Questions and Answers for Week 5

Question: Will the midterm have the same number of questions as the sample questions given on blackboard? If not, how many will it have?

Answer: The midterm will have 10 questions.

Question: How do we justify $E\{\cos(2\pi f_c t + 2\Theta)\} = 0$ if Θ is uniform over $[0, 2\pi]$?

Answer: The average of a (deterministic) sinusoidal waveform within one or multiple periods is zero. Mathematically, it can be also demonstrated using the following

$$E\{\cos(2\pi f_c t + 2\Theta)\}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \cos(2\pi f_c t + \theta)\, d\theta$$

$$= \frac{1}{2\pi} \{-\sin(2\pi f_c t + \theta)\}|_0^{2\pi}$$

$$= 0$$

Question: I wonder when we do the expectation of $E\{X(t)\} = E\{A cos(2\pi f_c t) + B\sin(2\pi f_c t)\}$, is it correct that A and B can be treated with variables, therefore they are independent of cos(2pit) and sine(2pit)? So that we can just take E(A) and E(B) out explicitly.

Answer: Since $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are deterministic function, they can be moved outside the expectation, such as of $E\{A cos(2\pi f_c t)\} = E(A)\cos(2\pi f_c t)$ and $E\{B\sin(2\pi f_c t)\} = E(B)\sin(2\pi f_c t)$.

Question: I cannot deduce the correct answer for Q2 in the sample midterm test, could you provide the detail deduction process?

Answer: From the question, we can have

$$\begin{aligned}
E\{X(t)X(t+\tau)\} &= E\{(A\cos(2\pi t) - B\sin(2\pi t))(A\cos(2\pi(t+\tau)) - B\sin(2\pi(t+\tau)))\} \\
&= E\{A\cos(2\pi t)A\cos(2\pi(t+\tau)) - A\cos(2\pi t)B\sin(2\pi(t+\tau)) \\
&\quad -B\sin(2\pi t)A\cos(2\pi(t+\tau)) + B\sin(2\pi t)B\sin(2\pi(t+\tau))\} \\
&= E\{A^2\}\cos(2\pi t)\cos(2\pi(t+\tau)) - E\{AB\}\cos(2\pi t)\sin(2\pi(t+\tau)) \\
&\quad - E\{BA\}\sin(2\pi t)\cos(2\pi(t+\tau)) + E\{B^2\}\sin(2\pi t)\sin(2\pi(t+\tau)) \\
&= 1\times\cos(2\pi t)\cos(2\pi(t+\tau)) - 0\times\cos(2\pi t)\sin(2\pi(t+\tau)) \\
&\quad - 0\times\sin(2\pi t)\cos(2\pi(t+\tau)) + 1\times\sin(2\pi t)\sin(2\pi(t+\tau)) \\
&= \cos(2\pi t)
\end{aligned}$$

Question: For Q16 in the sample midterm test, I am not sure how to confirm options a and c are correct, since the only equation in my mind for quantization is the SNR for the output. Is there any equation for quantization error and its average power?

Answer:

Q16 is a little bit tricky.

Option a is based on the expression of SNR_0 on slide 11 of Lecture 8, $SNR_o = 6n + \log_{10}\left(\frac{3P}{m_p^2}\right)$.

For many analogue message signals, such as speech signals, the probability to have a small amplitude is large while the probability to have a large amplitude is small. Non-uniform quantization (or companding) can reduce the quantization error for a sample with a small amplitude, which is with a large probability, even if it increases the quantization error for a sample with a large amplitude, which is with a small probability. As a result, the average power of the quantization error will be reduced. Therefore, Option c is correct.

For uniform quantization, quantization error and its average power are discussed on Slide 9 of Lecture 8. For non-uniform quantization, quantization error and its average power depend on the companders and the probability distribution of the sampling signals. There are no simple closed-form expressions.

Questions: could you give a brief explanation for why quantization error would decrease for small signal but increase for large signal, me and my classmates are confused with these problems?

Answer: As demonstrated on Slide 17 of Lecture, nonuniform quantization uses quantization levels of variable spacing, denser at small signal amplitudes, which reduces the quantization error and more often to happen, broader at large amplitudes, which even increase the quantization error but less often happen.