Digital modulation and demodulation (Lectures 10 & 11)

1. **(a) Consider a binary ASK modulated-carrier system, which employs coherent demodulation. Let the carrier amplitude at the detector input be 0.7 volts. Assume an additive white Gaussian noise channel with a standard deviation of 0.125 volts. If the binary source stream has equal probabilities of occurrence of a symbol 0 and a symbol 1, estimate the probability of detection error.**
   (b) If PSK was used instead, what is the probability of error?

2. **Consider the FSK system where symbol 0 and 1 are transmitted by frequency f_0 and f_1, respectively. The unmodulated carrier frequency is $f_c = (f_0 + f_1)/2$. In practice, $f_c T \gg 1$ where T is the symbol period. Define the frequency separation $\Delta f \triangleq |f_1 - f_0|$. Larger Δf means larger bandwidth. FSK using $\Delta f = 1/2T$ is known as minimum-shift keying (MSK). Show that $\Delta f = 1/2T$ is the minimum separation so that the two sinusoids are orthogonal over one symbol period.**

3. **Consider a binary source alphabet where a symbol 0 is represented by 0 volts, and a symbol 1 is represented by 1 volt. Assume these symbols are transmitted over a baseband channel having uniformly distributed noise with a probability density function:**

$$p(n) = \begin{cases} \frac{1}{2}, & |n| < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Assume that the decision threshold T is within the range of 0 to 1 volt. If the symbols are equally likely, derive an expression for the probability of error.

4. **Show that 16 QAM can be represented as a superposition of two four-phase constant envelope signals where each component is amplified separately before summing, i.e.**

$$s(t) = G(A_n \cos 2\pi f_c t - B_n \sin 2\pi f_c t) + (C_n \cos 2\pi f_c t - D_n \sin 2\pi f_c t)$$

where $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, $\{D_n\}$ are statistically independent binary sequences with elements from the $\{+1, -1\}$ and G is the amplifier gain. Thus, show that the resulting signal is equivalent to

$$s(t) = I_n \cos 2\pi f_c t - Q_n \sin 2\pi f_c t$$

and determine I_n and Q_n in terms of A_n, B_n, C_n and D_n.