DSB, SSB, and AM (Lectures 5 & 6)

1. **Consider a message signal with a bandwidth of 10 kHz and an average power of P =10 watts. Assume the transmission channel attenuates the transmitted signal by 40 dB, and adds noise with a power spectral density of:**

$$\mathbb{S}(f) = \begin{cases} N_o(1 - \dfrac{|f|}{200 \times 10^3}), & |f| < 200 \times 10^3 \\ 0, & otherwise \end{cases}$$

where $N_o = 10^{-9}$ watts/Hz.

What is the predetection SNR at the receiver if each of the following modulation schemes is used? Assume that a suitable filter is used at the input of the receiver to limit the out-of-band noise.

(a) **Baseband**

(b) **DSB-SC with a carrier frequency of 100kHz and a carrier amplitude of $A_c = 1V$.**

(c) **DSB-SC with a carrier frequency of 150kHz and a carrier amplitude of $A_c = 1V$.**

Solution:

In this question, it is very important to note that the channel noise is not white Gaussian noise. Therefore, we can not directly use the expressions derived in the lecture since these are valid only for white noise. Also note that we are considering the predetection SNR at the receiver (not the output SNR). If the channel attenuates the transmitted signal by 40 dB, then the received signal power will be 10^{-4} times the transmitted signal power.

(a) For baseband, the transmitted signal power is $P_T = 10$ W (i.e., the same as the message power). The received signal power is therefore $P_R = 1$ mW.

The noise power is found by integrating the noise PSD over the transmission bandwidth:

$$P_N = 2\int_0^{10^4} \mathbb{S}(f)df = 2N_o\int_0^{10^4}\left(1 - \frac{f}{200 \times 10^3}\right)df$$

$$= 2N_o \times 10^4 \times 0.975 = 19.5\mu W$$

This gives an SNR at the receiver input of

$$SNR = \frac{1 \times 10^{-3}}{19.5 \times 10^{-6}} = 17.1dB.$$

(b) For DSB-SC, the transmitted signal power is $P_T = \frac{A_c^2 P}{2} = 5$W. The received signal power is therefore $P_R = 0.5$ mW.

The noise power is:

$$P_N = 2\int_{f_c - 10^4}^{f_c + 10^4} \mathbb{S}(f)df = 2N_o \times 0.5 \times 20 \times 10^3$$

$$= 20\mu W$$

giving a receiver input SNR of

$$SNR = \frac{0.5 \times 10^{-3}}{20 \times 10^{-6}} = 14dB.$$

(c) At this carrier frequency, the noise power becomes:

$$P_N = 2N_o \times 0.25 \times 20 \times 10^3 = 10\mu W,$$

and the receiver input SNR is

$$SNR = \frac{0.5 \times 10^{-3}}{10 \times 10^{-6}} = 17dB.$$

Observe that these results are quite different from what one would obtain if the channel had white Gaussian noise. If the noise PSD were flat, the SNR of DSB-SC would be independent of the carrier frequency.

2. Consider the standard AM modulation, where the transmitted signal is given by

$$\boldsymbol{s(t) = [A + m(t)]\cos(2\pi f_c t),}$$

where $m(t)$ is the message signal. Assume that the modulating wave is a sinusoidal wave, i.e., single-tone modulation,

$$\boldsymbol{m(t) = A_m cos(2\pi f_m t).}$$

Given the baseband signal-to-noise ratio $SNR_{Baseband}$, consider an AM envelope detector when the noise power is small. Compute the output SNR in terms of the modulation index μ, which is defined as $\mu \triangleq m_p/A$, where m_p is the peak value of the message signal. What value of μ gives the maximum output SNR?

Solution:

In the lecture, the output SNR of an envelope detector (for small noise) was shown to be

$$SNR = \frac{P}{A^2 + P} SNR_{baseband}.$$

For single-tone modulation, we have $P = \frac{1}{2}m_p^2$. Substitution gives

$$SNR = \frac{\frac{1}{2}m_p^2}{A^2 + \frac{1}{2}m_p^2} SNR_{baseband} = \frac{\mu^2}{2 + \mu^2} SNR_{baseband}.$$

We may rewrite it as

$$SNR = \left(1 - \frac{2}{2 + \mu^2}\right) SNR_{baseband}.$$

Now, increasing the value of μ will decrease the second term, thereby increasing the SNR. But, μ cannot be increased arbitrarily, since for an envelope detector to operate the modulation index should satisfy $\mu \leq 1$. Thus, the value of μ that gives the maximum SNR is $\mu = 1$. The resulting SNR expression is

$$SNR = \frac{1}{3} SNR_{baseband}.$$

3. For each of the baseband signals: (i) $m(t) = \cos 1000\pi t$; (ii) $m(t) = 2\cos 1000\pi t + \sin 2000\pi t$; (iii) $m(t) = \cos 1000\pi t \cos 3000\pi t$, do the following.

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10000\pi t$.

(c) Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

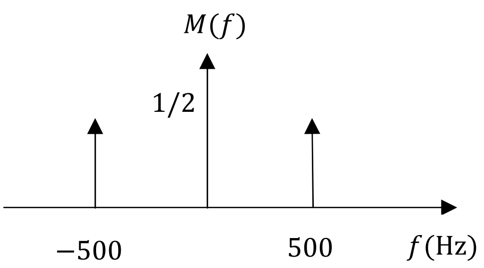
(d) Identify the frequencies in the baseband, and the corresponding frequencies in the

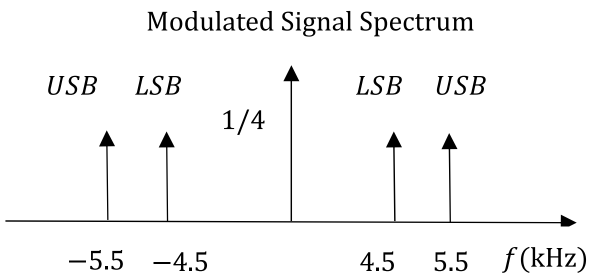
DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

Solution:

(i) $m(t) = \cos w_m t = \cos 2\pi f_m t = \cos 1000\pi t \Rightarrow f_m = 500\text{Hz}.$

$M(f) = \frac{1}{2}\delta(f-500) + \frac{1}{2}\delta(f+500).$



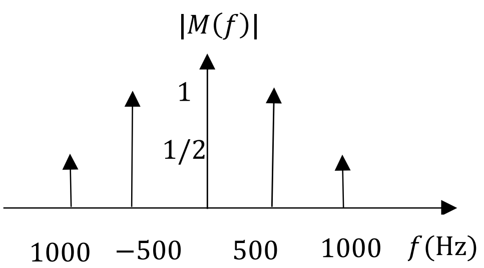


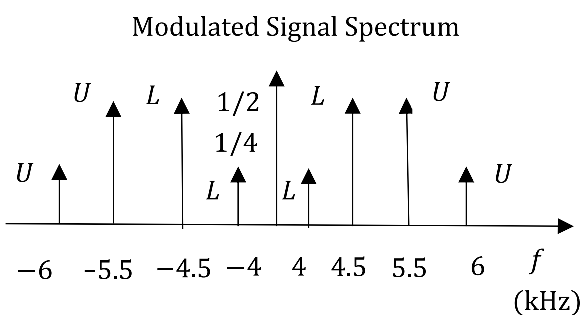
(ii) $m(t) = 2\cos w_{m,1}t + \sin w_{m,2}t = 2\cos 2\pi f_{m,1}t + \sin 2\pi f_{m,2}t = 2\cos 1000\pi t + \sin 2000\pi t$

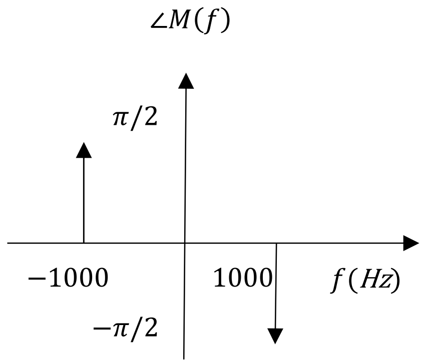
$M(f) = \delta(f-500) + \delta(f+500) + \frac{j}{2}\big(\delta(f+1000) - \delta(f-1000)\big).$

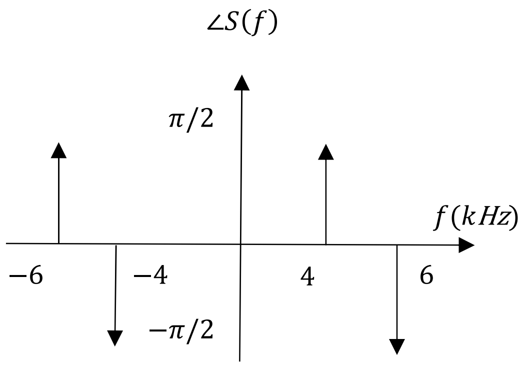
$|M(f)| = \delta(f-500) + \delta(f+500) + \frac{1}{2}\big(\delta(f+1000) + \delta(f-1000)\big).$

$$\angle M(f) = \begin{cases} -\pi/2, & f = 1000 \\ \pi/2, & f = -1000. \\ 0, & else \end{cases}$$



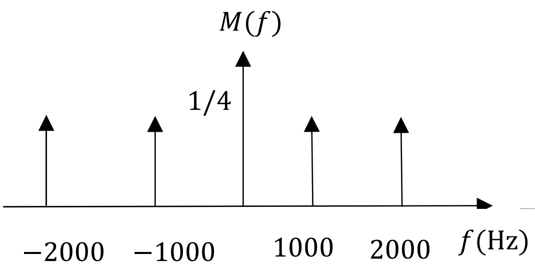


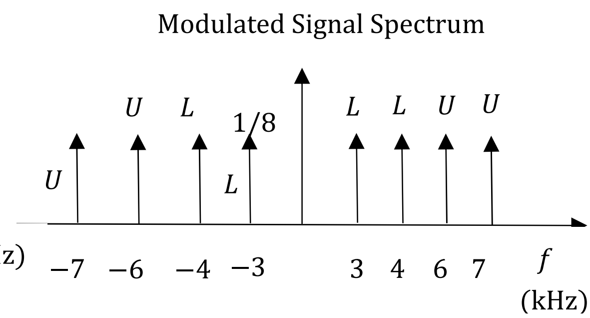




(iii) $m(t) = \cos w_{m,1}t \cos w_{m,2}t = \cos 1000\pi t \cos 3000\pi t = \frac{1}{2}\left(\cos 2\pi f_{m,1}t + \cos 2\pi f_{m,2}t\right) =$

$\frac{1}{2}(\cos 2000\pi t + \cos 4000\pi t) \rightarrow f_{m,1} = 1000Hz\ f_{m,2} = 2000Hz$

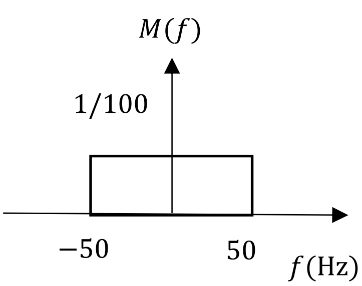


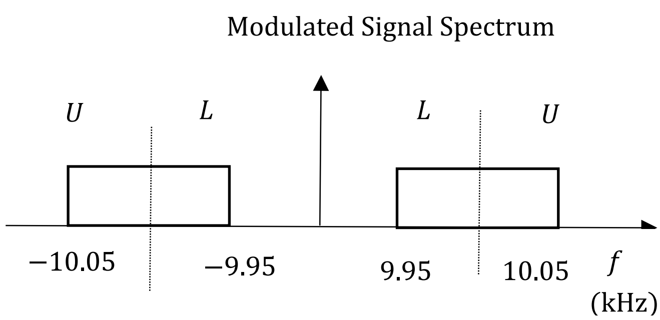


4. **Repeat Prob. 3 [parts (a), (b), and (c) only] if (i) $m(t) = \text{sinc}(100t)$; (ii) $m(t) = e^{-|t|}$; (iii) $m(t) = e^{-|t-1|}$. Observe that $e^{-|t-1|}$ is $e^{-|t|}$ delayed by 1 second. For the last case, you need to consider both the amplitude and the phase spectra.**

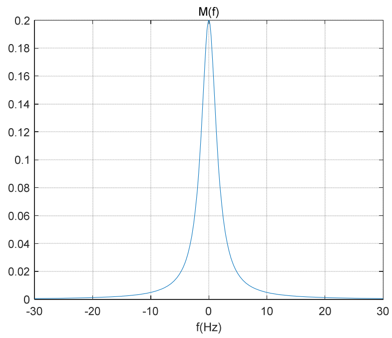
Solution:

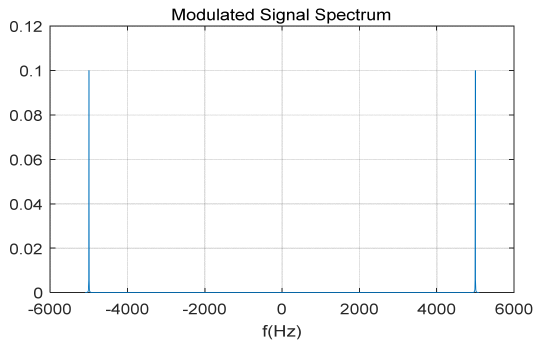
(i) $\mathcal{F}(m(t)) = \frac{1}{100} rect\left(\frac{f}{100}\right) = \begin{cases} 1/100, & |f| < 50 \\ 0, & otherwise \end{cases}$





(ii) $\mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2}$, $\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2} + \frac{10}{100+4\pi^2(f+5000)^2}$



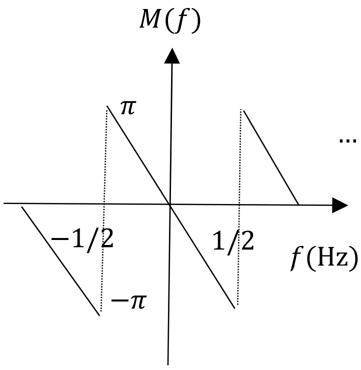


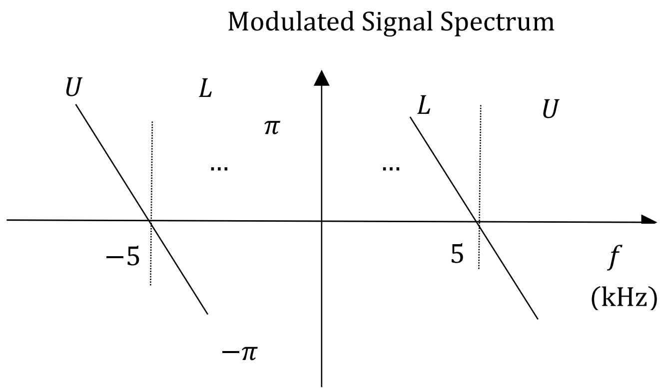
(iii) $\mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2} e^{-j2\pi f}$, $\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2} e^{-j2\pi(f-5000)} +$

$\frac{10}{100+4\pi^2(f+5000)^2} e^{-j2\pi(f+5000)}$

The amplitude spectra is the same as that in (ii), while the phase spectral is:

$\angle M(f) = -2\pi f$.





5. Sketch the AM signal $[A + m(t)]\cos(2\pi f_c t)$ for the periodic triangle signal $m(t)$ shown in Fig. P4.3-2 corresponding to the modulation indices (a) $\mu = 0.5$; (b) $\mu = 1$; (c) $\mu = 2$; (d) $\mu = \infty$; How do you interpret the case of $\mu = \infty$?

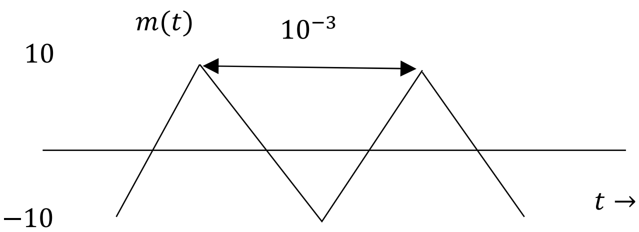
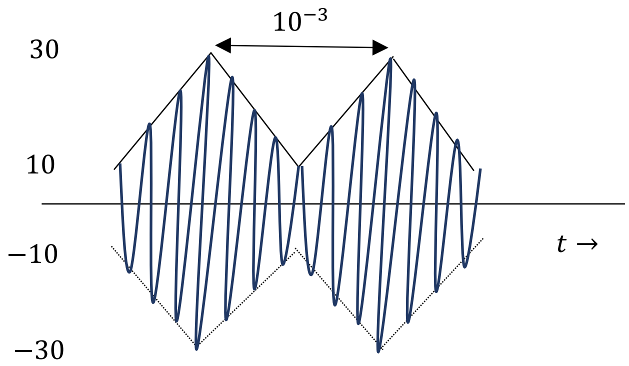


Figure P.4.3-2

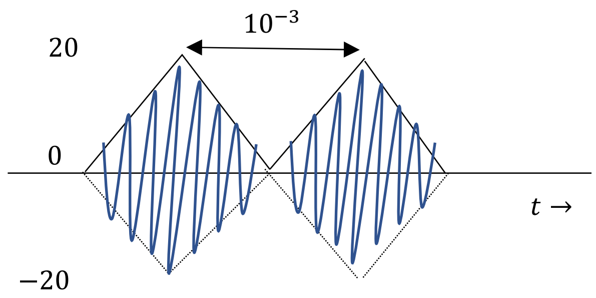
Solution:

As $m_{\min} = -m_{\max} = -10$, we have $\mu = \frac{m_p}{A}$.

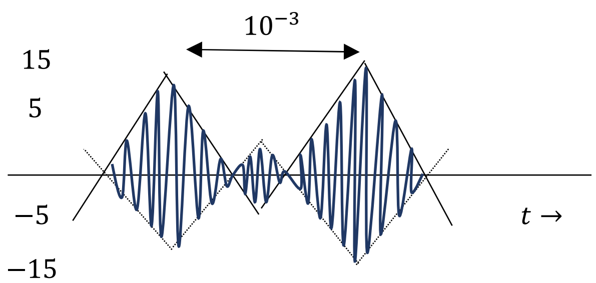
(a) $\mu = \frac{m_p}{A} = 0.5 \rightarrow A = \frac{m_p}{\mu} = \frac{10}{0.5} = 20$



(b) $\mu = \frac{m_p}{A} = 1 \rightarrow A = \frac{m_p}{\mu} = \frac{10}{1} = 10$



(c) $\mu = \frac{m_p}{A} = 2 \rightarrow A = \frac{m_p}{\mu} = \frac{10}{2} = 5$



(d) $\mu = \frac{m_p}{A} = \infty \rightarrow A = \frac{m_p}{\mu} = \frac{10}{\infty} = 0$

This means that $\mu = \infty$ represents the DSB-SC case.

6. For the AM signal with $m(t)$ shown in Fig. P.4.3-2 and $\mu = 0.8$:

(a) Find the amplitude and power of the carrier.

(b) Find the sideband power and the power efficiency η.

Solution:

(a) The carrier amplitude is $A = \frac{m_p}{\mu} = \frac{10}{0.8} = 12.5$. The carrier power is $P_c = \frac{A^2}{2} = \frac{12.5^2}{2} = 78.125$

(b) The sideband power is $\overline{m^2(t)}/2$. Because of symmetry of amplitude values every quarter cycle, the power of $m(t)$ may be computed by averaging the signal energy over a quarter cycle only. Over a quarter cycle $m(t)$ can be represented as $m(t) = 40t/T_0$. Note that $T_0 = 10^{-3}$ Hence,

$$\overline{m^2(t)} = \frac{1}{T_0/4}\int_0^{T_0/4} (\frac{40t}{T_0})^2 dt = 33.34$$

The sideband power is

$$P_s = \frac{\overline{m^2(t)}}{2} = 16.67$$

The efficiency is

$$\eta = \frac{P_s}{P_c+P_s} = \frac{16.67}{78.125 + 16.67} = 17.59\%$$