Solution 2.1

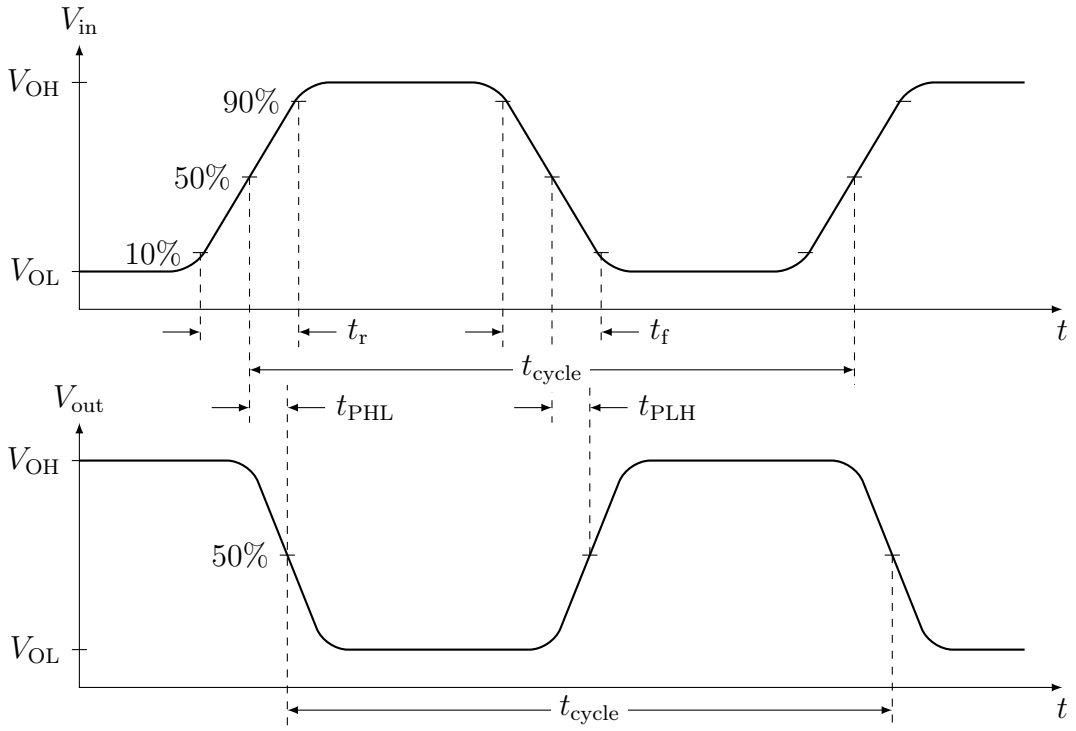

Figure 2.2: Definition of inverter delay

The time interval between the 50% values of successive voltages results in the delay t_{PHL} and t_{PLH} at the output of the inverter for rising and falling edges at the input of the inverter respectively. The average delay time is $t_{\text{P}} = (t_{\text{PHL}} + t_{\text{PLH}})/2$. For symmetrical inverters $t_{\text{PHL}} = t_{\text{PLH}}$ applies. Each inverter delays a signal by t_{P}.

a) Delay from A to B:

$$t_{\text{P}} = n \cdot t_0 = 5 \cdot 2\text{ns} = 10\text{ns}$$

b) Period of the ring oscillator:

$$T = 2 \cdot n \cdot t_0 = 2 \cdot 5 \cdot 2\text{ns} = 20\text{ns}$$

Frequency of the ring oscillator:

$$f = 1/T = 1/20\text{ns} = 50\text{MHz}$$

Solution 2.2

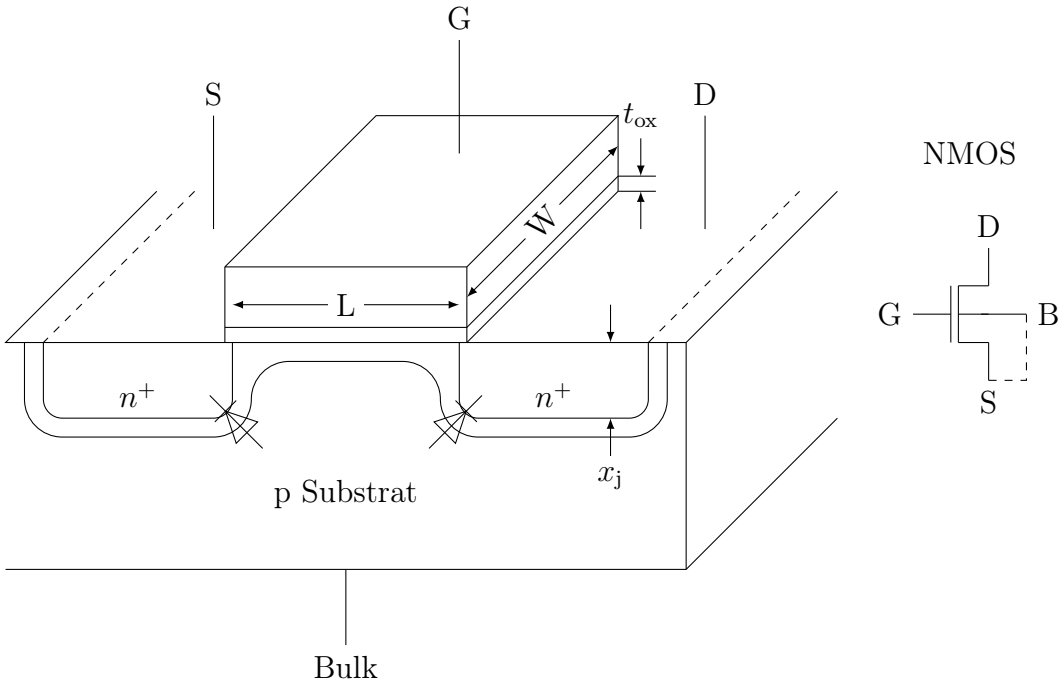

Figure 2.3: NMOS transistor structure and symbol

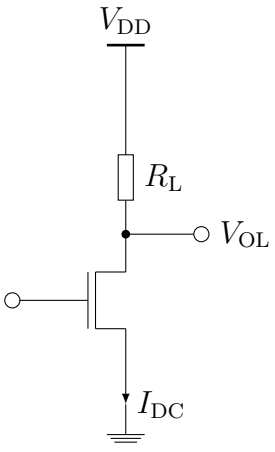

Figure 2.4: Cross current $I_{DC} = (V_{DD} - V_{OL})/R_L$

Solution 2.3

a) Φ_F: < 0 for p-typ semiconductor, > 0 for n-typ semiconductor:

$$2|\Phi_\mathrm{Fp}| = \frac{2\mathrm{kT}}{\mathrm{q}}\left|\ln\frac{n_\mathrm{i}}{p}\right| = 2(0.026\mathrm{V})\left|\ln\frac{1.45\cdot10^{10}\mathrm{cm}^{-3}}{3\cdot10^{17}\mathrm{cm}^{-3}}\right| = 0.88\mathrm{V}$$

b) Width X_d and negative charge Q_B0 of the space charge region:

$$X_\mathrm{d} = \left(\frac{2\epsilon_\mathrm{si}|\Phi_\mathrm{s}-\Phi_\mathrm{F}|}{\mathrm{q}N_\mathrm{A}}\right)^{1/2} = \sqrt{\frac{2(11.7)8.85\cdot10^{-14}\mathrm{AsV}^{-1}\mathrm{cm}^{-1}(0.88\mathrm{V})}{(1.6\cdot10^{-19}\mathrm{As})(3\cdot10^{17}\mathrm{cm}^{-3})}} = 60\mathrm{nm}$$

$$\begin{aligned}
Q_\mathrm{B0} &= -\sqrt{(2\mathrm{q}N_\mathrm{A}\epsilon_\mathrm{si})|-2\Phi_\mathrm{F}|} \\
&= -\sqrt{2(1.6\cdot10^{-19}\mathrm{As})(3\cdot10^{17}\mathrm{cm}^{-3})(1.0\cdot10^{-12}\mathrm{AsV}^{-1}\mathrm{cm}^{-1})|-0.88\mathrm{V}|} \\
&\approx -3\cdot10^{-7}\mathrm{C/cm}^2
\end{aligned}$$

c) Oxide capacity C_ox and body factor γ:

$$C_\mathrm{ox} = \frac{4\epsilon_\mathrm{o}}{t_\mathrm{ox}} = \frac{4(8.85\cdot10^{-14}\mathrm{AsV}^{-1}\mathrm{cm}^{-1})}{22\text{Å}} = 1.6\cdot10^{-6}\mathrm{F/cm}^2$$

$$\begin{aligned}
\gamma &= \frac{1}{C_\mathrm{ox}}\sqrt{2\mathrm{q}\epsilon_\mathrm{si}N_\mathrm{A}} \\
&= \frac{\sqrt{2(1.6\cdot10^{-19}\mathrm{As})11.7(8.85\cdot10^{-14}\mathrm{AsV}^{-1}\mathrm{cm}^{-1})(3\cdot10^{17}\mathrm{cm}^{-3})}}{1.6\cdot10^{-6}\mathrm{F/cm}^2} \\
&\approx 0.2\mathrm{V}^{1/2}
\end{aligned}$$

d) Threshold voltage V_T0:

$$\Phi_\mathrm{GC} = \Phi_\mathrm{Fp} - \Phi_\mathrm{G(Gate)} = -0.44\mathrm{V} - 0.55\mathrm{V} = -0.99\mathrm{V}$$

$$\frac{Q_\mathrm{B0}}{C_\mathrm{ox}} = \frac{-3\cdot10^{-7}\mathrm{C/cm}^2}{1.6\cdot10^{-6}\mathrm{F/cm}^2} = -0.188\mathrm{V}$$

$$\frac{Q_\mathrm{ox}}{C_\mathrm{ox}} = \frac{(2\cdot10^{10}\mathrm{cm}^{-2})(1.6\cdot10^{-19}\mathrm{As})}{1.6\cdot10^{-6}\mathrm{F/cm}^2} = 0.002\mathrm{V}$$

$$\begin{aligned}
V_\mathrm{T0} &= \Phi_\mathrm{GC} - 2\Phi_\mathrm{FP} - \frac{Q_\mathrm{B0}}{C_\mathrm{ox}} - \frac{Q_\mathrm{ox}}{C_\mathrm{ox}} \\
&= -0.99\mathrm{V} - (-0.88\mathrm{V}) - (-0.188\mathrm{V}) - 0.002\mathrm{V} \\
&= 0.08\mathrm{V}
\end{aligned}$$

In the above solution we assume that the n^+ gate doping is so high that the Fermi level in the gate coincides with the conduction band. An electrostatic potential of $\Phi_{\mathrm{G(Gate)}} = 0.55\mathrm{V}$ is implied. If the gate were p^+-doped, the value of V_{T0} would be 1.18V. Unfortunately, this value is above the desired target level. Therefore, the doping of the poly-gate must be n^+ to keep the value below the target level. For the same reason the poly-gate of p-channel transistors is doped with p^+. Of course, the above calculated values are not really applicable for NMOS devices, but only an approximation model, which should represent the trend. We need a value closer to $V_{\mathrm{T0}} = 0.4\mathrm{V}$.

e) V_{T} is shifted by $\dfrac{Q_{\mathrm{I}}}{C_{\mathrm{ox}}} = \dfrac{\mathrm{q}N_{\mathrm{I}}}{C_{\mathrm{ox}}}$.

$$N_{\mathrm{I}} = \frac{Q_{\mathrm{I}}}{\mathrm{q}} = \frac{C_{\mathrm{ox}}\Delta V}{\mathrm{q}} = \frac{1.6 \cdot 10^{-6}\mathrm{F/cm^2}(0.4\mathrm{V} - (0.08\mathrm{V}))}{1.6 \cdot 10^{-19}\mathrm{As}} = 3.2 \cdot 10^{12}\frac{\mathrm{ions}}{\mathrm{cm^2}}$$