

Ex.

large-scale fading
(shadowing)

$$S_{dB} \sim N(0, \sigma_s^2) \\ = 10 \log_{10} S$$

$$M: \pi = 10 \log_{10} S \sim N(0, \sigma_s^2)$$

calculate $S = 10^{\frac{\pi}{10}}$ \rightarrow shadowing
realisation

multiple for $i = 1:1000$
realisation $x = \text{randn}(0, \sigma_s^2)$;
of $S[i] = 10^{\frac{x}{10}}$;
shadowing
end

Plot(S)

$$\sigma_s^2 = 8 \text{ dB} \quad \sigma_s^2 = 10^{\frac{8}{10}}$$

Ex 2:

$$h_i \sim \mathcal{CN}(0, 1)$$

↓ ↘
real imag

$$h_i = h_{\text{real}} + i h_{\text{imag}}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \mathcal{CN}(0, 1) & \mathcal{N}(0, \frac{1}{2}) & \mathcal{N}(0, \frac{1}{2}) \\ & = \frac{\mathcal{N}(0, 1)}{\sqrt{2}} & \end{array}$$

M:

for i = 1 : 1000

$$h_R = \frac{\mathcal{N}(0, 1)}{\sqrt{2}} \sim \frac{1}{\sqrt{2}} \times \text{randn}$$

$$h_I = \frac{\mathcal{N}(0, 1)}{\sqrt{2}}$$

$$h = h_R + i h_I$$

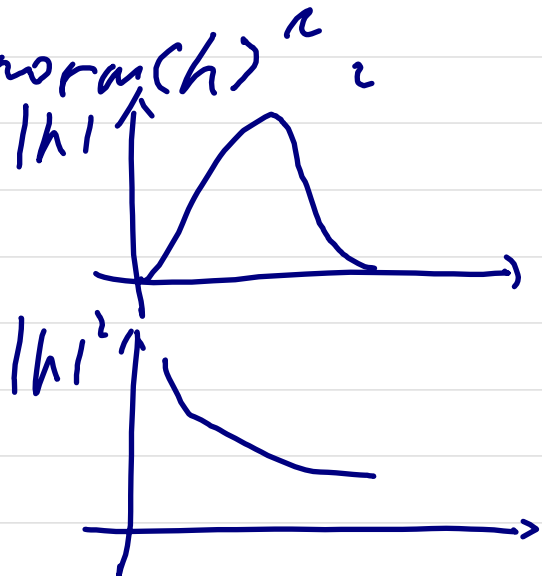
$$\text{norm}[i] = \text{abs}(h)$$

$$\text{square abs.}[i] = \text{norm}(h)^2$$

end

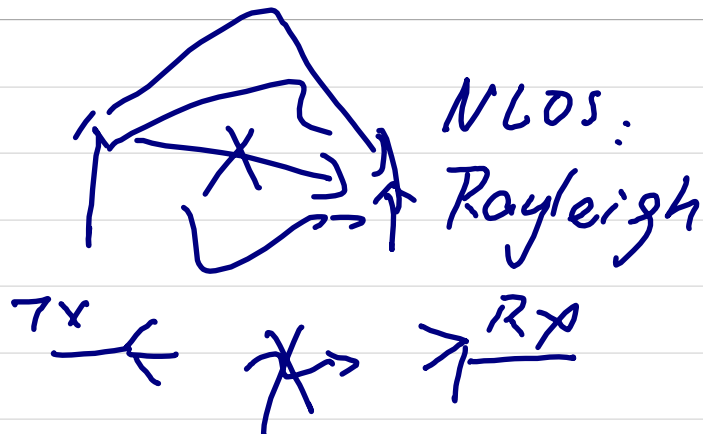
plot (norm)

plot (square abs)



① Ex2

$$h \sim (\mathcal{N}(0,1))$$



② Ex3

$$h = h_{\text{LOS}} + h_{\text{Rayleigh}}$$



$$h = \sqrt{\frac{k}{k+1}} \bar{h} + \sqrt{\frac{1}{1+k}} \tilde{h}$$

k : how much ^{energy} is in LOS
Ricean factor

$$k \rightarrow 0 \Rightarrow h = \tilde{h} \text{ (Rayleigh)}$$

$$k \rightarrow \infty \Rightarrow h \approx \bar{h} = e^{j\phi}$$

large

fix k

for i = 1:10000

$$h_R = \frac{N(0,1)}{\sigma_R}$$

$$h_I = \frac{N(0,1)}{\sigma_I}$$

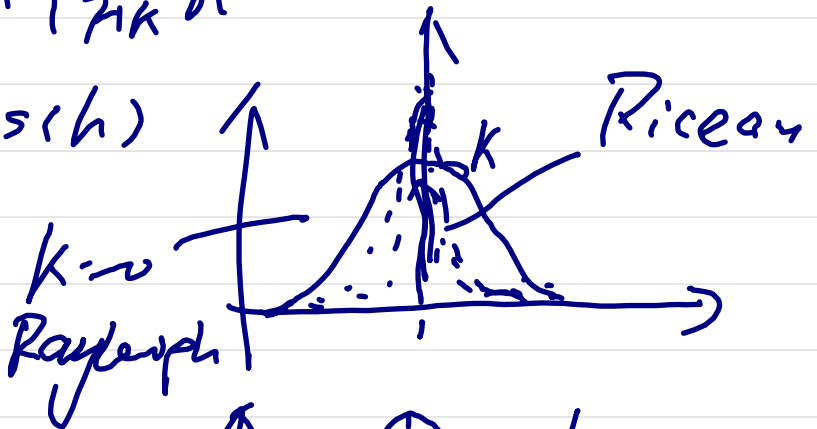
$$\tilde{h} = h_R + i h_I \rightarrow \text{Rayleigh}$$

$$\bar{h} = e^{j\phi} = 1$$

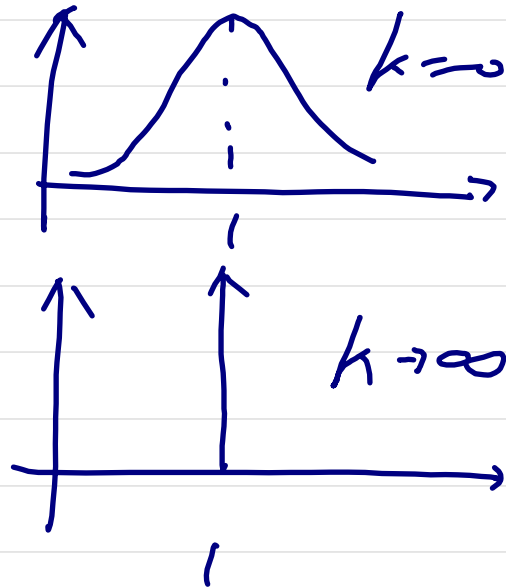
$$h = \sqrt{\frac{k}{1+k}} \bar{h} + \sqrt{\frac{1}{1+k}} \tilde{h}$$

$$\text{norm}[i] = \text{abs}(h)$$

end
plot(norm)



Correlated
uncorrelated

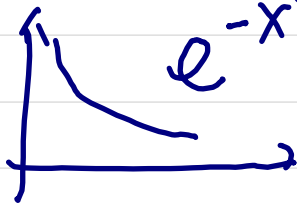


Ex 4:

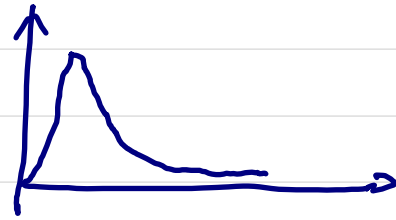
n complex normal RV

$$X_1, X_2, \dots, X_n \Rightarrow Y = |X_1|^2 + |X_2|^2 + \dots + |X_n|^2$$

$n=1 \Rightarrow |X_1|^2$ (orig. RV)



$n=3$



$n=3$

for $i = 1:100$

$$h_R = \frac{N(0,1)}{R}$$

$$h_I = \frac{N(0,1)}{R}$$

$$h_1 = h_R + i h_I$$

$$\vdots$$
$$h_2 = h_R + i h_I$$

sum_wom_h[i] = abs(h1)^2 + abs(h2)^2
plot(sum_wom_h)

for $i = 1 : 100$

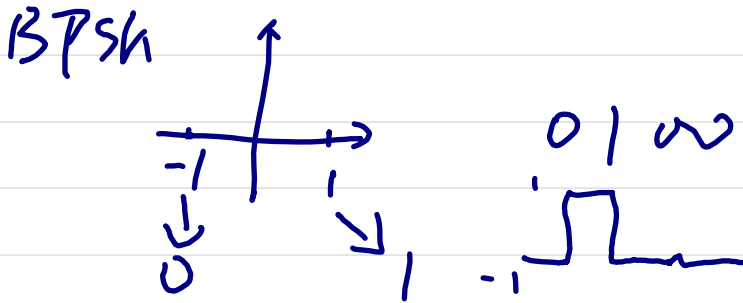
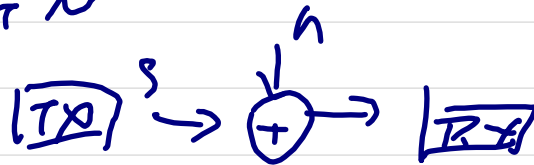
for $j = 1 : n$

$h = h_R + j h_z$

$\text{sum_norm} \cdot h[i] = \text{sum_norm}[i-1]$
 $+ \text{abs}(h)^2$

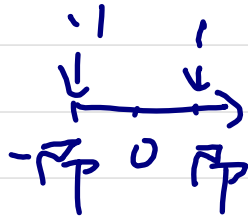
Ex 5:

BPSK $[TX] \rightarrow \text{modulator} \rightarrow [RX]$
 ① Analog

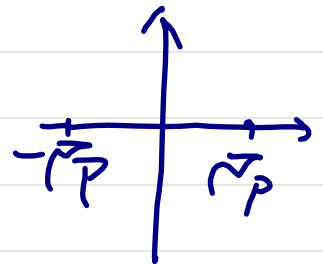


$$\text{Power} = |s|^2$$

$$110100 \rightarrow \{1, 1, -1, 1, -1, -1\}$$

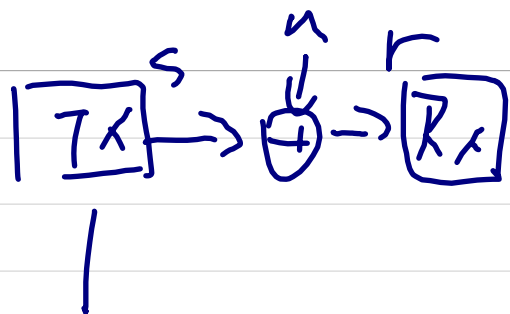


BPSK with power $P \Rightarrow$ symbols



$$P = 10 \text{ dB} \Rightarrow 10 \Rightarrow 2 \dots$$

$$P = 20 \text{ dB} \Rightarrow 100 \Rightarrow 10$$

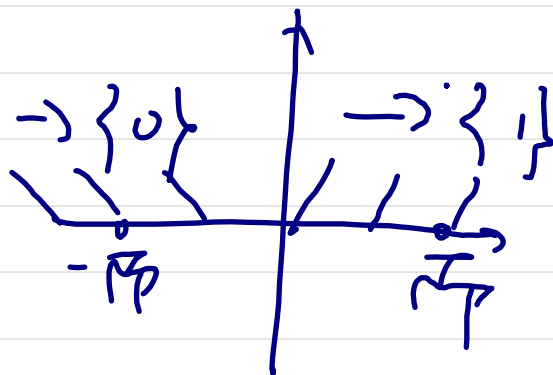
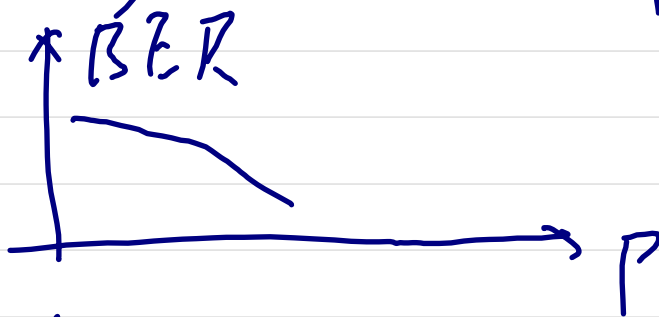


generate 1000 random bits

$$0 \rightarrow (-1) \rightarrow -\sqrt{P}$$

$$1 \rightarrow \sqrt{P}$$

$$r = s + n \sim \mathcal{N}(0, 1)$$



$$b_i \in \{0, 1\} \mapsto s_i \in \{-\sqrt{P}, \sqrt{P}\}$$

$$\oplus \leftarrow n$$

$$\begin{matrix} 70 \rightarrow 1 \\ 00 \rightarrow 0 \end{matrix} \leftarrow \text{Re}\{y_i\} \leftarrow y_i = s_i + n$$

Ex 5. 1)

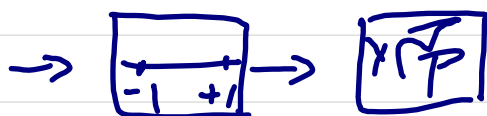
BPSK/QPSK on AWGN channel

① BPSK

bit/data generation

101...011
1e4

mapping power=SNR

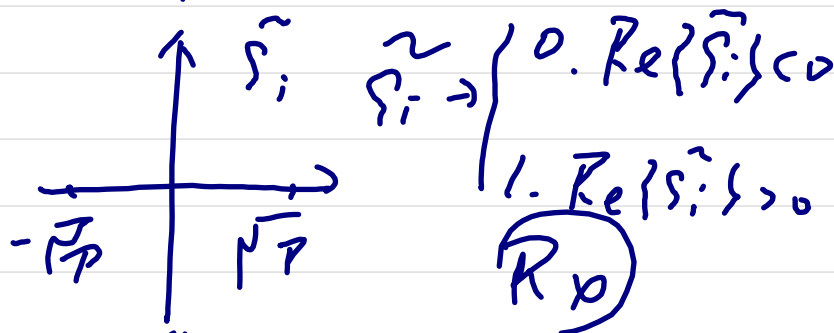


eg: 1 → 1 → \sqrt{P}
0 → -1 → $-\sqrt{P}$

(7v)



$s_i \sim \mathcal{CN}(0, 1)$



$$P_e = \frac{\# \text{error}}{\# \text{bits}}$$

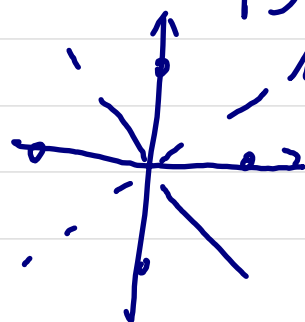
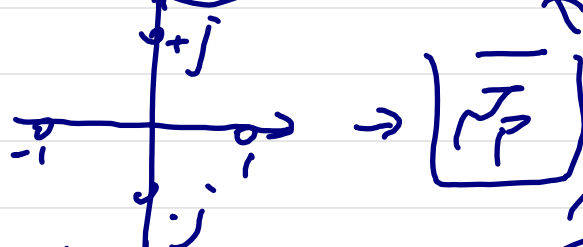
② QPSK

00 10 01 00 10
1 -j +j 1 -j

$\mathcal{CN}(0, 1)$



$\{R_x \text{ knows } h_i\}$

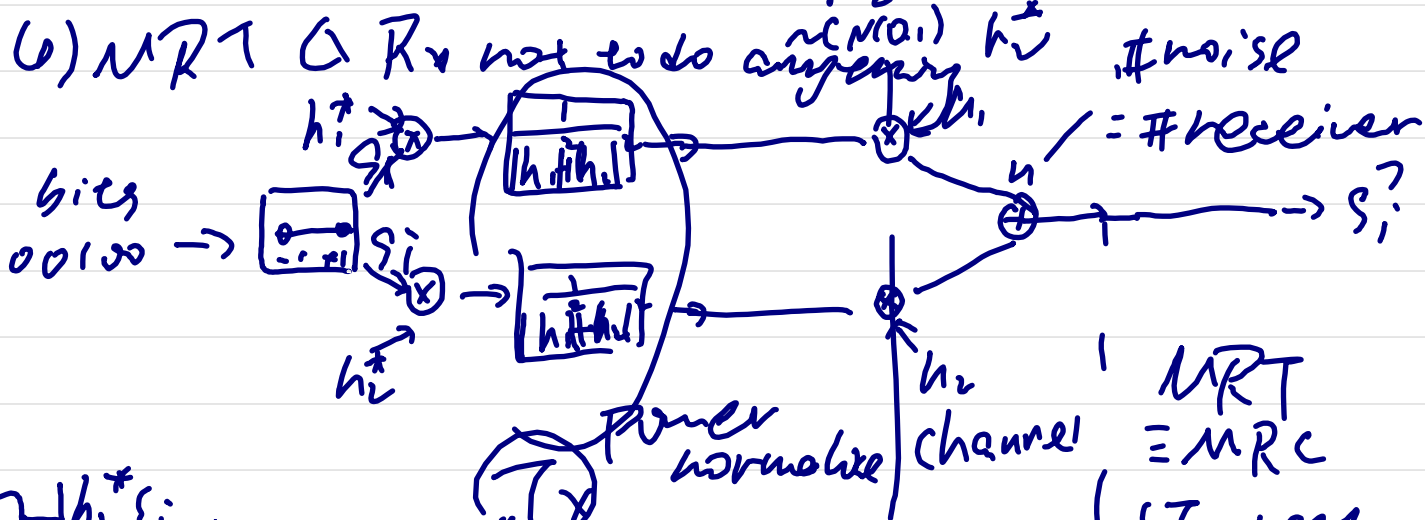
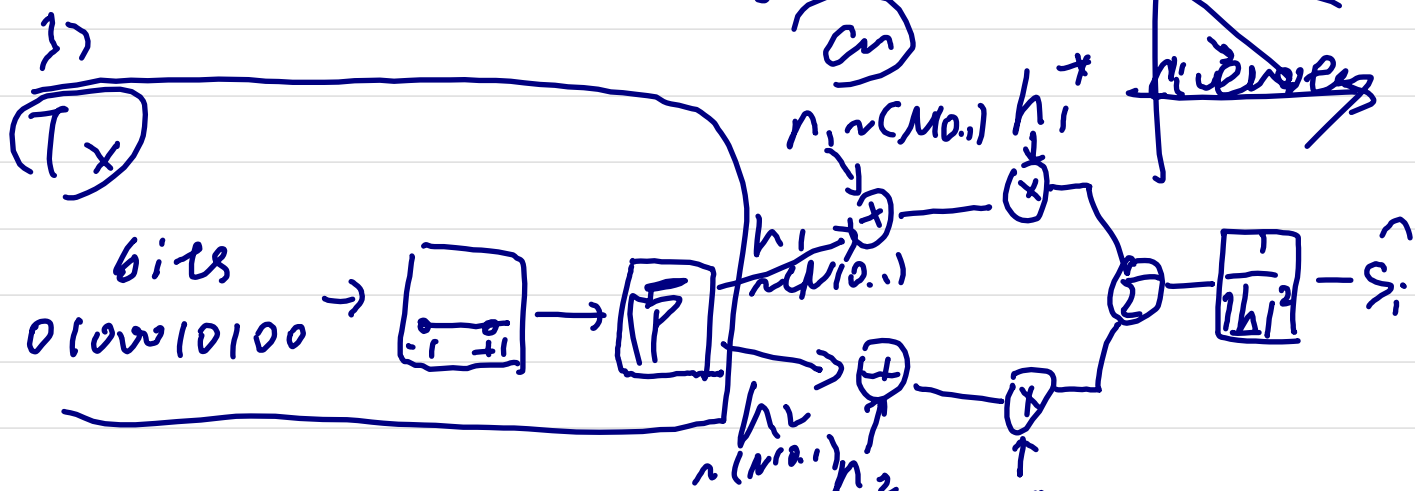


ML decoding
 $\hat{s}_i = \frac{s_i}{h_i}$
 $\hat{s}_i = s_i + h_i$
 $h_i \in \{j\}$

$$P_e(\text{symbol}) = \frac{\# \text{ wrong symbol}}{\# T_x \text{ symbol}}$$

2) channel: $h_i s_i + \text{CN}(0,1)$. $h_i \sim \text{CN}(0,1)$

change after several symbols.



$$\begin{aligned} & \begin{bmatrix} h_1^* s_i \\ h_2^* s_i \end{bmatrix} \rightarrow \text{Power Normalizer} \rightarrow \text{channel} \rightarrow \text{MRT} \equiv \text{MRC} \\ & \text{Tx needs to know CN} \\ & |h_1^* s_i|^2 + |h_2^* s_i|^2 \\ & = P(E_g)(|h_1|^2 + |h_2|^2) \end{aligned}$$

5)

$$\begin{array}{c} \underline{000110111010} \\ s_6 \ s_5 \ s_4 \ s_3 \ s_2 \ s_1 \end{array} \left| \begin{array}{l} 2 \times \text{symbols} \\ \text{each time} \\ (s_2, s_1) \end{array} \right.$$

Alamouti: get diversity gain w/o CSI

$$s_1, s_2 \rightarrow \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{array}{l} \xrightarrow{h_1} \\ \xrightarrow{h_2} \end{array} \begin{array}{l} y^{\textcircled{1}} = \frac{1}{\sqrt{2}} (h_1 s_1 + h_2 s_2) + n^{\textcircled{1}} \\ y^{\textcircled{2}} = \frac{1}{\sqrt{2}} (h_1 (-s_2^*) + h_2 s_1^*) + n^{\textcircled{2}} \end{array}$$

$$\begin{bmatrix} s_1 & s_2 \end{bmatrix} \begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix}^* = 0 \Rightarrow \begin{array}{l} y^{\textcircled{1}} = \frac{1}{\sqrt{2}} (h_1 s_1 + h_2 s_2) + n^{\textcircled{1}} \\ y^{\textcircled{2}*} = \frac{1}{\sqrt{2}} (-h_1^* s_2 + h_2^* s_1) + n^{\textcircled{2}} \end{array}$$

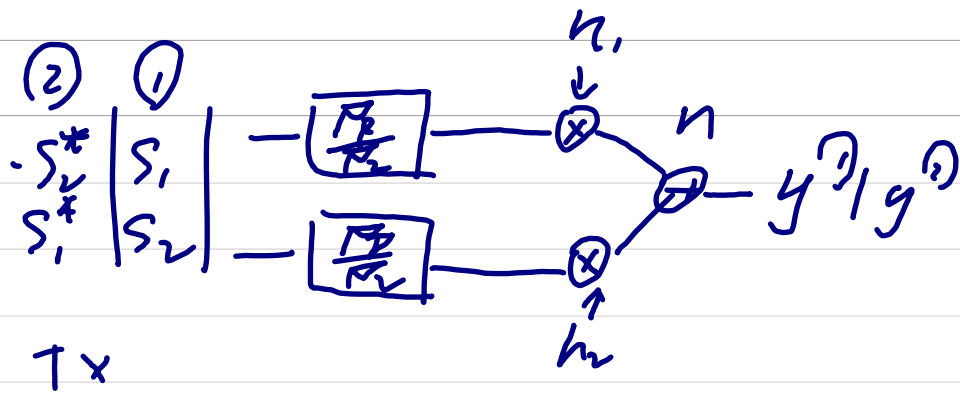
orthogonal

$$\begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}*} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n^{\textcircled{1}} \\ n^{\textcircled{2}} \end{bmatrix}$$

$$s_1 = \begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) s_1$$

$$s_2 = \begin{bmatrix} h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} y^{\textcircled{1}} \\ y^{\textcircled{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) s_2$$

diversity gain
 $\frac{1}{\sqrt{2}} \rightarrow 3 \text{ dB}$



$$\Rightarrow \begin{pmatrix} \hat{s}_1^{pre} = (h_1^* \ h_2) \\ \hat{s}_2^{pre} = (h_2^* \ h_1) \end{pmatrix} (y^D, y^D)$$

$$\Rightarrow \hat{s}_1 = \frac{\hat{s}_1^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

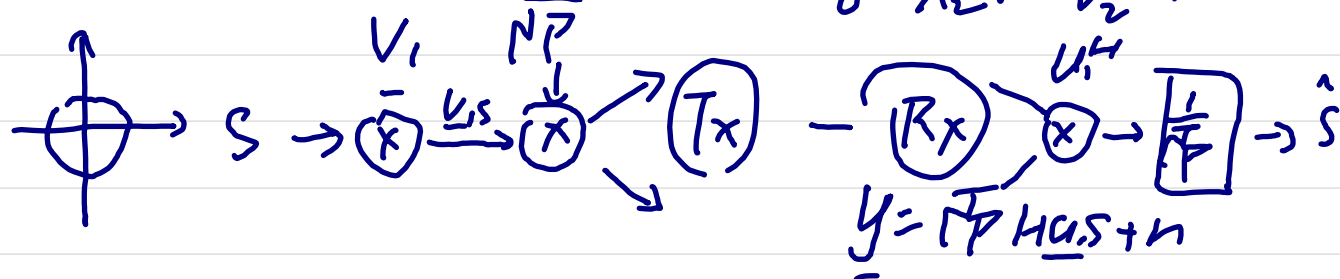
$$\hat{s}_2 = \frac{\hat{s}_2^{pre} \times 2}{|h_1|^2 + |h_2|^2}$$

6. ^a Dominant eigenmode transmission (DET)

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

$$\stackrel{\text{SVD}}{=} U \Lambda V^H$$

$$R_x \quad T_x = (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \underline{v}_1^H \\ \underline{v}_2^H \end{pmatrix}$$



$$\underline{y} = \underline{F}^H H \underline{u}_1 s + \underline{n}$$

2x2 DET:

$$\begin{matrix} R_{x1} \\ R_{x2} \end{matrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underline{F}^H U \Lambda V^H \underline{v}_1 s + \underline{n}$$

array gain = 4

dir. gain = 4

$$= \frac{\underline{v}_1^H}{\underline{v}_1^H \underline{v}_1} \underline{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad 2 \times 2 \text{ Alamouti} \dots$$

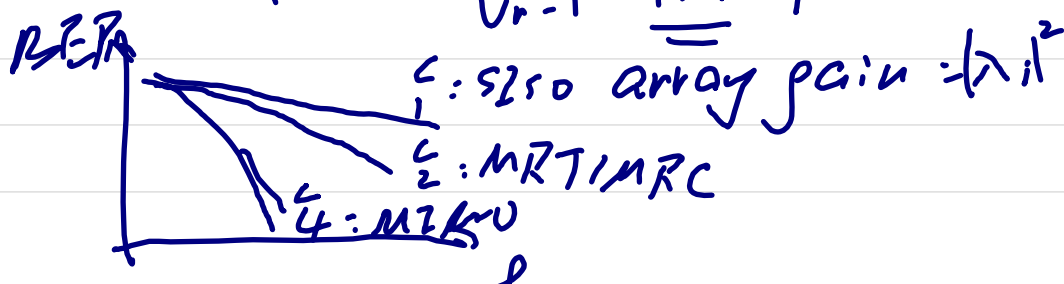
$$= \underline{F}^H (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} s \quad \text{array gain} = 2 \quad (\text{MRT/MSI: 3 dB loss})$$

$$= \underline{F}^H (\underline{u}_1 | \underline{u}_2) \begin{pmatrix} \lambda_1 s \\ 0 \end{pmatrix} = \underline{F}^H \underline{u}_1 \lambda_1 s \quad \text{d.v. gain} = 4$$

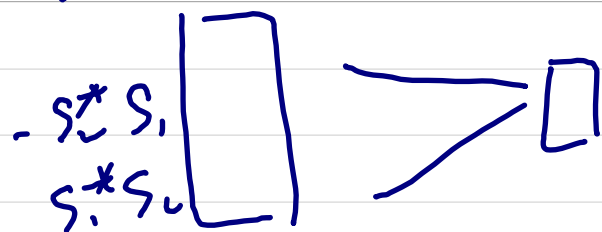
$$R_x: \underline{y} = \underline{F}^H H \underline{u}_1 s + \underline{n}$$

$$\underline{u}_1^H \underline{y} = \underline{F}^H H \underline{u}_1^H \underline{u}_1 \lambda_1 s + \underline{u}_1^H \underline{n} = \underline{F}^H \lambda_1 s + \underline{n}$$

$$SNR = |\lambda_1|^2 \frac{P}{\sigma_n^2} = |\lambda_1|^2 \rho$$



b)

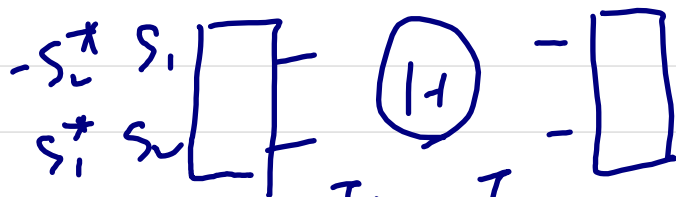


$$\begin{aligned} y(1) &= h_1 s_1 + h_2 s_2 + n \\ y(2) &= -h_1 s_2^* + h_2 s_1^* + n \end{aligned} \Rightarrow \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

y

$$\hat{s}_1 = [h_1^* \ h_2^T] y \quad \text{div. gain} = 2$$

$$\hat{s}_2 = [h_2^* \ -h_1^T] y$$



$$H = \begin{pmatrix} \begin{matrix} \text{Tx}_1 \\ h_{11} & h_{12} \end{matrix} \\ \begin{matrix} \text{Rx}_2 \\ h_{21} & h_{22} \end{matrix} \end{pmatrix} = (\underline{h}_1 | \underline{h}_2)$$

$$\underline{y}(1) = \underline{h}_1 s_1 + \underline{h}_2 s_2 + \underline{n}(1)$$

$$\underline{y}(2) = -\underline{h}_1 s_2^* + \underline{h}_2 s_1^* + \underline{n}_2$$

$$\Rightarrow \begin{bmatrix} \underline{y}(1) \\ \underline{y}(2)^* \end{bmatrix} = \begin{bmatrix} \underline{h}_1 & \underline{h}_2 \\ \underline{h}_2^* & -\underline{h}_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \underline{n}(1) \\ \underline{n}_2 \end{bmatrix}$$

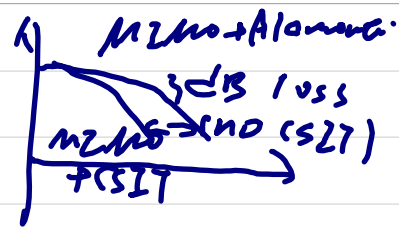
$$\hat{s}_1 = [\underline{h}_1^* \ \underline{h}_2^T] \underline{y}$$

$$\hat{s}_2 = [\underline{h}_2^* \ -\underline{h}_1^T] \underline{y}$$

$$\Rightarrow \text{SNR} = \frac{|\underline{h}_1|^2 + |\underline{h}_2|^2}{2}$$

array gain = 1

diversity gain = 4



$$s_1^* s_1 = \left[\frac{1}{\sqrt{2}} \right] \left[\frac{1}{\sqrt{2}} \right] = 1$$

$$s_2^* s_2 = \left[\frac{1}{\sqrt{2}} \right] \left[\frac{1}{\sqrt{2}} \right] = 1$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix}$$

$$\begin{matrix} \text{---} & y(1) & | & y(2) \\ \text{---} & \underline{2 \times 1} & & \underline{2 \times 1} \end{matrix}$$

$$P_{\text{tot}} = \left(\frac{\sqrt{P}}{2} \right)^2 \cdot 2 = P$$

$$\underline{y} = \begin{pmatrix} y(1) \\ y(2) \end{pmatrix} \quad 4 \times 1$$

$$\hat{s}_1 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_1^H \ \underline{h}_2^T] \underline{y}$$

$$\hat{s}_2 = \frac{\sqrt{2}}{\sqrt{P}} [\underline{h}_2^H \ -\underline{h}_1^T] \underline{y}$$

7.8

$$\begin{array}{c} \downarrow \\ \text{H} \\ \downarrow \end{array} \quad \begin{array}{c} \downarrow \\ \text{H} \\ \downarrow \end{array} \quad \text{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

spacing $> \frac{\lambda}{2}$: h uncorrelated.

$< \frac{\lambda}{2}$: h correlated.

How correlation impact the performance?

Kronecker model correlation at R_x, T_x (1.2)

$$R_r = \begin{bmatrix} 1 & r^* \\ r & 1 \end{bmatrix} \quad R_t = \begin{bmatrix} 1 & t^* \\ t & 1 \end{bmatrix}$$

$$H = R_r^H H_w R_t^H$$

no R_x correlation. \Rightarrow $R_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R_r^H$
 only T_x correlation $\Rightarrow R_t = \begin{bmatrix} 1 & t \\ t^* & 1 \end{bmatrix}$

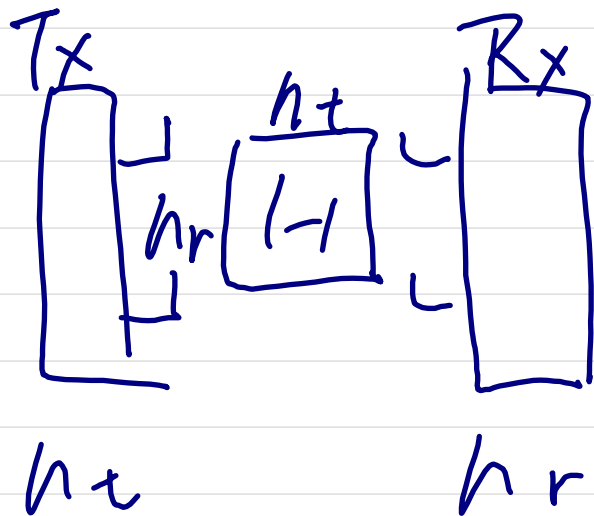
$$H = H_w R_t^H$$

task: repeat the previous exercise with channel $H = H_w R_t^H$ ($t = 0, 0.9$)

$$E\{H H^H\} = R_r \otimes R_t$$

$$R_t^H = \begin{pmatrix} 1 & t^* \\ t & 1 \end{pmatrix}$$

ensure at least 100 errors



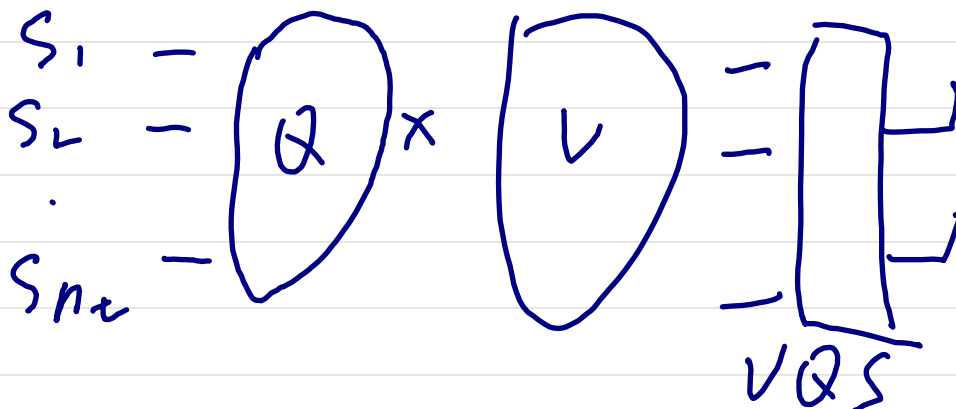
$$J(H, Q) = (\log_2 \det(I + P H Q H^H))$$

$$C = \max_{Q \text{ (obtained by WZF)}} J(H, Q)$$

$$|h| = U \Lambda V^H$$

$n_r \times n_t$ $n_r \times n_r$ $n_t \times n_t$

$$Q = \begin{bmatrix} Q_1 & \dots & Q_{n_t} \end{bmatrix} \quad \text{Tr}(Q) = P$$



$$\underline{J} = U \Lambda V^H V Q S = U \Lambda Q S$$

$$\underbrace{(U^H)}_{T \times I} \underbrace{U \Lambda V^H V}_{I} Q S + n = \Lambda Q S + n$$

$$C = \sum_{i=1}^{n_t} \log_2(1 + \lambda_i^2 q_i P) \quad \begin{pmatrix} \lambda_1 q_1 s_1 \\ \lambda_2 q_2 s_2 \\ \vdots \\ \lambda_{n_t} q_{n_t} s_{n_t} \end{pmatrix} \quad \begin{pmatrix} q_1 & \dots & q_{n_t} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_{n_t} \end{pmatrix}$$

$$H = U \Lambda V^H$$

$$H H^H = U \Lambda \underbrace{V^H V}_I \Lambda^H U^H = U \Lambda^2 U^H$$

$$\begin{pmatrix} T \times \rightarrow V \\ R \times \rightarrow U^H \end{pmatrix}$$

2×2

2×4

4×4

$$C = (\log(1 + \lambda_1^2 q_1)) + \log(1 + \lambda_2^2 q_2)$$

$$H = U \Lambda U^H$$

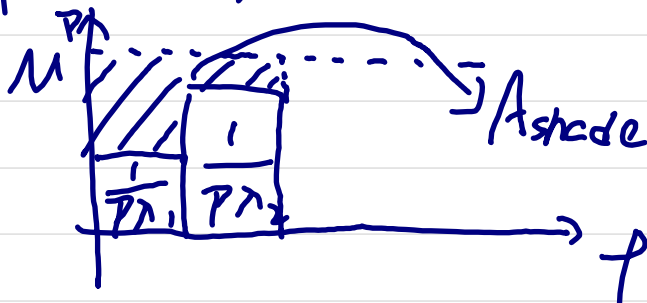
\downarrow
 (λ_1, λ_2)

$$\begin{aligned} q_1 + q_2 &= P \\ \Rightarrow q_1 &= x_1 P \\ q_2 &= x_2 P \\ x_1 + x_2 &= 1 \end{aligned}$$

$$N_s = \min(n_t, n_r)$$

$= 2$

Asynchronous



$$\frac{\text{SVD}(H) : 2}{\text{SVD}(H^H H) : \text{no square}}$$



$$\left(\mu_i - \frac{1}{p\lambda_1} \right) + \left(\mu_i - \frac{1}{p\lambda_2} \right) = 1$$

μ^* such that

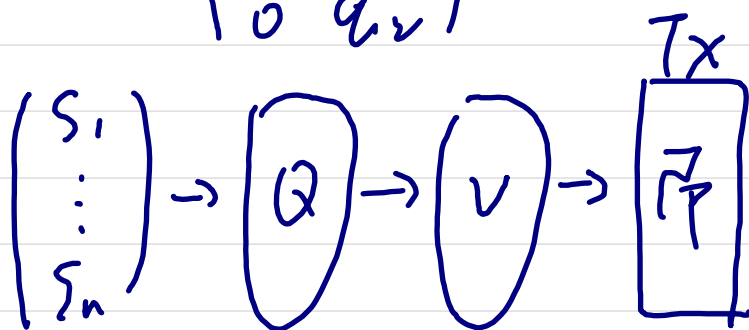
$$\max \left\{ \mu^* - \frac{1}{p\lambda_1}, 0 \right\} + \max \left\{ \mu^* - \frac{1}{p\lambda_2}, 0 \right\} = 1$$

$$H = U \Sigma V^H$$

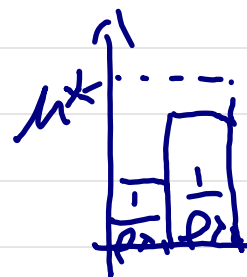
$$\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

↓

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \quad q_1 + q_2 = 1$$



$$C = \log_2(1 + \underbrace{P q_1 \sigma_1^2}_{\lambda_1}) + \log_2(1 + \underbrace{P q_2 \sigma_2^2}_{\lambda_2})$$




μ^* such that

$$\left\{ \mu^* - \frac{1}{p\lambda_1} \right\}^+ + \left\{ \mu^* - \frac{1}{p\lambda_2} \right\}^+ = 1$$

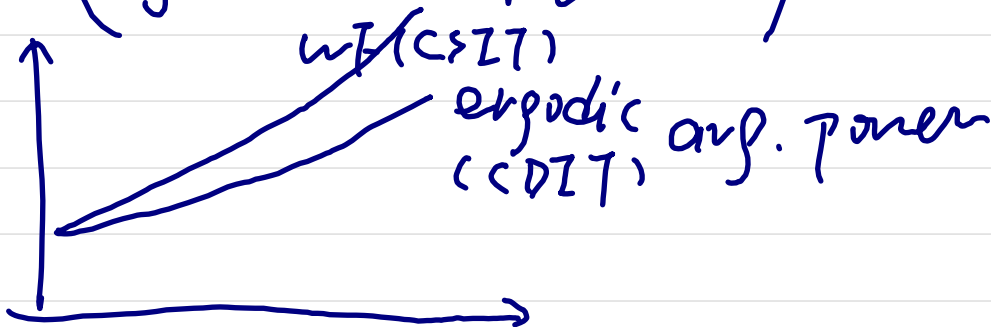
① generate matrix H

② SVD $H = U \Sigma V^H$ ③ $\Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \rightarrow \lambda = (\lambda_1 \lambda_2)$

④  $(M^* - \frac{1}{p_{\lambda_1}})^T + (M^* - \frac{1}{p_{\lambda_2}})^T = I$

⑤ calculate $C = \log_2(1 + p_{\lambda_1} S_1) + \log_2(1 + p_{\lambda_2} S_2)$

$$C = E \left(\log_2 \det \left(I + \frac{P}{h_t} H H^H \right) \right)$$



11.

2) ML receiver

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$\{\hat{s}_1, \hat{s}_2\}$: all possible combinations

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = H \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} \quad \arg \min_{\hat{s}_1, \hat{s}_2} \|y - \hat{y}\|^2$$

3) ZF

$$y = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{pmatrix} = H^{-1} y$$

4) MMSE

$$\text{ZF } T = H^{-1}$$

$$\tilde{s} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = T y \quad \text{MMSE } T = \sqrt{\frac{2}{E_s}} H^H (H H^H + \frac{2}{P} I)^{-1}$$

5) unordered

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = (h_1 | h_2) \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + n$$

$= h_1 s_1 + h_2 s_2 + n$

z7-s7c: estimate one, remove one...

$$G = H^{-1} \quad G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

$$\tilde{s}_1 = G(1,:) \underline{y}$$

$$\tilde{s}_1 \rightarrow \hat{s}_1$$

$$\underline{\hat{y}} = \underline{y} - \underline{h}_1 \hat{s}_1 = \underline{h}_2 s_2 + n \stackrel{MRL}{\Rightarrow} \hat{s}_2 = \frac{\underline{h}_2^T}{\|\underline{h}_2\|} \hat{\underline{y}}$$

6) ordered

$$G = \begin{pmatrix} G(1,:) \\ G(2,:) \end{pmatrix}$$

Choose $G(i,:)$ such that $\|G(i,:)\|$ is min
 $\min\{\|G(1,:)\|, \|G(2,:)\|\}$

7) Alamouti

$$\begin{bmatrix} \\ \end{bmatrix} \xrightarrow{(h_1, h_2)} \begin{bmatrix} \\ \end{bmatrix}$$

$$y(1) = H(\underline{s}_1) = h_1 s_1 + h_2 s_2 + n_1$$

$$y(2) = H(\underline{s}_1^*) = -h_1 s_1^* + h_2 s_1^* + n_2$$

$$\begin{pmatrix} -s_1^* & s_1 \\ s_1^* & s_2 \end{pmatrix} \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_1^* & -h_2^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$\uparrow \quad \uparrow$
 $\underline{2} \quad \underline{1}$

$$\tilde{s}_1 = [h_1^H \ h_2^T] \begin{bmatrix} y(1) \\ y(2)^* \end{bmatrix}$$

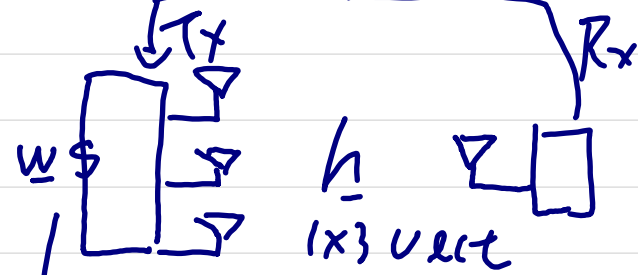
$$\tilde{s}_2 = [h_2^H \ -h_1^T] \begin{bmatrix} y(1) \\ y(2) \end{bmatrix}$$

12.

$$H = R_{Tx}^i H_w R_{Rx}^i$$

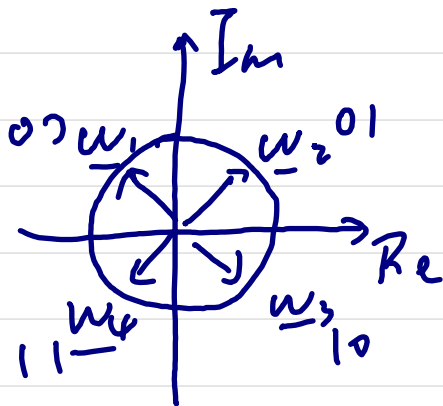
418 bits

13. CSI feedback



CSIT: how possible?

- Rx can estimate \underline{h} .
- Rx has to feedback \underline{h} to Tx.



Rx can only feedback 2 bits

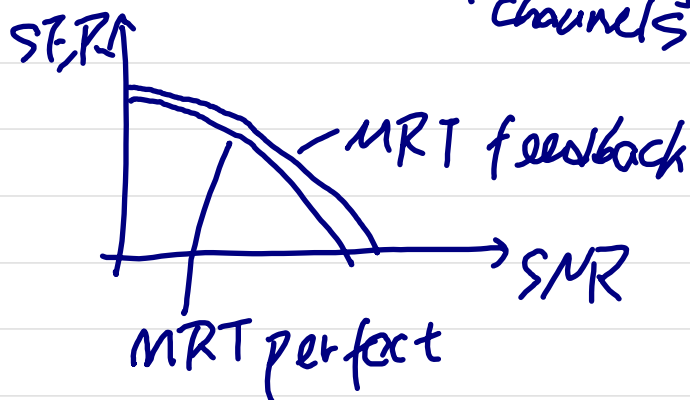
1. Rx estimate \underline{h}
2. Rx choose $\hat{\underline{h}}$

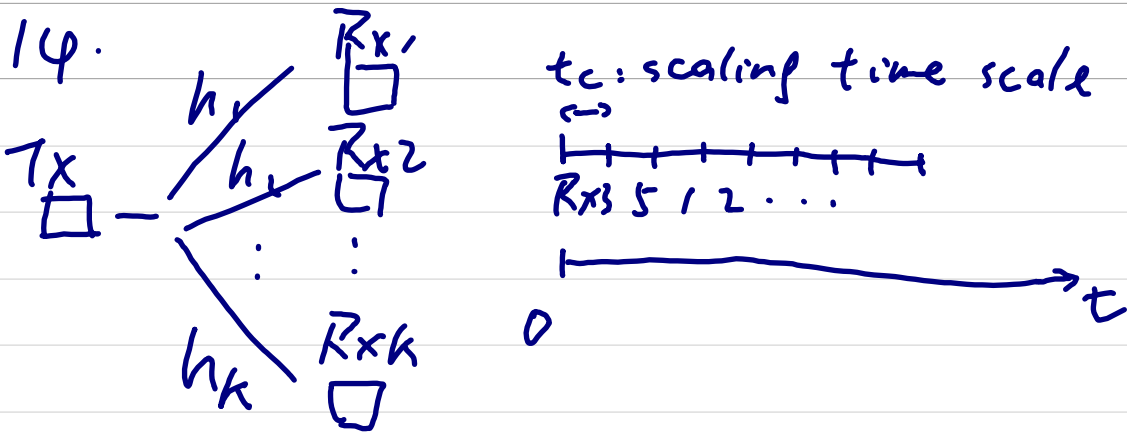
$$\underline{w} = \frac{\underline{h}}{|\underline{h}|}$$

Quantised precoding

$\underline{w}_1 \dots \underline{w}_{np}$ n_p precoding channels

$$\underline{w} = \max_{i \in \{1, 2, \dots, n_p\}} \|\underline{h}_{w_i}\|^2$$





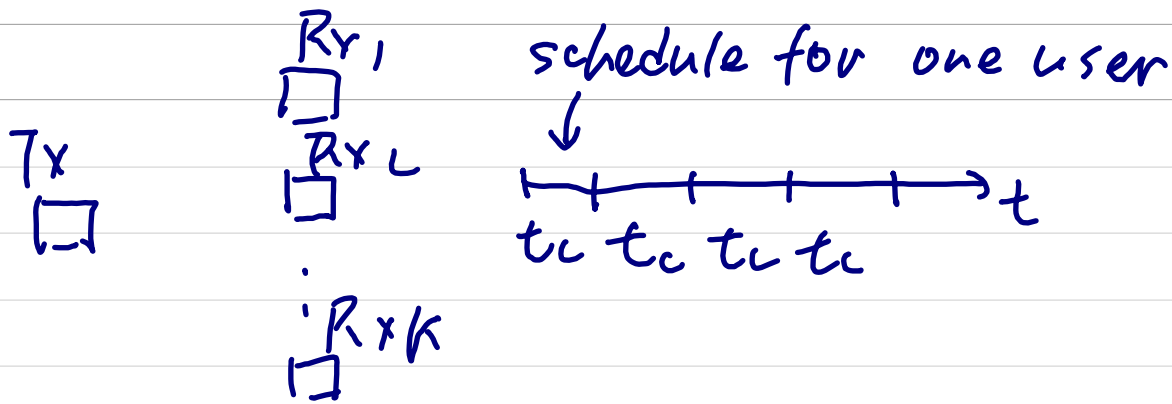
$$Q \in \{1, 2, \dots, k\}$$

$$Q^* = \underset{Q \in \{1, \dots, k\}}{\operatorname{argmax}} U_Q \quad \text{target func.}$$

$$\text{exp. } Q^* = \underset{Q}{\operatorname{argmax}} R_Q \quad \text{rate} \sim [c, 1]$$

$$Q^* = \underset{Q, S}{\operatorname{argmax}} V_Q \frac{R_Q}{\bar{R}_Q} \quad \text{long-term avg.}$$

proportional fair scheduling



For each user in each slot

3 params: $\begin{cases} \delta_q: \text{QoS} \\ R(t): \text{current rate} \\ \overline{R(t)}: \text{long-term rate} \end{cases}$

r_q : given

$$R(t) = \log_2(1 + |h_q|^2 p)$$

$$\overline{R(t)} = \begin{cases} (1 - \frac{1}{t_c}) \overline{R_q(t-1)} + \frac{1}{t_c} R_q(t) & \text{if } q \text{ scheduled} \\ (1 - \frac{1}{t_c}) \overline{R_q(t-1)} & \text{if } q \text{ not scheduled} \end{cases}$$

$$q^* = \arg \max_q r_q \frac{R_q}{\overline{R_q}}$$

$$h_k(t) = \epsilon h_k(t-1) + \sqrt{1 - \epsilon^2} N_k' \text{ noise } \mathcal{CN}(0, 1)$$

trade off by slot length.

$t_i \rightarrow T$: max rate

t_{i+1} : round \bar{R}_i (max fair)

for $t = 1 : 1/\epsilon^2$

① generate the channel of each user

$$q \in \{1 \dots K\}$$

$$h_q(t) = \epsilon h_q(t-1) + \sqrt{1-\epsilon^2} n$$

② calculate the instantaneous rate
and long-term rate of each user

$$R_q(t) = \log_2(1 + |h_q(t)|^2 \rho)$$

$$\text{select } q^* = \underset{q}{\text{argmax}} \gamma_q \frac{R_q(t)}{\bar{R}_q(t-1)}$$

update the long-term rate $\bar{R}_q(t)$,

in each slot t

① generate channel for each user

② calculate the inst. rate of user k

$$R_k(t) = \log_2(1 + |h_k(t)|^2 \rho)$$

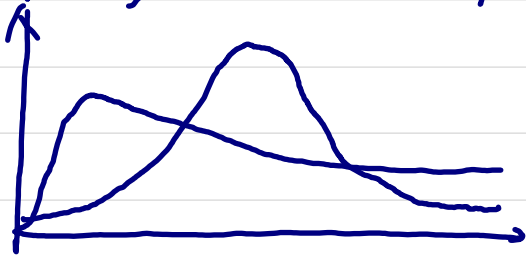
③ choose user q that maximise

$$q^* = \underset{q}{\text{argmax}} \gamma_q \frac{R_q(t)}{\bar{R}_q(t-1)}$$

⑤ iterate.

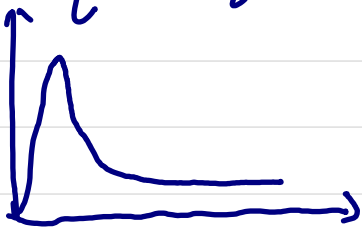
④ update long-term rate $\bar{R}_q(t)$

$$1. X \sim \mathcal{N}(\mu, \sigma^2) \rightarrow X = \mu + \sigma \cdot \text{randn}$$



log-normal: $\log S$ is in normal distribution
 $db \sim \text{pow}(\cdot)$

$$2. \sqrt{\frac{1}{2}} \cdot (\text{randn} + i \cdot \text{randn})$$

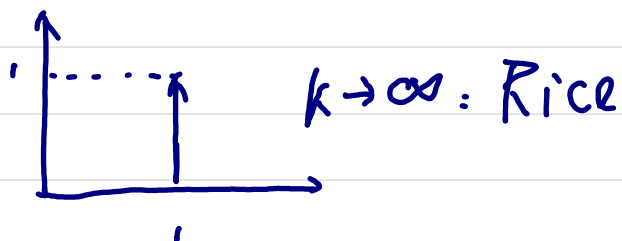
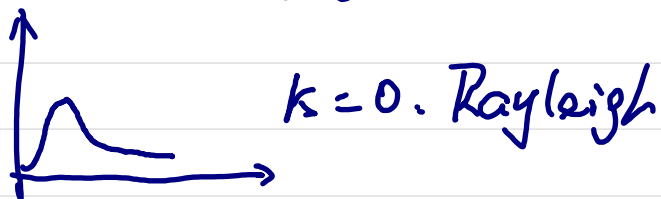


$$3. \quad h = \sqrt{\frac{k}{1+k}} \tilde{h} + \sqrt{\frac{1}{1+k}} \bar{h}$$

Ricean factor $k \in \mathbb{R}_{++}$

how much energy is in LOS path

$|h|$ distribution:

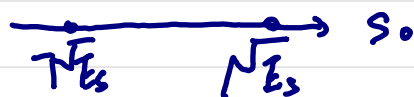


4. $n = 1$: $|h|^2 \sim \text{Exp}(\frac{1}{\sigma^2})$

5. AWGN channel: $T_x \rightarrow \oplus \rightarrow R_x$
 $y = x + n$

fading channel: $T_x \rightarrow \otimes \rightarrow \oplus \rightarrow R_x$
 $y = hx + n$

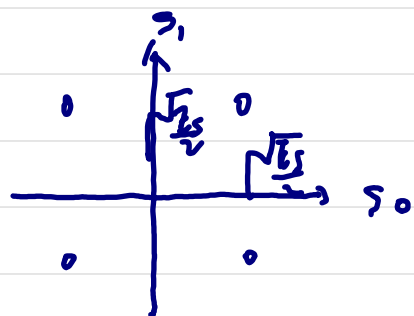
BPSK:



bits: 01001

↓
 symbols { BPSK:
 QPSK:

QPSK:



W1: encode / rate
BER

noise $\rightarrow R_x$

MRT vs Alamouti: normalize WC

diversity: information symbols pass through multiple independent fading channels.

① repetition coding^(t.f.1): same symbol over several paths

- max diversity gain
- no coding gain

② Alamouti:

array gain - SNR

diversity gain - # independent fading paths

multiplexing gain - (sum) rate

coding gain - tx power (to achieve same product distance)

