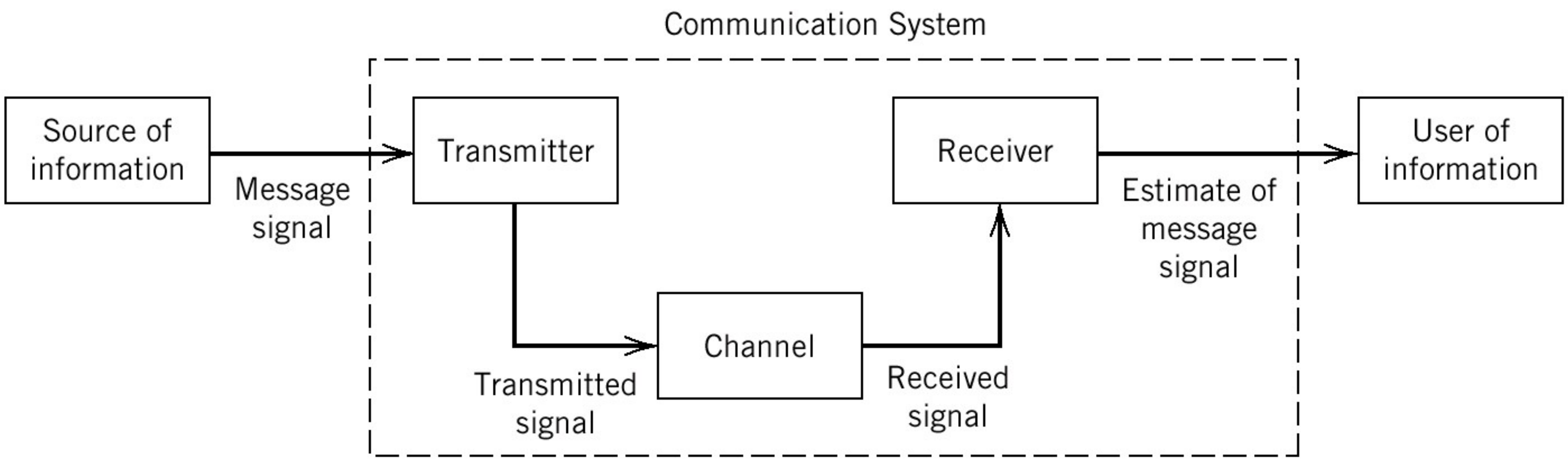
What is Communication?

- **Communication**: transmission of information from one point to another.
- **Four Basic Elements**
  - **Information source:** voice, music, picture, video, …
  - **Transmitter**: converts information in the source into a form suitable for transmission over the channel
  - **Channel**: the physical medium, introduces distortion, noise, interference
  - **Receiver**: reconstruct a recognizable form of the source signal



Energy, Average Power, Bandwidth

Energy:

$$\boxed{E = \int_{-\infty}^{+\infty} |s(t)|^2 \, dt = \int_{-\infty}^{+\infty} |S(f)|^2 \, df}$$

Average Power = time average of energy, computed over a large interval

$$P = \frac{1}{T} \int_{-T/2}^{+T/2} |s(t)|^2 \, dt$$

Average Power of a Random Signal: Assemble average as following

$$P = E\left\{s^2(t)\right\} = \int_{-\infty}^{\infty} x^2 f_{s(t)}(x,t) dx$$

Average Power of a Signal consisting of random and deterministic components:
Both time and assemble average

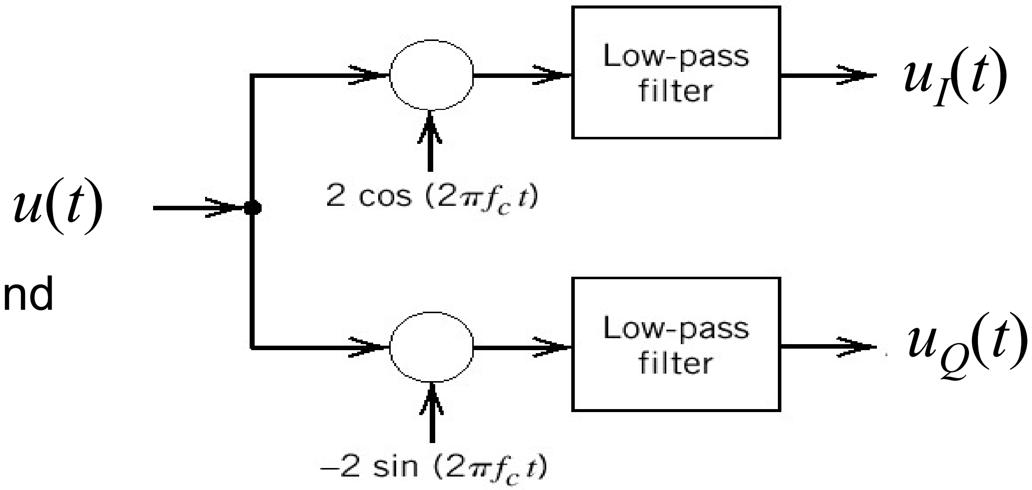
One-sided bandwidth: only consider positive frequencies when computing bandwidth for ***physical*** **(real) signals**

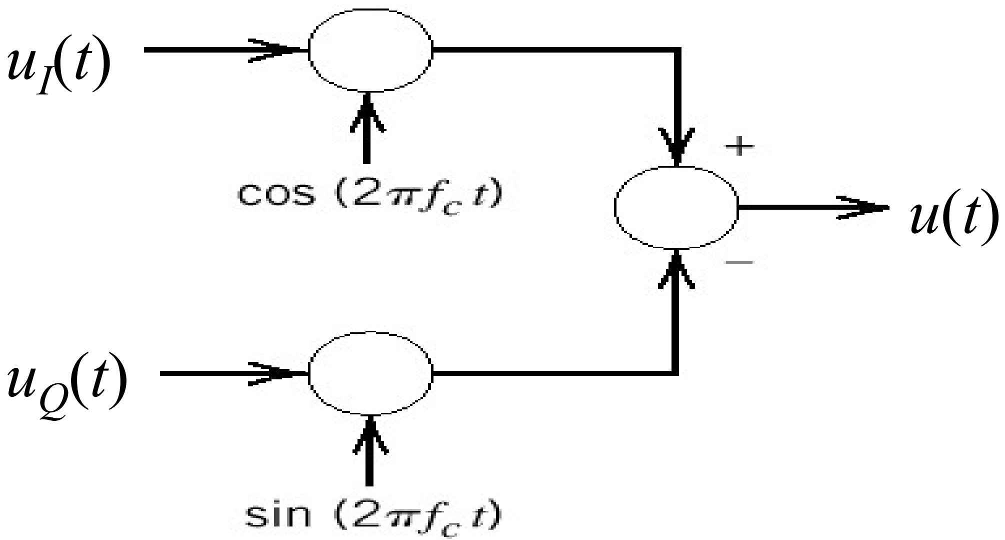
Complex-valued (in the time domain) signals: Complex envelope of a real-valued passband signal. The two-sided bandwidth of the complex envelope equals the physical (one-sided) bandwidth of the passband signal

Baseband and Passband Signals/Channels

Up-Conversion:

baseband → passband



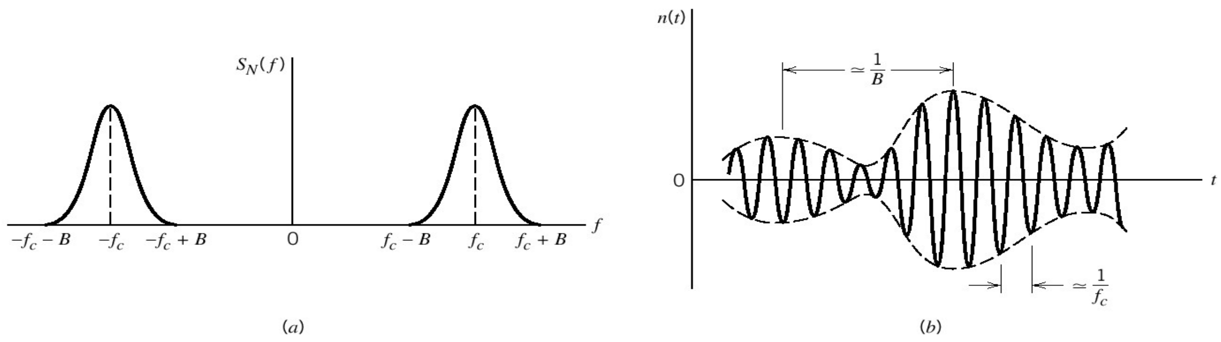


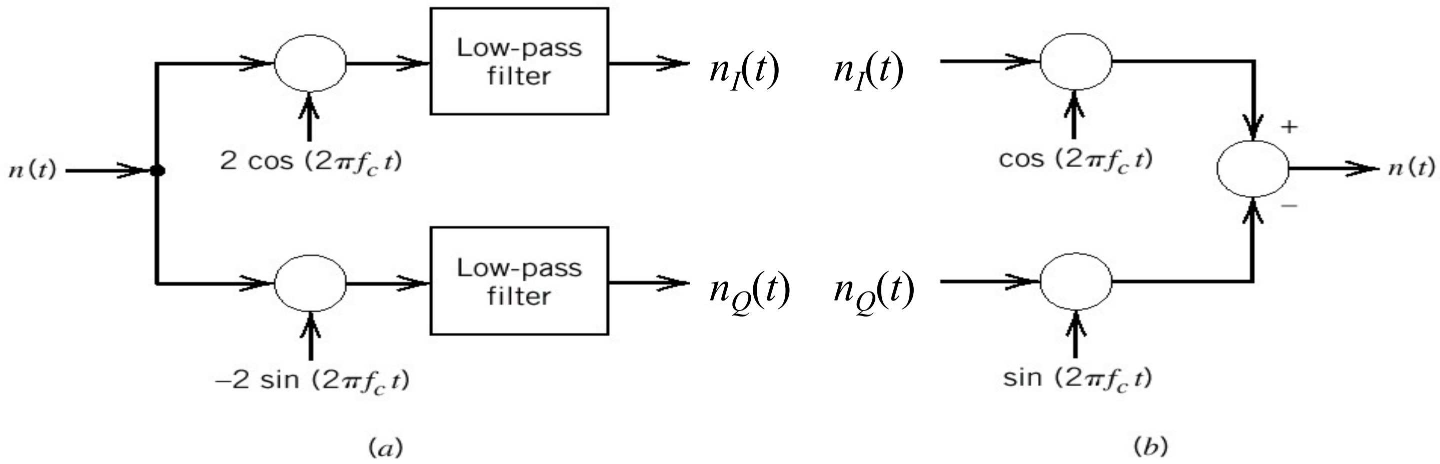
Down-Conversion:

passband → baseband

Band-pass Noise

- n(t) in canonical form: $n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$
- $n_I(t)$ and $n_Q(t)$ are fully representative of the band-pass noise.
  - Given band-pass noise, one may extract in-phase and quadrature components (using LPF of bandwidth *B*).
  - Given the two components, one may generate band-pass noise. This is useful in computer simulation.





Properties of Baseband Noise

- Usually noise $n(t)$ has zero mean, then so do $n_I(t)$ and $n_Q(t)$.

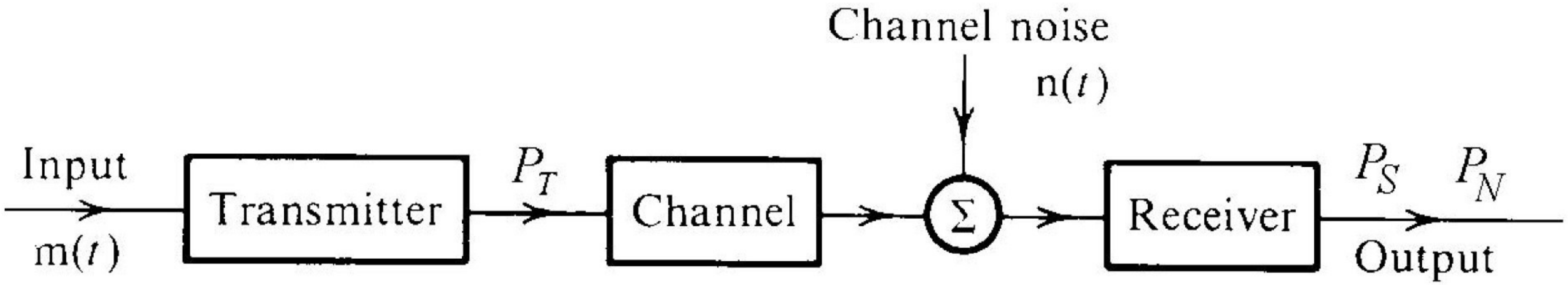
- $n_I(t)$ and $n_Q(t)$ have the same variance (i.e., same power) as $n(t)$

- If noise $n(t)$ is Gaussian, then so are $n_I(t)$ and $n_Q(t)$ and
  the envelope $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$ is Rayleigh distribution.

- Both in-phase and quadrature components have the same PSD:

$$S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & |f| \leq B \\ 0, & \text{otherwise} \end{cases}$$

Analog Communication Systems



Model of an analog communication system

Signal-to-Noise Ratio (SNR) at the output of the receiver:

$$SNR_o \equiv \frac{\text{average power of message signal at the receiver output}}{\text{average power of noise at the receiver output}} = \frac{P_S}{P_N}$$

Normally expressed in decibels (dB): SNR (dB) = 10 $\log_{10}$(SNR)

A Baseband Communication System

- It does not use modulation
- Transmit power is identical to message power:

$$P_T = P$$

- If the unit channel gain or no propagation loss, then

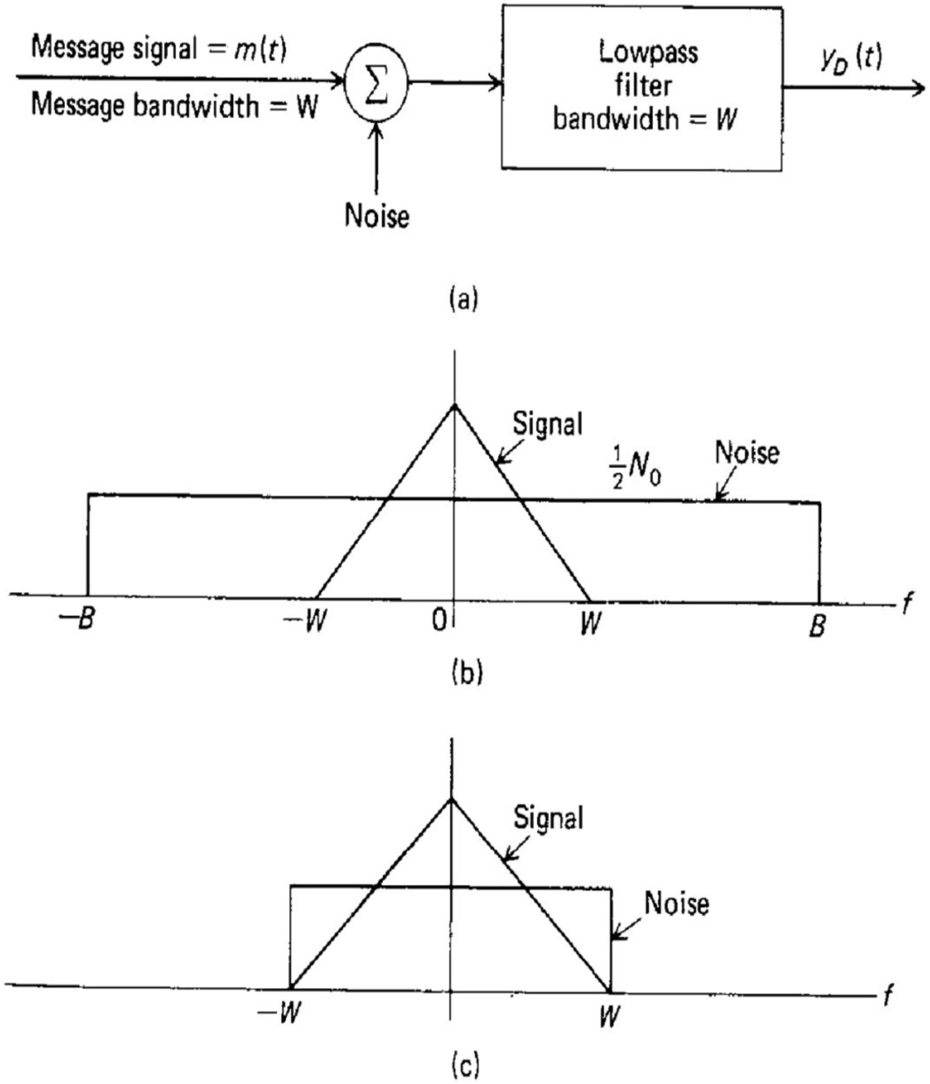
$$P_S = P_T = P$$

- Average noise power at receiver

$$P_N = 2W \times N_0/2 = WN_0$$

- SNR at receiver output:

$$\boxed{SNR_{\text{baseband}} = \frac{P_T}{N_0 W}}$$



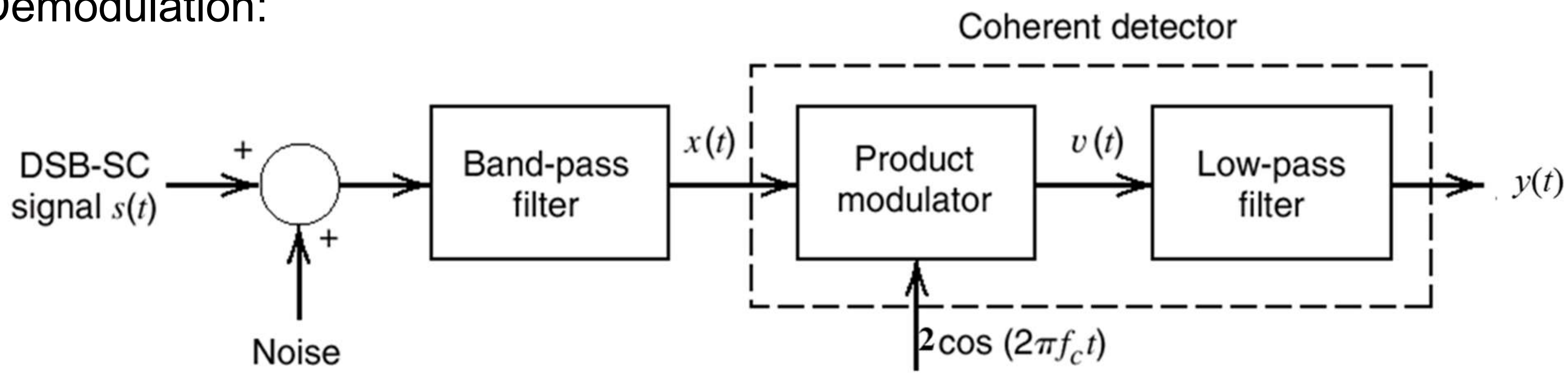
Double Sideband-Suppressed Carrier (DSB-SC) Modulation

- General form of a DSB-SC signal (suppressed carrier AM):

$$s(t) = m(t) A \cos(2\pi f_c t)$$

  - A: amplitude of the carrier; f_c: carrier frequency
  - $m(t)$: message signal with bandwidth W and average power, P
  - s(t): DSB modulated signal: bandwidth $2\mathrm{W}$, average power $P_T = \frac{1}{2} A^2 P$

- Demodulation:



  - Bandwidth of band-pass filter: $2\mathrm{W}$
  - Bandwidth of the low-pass filter: W
  - SNR at the receiver output:

$$SNR_{baseband} = \frac{P_T}{N_0 W} \quad \Rightarrow \quad SNR_{DSB-SC} = SNR_{baseband}$$

Single Sideband (SSB) Modulation

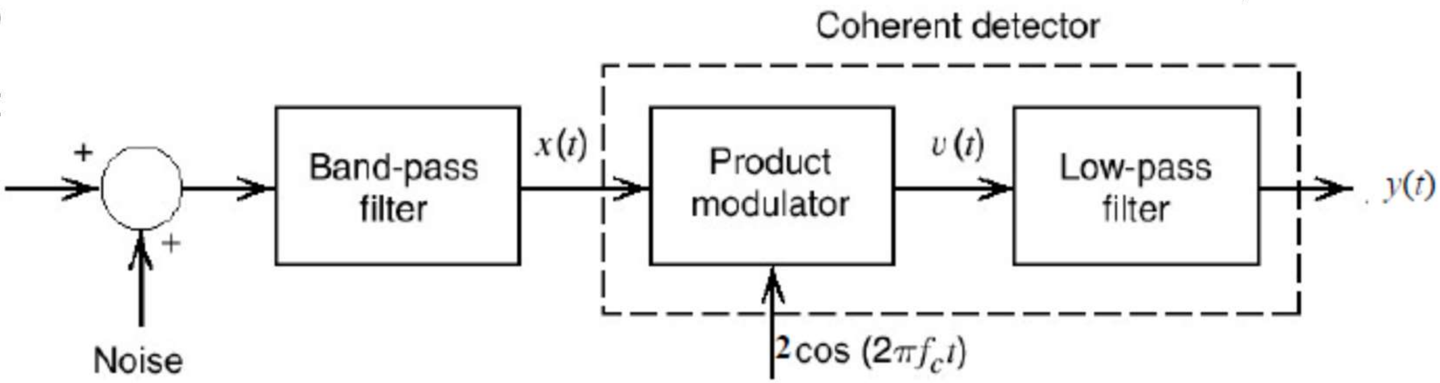
- General form of a SSB-SC signal:

$$s(t) = \frac{A}{2} m(t)\cos(2\pi f_c t) - \frac{A}{2}\hat{m}(t)\sin(2\pi f_c t)$$

  - A: amplitude of the carrier; f_c: carrier frequency
  - $m(t)$: message signal with bandwidth W and average power, P
  - s(t): SSB modulated signal: bandwidth W and average power $P_T = \frac{1}{4}A^2 P$
- Demodulatio

SSB Modulat

Signal s(t)



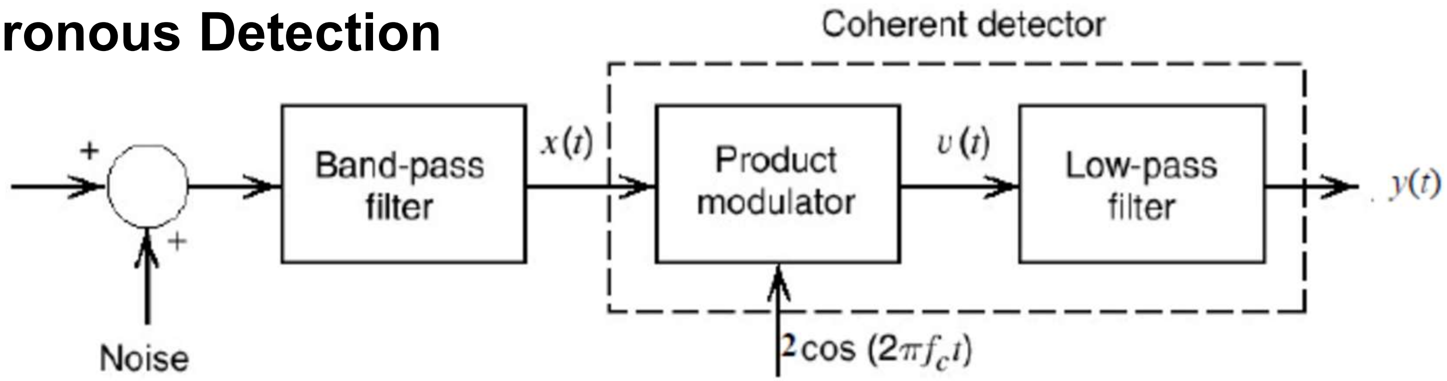
  - Bandwidth of band-pass filter: W
  - Bandwidth of the low-pass filter: W
  - SNR at the receiver output:

$$SNR_{SSB} = SNR_{baseband} = SNR_{DSB-SC}$$

Standard AM

- **Standard AM**: $s_{AM}(t) = [A + m(t)]\cos(2\pi f_c t)$
  - Usually $A \geq \mathrm{m_p} = \max|m(t)|$, modulation index $\mu = \frac{m_p}{A} \leq 1$
  - $m(t)$: message signal with bandwidth W and average power, P
  - AM modulated signal: bandwidth $2\mathrm{W}$ and average power $\dfrac{A^2 + P}{2}$

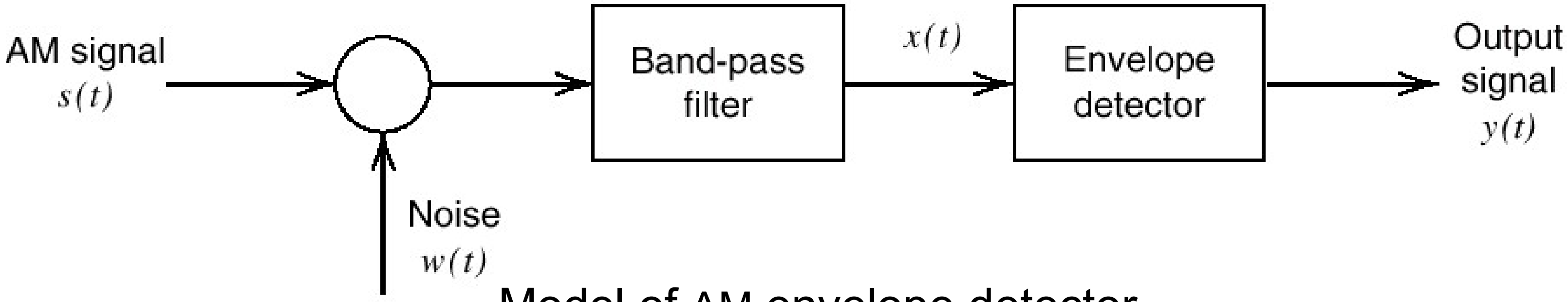
- **Synchronous Detection**



  - Bandwidth of band-pass filter: $2\mathrm{W}$
  - Bandwidth of the low-pass filter: W
  - SNR at the receiver output: $SNR_{AM} = \dfrac{P}{A^2 + P} SNR_{baseband}$

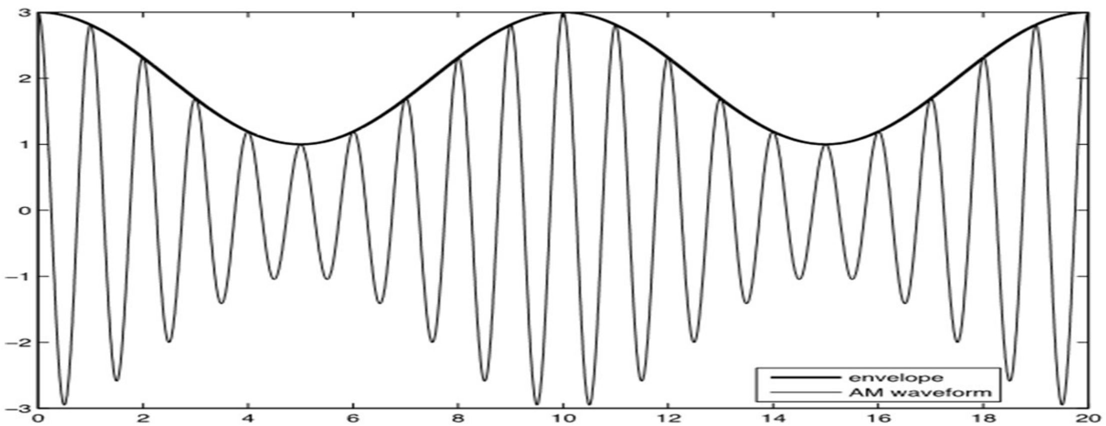
Non-coherent Receiver

- Receiver:



Model of AM envelope detector

  – Bandwidth of band-pass filter: 2W
  – $\mu = \frac{m_p}{A} \leq 1$, or $m_p \leq A$
  – SNR at the receiver output for small noise case: $SNR_{env} \approx \frac{P}{A^2 + P} SNR_{baseband}$



Frequency Modulation

- Instantaneous frequency is varied linearly with message:

$$f_i(t) = f_c + k_f m(t)$$

  - k_f is the frequency sensitivity of the modulator.
  - frequency deviation: $\Delta f = k_f m_p$
  - deviation ratio/modulation index:

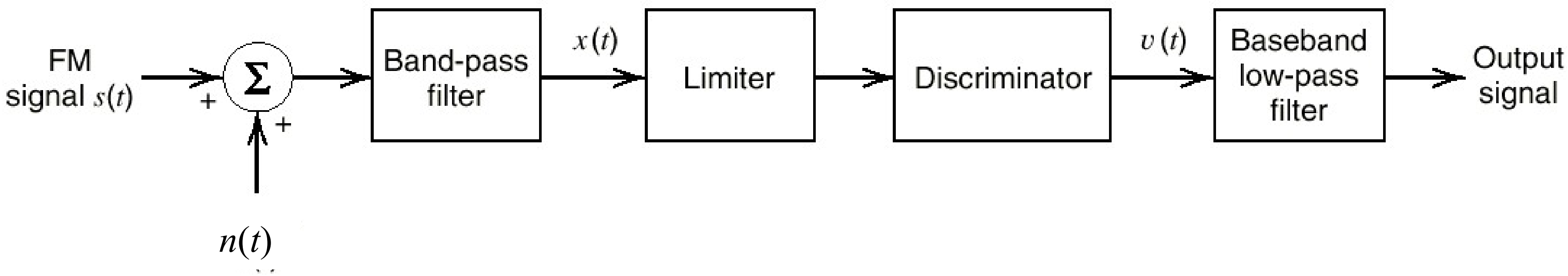
$$\beta = \Delta f / W,$$ W: message bandwidth

- Modulated signal:

$$s(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

  - The envelope is constant
  - Signal s(t) is a non-linear function of the message signal m(t)
  - Bandwidth using Carson's rule of thumb: $B_T = 2W(\beta+1) = 2(\Delta f + W)$

FM Receiver

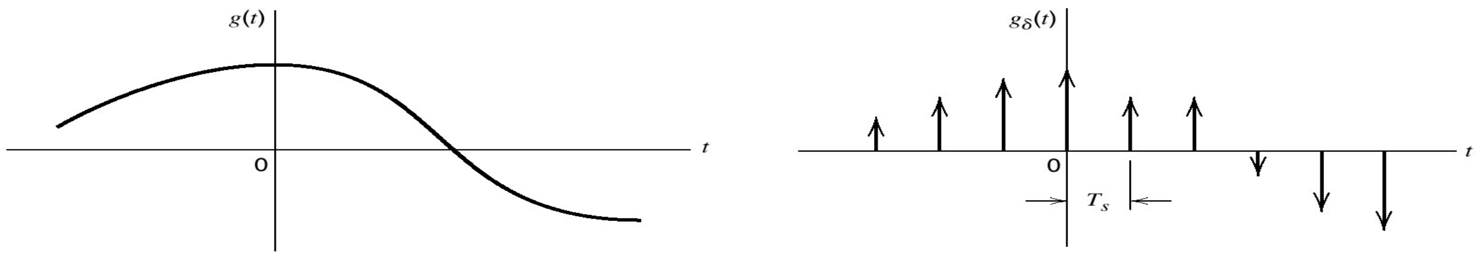


- Bandwidth of bandpass filter: $B_T = 2W(\beta+1) = 2(\Delta f + W)$
- Discriminator: instantaneous amplitude is proportional to instantaneous frequency
- Bandwidth of baseband low-pass filter: W
- SNR at the receiver output:

$$SNR_{FM} = 3\beta^2 \frac{P}{m_p^2} SNR_{baseband}$$

Sampling, Quantization, PCM

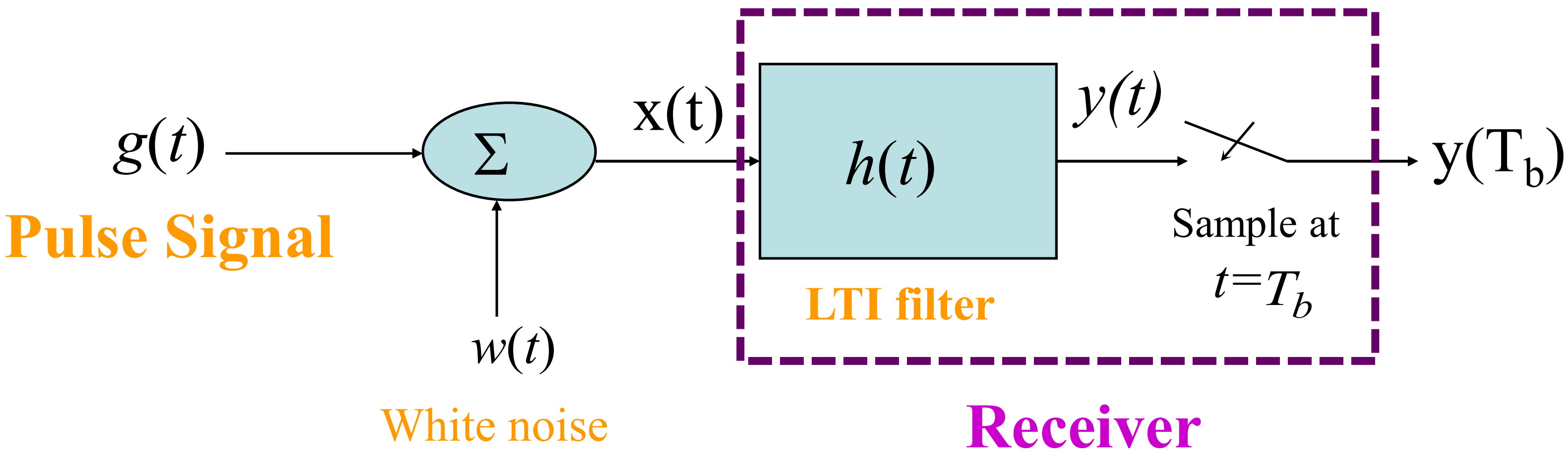
- Sampling Theorem: A signal whose spectrum is band-limited to W Hz, can be reconstructed exactly from its samples if they are taken uniformly at a rate of $R \geq 2W$ Hz.
- Nyquist frequency: $f_s = 2W$ Hz



(a) (b)

- Quantization
  - Uniform quantization: $SNR_o(\text{dB}) = 6n + 10\log_{10}\left(\dfrac{3P}{m_p^2}\right)$ (dB)
  - Non-uniform quantization: improving SNR
- PCM: Pulse code modulation
- Grey Mapping
- Line Code

Matched Filter



$$x(t) = g(t) + w(t) \qquad 0 \le t \le T_b$$

- w(t): white noise with zero mean and PSD $\dfrac{N_o}{2}$
- Receiver wants to detect the pulse in presence of additive noise
  - Receiver knows what pulse shape it is looking for
- Goal: Design a receive (linear) filter that minimizes the effect of noise
  - Optimize the design of the filter

Properties of Matched Filters

- Impulse response is

$$h_{\text{opt}}(t) = k\, g(T_b - t)$$

T_b **:** symbol period,

$g(t)$**:** transmitter pulse shape, k: gain

  - scaled, time-reversed and shifted version of $g(t)$
  - duration and shape determined by pulse shape $g(t)$

- Maximizes peak pulse SNR

$$\eta_{\max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2\, df \;=\; \frac{2}{N_0} \int_{-\infty}^{\infty} |g(t)|^2\, dt = \frac{2E}{N_0} = \text{SNR}$$

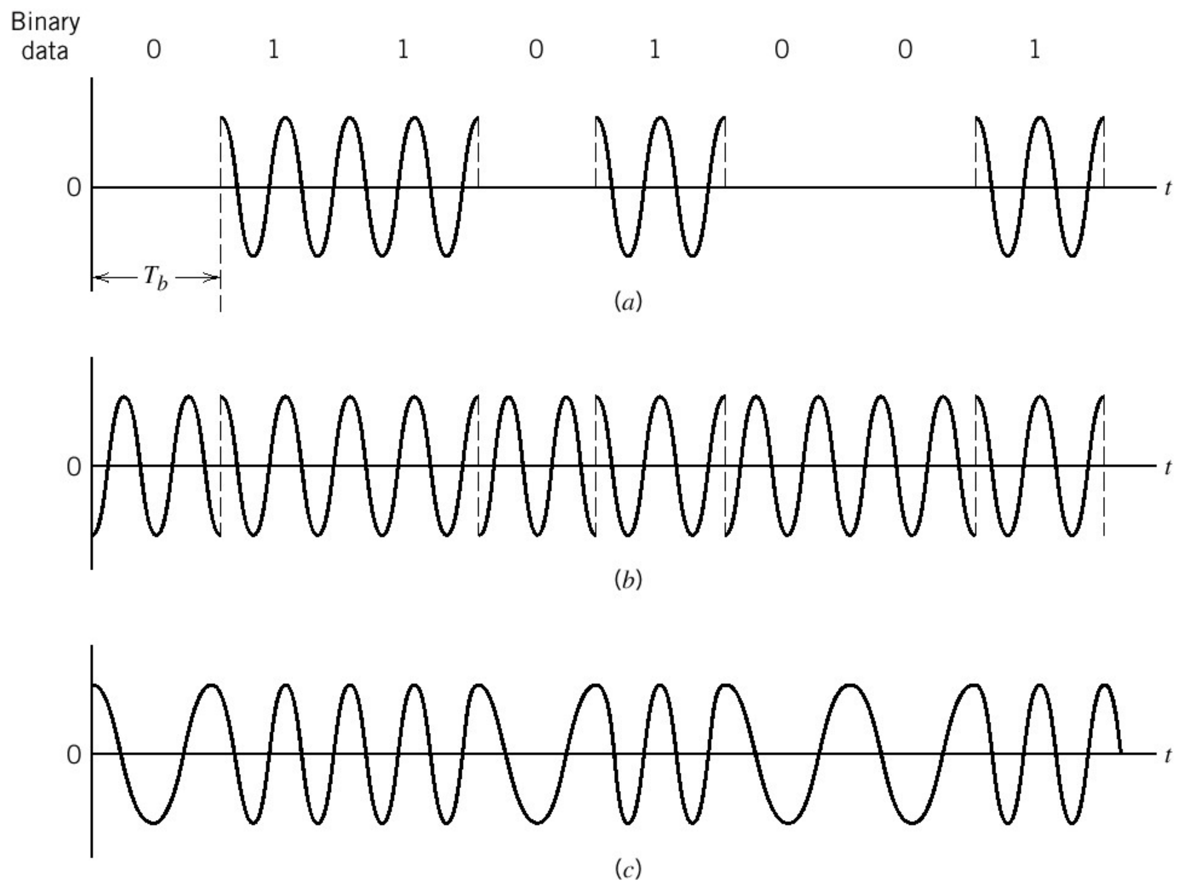
  - does not depend on pulse shape $g(t)$
  - proportional to signal energy (energy per bit) E
  - inversely proportional to noise power spectral density

Basic Forms: ASK, PSK, and FSK

(a) Amplitude-shift keying (ASK).

(b) Phase-shift keying (PSK).

(c) Frequency-shift keying (FSK).



Performance Comparison

Scheme	Bit-Error Rate (BER)
Coherent ASK	$Q\left(\frac{A}{2\,\sigma}\right)=Q(\sqrt{\frac{E_b}{N_o}})$
Coherent FSK	$Q\left(\frac{A}{\sqrt{2}\sigma}\right)=Q(\sqrt{\frac{E_b}{N_o}})$
Coherent PSK	$Q\left(\frac{A}{\sigma}\right)=Q(\sqrt{\frac{2E_b}{N_o}})$
Noncoherent ASK	$\frac{1}{2}\exp\left(-\frac{A^2}{8\sigma^2}\right)=\frac{1}{2}\exp\left(-\frac{E_b}{2N_o}\right)$
Noncoherent FSK	$\frac{1}{2}\exp\left(-\frac{A^2}{4\sigma^2}\right)=\frac{1}{2}\exp\left(-\frac{E_b}{2N_o}\right)$
DPSK	$\frac{1}{2}\exp\left(-\frac{A^2}{2\sigma^2}\right)=\frac{1}{2}\exp\left(-\frac{E_b}{N_o}\right)$

ASK: $\frac{E_b}{N_o}=\frac{A^2}{4\sigma^2}$; FSK: $\frac{E_b}{N_o}=\frac{A^2}{2\sigma^2}$; PSK: $\frac{E_b}{N_o}=\frac{A^2}{2\sigma^2}$

Source Entropy

- If symbol s_k has occurred, this corresponds to

$$I(s_k) = \log_2 \frac{1}{p_k} = -\log_2 p_k$$

bits of information.

- Expected value of $I(s_k)$ over the source alphabet

$$E\{I(s_k)\} = \sum_{k=1}^{K} p_k I(s_k) = -\sum_{k=1}^{K} p_k \log_2 p_k$$

- **Source entropy:** average amount of information per source symbol:

$$\boxed{H(S) = -\sum_{k=1}^{K} p_k \log_2 p_k}$$

- Units: bits/symbol.

Average Code Length and Coding Theorem

Average Length: For system/source with N symbols, $\{a_n\}_{n=1}^{N}$, the average length of a coding will be

$$\bar{L} = \sum_{n=1}^{N} l_n p_n$$

where l_n is the length of the codeword corresponding to a_n and p_n is the probability of a_n.

Coding Theorem

Given a discrete memoryless source of entropy $H(S)$, average codeword length for any uniquely decodable source coding scheme, $\bar{L}$, is bounded by $H(S)$, that is, $\bar{L} \geq H(S)$

Huffman Code

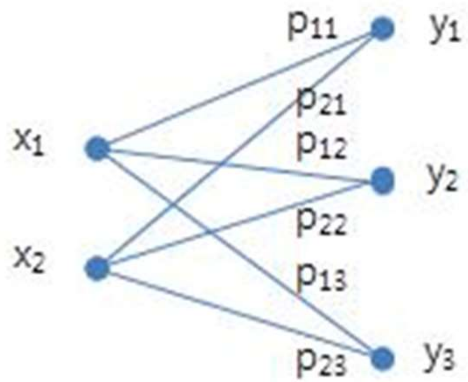
Average Length of the Huffman Code, $\bar{L}$: $\mathrm{H}(S) \leq \bar{L} < H(S) + 1$

Conditional Entropy

- **Conditional Entropy**

$$H(X \mid Y = y_k) = \sum_{j=0}^{J-1} p(x_j \mid y_k) \log_2 \left[\frac{1}{p(x_j \mid y_k)} \right]$$

$$H(X|Y) = \sum_{k=1}^{K-1} p(y_k) H(X|Y = y_k)$$



- **Mutual Information**

$$I(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = I(Y,X)$$

- **Channel Capacity:** For a given channel, $p(x_j|y_k)$, its capacity will be

$$C = \max_{\{p(x_j)\}} I(X,Y)$$

- **Channel Coding Theorem:** If the transmission rate $R \leq C$, then there exists a coding scheme such that R bits per channel use can be transmitted over the channel with an arbitrarily small probability of error.

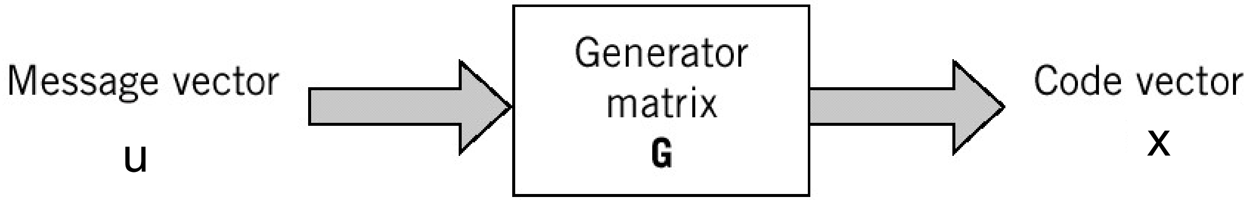
  Conversely, if $R > C$, error probability is always bounded above zero when the transmission rate is above the capacity.

Linear Block Codes

- An (n, k) binary linear block code takes a block of k bits of source data and encodes them using n bits.
  - Ratio between the number of source bits and the number of bits used in the code, $R=k/n$, is referred to as the **code rate**.

- G is a $k \times n$ matrix (k rows, n columns), that takes a source block u (a binary vector of length k), to a code word x (a binary vector of length n),

$$x = u \cdot G$$



- Hamming distance, Hamming weight, and error detection, error correction