Modulation + detection	Average error probability
Coherent ASK	$Q\left(\frac{A}{2\sigma}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
coherent FSK	$Q\left(\frac{A}{\sqrt{2}\sigma}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
coherent PSK	$Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
Noncoherent ASK	$\frac{1}{2}e^{-\frac{A^2}{8\sigma^2}} = \frac{1}{2}e^{-\frac{E_b}{2N_0}}$
Noncoherent FSK	$\frac{1}{2}e^{-\frac{A^2}{4\sigma^2}} = \frac{1}{2}e^{-\frac{E_b}{2N_0}}$
DPSK	$\frac{1}{2}e^{-\frac{A^2}{2\sigma^2}} = \frac{1}{2}e^{-\frac{E_b}{N_0}}$

ASK: $\frac{E_b}{N_0} = \frac{A^2}{4\sigma^2}$ FSK. PSK: $\frac{E_b}{N_0} = \frac{A^2}{2\sigma^2}$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-\frac{t^2}{2}}dt \le \begin{cases} \frac{1}{\sqrt{2\pi}x}e^{-\frac{x^2}{2}}, & \text{large } x \geq 0 \\ \frac{1}{2}e^{-\frac{x^2}{2}}, & \text{small } x \geq 0 \end{cases}$$

1. a) $P_{e.c.ASK} = Q\left(\frac{A}{2\sigma}\right) \le \frac{1}{\sqrt{2\pi}\frac{A}{2\sigma}} e^{-\frac{A^2}{8\sigma^2}} = 2.8 \times 10^{-3}$

b) $P_{e.c.psk} = Q\left(\frac{A}{\sigma}\right) \le \frac{1}{\sqrt{2\pi}\frac{A}{\sigma}} e^{-\frac{A^2}{2\sigma^2}} = 1.1 \times 10^{-8}$

2. Orthogonal $\Rightarrow$ zero correlation.

$$\mathrm{Corr}(\cos 2\pi f_0 t, \cos 2\pi f_1 t)$$
$$= \frac{1}{T}\int_0^T \cos 2\pi f_0 t \cos 2\pi f_1 t \, dt$$
$$= \frac{1}{2T}\int_0^T \cos 2\pi(f_1+f_0)t + \cos 2\pi(f_1-f_0)t \, dt$$
$$= \frac{1}{2T}\int_0^T \cos 2\pi 2f_c t + \cos 2\pi\Delta f t \, dt$$
$$= \frac{1}{2}\left(\frac{\sin 4\pi f_c T}{4\pi f_c T} + \frac{\sin 2\pi\Delta f T}{2\pi\Delta f T}\right)$$

$\mathrm{sinc}(4\pi f_c T) \approx 0$ because $f_c T \gg 1$

$$\approx \frac{1}{2}\frac{\sin 2\pi\Delta f T}{2\pi\Delta f T}$$

$2\pi\Delta f T = n\pi \Rightarrow \Delta f = \frac{n}{2T}$

$\therefore \Delta f_{min} = \frac{1}{2T}$

3. The noise is uniformly distributed within $(-1, 1)$

$$x \rightarrow \underset{\substack{\uparrow \\ n}}{\oplus} \rightarrow y \rightarrow \boxed{\text{Detection}} \rightarrow \hat{x}$$

$$P(y \mid x=0) = \begin{cases} \frac{1}{2}, & -1 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$P(y \mid x=1) = \begin{cases} \frac{1}{2}, & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

$$P(\hat{x}=1 \mid x=0) = \int_T^1 \frac{1}{2}\,dt = \frac{1}{2}(1-T)$$

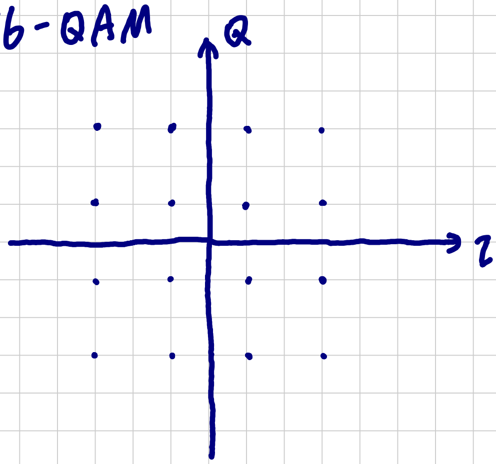
$$P(\hat{x}=0 \mid x=1) = \int_0^T \frac{1}{2}\,dt = \frac{1}{2}T$$

$$\therefore P_e = \frac{1}{2} P(\hat{x}=1 \mid x=0) + \frac{1}{2} P(\hat{x}=0 \mid x=1)$$

$$= 0.25.$$

4.

16-QAM



$$s(t) = G(A_n \cos 2\pi f_c t - B_n \sin 2\pi f_c t) + C_n \cos 2\pi f_c t - D_n \sin 2\pi f_c t$$

$$= (GA_n + C_n)\cos 2\pi f_c t - (GB_n + D_n)\sin 2\pi f_c t$$

16-QAM requires $Q_n, I_n \in \{\pm 1, \pm 3\}$.

Since $\{A_n\}, \{B_n\}, \{C_n\}, \{D_n\} \in \{\pm 1\}$,

we choose $G = \pm 2$.

$$\therefore \begin{cases} I_n = \pm 2 A_n + C_n \\ Q_n = \pm 2 B_n + D_n \end{cases}.$$