1. FM signal: $x(t) = A\cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$

$= 10\cos(10000\pi t + 4000\pi \int_0^t m(\tau)d\tau)$

$\therefore \int_0^t m(\tau)d\tau = \frac{5}{4}t \Rightarrow m(t) = \frac{5}{4}$.

2. $SNR_{FM} = 3\beta^2 \frac{P}{m_p^2} SNR_{baseband}$

$m(t) = A_m \cos(2\pi f_m t)$

a) $m_p = A_m$. $P = \frac{1}{2}A_m^2 \Rightarrow SNR_{FM} = \frac{3}{2}\beta^2 SNR_{baseband}$

b) $SNR_{env} = \frac{M^2}{2+M^2} SNR_{baseband} \overset{M=1}{=\!=} \frac{1}{3} SNR_{baseband}$

$\frac{3}{2}\beta^2 > \frac{1}{3} \Rightarrow \beta > \frac{\sqrt{2}}{3}$

3. $P = \sigma_m^2$. $m_p = 4\sigma_m$.

$SNR_{FM} = 3\beta^2 \frac{P}{m_p^2} SNR_{baseband}$

$= \frac{3}{16}\beta^2 SNR_{baseband}$

4. $f_s \geq 2f_{max} = 20\,kHz$.

bit rate $= 16 \times 20 = 320\,kbps$

$nbits = 60^2 \times 320 = 1152\,Mbits = 144\,Mbytes$.

5. $\sigma_q^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q_n^2 \frac{1}{\Delta} dq_n = \frac{\Delta^2}{12}$ Uniform quantizer has $MSE = \frac{\Delta^2}{12}$.

a) need to quantize $\pm 4\sigma$ into 2^n levels

$$\Delta = \frac{8\sigma}{2^n}$$

$$\therefore P_N = \frac{1}{12}\left(\frac{8\sigma}{2^n}\right)^2 = \frac{16}{3}\frac{\sigma^2}{2^{2n}}$$

$$P_S = \sigma^2 \quad \therefore SNR = \frac{P_S}{P_N} = \frac{3}{16}2^{2n}$$

$$SNR_{dB} = 10\log_{10}\frac{3}{16}2^{2n} = -7.3 + 6.02n$$

b) $P(|X| > 4\sigma) = 2\int_{4\sigma}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx = 2Q\left(\frac{4\sigma}{\sigma}\right) = 6.3 \times 10^{-5}$

6. Consider first quarter.

$$P_S = \frac{1}{\frac{T}{4}}\int_0^{\frac{T}{4}} t^2 dt = \frac{1}{3}$$

$$P_n = \frac{\Delta^2}{12} = \frac{1}{12}\left(\frac{2}{2^n}\right)^2 = \frac{2^{-2n}}{3}$$

$$\therefore SNR = \frac{P_S}{P_n} = 2^{2n}$$

$$SNR_{dB} = 10\log_{10} 2^{2n} = 6.02n$$