1. a) $P_T = 10W$. $P_R = \frac{10}{10^4} = 10^{-3} W$

$$P_N = 2\int_0^{10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 2N_0\cdot10^4 - 2N_0\cdot\frac{1}{40}\cdot10^4 = 1.95\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 51.28 = 17.1 dB$$

b) $P_T = \frac{A_c^2 P}{2} = 5W$. $P_R = 5\times10^{-4} W$.

$$P_N = 2\int_{f_c-10^4}^{f_c+10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 4N_0\cdot10^4 - 2N_0\cdot10^4 = 2\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 25 = 14 dB.$$

c) $$P_N = 2\int_{f_c-10^4}^{f_c+10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 4N_0\cdot10^4 - 3N_0\cdot10^4 = 1\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 50 = 17 dB.$$

2. For a small noise,

$$SNR_{env} \approx \frac{P}{A^2+P} SNR_{baseband}$$

$$= \frac{\frac{1}{2}m_p^2}{A^2+\frac{1}{2}m_p^2} SNR_{baseband}$$

$$= \frac{\mu^2}{2+\mu^2} SNR_{baseband}$$

modulation index $\mu = \frac{m_p}{A} \leq 1$

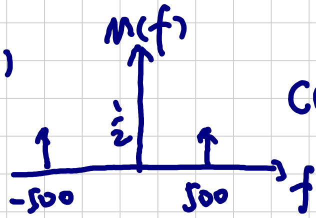
$$\begin{cases} (SNR_{env})_{max} = \frac{1}{3} SNR_{baseband}, & \mu = 1 \\ SNR_{env} \to 0, & \mu \to 0 \end{cases}$$

3.

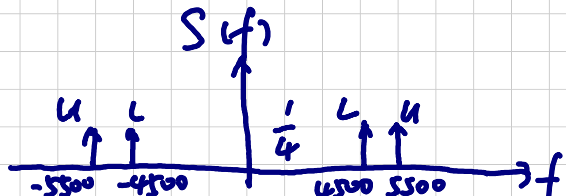
$$\cos 2\pi f_c t \Longleftrightarrow \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

$$\sin 2\pi f_c t \Longleftrightarrow \frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)] = \frac{1}{2}[-j\delta(f-f_c)+j\delta(f+f_c)]$$

(i)

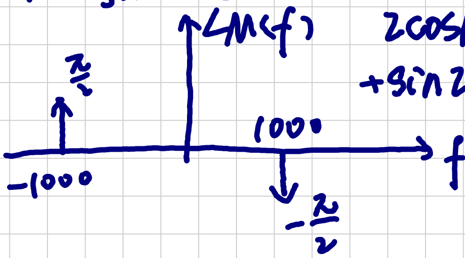


$\cos 1000\pi t$



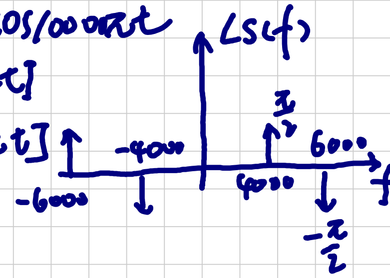
$$\cos 1000\pi t \cos 10000\pi t = \frac{1}{2}[\cos 11000\pi t + \cos 9000\pi t]$$

(ii)

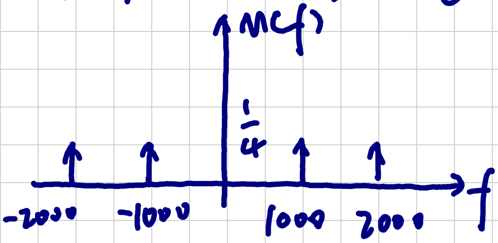


$2\cos 1000\pi t + \sin 2000\pi t$

$$(2\cos 1000\pi t + \sin 2000\pi t)\cos 10000\pi t$$
$$= [\cos 11000\pi t + \cos 9000\pi t]$$
$$+ \frac{1}{2}[\sin 12000\pi t - \sin 8000\pi t]$$

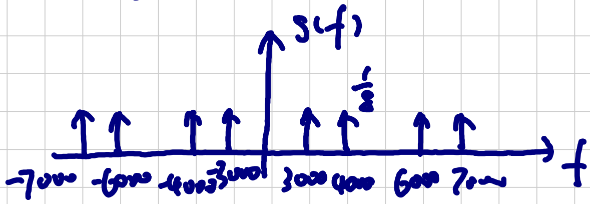


(iii) $\cos 1000\pi t \cos 3000\pi t = \frac{1}{2}[\cos 4000\pi t + \cos 2000\pi t]$



$$\frac{1}{2}[\cos 4000\pi t + \cos 2000\pi t]\cos 10000\pi t$$

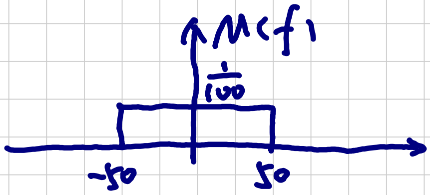
$$= \frac{1}{4}[\cos 14000\pi t + \cos 6000\pi t + \cos 12000\pi t + \cos 8000\pi t]$$

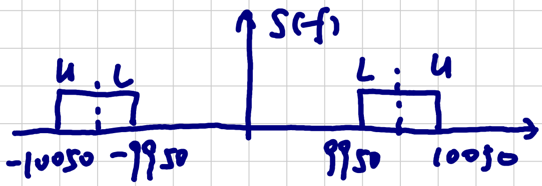


4.

(i) $m(t) = \text{sinc}\,100t$

$$\mathcal{F}(m(t)) = \frac{1}{100}\,\text{rect}\left(\frac{f}{100}\right) = \begin{cases} \frac{1}{100}, & |f| < 50 \\ 0, & \text{otherwise} \end{cases}$$





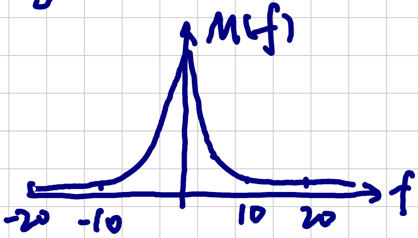
(ii) $m(t) = e^{-10t}u(t) + e^{10t}u(-t)$

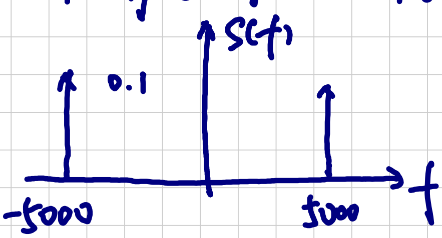
$$\mathcal{F}(m_1(t)) = \int_0^\infty e^{-(10+j2\pi f)t}\,dt = -\frac{1}{10+j2\pi f}e^{-(10+j2\pi f)t}\Big|_0^\infty = \frac{1}{10+j2\pi f}$$

$$\mathcal{F}(m_2(t)) = \int_{-\infty}^0 e^{-(j2\pi f-10)t}\,dt = -\frac{1}{j2\pi f-1}e^{-(j2\pi f-10)t}\Big|_0^\infty = \frac{1}{10-j2\pi f}$$

$$\therefore \mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2}$$

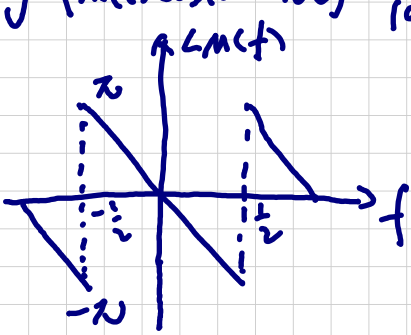
$$\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2} + \frac{10}{100+4\pi^2(f+5000)^2}$$

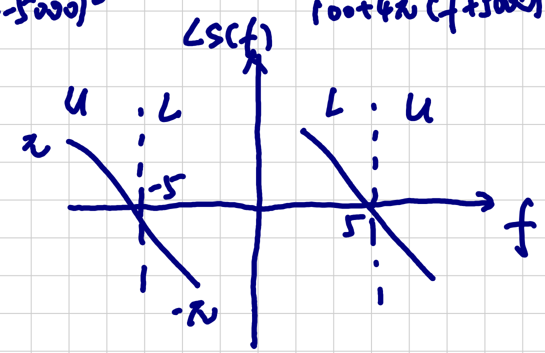




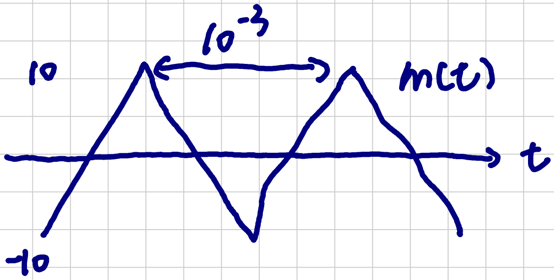
(iii) $\mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2}\cdot e^{-j2\pi f}$

$$\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2}e^{-j2\pi(f-5000)} + \frac{10}{100+4\pi^2(f+5000)^2}e^{-j2\pi(f+5000)}$$

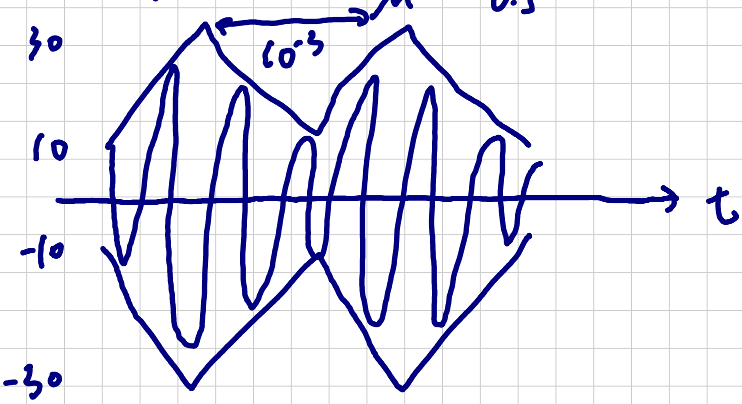




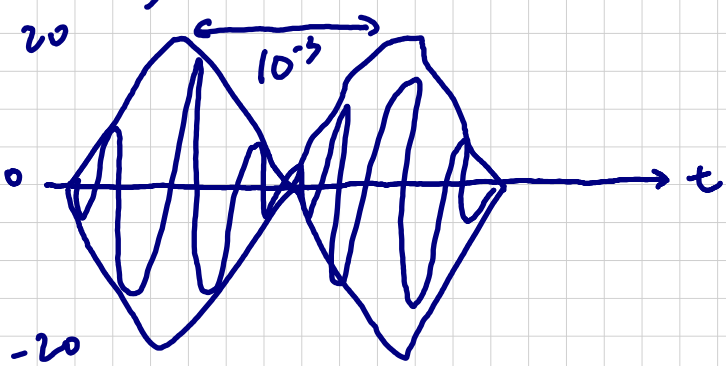
5.



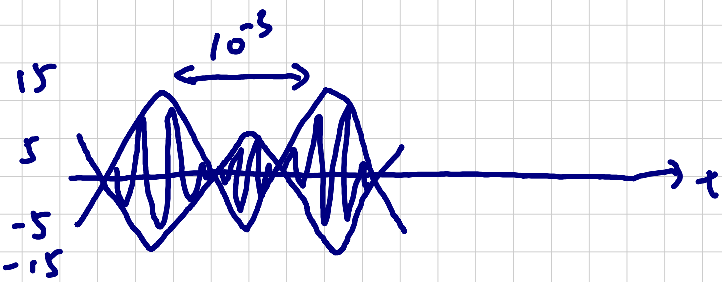
a) $\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu} = \frac{10}{0.5} = 20$



b) $A = \frac{m_p}{\mu} = 10$



c) $A = \frac{m_p}{\mu} = 5$



d) $A = \frac{m_p}{\mu} \to 0$

$(A + m(t))\cos 2\pi f_c t \to m(t)\cos 2\pi f_c t$: DSB-SC

6. a) $A = \frac{m_p}{\mu} = \frac{10}{0.8} = 12.5$

$P_c = \frac{A^2}{2} = 78.125$

b) In the first $\frac{1}{4}$ period, we have $m(t) = \frac{40t}{T_0}$

$\overline{m^2(t)} = \frac{1}{\frac{T_0}{4}} \int_0^{\frac{T_0}{4}} \left(\frac{40t}{T_0}\right)^2 dt = 33.34$

Sideband power $P_s = \frac{1}{2}\overline{m^2(t)} = 16.67$

$\eta = \frac{P_s}{P_c + P_s} = \frac{16.67}{78.125 + 16.67} = 17.59\%$