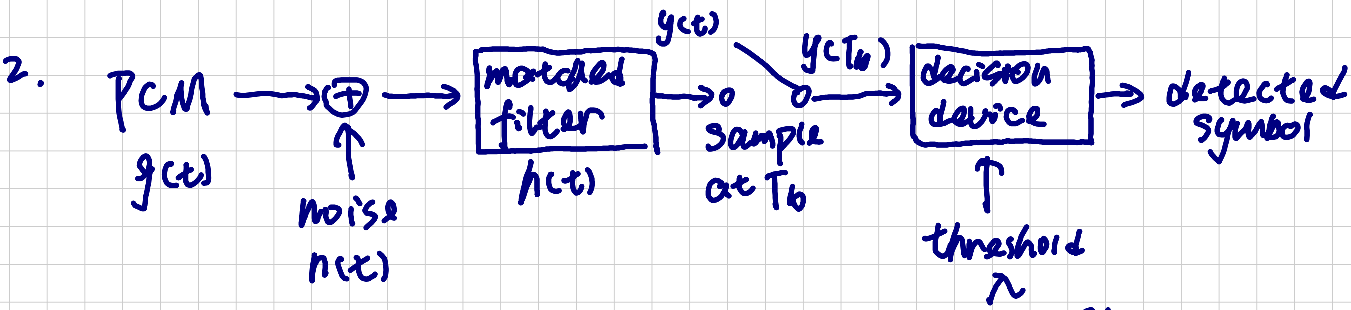
1. $P(X=0)=0.7,\ P(X=1)=0.3.$

$$\lambda^* = -\frac{\sigma^2}{A}\log\frac{P_1}{1-P_1}+\frac{A}{2}$$

$$= -\frac{9}{5}\log\frac{0.3}{0.7}+\frac{5}{2} = 4.025.$$

2.



$$y(t) = (g(t)+n(t)) * h(t) = \underbrace{\int_{-\infty}^{+\infty} g(\tau)h(T-\tau)d\tau}_{g_0(t)} + \underbrace{\int_{-\infty}^{+\infty} n(\tau)h(T-\tau)d\tau}_{n_0(t)}$$

SNR at $t=T_b$, $\eta = \dfrac{|g_0(T_b)|^2}{E[n_0^2(T_b)]}$ ← instantaneous signal power / ← average noise power

signal:

$$g_0(t) = \int_{-\infty}^{+\infty} H(f)G(f)e^{j2\pi ft}df$$

$$\Rightarrow |g_0(T_b)|^2 = \left|\int_{-\infty}^{+\infty} H(f)G(f)e^{j2\pi fT_b}df\right|^2$$

$X(t) \rightarrow \boxed{\text{Filter}}_{h(t)} \rightarrow Y(t)$

if $X(t)$ is WSS and filter is linear,

$S_Y(f) = |H(f)|^2 S_X(f)$

noise:

$$S_{N_0}(f) = S_N(f)\,S_H(f) = S_N(f)|H(f)|^2$$

$$\Rightarrow E[n^2(t)] = \int_{-\infty}^{+\infty} S_{N_0}(f)df = \int_{-\infty}^{+\infty} S_N(f)|H(f)|^2df$$

$$\therefore \eta = \frac{\left|\int_{-\infty}^{+\infty} H(f)G(f)e^{j2\pi fT_b}df\right|^2}{\int_{-\infty}^{+\infty} S_N(f)|H(f)|^2df}$$

$$\left|\int_{-\infty}^{+\infty}\phi_1(x)\phi_2^*(x)dx\right|^2 \le \int_{-\infty}^{+\infty}|\phi_1(x)|^2dx \int_{-\infty}^{+\infty}|\phi_2(x)|^2dx$$

with equality iff

$\phi_1(x) = k\phi_2(x)$

Let $X(f) = H(f)\sqrt{S_N(f)}$

$$Y^*(f) = \frac{G(f)e^{j2\pi fT_b}}{\sqrt{S_N(f)}}$$

$$\therefore \eta = \frac{\left|\int_{-\infty}^{+\infty} X(f) Y^*(f) df\right|^2}{\int_{-\infty}^{+\infty} |X(f)|^2 df}$$

$$\overset{C-S}{\leq} \frac{\int_{-\infty}^{+\infty} |X(f)|^2 df \int_{-\infty}^{+\infty} |Y(f)|^2 df}{\int_{-\infty}^{+\infty} |X(f)|^2 df}$$

$$= \int_{-\infty}^{+\infty} \frac{|G(f)|^2}{S_N(f)} df$$

with equality iff $X(f) = kY(f)$, namely

$$H(f)\sqrt{S_N(f)} = k\frac{G(-f)e^{-j2\pi f T_0}}{\sqrt{S_N(f)}}$$

$$\Rightarrow H(f) = k\frac{G(-f)e^{-j2\pi f T_0}}{S_N(f)}$$

$$G(f) = \frac{1}{2\pi}\int_0^{\infty} e^{-\alpha t} e^{-j2\pi f t} dt = -\frac{1}{\alpha + 2\pi f} e^{-(\alpha + 2\pi f)t}\Big|_0^{\infty} = \frac{1}{\alpha + 2\pi f}$$

$$\therefore H(f) = k\frac{\frac{1}{\alpha - 2\pi f} e^{-j2\pi f T_0}}{\frac{1}{\alpha^2 + (2\pi f)^2}} = k(\alpha + j2\pi f) e^{-j2\pi f T_0}$$