Questions and Answers for Lectures 1 and 2

1) I noticed that the order in the notes is different than that in the slides, in the sense that the first two lectures didn't cover everything that was in the notes, for example some parts about noise, will we cover those later on or should we study them on our own?

Yes. I will cover the noise part next week. But, noise can be regarded as a special random process, which is why it is placed here in the note. You can read this part this week or next week. In general, Lectures 1 and 2 are just overview of communication history and review of random process. We can only select some materials from the notes due to limited time. You are expected to study related materials in the notes.

2) I am not sure I understand the difference between a random variable and a random process? Doesn't a random variable also do this mapping?

They both are mapping from (random) sample space. Random variable is mapping from a (random) sample into a real number while random process into a real function of time. For example, if you toss a coin, you get one of two samples, H (head) or T (tail), with Prob{H}=0.4 and Prob{T}=0.6. Your sample space is S={H,T}. If I have a map that maps T into 0 and H into 1, then you get a random variable, called X. You will have Pr{X=0}=0.4, Pr{X=1}=0.6. If you map H into sin(t) and T into cos(t), you get a random process, called X(t). For a given time, such as t=1, X(1) turns into a random variable, Prob{X(1)=sin(1)}=0.4 and Prob{X(1)=cos(1)}=0.6. If you fix the sample, such as H, then X(t) = sin(t), a deterministic function.

3) How exactly do we characterize the Gaussian process, apart from the fact that the distribution is normal?

For the Gaussian process, any order of probability distribution function is joint Gaussian/normal. Since joint Gaussian/normal distribution can be expressed by its average and correlation matrix, Gaussian/normal process has many special characteristics, which makes it very **abnormal**.

4) What do you mean by a complex-valued LTI system since we only deal with real systems?

When we study LTI, we have presumed that the system parameters (impulse response), input, and output can all be complex numbers/signals. While the random process is defined as a real function at the very beginning. But, we can easily extend to a complex random process, Z(t)=X(t)+jY(t) using two real random processes X(t) and Y(t). In many situations in communication systems, we need to address complex random processes. Therefore, we emphasize here that we are discussing a general (complex) LTI system with input and output that could be complex functions in general.

5) To find the spectral density power of the output signal, we have to do convolution twice right? But won't this be a too long approach? Isn't there a shorter way?

Yes. In the time domain, you need to do convolution twice, which is seldom used. Instead, you can use the frequency domain relationship. In that case, $S_o(\omega) = |H(\omega)|^2 S_i(\omega)$ or $S_o(2\,\pi f) = |H(2\,\pi f)|^2 S_i(2\,\pi f)$ where $S_o(2\,\pi f)$ and $S_i(2\,\pi f)$ are PSD of the output and input random processes, respectively, and $H(\omega)$ is the frequency response of the LTI system.