1. a) $P_T = 10W$. $P_R = \dfrac{10}{10^4} = 10^{-3} W$

$P_N = 2 \int_0^{10^4} N_o \left(1 - \dfrac{f}{200 \times 10^3}\right) df = 2N_o \cdot 10^4 - 2N_o \cdot \dfrac{1}{40} \cdot 10^4 = 1.95 \times 10^{-5} W$

$SNR = \dfrac{P_R}{P_N} = 51.28 = 17.1 dB$

b) $P_T = \dfrac{A_c^2 P}{2} = 5W$. $P_R = 5 \times 10^{-4} W$.

$P_N = 2 \int_{f_c - 10^4}^{f_c + 10^4} N_o \left(1 - \dfrac{f}{200 \times 10^3}\right) df = 4N_o \cdot 10^4 - 2N_o \cdot 10^4 = 2 \times 10^{-5} W$

$SNR = \dfrac{P_R}{P_N} = 25 = 14 dB.$

c) $P_N = 2 \int_{f_c - 10^4}^{f_c + 10^4} N_o \left(1 - \dfrac{f}{200 \times 10^3}\right) df = 4N_o \cdot 10^4 - 3N_o \cdot 10^4 = 1 \times 10^{-5} W$

$SNR = \dfrac{P_R}{P_N} = 50 = 17 dB.$

2. For a small noise,

$$SNR_{env} \approx \frac{P}{A^2 + P} SNR_{baseband}$$

$$= \frac{\frac{1}{2}m_p^2}{A^2 + \frac{1}{2}m_p^2} SNR_{baseband} \qquad \mu = \frac{m_p}{A} \leq 1$$

$$\Rightarrow m_p^2 = A^2\mu^2$$

$$= \frac{\mu^2}{2 + \mu^2} SNR_{baseband}$$
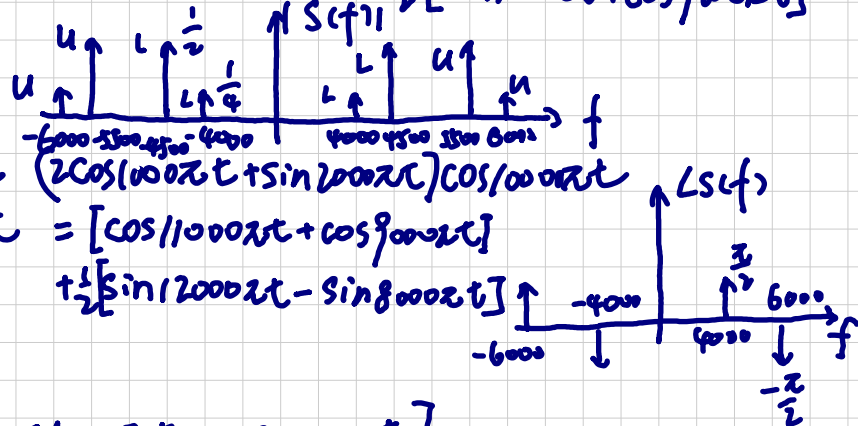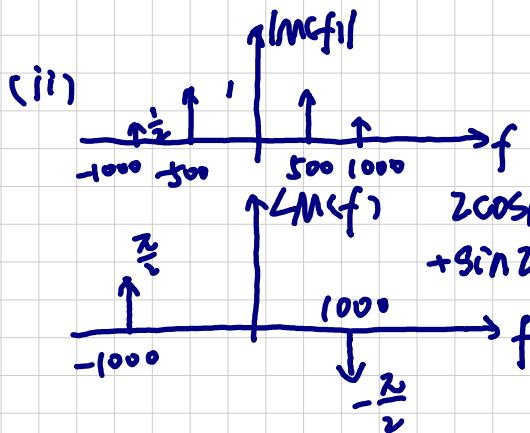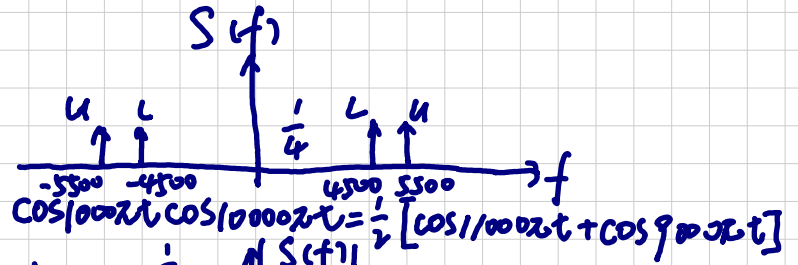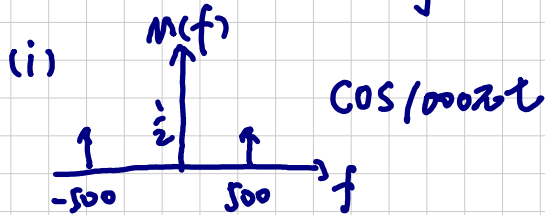
Modulation index $\mu = \frac{m_p}{A} \leq 1$

$$\begin{cases} (SNR_{env})_{max} = \frac{1}{3} SNR_{baseband}. & \mu = 1 \\ SNR_{env} \to 0 & , \ \mu \to 0 \end{cases}$$
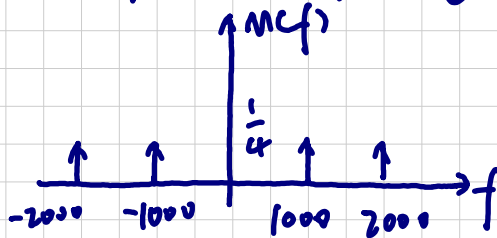
3.

$$\cos 2\pi f_c t \iff \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$$

$$\sin 2\pi f_c t \iff \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)] = \frac{1}{2}[-j\delta(f-f_c) + j\delta(f+f_c)]$$

(i)

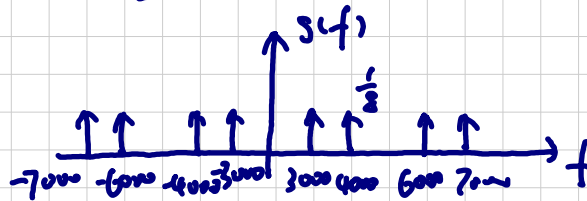$M(f)$

$\cos 1000\pi t$

$\frac{1}{2}$

-500 500 f

$S(f)$

U L $\frac{1}{4}$ L U

-5500 -4500 4500 5500 f

$\cos 1000\pi t \cos 10000\pi t = \frac{1}{2}[\cos 11000\pi t + \cos 9000\pi t]$

(ii)

$|M(f)|$

$\frac{1}{2}$ 1

-1000 -500 500 1000 f

$|S(f)|$

U L $\frac{1}{2}$ L U

$\frac{1}{4}$

-6000 -5500 -4500 -4000 4000 4500 5500 6000 f

$\angle M(f)$

$\frac{\pi}{2}$

-1000 1000 f

$-\frac{\pi}{2}$

$2\cos 1000\pi t + \sin 2000\pi t$

$(2\cos 1000\pi t + \sin 2000\pi t)\cos 10000\pi t$

$= [\cos 11000\pi t + \cos 9000\pi t]$
$+ \frac{1}{2}[\sin 12000\pi t - \sin 8000\pi t]$

$\angle S(f)$

$\frac{\pi}{2}$

-6000 -4000 4000 6000 f

$-\frac{\pi}{2}$

(iii) $\cos 1000\pi t \cos 3000\pi t = \frac{1}{2}[\cos 4000\pi t + \cos 2000\pi t]$

$M(f)$

$\frac{1}{4}$

-2000 -1000 1000 2000 f

$\frac{1}{2}[\cos 4000\pi t + \cos 2000\pi t]\cos 10000\pi t$

$= \frac{1}{4}[\cos 14000\pi t + \cos 6000\pi t + \cos 12000\pi t + \cos 8000\pi t]$

$S(f)$

$\frac{1}{8}$

-7000 -6000 -4000 -3000 3000 4000 6000 7000 f

4.

(i) $m(t) = \text{sinc}(100\pi t$

$$F(m(t)) = \frac{1}{100}\,\text{rect}(\frac{f}{100}) = \begin{cases} \frac{1}{100}, & |f| < 50 \\ 0, & \text{otherwise} \end{cases}$$
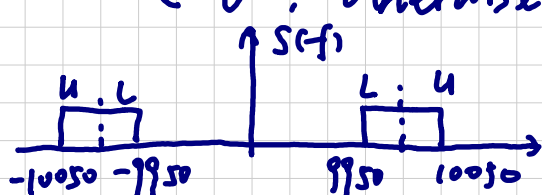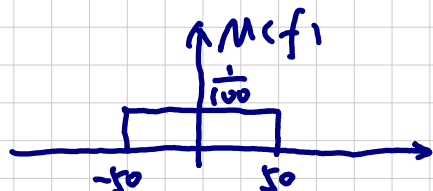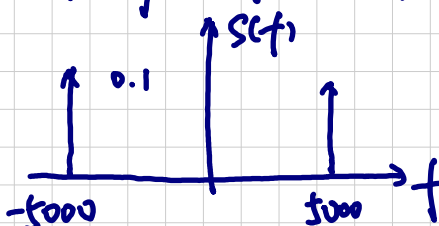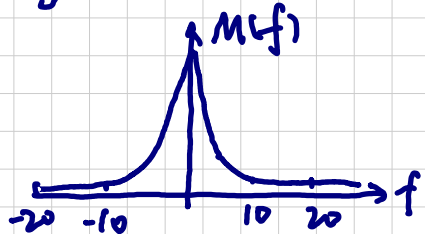

(ii) $m(t) = e^{-10t}u(t) + e^{10t}u(-t)$

$$F(m_1(t)) = \int_0^\infty e^{-(10+j2\pi f)t}\,dt = -\frac{1}{10+j2\pi f}e^{-(10+j2\pi f)t}\Big|_0^\infty = \frac{1}{10+j2\pi f}$$

$$F(m_2(t)) = \int_{-\infty}^0 e^{-(j2\pi f-10)t}\,dt = -\frac{1}{j2\pi f-1}e^{-(j2\pi f-10)t}\Big|_{-\infty}^0 = \frac{1}{10-j2\pi f}$$
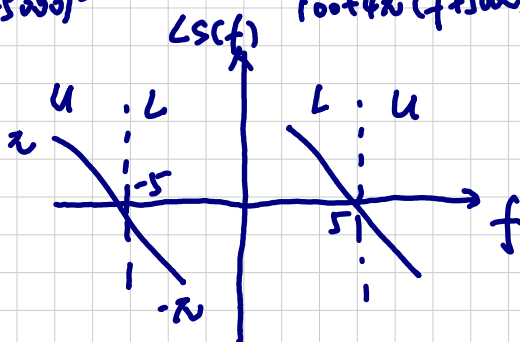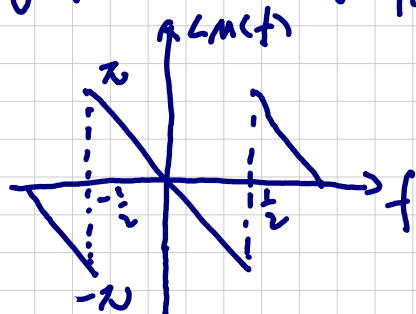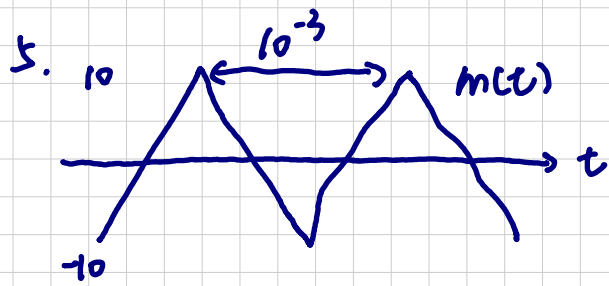
$$\therefore F(m(t)) = \frac{20}{100+4\pi^2 f^2}$$

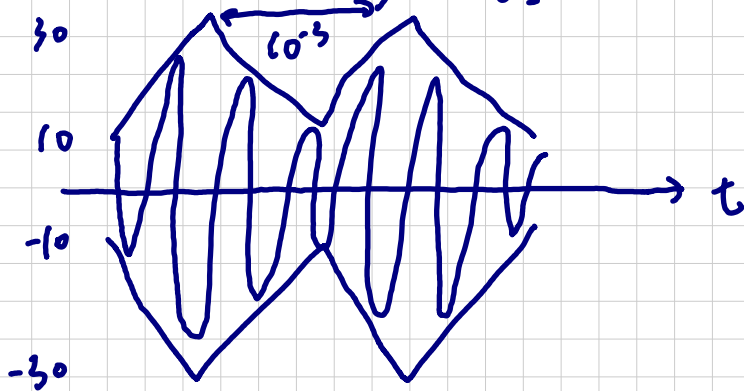$$F(m(t)\cos(10000\pi t)) = \frac{10}{100+4\pi^2(f-5000)^2} + \frac{10}{100+4\pi^2(f+5000)^2}$$

(iii) $F(m(t)) = \frac{20}{100+4\pi^2 f^2}\,e^{-j2\pi f}$

$$F(m(t)\cos(10000\pi t)) = \frac{10}{100+4\pi^2(f-5000)^2}e^{-j2\pi(f-5000)} + \frac{10}{100+4\pi^2(f+5000)^2}e^{-j2\pi(f+5000)}$$
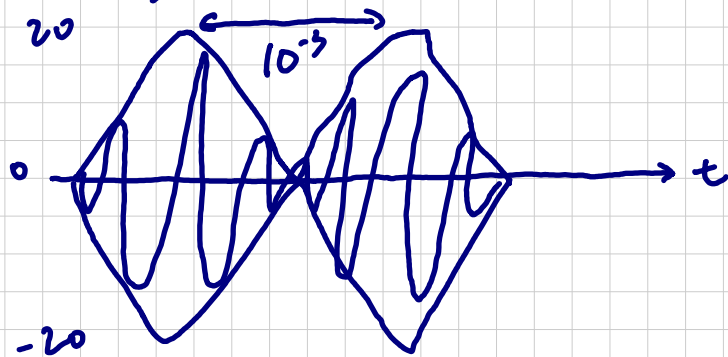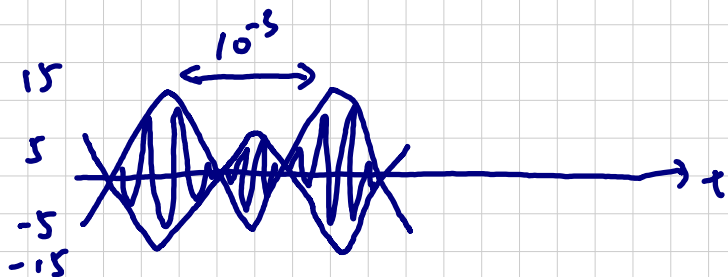
5.

$m(t)$ triangular wave, amplitude 10 to -10, period 10^{-3}

a) $\mu = \dfrac{m_p}{A} \Rightarrow A = \dfrac{m_p}{\mu} = \dfrac{10}{0.5} = 20$

b) $A = \dfrac{m_p}{\mu} = 10$

c) $A = \dfrac{m_p}{\mu} = 5$

d) $A = \dfrac{m_p}{\mu} \to 0$

$(A + m(t))\cos 2\pi f_c t \to m(t)\cos 2\pi f_c t : DSB-SC$

6. a) $A = \dfrac{m_p}{\mu} = \dfrac{10}{0.8} = 12.5$

$P_c = \dfrac{A^2}{2} = 78.125$

b) In the first $\frac{1}{4}$ period, we have $m(t) = \dfrac{40t}{T_0}$

$\overline{m^2(t)} = \dfrac{1}{\frac{T_0}{4}} \int_0^{\frac{T_0}{4}} \left(\dfrac{40t}{T_0} \right)^2 dt = 33.34$

Sideband power $P_s = \dfrac{1}{2} \overline{m^2(t)} = 16.67$

$\eta = \dfrac{P_s}{P_c + P_s} = \dfrac{16.67}{78.125 + 16.67} = 17.5\%$