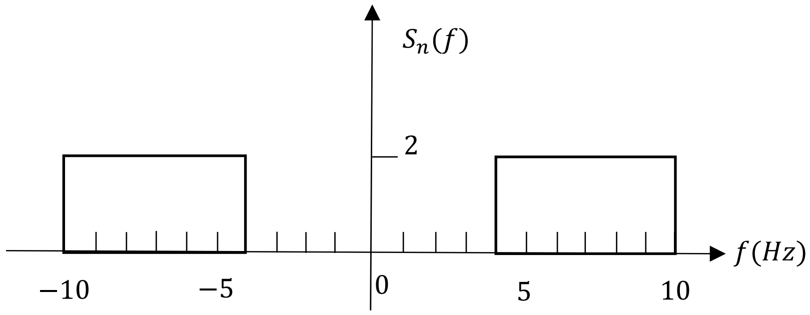
Baseband and Passband Signal and Noise Effects (Lectures 3 & 4)

1. **Consider a bandpass noise signal having the power spectral density shown below. Draw the power spectral density (PSD) of $n_I(t)$ if the center frequency is chosen as:**
   (a) $f_c = 7\ Hz$
   (b) $f_c = 5\ Hz$



2. **Let**

$$f_k(t) \triangleq \begin{cases} e^{-\frac{t}{k}}, & if\ t > 0, \\ 0, & if\ t = 0, \\ -e^{-\frac{t}{k}}, & if\ t < 0, \end{cases}$$

Find $F_k(f)$, the Fourier transform of $f_k(t)$. Letting $k \to \infty$, find the Fourier transform of function $sgn(t)$, defined as

$$sgn(t) \triangleq \begin{cases} 1, & if\ t > 0, \\ 0, & if\ t = 0, \\ -1, & if\ t < 0. \end{cases}$$

Using this, find the Fourier transform of unit step function

$$u(t) \triangleq \begin{cases} 1, & if\ t > 0, \\ 1/2, & if\ t = 0, \\ 0, & if\ t < 0. \end{cases}$$

3. **Hilbert transform of a signal $g(t)$ is defined as**

$$\hat{g}(t) = g(t) * \frac{1}{\pi t}$$

Using the result of the previous exercise, find $\hat{G}(f)$, the Fourier transform of $\hat{g}(t)$.

4. **Prove the following properties of Hilbert transforms:**
   a) **If $x(t) = x(-t)$, then $\hat{x}(t) = -\hat{x}(-t)$.**
   b) **If $x(t) = -x(-t)$, then $\hat{x}(t) = \hat{x}(-t)$.**
   c) **If $x(t) = \cos(2\pi f_0 t)$, then $\hat{x}(t) = \sin(2\ \pi f_0 t)$.**
   d) **If $x(t) = \sin(2\pi f_0 t)$, then $\hat{x}(t) = -\cos(2\ \pi f_0 t)$.**
   e) **$\hat{\hat{x}}(t) = -x(t)$.**
   f) **$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} \hat{x}^2(t)dt.$**
   g) **$\int_{-\infty}^{\infty} x(t)\hat{x}(t)dt = 0.$**