1\. a) $P_T = 10W$. $P_R = \frac{10}{10^4} = 10^{-3} W$

$$P_N = 2\int_0^{10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 2N_0\cdot10^4 - 2N_0\cdot\frac{1}{40}\cdot10^4 = 1.95\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 51.28 = 17.1 dB$$

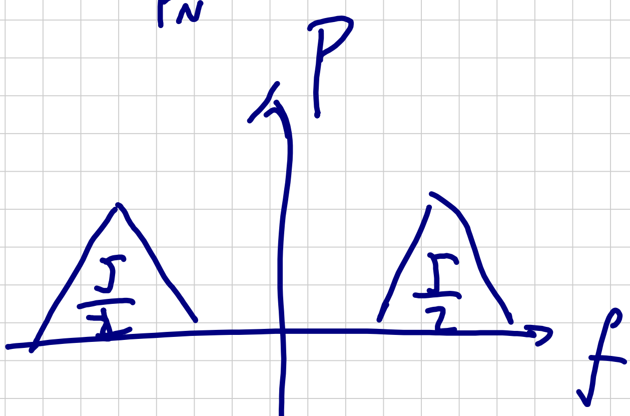
b) $P_T = \frac{A_c^2 P}{2} = 5W$. $P_R = 5\times10^{-4} W$.

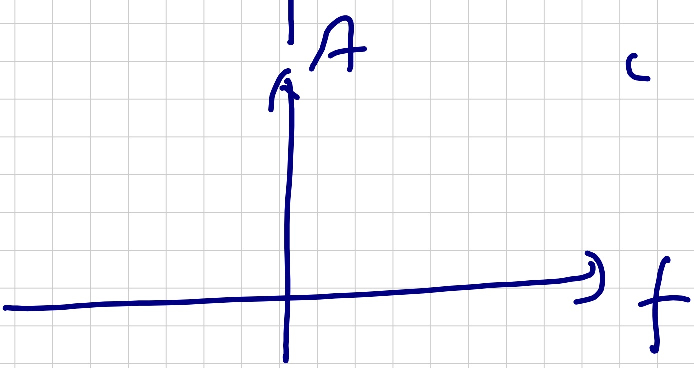
$$P_N = 2\int_{f_c-10^4}^{f_c+10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 4N_0\cdot10^4 - 2N_0\cdot10^4 = 2\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 25 = 14 dB.$$

c) $$P_N = 2\int_{f_c-10^4}^{f_c+10^4} N_0 \left(1 - \frac{f}{200\times10^3}\right) df = 4N_0\cdot10^4 - 3N_0\cdot10^4 = 1\times10^{-5} W$$

$$SNR = \frac{P_R}{P_N} = 50 = 17 dB.$$





2. For a small noise,

$$SNR_{env} \approx \frac{P}{A^2+P} SNR_{baseband}$$

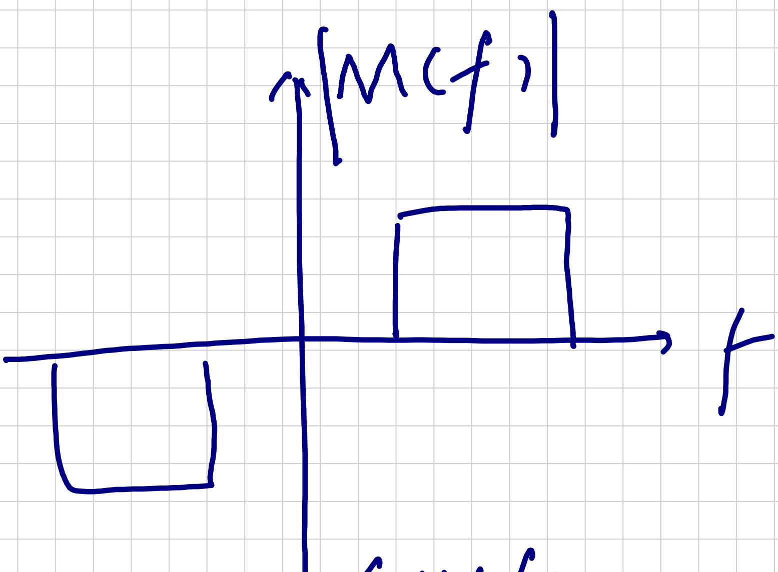
$$= \frac{\frac{1}{2}m_p^2}{A^2+\frac{1}{2}m_p^2} SNR_{baseband} \qquad \mu = \frac{m_p}{A} \le 1$$

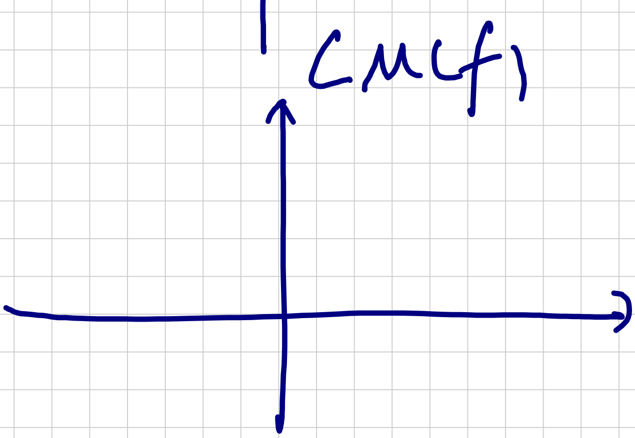
$$\Rightarrow m_p^2 = A^2\mu^2$$

$$= \frac{\mu^2}{2+\mu^2} SNR_{baseband}$$

Modulation index $\mu = \frac{m_p}{A} \le 1$

$$\begin{cases} (SNR_{env})_{max} = \frac{1}{3} SNR_{baseband}. & \mu = 1 \\ SNR_{env} \to 0 & , \mu \to 0 \end{cases}$$





3.

$$\cos 2\pi f_c t \iff \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

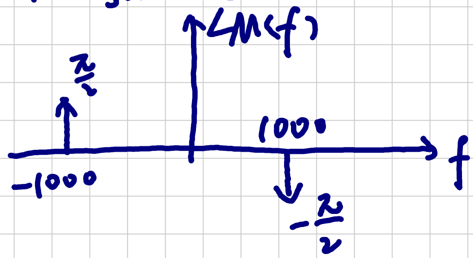
$$\sin 2\pi f_c t \iff \frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)] = \frac{1}{2}[-j\delta(f-f_c)+j\delta(f+f_c)]$$

a)

$$m(t) = 2\cos 1000\pi t + \sin 2000\pi t$$

$$M(f) = \delta(f-500)+\delta(f+500) + \frac{1}{2}\left\{-j\delta(f-1000)+j\delta(f+1000)\right\}$$

$$\Rightarrow \angle M(f) = \begin{cases} -\frac{\pi}{2}, & f=1000 \\ \frac{\pi}{2}, & f=-1000 \\ 0, & \text{otherwise} \end{cases}$$



b) DSB-SC: $x_D(t) = m(t)\cos 2\pi f_c t$

AM: $x_A(t) = (A+m(t))\cos 2\pi f_c t$

$x(t)$:

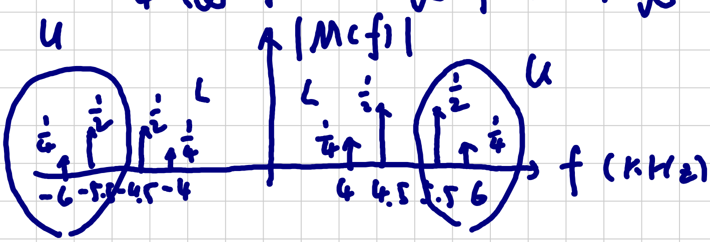
$$m(t)\cos 10000\pi t = 2\cos 1000\pi t\cdot\cos 10000\pi t + \sin 2000\pi t\cos 10000\pi t$$

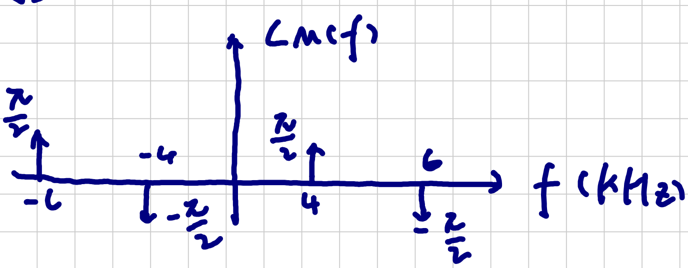
$$= \cos 11000\pi t + \cos 9000\pi t + \frac{1}{2}(\sin 12000\pi t + \sin(-8000\pi t))$$

$$-\sin 8000\pi t$$

$$X(f) = \frac{1}{2}(\delta(f-5500)+\delta(f+5500)+\delta(f-4500)+\delta(f+4500))$$

$$+\frac{1}{4}(-j\delta(f-6000)+j\delta(f+6000)+j\delta(f-4000)-j\delta(f+4000))$$

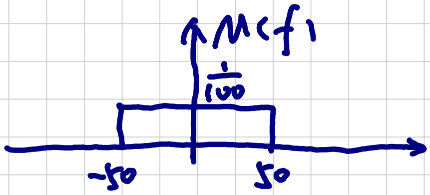


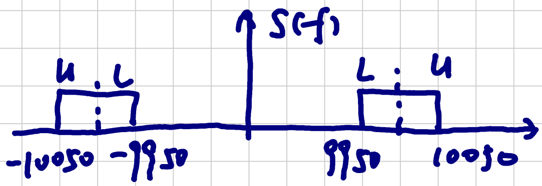


4.

(i) $m(t) = \text{sinc}\, 100t$

$$\mathcal{F}(m(t)) = \frac{1}{100}\,\text{rect}\left(\frac{f}{100}\right) = \begin{cases} \frac{1}{100}, & |f| < 50 \\ 0, & \text{otherwise} \end{cases}$$





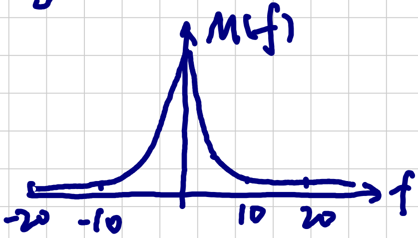
(ii) $m(t) = e^{-10t}u(t) + e^{10t}u(-t)$

$$\mathcal{F}(m_1(t)) = \int_0^\infty e^{-(10+j2\pi f)t}\,dt = -\frac{1}{10+j2\pi f}e^{-(10+j2\pi f)t}\Big|_0^\infty = \frac{1}{10+j2\pi f}$$

$$\mathcal{F}(m_2(t)) = \int_{-\infty}^0 e^{-(j2\pi f-10)t}\,dt = -\frac{1}{j2\pi f-1}e^{-(j2\pi f-10)t}\Big|_0^\infty = \frac{1}{10-j2\pi f}$$

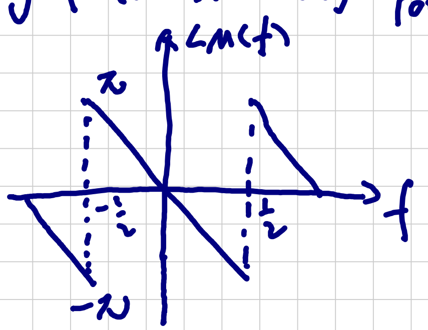
$$\therefore \mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2}$$

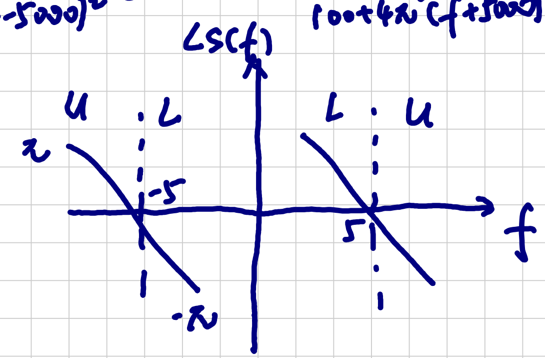
$$\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2} + \frac{10}{100+4\pi^2(f+5000)^2}$$



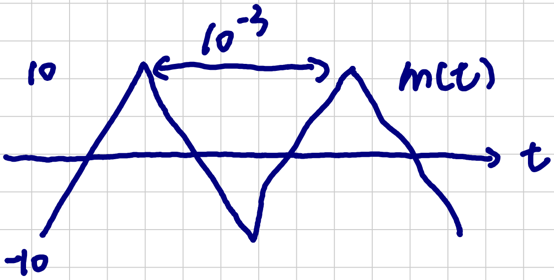
(iii) $\mathcal{F}(m(t)) = \frac{20}{100+4\pi^2 f^2}\cdot e^{-j2\pi f}$

$$\mathcal{F}(m(t)\cos 10000\pi t) = \frac{10}{100+4\pi^2(f-5000)^2}e^{-j2\pi(f-5000)} + \frac{10}{100+4\pi^2(f+5000)^2}e^{-j2\pi(f+5000)}$$

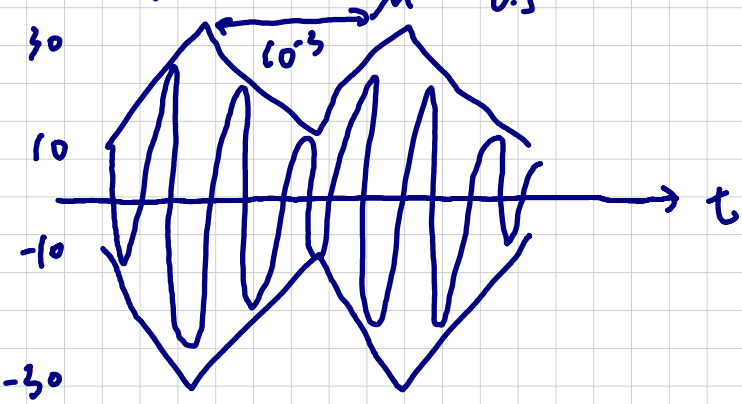




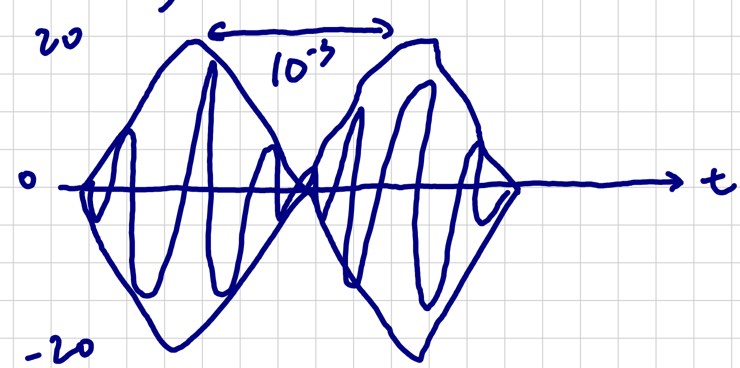
5.



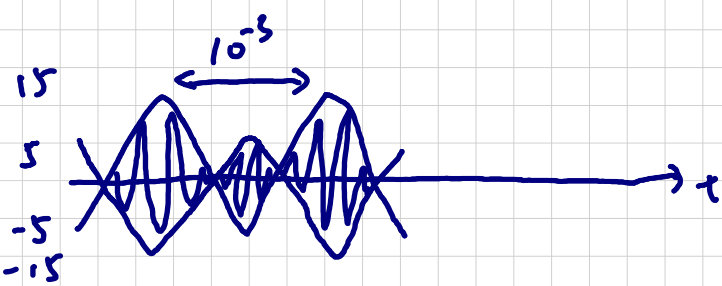
a) $\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu} = \frac{10}{0.5} = 20$



b) $A = \frac{m_p}{\mu} = 10$



c) $A = \frac{m_p}{\mu} = 5$



d) $A = \frac{m_p}{\mu} \to 0$

$(A+m(t))\cos 2\pi f_c t \to m(t)\cos 2\pi f_c t$: DSB-SC

6. a) $A = \frac{m_p}{\mu} = \frac{10}{0.8} = 12.5$

$P_c = \frac{A^2}{2} = 78.125$

b) In the first $\frac{1}{4}$ period, we have $m(t) = \frac{40t}{T_0}$

$$\overline{m^2(t)} = \frac{1}{\frac{T_0}{4}} \int_0^{\frac{T_0}{4}} \left(\frac{40t}{T_0}\right)^2 dt = 33.34$$

Sideband power $P_s = \frac{1}{2}\overline{m^2(t)} = 16.67$

$$\eta = \frac{P_s}{P_c + P_s} = \frac{16.67}{78.125 + 16.67} = 17.59\%$$

$\hat{u}(t)$

-90° shift to $u(f)$, $f>0$

90° - - - to $f<0$

$\boxed{j \cdot \hat{u}(t)}$

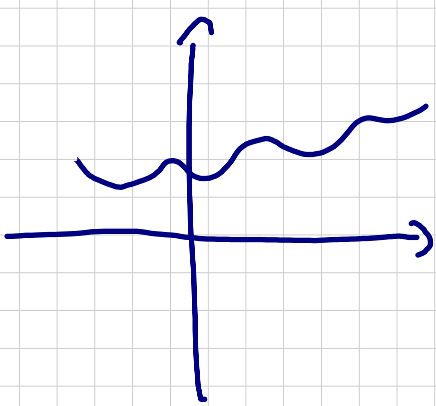
0° shift to $u(f)$, $f>0$

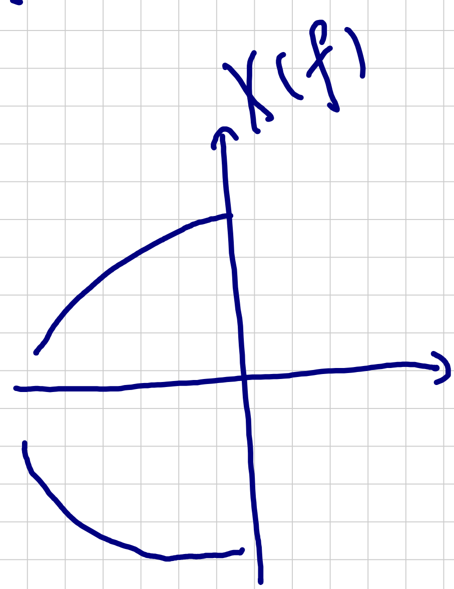
180° - - - - - $f<0$

$$\underbrace{u(t)}+j\underbrace{\hat{u}(t)} = u_+(t)$$

doubled

$f>0$





$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$

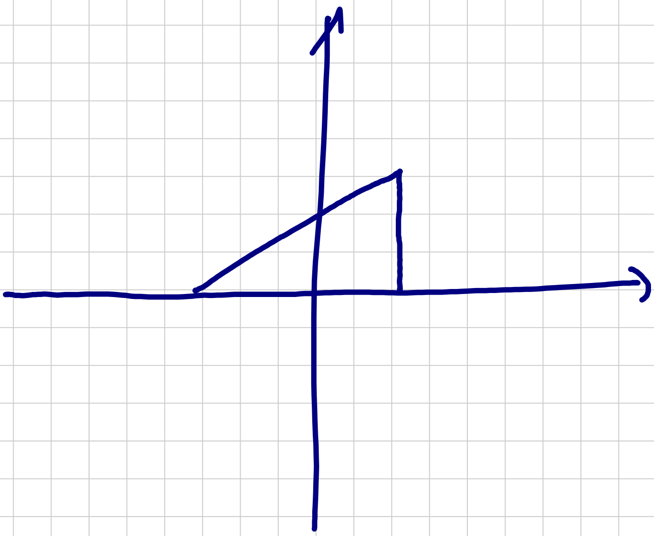
$$j \cdot H(f) = \begin{cases} -j^2 = 1, & f > 0 \\ j^2 = -1, & f < 0 \end{cases}$$

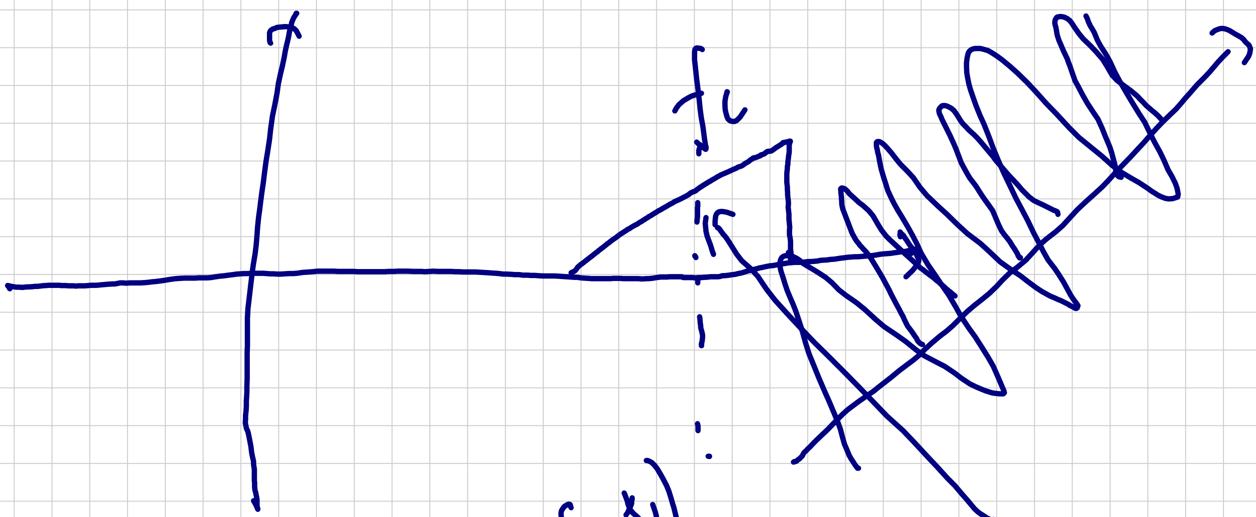
$$U_+(f) = U(f) + j \cdot \underbrace{H(f) \cdot U(f)}_{\hat{U}(f)}$$

$$U_+(f) = U(f) + \operatorname{sgn}(f) \cdot U(f)$$

$u_+(t)$

$u(t)\cdot \exp(j2\pi f_c t)$





DSB $\rightarrow m(t)\cdot\cos(2\pi f_c t)$

AM: $\boxed{(A+m(t))}\cdot\cos(2\pi f_c t)$

$$R_{XY}(t,u) = \mathbb{E}\{X(t)\,Y(u)\}$$

$$= \mathbb{E}\left\{X(t)\int X(u-\tau_2)\,h(\tau_2)\,d\tau_2\right\}$$

$$= \int \mathbb{E}\{X(t)\,X(u-\tau_2)\}\,h(\tau_2)\,d\tau_2$$

$$= \int R_{XX}(t,\,u-\tau_2)\,h(\tau_2)\,d\tau_2$$

$$= R_{XX}(t,u) * h(u)$$

$$R_{YY}(t,u) = \mathbb{E}\{Y(t)\,Y(u)\}$$

$$= \mathbb{E}\left\{\int X(t-\tau_1)\,h(\tau_1)\,d\tau_1\,Y(u)\right\}$$

$$= \int \mathbb{E}\{X(t-\tau_1)\,Y(u)\}\,h(\tau_1)\,d\tau_1$$

$$= \int R_{XY}(t-\tau_1,\,u)\,h(\tau_1)\,d\tau_1$$

$$= R_{XY}(t,u) * h(t)$$

$$= R_{XX}(t,u) * h(u) * h(t)$$

$$\underbrace{\iint R_{xx}(\tau_1,\tau_2)\,h(t-\tau_1)\,d\tau_1}_{R_{xx}(t,\tau_2)*h(t)}\,h(u-\tau_2)\,d\tau_2$$

$$=\underbrace{\int R_{xx}(t,\tau_2)\,h(u-\tau_2)\,d\tau_2}_{R_{xx}(t,u)*h(u)} * h(t)$$

$$f(t)*g(t) = \int f(\tau)\,g(t-\tau)\,d\tau$$

$$= \int f(t-\tau)\,g(\tau)\,d\tau$$