Elementary Information Theory (Lectures 12 & 13)

1. **Consider a source having an *M*=4 symbol alphabet where $P(x_1) = 1/2;\ P(x_2) = 1/4\ P(x_3) = P(x_4) = 1/8$ and symbols are statistically independent.**
   a) **Calculate the information conveyed by the receipt of the symbol $x_1, x_2\ , x_3$. and x_4.**
   b) **What is the source entropy?**

2. **For the source in Problem 1, find the corresponding codewords for different symbols if Huffman coding is used.**

3. **Find the capacity of an AWGN channel with a bandwidth B = 1 MHz, signal power of 10W and noise power-spectral density of N0/2 = 10^{-9} W/Hz.**

4. **A binary channel matrix is given by**

$$\begin{array}{cc} & \text{outputs} \\ & \begin{array}{cc} y_1 & y_2 \end{array} \\ \text{inputs} \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{pmatrix} \end{array}$$

   This means $P_{y|x}(y_1|x_1) = 2/3$, $P_{y|x}(y_2|x_1) = 1/3$, etc. You are also given that $P_x(x_1) = 1/3$ and $P_x(x_2)=2/3$.
   a. **Determine $H(x)$, $H(x|y)$, $H(y)$, $H(y|x)$, and $I(x;y)$.**
   b. **How to determine the capacity of the channel?**

5. **Obtain the codewords of the (7; 4) Hamming code using the following generator matrix**

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

   and show that the minimum distance of this code is dmin = 3.