the incenter of triangle XYZ.

9. ★ (Euler's theorem) Let ABC be a triangle. Let O denote the circumcenter and let R denote the circumradius of ABC. Let I denote the incenter and r denote the inradius of ABC. Let $d \coloneqq OI$. Prove that
$$\frac{1}{R+d} + \frac{1}{R-d} = \frac{1}{r}.$$
   Deduce that
$$R \geq 2r$$
   with equality if and only if ABC is an equilateral triangle.

10. ★ Let ABC be a triangle with $AB > AC$. Its circumcircle is Γ and its incentre is I. Let D be the contact point of the incircle of ABC with BC. Let K be the point on Γ such that $\sphericalangle AKI = 90°$. Prove that AI and KD meet on Γ.